Course 6 Project - Statistical Inference - PART 1

majusus

April 7, 2019

Git repository

URL :
https://github.com/majusus/datasciencecoursera/tree/master/Course6Assignment1

Synopsis

This is a project for the Coursera Statistical Inference Class. The project consists of two parts:

```
1.Simulation Exercise to explore inference
2.Basic inferential analysis using the ToothGrowth data in the R datasets package
```

Part 1 - Simulation Exercise

Overview

Investigate the exponential distribution in R and compare it with the Central Limit Theorem. The exponential distribution can be simulated in R with rexp(n, lambda) where lambda is the rate parameter. The mean of exponential distribution is 1/lambda and the standard deviation is also 1/lambda. Set lambda = 0.2 for all of the simulations. You will investigate the distribution of averages of 40 exponentials. Note that you will need to do a thousand simulations.

Instructions

1. Show the sample mean and compare it to the theoretical mean of the distribution.
2. Show how variable the sample is (via variance) and compare it to the theoretical variance of the distribution.
3. Show that the distribution is approximately normal.

Prepare Environment

Load Libraries and set Global Options.

```
#To suppress loading messages set *message = FALSE*.
#Set global options *echo = TRUE* so others will be able to read the code and
set *results = hold* to hold & push output to end of chunk.
library(knitr)
opts_chunk$set(echo = TRUE, results = 'hold')
library(data.table)
library(ggplot2)
```

Set variables as defined in the problem.

```
n <- 40 # number of exponentials (sample size)
lambda <- 0.2 # lambda for rexp (limiting factor) (rate)
nosim <- 1000 # number of simulations
quantile <- 1.96 # 95th % quantile to be used in Confidence Interval
set.seed(234) # set the seed value for reproducibility
```

Create a matrix with 1000 simulations each with 40 samples drawn from the exponential distribution.

```
# Use rexp() and matrix() to generate 40 samples creating a matrix with 1000
rows and 40 columns.
simData <- matrix(rexp(n * nosim, rate = lambda), nosim)
```

Calculate the averages across the 40 values for each of the 1000 simulations.

```
simMeans <- rowMeans(simData) # Matrix Mean
```

Mean Comparison

Show the sample mean and compare it to the theoretical mean of the distribution.

Sample Mean

Calculate the actual mean of sample data; the average sample mean of 1000 simulations of 40 randomly sampled exponential distributions.

```
sampleMean <- mean(simMeans) # Mean of sample means
sampleMean

## [1] 5.001573
```

Theoretical Mean

Calculate the theoretical mean; the expected mean of the exponential distribution of rate = 1/lambda.

```
theoMean <- 1 / lambda # Theoretical Mean
theoMean

## [1] 5
```

The distribution of the mean of the sample means is centered at **5.001573** and the theoretical mean is centered at **5**. The mean of the sample means and the theoretical mean (expected mean) are very close.

Variance Comparison

Show how variable the sample is and compare it to the theoretical variance of the distribution.

Sample Variance

Calculate the Actual Variance.

```
sampleVar <- var(simMeans)
sampleVar

## [1] 0.6631504
```

Theoretical Variance

Calculate the theoretical variance (expected variance).

```
theoVar  <- (1 / lambda)^2 / (n)
theoVar

## [1] 0.625
```

The variance of the sample means is **0.6631504** and the thoeretical variance of the distribution is **0.625**. Both variance values are very close to each other.

Sample Standard of Deviation

Calculate the sample means standard of deviation.

```
sampleSD <- sd(simMeans)
sampleSD

## [1] 0.8143405
```

Theoretical Standard of Deviation

Calculate the theoretical standard of deviation.

```
theoSD <- 1/(lambda * sqrt(n))
theoSD

## [1] 0.7905694
```

The sample means standard of deviation is **0.8143405** and the thoeretical means of standard deviation is **0.7905694**. Again, the values are close.

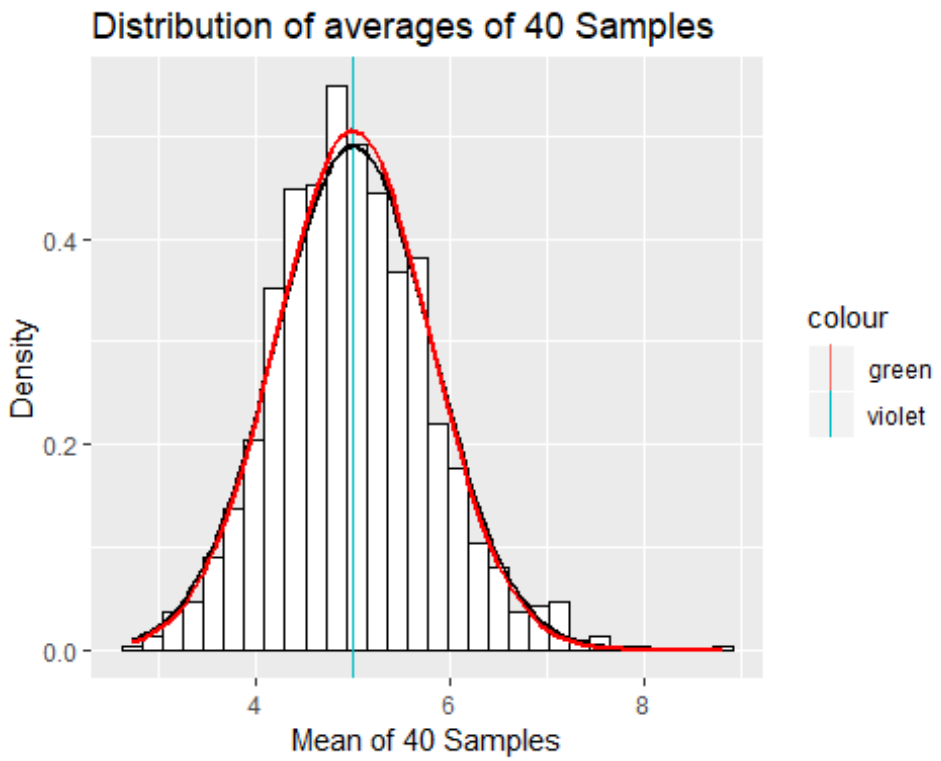
RESULTS

Show that the distribution is approximately normal.

Display the results to visually compare the actual (sample) values versus the theoretical values.

```
plotdata <- data.frame(simMeans)
m <- ggplot(plotdata, aes(x =simMeans))
m <- m + geom_histogram(aes(y=..density..), colour="black",
                        fill = "white")
m <- m + labs(title = "Distribution of averages of 40 Samples", x = "Mean of
40 Samples", y = "Density")
m <- m + geom_vline(aes(xintercept = sampleMean, colour = "green"))
m <- m + geom_vline(aes(xintercept = theoMean, colour = "violet"))
m <- m + stat_function(fun = dnorm, args = list(mean = sampleMean, sd =
sampleSD), color = "black", size = 1.0)
m <- m + stat_function(fun = dnorm, args = list(mean = theoMean, sd =
theoSD), colour = "red", size = 1.0)
m

## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



The density of the actual data is shown by the light blue bars. The theoretical mean and the sample mean are so close that they overlap. The “red” line shows the normal curve formed

by the the theoretical mean and standard deviation. The “royal blue” line shows the curve formed by the sample mean and standard deviation.

As you can see from the graph, the distribution of averages of 40 exponential distributions is close to the normal distribution with the expected theoretical values based on the given lambda.

Confidence Interval Comparison

Check the confidence interval levels to see how they compare.

Sample CI

Calculate the sample confidence interval; sampleCI = mean of x plus or minus the .975th normal quantile times the standard error of the mean standard deviation of x divided by the square root of n (the length of the vector x).

```
sampleConfInterval <- round (mean(simMeans) + c(-
1,1)*1.96*sd(simMeans)/sqrt(n),3)
sampleConfInterval

## [1] 4.749 5.254
```

Theoretical CI

Calculate the theoretical confidence interval; theoCI = theoMean of x plus or minus the .975th normal quantile times the standard error of the mean standard deviation of x divided by the square root of n (the length of the vector x).

```
theoConfInterval <- theoMean + c(-1,1) * 1.96 * sqrt(theoVar)/sqrt(n)
theoConfInterval

## [1] 4.755 5.245
```

The sample confidence interval is **4.749 5.254** and the theoretical confidence level is **4.755 5.245**. The confidence levels also match closely. Again, proving the distribution is approximately normal.

Conclusion

It is determined that the distribution does indeed demonstrate the Central Limit Theorem; a bell curve. After graphing all the values above and comparing the confidence intervals the distribution is approximately normal.