1.1 $\quad f(x) = (x^2+3x-1)^7 + (3-\sqrt{x})^6$

$$a(x) = x^7 \qquad b(x) = x^6$$
$$p(x) = x^2+3x-1 \qquad q(x) = 3-\sqrt{x}$$

$$f(x) = a(p(x))b(q(x))$$
$$f'(x) = a(p(x))[b(q(x))]' + [a(p(x))]'b(q(x))$$
$$\boldsymbol{f'(x) = (x^2+3x-1)^7[6(-x^{\frac{1}{2}}+3)^5\left[\frac{-1}{2\sqrt{x}}\right]] \quad + \quad (3-\sqrt{x})^6[7(x^2+3x-1)^6[2x+3]]}$$

$$f'(1) = ((1)^2+3(1)-1)^7[6(-(1)^{\frac{1}{2}}+3)^5\left[\frac{-1}{2\sqrt{1}}\right]] + (3-\sqrt{1})^6[7(1^2+3(1)-1)^6[2(1)+3]]$$
$$f'(1) = (3^7)(6)(2^5)\left(\frac{-1}{2}\right) + (2^6)(7)(3^6)(5)$$
$$\boldsymbol{f'(1) = 1423008}$$

1.2 $\quad f(x) = \dfrac{(3x^2-x)^5}{(4x^3-x^2-1)^8}$

$$a(x) = x^5 \qquad b(x) = x^8$$
$$p(x) = 3x^2-x \qquad q(x) = 4x^3-x^2-1$$

$$f(x) = \frac{a(p(x))}{b(q(x))}$$
$$f'(x) = \frac{[a(p(x))]'b(q(x)) - a(p(x))[b(q(x))]'}{b(q(x))^2}$$
$$\boldsymbol{f'(x) = \frac{5(3x^2-x)^4(6x-1)(4x^3-x^2-1)^8 - (3x^2-x)^5(8)(4x^3-x^2-1)^7(12x^2-2x)}{(4x^3-x^2-1)^{16}}}$$

$$f'(1) = \frac{5(3(1)^2-1)^4(6(1)-1)(4(1)^3-(1)^2-1)^8 - (3(1)^2-1)^5(8)(4(1)^3-(1)^2-1)^7(12(1)^2-2(1))}{(4(1)^3-(1)^2-1)^{16}}$$
$$f'(1) = \frac{5(2^4)(5)(2^8) - (2^5)(8)(2^7)(10)}{2^{16}}$$
$$\boldsymbol{f'(1) = -\frac{55}{16}}$$

2. $\quad g(x) = \sqrt[3]{x}f(x), \quad f(4) = 3, \quad f'(4) = -5$

$$g'(x) = \frac{1}{3}x^{\frac{-2}{3}}f(x) + \sqrt[3]{x}f'(x)$$
$$g'(4) = \frac{1}{3}4^{\frac{-2}{3}}f(4) + \sqrt[3]{4}f'(4)$$
$$g'(4) = \frac{1}{3}4^{\frac{-2}{3}}(3) + \sqrt[3]{4}(-5)$$
$$g'(4) = 4^{\frac{-2}{3}} - 5\sqrt[3]{4}$$
$$\boldsymbol{g'(4) \approx -7.54}$$

3. $f(x) = \begin{cases} g(x) & ,x \leq 1 \\ x^3 + 2x & ,x > 1 \end{cases}$ is continuous at $x = 1$, $(f \circ g)'(1) = 58$

$$f(x) \text{ is continuous at } x = 1 \iff \lim_{a \to 1^-} f(a) = \lim_{a \to 1^+} f(a) = f(a)$$
$$g(1) = (1)^3 + 2(1)$$
$$g(1) = 3$$

$$(f \circ g)'(1) = 58$$
$$f'(g(1))g'(1) = 58$$
$$f'(3)g'(1) = 58$$
$$[3(3)^2 + 2]g'(1) = 58$$
$$\boldsymbol{g'(1) = 2}$$

4. $G(x) = f(xf(x^{-1}f(x))),\ f(1) = 2,\ f(2) = 3,\ f'(1) = 4,\ f'(2) = 5,\ f'(3) = 6$

$$a(x) = f(x^{-1}f(x))$$
$$G(x) = f(xa(x))$$

$$G'(x) = f'(xa(x))[x(a(x))]'$$
$$G'(x) = f'(xa(x))[a(x) + xa'(x)]$$
$$G'(x) = f'(xa(x))[a(x) + x[f'(x^{-1}f(x))[-x^{-2}f(x) + f'(x)x^{-1}]]]$$
$$G'(x) = f'(xf(\frac{f(x)}{x}))\left[f(\frac{f(x)}{x}) + x\left[f'(\frac{f(x)}{x})\left[\frac{-f(x)}{x^2} + \frac{f'(x)}{x}\right]\right]\right]$$

$$G'(1) = f'(1f(\frac{2}{1}))\left[f(\frac{2}{1}) + 1\left[f'(\frac{2}{1})\left[\frac{-2}{1^2} + \frac{4}{1}\right]\right]\right]$$
$$G'(1) = 6[3 + 5[-2 + 4]]$$
$$\boldsymbol{G'(1) = 78}$$

5. $f, g, s : \mathbb{R} \to \mathbb{R},\ f(x) = x + 1,\ g(f(x)) = x^2 + 2x - 1,\ s(x) = \lim_{h \to 0} \dfrac{g(x+h)^2 - g(x)^2}{h}$

$$g(x+1) = x^2 + 2x - 1$$
$$g(x+1) = (x+1)^2 - 2$$
$$g(x) = x^2 - 2$$

$$s(x) = \frac{dg(x)^2}{dx}$$
$$s(x) = \frac{d[x^4 - 4x^2 + 4]}{dx}$$
$$s(x) = 4x^3 - 8x$$

$$sg(x) = (4x^3 - 8x)(x^2 - 2)$$

$$sg(1) = (4(1)^3 - 8(1))(1^2 - 2)$$
$$\boldsymbol{sg(1) = 4}$$