

EXERCISES

CHAPTER 9

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1. Redacted

Definition Extended Rules for λD_0

$$\frac{\Delta; \Gamma \vdash K : L \quad \Delta; \bar{a} : \bar{M} \vdash M : N}{\Delta, (\bar{x} : \bar{A} \triangleright a(\bar{x}) := M : N); \Gamma \vdash K : L} \text{def}$$
$$\frac{\Delta, \mathcal{D}; \Gamma \vdash * : \square \quad \Delta, \mathcal{D}; \Gamma \vdash \bar{U} : A \quad \overline{[\bar{x} := \bar{U}]} \quad \mathcal{D} \equiv \bar{x} : \bar{A} \triangleright a(\bar{x}) := M : N}{\Delta, \mathcal{D}; \Gamma \vdash a(\bar{U}) : N \quad [\bar{x} := \bar{U}]} \text{inst}$$
$$\frac{\Delta; \Gamma \vdash x : A \quad \Delta; \Gamma \vdash A : s \quad A \stackrel{\Delta, \beta}{=} B}{\Delta; \Gamma \vdash x : B} \text{conv}$$

Lemma 1. Given $\mathcal{D} \equiv \bar{x} : \bar{A} \triangleright a(\bar{x}) := M : N$ and $a \notin \Delta$

$$\frac{\Delta; \bar{x} : \bar{A} \vdash M : N}{\Delta, \mathcal{D}; \bar{x} : \bar{A} \vdash a(\bar{x}) : N} \text{par}$$

Problem

(9.1) Given

$$(\mathcal{D}_1) \quad x : \mathbb{Z}, y : \mathbb{Z} \triangleright a(x, y) := x^2 + y^2 : \mathbb{Z}$$

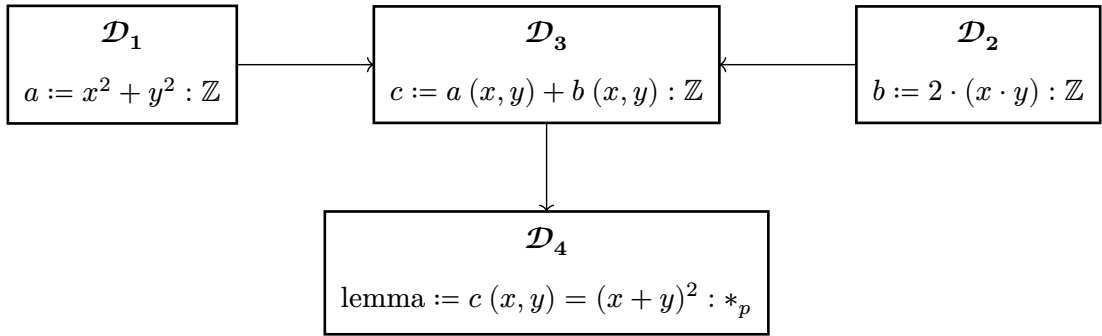
$$(\mathcal{D}_2) \quad x : \mathbb{Z}, y : \mathbb{Z} \triangleright b(x, y) := 2 \cdot (x \cdot y) : \mathbb{Z}$$

$$(\mathcal{D}_3) \quad x : \mathbb{Z}, y : \mathbb{Z} \triangleright c(x, y) := a(x, y) + b(x, y) : \mathbb{Z}$$

$$(\mathcal{D}_4) \quad x : \mathbb{Z}, y : \mathbb{Z} \triangleright \text{lemma}(x, y) := c(x, y) = (x + y)^2 : *_p$$

Consider $\Delta \equiv \mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3, \mathcal{D}_4$. Describe the dependencies between the four definitions and give all possible linearizations of the corresponding partial order.

Solution. Hasse diagram given below



The only unordered pair is $(\mathcal{D}_1, \mathcal{D}_2)$. Therefore there are two possible linearizations:

$$(1) \quad \mathcal{D}_1 \leq \mathcal{D}_2 \leq \mathcal{D}_3 \leq \mathcal{D}_4$$

$$(2) \quad \mathcal{D}_2 \leq \mathcal{D}_1 \leq \mathcal{D}_3 \leq \mathcal{D}_4$$