

EXERCISES

CHAPTER 5

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1. Reduced

Definition Some rules for reference.

λC Calculus Rules

$$\begin{array}{c} \frac{}{\emptyset \vdash * : \square} \text{Sort} \qquad \frac{\Gamma \vdash A : s}{\Gamma, x : A \vdash x : A} \text{Var} \qquad \frac{\Gamma \vdash A : B \quad \Gamma \vdash C : s}{\Gamma, x : C \vdash A : B} \text{Weak} \\[10pt] \frac{\Gamma \vdash A : s_1 \quad \Gamma, x : A \vdash B : s_2}{\Gamma \vdash \Pi x : A . B : s_2} \text{Form} \\[10pt] \frac{\Gamma \vdash M : \Pi x : A . B \quad \Gamma \vdash N : A}{\Gamma \vdash M N : B [x := N]} \text{App} \\[10pt] \frac{\Gamma, x : A \vdash M : B \quad \Gamma \vdash \Pi x : A . B : s}{\Gamma \vdash \lambda x : A . M : \Pi x : A . B} \text{Abst} \\[10pt] \frac{\Gamma \vdash A : B \quad B \stackrel{\beta}{=} B' \quad \Gamma \vdash B' : s}{\Gamma \vdash A : B'} \text{Conv} \end{array}$$

Problem

(6.1 a) Give a complete derivation in tree format showing that

$$\perp \equiv \Pi \alpha : * . \alpha$$

is legal in λC .

Solution. Here we will show that there exists $s \in \text{sort}$ and Γ such that $\Gamma \vdash \perp : s$.

Proof.

$$\frac{\frac{\vdash * : \square}{\alpha : * \vdash \alpha : *} \text{Var}}{\vdash \Pi \alpha : * . \alpha : *} \text{Form}$$

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Problem

(6.1 a) Give a complete derivation in tree format showing that $\perp \rightarrow \perp$ is legal in λC where

$$\perp \equiv \Pi \alpha : * . \alpha$$

Solution. Here we will show that there exists $s \in \text{sort}$ and Γ such that $\Gamma \vdash \perp \rightarrow \perp : s$.

Proof.

$$\frac{(6.1 \text{ a}) \frac{\vdash \perp : *}{x : \perp \vdash \perp : *} \text{Form} \quad (6.1 \text{ a}) \frac{\vdash \perp : *}{\vdash \perp : *} \text{Weak}}{\vdash \Pi x : \perp . \perp : *} \text{Form}$$

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Problem

(6.1 c) To which systems of the λ -cube does \perp belong? And $\perp \rightarrow \perp$?

Solution. The set of (s_1, s_2) pairs in formation rules of the derivation of \perp is $\{(\square, *)\}$. The minimal system corresponding is $\lambda 2$. The same for $\perp \rightarrow \perp$. Therefore \perp and $\perp \rightarrow \perp$ belongs to $\lambda 2$, $\lambda \omega$, λP and λC .