

# EXERCISES

## CHAPTER 9

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1. Redacted

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**Definition** Extended Rules for  $\lambda D_0$

$$\frac{\Delta; \Gamma \vdash K : L \quad \Delta; \bar{a} : \bar{M} \vdash M : N}{\Delta, (\bar{x} : \bar{A} \triangleright a(\bar{x}) := M : N); \Gamma \vdash K : L} \text{ def}$$

$$\frac{\Delta, \mathcal{D}; \Gamma \vdash * : \square \quad \Delta, \mathcal{D}; \Gamma \vdash \bar{U} : \bar{A} [\bar{x} := \bar{U}] \quad \mathcal{D} \equiv \bar{x} : \bar{A} \triangleright a(\bar{x}) := M : N}{\Delta, \mathcal{D}; \Gamma \vdash a(\bar{U}) : N [\bar{x} := \bar{U}]} \text{ inst}$$

$$\frac{\Delta; \Gamma \vdash x : A \quad \Delta; \Gamma \vdash A : s \quad A \stackrel{\Delta, \beta}{=} B}{\Delta; \Gamma \vdash x : B} \text{ conv}$$

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*Lemma 1.* Given  $\mathcal{D} \equiv \bar{x} : \bar{A} \triangleright a(\bar{x}) := M : N$  and  $a \notin \Delta$

$$\frac{\Delta; \bar{x} : \bar{A} \vdash M : N}{\Delta, \mathcal{D}; \bar{x} : \bar{A} \vdash a(\bar{x}) : N} \text{ par}$$

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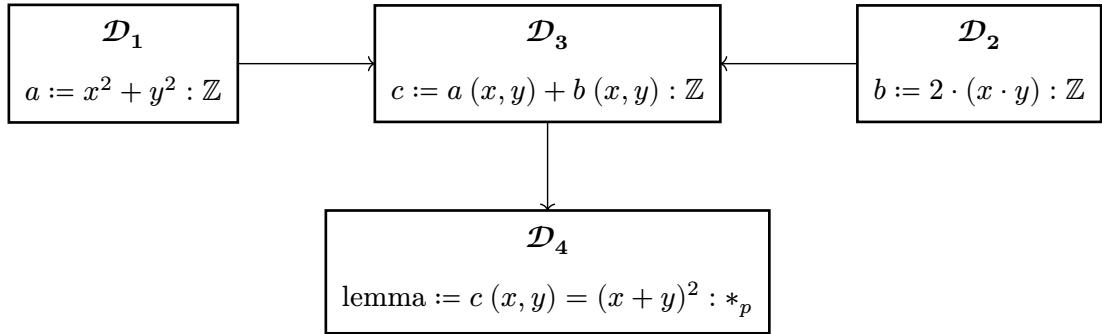
### Problem

(9.1) Given

$$\begin{aligned}
 (\mathcal{D}_1) \quad x : \mathbb{Z}, y : \mathbb{Z} \quad & \triangleright a(x, y) := x^2 + y^2 : \mathbb{Z} \\
 (\mathcal{D}_2) \quad x : \mathbb{Z}, y : \mathbb{Z} \quad & \triangleright b(x, y) := 2 \cdot (x \cdot y) : \mathbb{Z} \\
 (\mathcal{D}_3) \quad x : \mathbb{Z}, y : \mathbb{Z} \quad & \triangleright c(x, y) := a(x, y) + b(x, y) : \mathbb{Z} \\
 (\mathcal{D}_4) \quad x : \mathbb{Z}, y : \mathbb{Z} \quad & \triangleright \text{lemma}(x, y) := c(x, y) = (x + y)^2 : *_p
 \end{aligned}$$

Consider  $\Delta \equiv \mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3, \mathcal{D}_4$ . Describe the dependencies between the four definitions and give all possible linearizations of the corresponding partial order.

*Solution.* Hasse diagram given below



The only unordered pair is  $(\mathcal{D}_1, \mathcal{D}_2)$ . Therefore there are two possible linearizations:

- (1)  $\mathcal{D}_1 \leq \mathcal{D}_2 \leq \mathcal{D}_3 \leq \mathcal{D}_4$
- (2)  $\mathcal{D}_2 \leq \mathcal{D}_1 \leq \mathcal{D}_3 \leq \mathcal{D}_4$