Excercises

Chapter 4

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1. Reducted

Definition Some rules for reference.

$$\frac{}{\emptyset \vdash * : \square} \text{ Sort} \qquad \frac{\Gamma \vdash A : s \quad x \notin \text{dom } \Gamma}{\Gamma, x : A \vdash x : A} \text{ Var}$$

$$\frac{\Gamma \vdash A : B \quad \Gamma \vdash C : s \quad x \notin \text{dom } \Gamma}{\Gamma, x : C \vdash A : B} \text{ Weak} \qquad \frac{\Gamma \vdash A : s \quad \Gamma \vdash B : s}{\Gamma \vdash A \to B : s} \text{ Form}$$

$$\frac{\Gamma \vdash M : A \to B \quad \Gamma \vdash N : A}{\Gamma \vdash M\,N : B} \text{ App}$$

$$\frac{\Gamma, x : A \vdash M : B \quad \Gamma \vdash A \to B : s}{\Gamma \vdash \lambda x : A \,.\, M : A \to B} \text{ Abst}$$

$$\frac{\Gamma \vdash A : B \quad \Gamma \vdash B' : s \quad B \overset{\beta}{=} B'}{\Gamma \vdash A : B'} \text{ Conv}$$

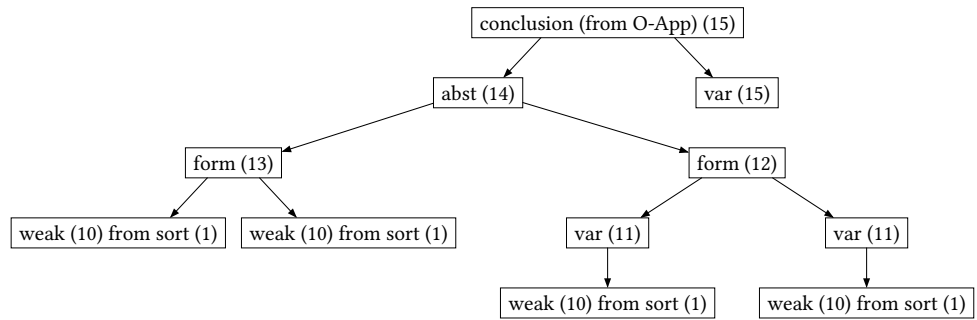Previously an alternative version of the flag derivation was used, only putting up a flag for a local premise (abstraction unwrapping) to save horizontal space. Currently, the standard flag derivation format will be used since now single lines will not be as long.

(4.1) Give a complete tree diagram of the derivation in section 4.5 (95)

Solution.

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                              ┌──────────────────────────────┐
                              │  conclusion (from O-App) (15) │
                              └──────────────────────────────┘
                               /                          \
                    ┌───────────┐                   ┌──────────┐
                    │  abst (14) │                   │ var (15) │
                    └───────────┘                   └──────────┘
                     /           \
          ┌───────────┐          ┌───────────┐
          │ form (13) │          │ form (12) │
          └───────────┘          └───────────┘
            /        \            /          \
┌────────────────────┐ ┌────────────────────┐ ┌──────────┐ ┌──────────┐
│ weak (10) from sort │ │ weak (10) from sort │ │ var (11) │ │ var (11) │
│         (1)         │ │         (1)         │ └──────────┘ └──────────┘
└────────────────────┘ └────────────────────┘      │             │
                                      ┌──────────────────────┐ ┌──────────────────────┐
                                      │ weak (10) from sort (1)│ │ weak (10) from sort (1)│
                                      └──────────────────────┘ └──────────────────────┘
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(4.2 a) Give a complete $\lambda\underline{\omega}$ derivation in flag format of

$$\emptyset \vdash (* \to *) \to * : \square$$

Solution.

1. $* : \square$ **Sort**
2. $* \to * : \square$ **1,1 Form**
3. $(* \to *) \to * : \square$ **2,1 Form**

(4.2 b) Give a complete $\lambda\underline{\omega}$ derivation in flag format of

$$\alpha : *, \beta : * \vdash (\alpha \to \beta) \to \alpha : *$$

Solution.

1. $\emptyset \vdash * : \square$ **Sort**

2. $\alpha : *$

3. $\quad | \quad \alpha : *$ **1 Var**

4. $\quad | \quad * : \square$ **1,1 Weak**

5. $\quad | \quad \beta : *$

6. $\quad | \quad | \quad \alpha : *$ **3,4 Weak**

7. $\quad | \quad | \quad \beta : *$ **4 Var**

8. $\quad | \quad | \quad \alpha \rightarrow \beta : *$ **6,7 Form**

9. $\quad | \quad | \quad (\alpha \rightarrow \beta) \rightarrow \alpha : *$ **8,6 Form**