

EXERCISES

CHAPTER 5

SEAN LI ¹

1. Reducted

Definition Some rules for reference.

$$\begin{array}{c} \frac{}{\emptyset \vdash * : \square} \text{Sort} \quad \frac{\Gamma \vdash A : s}{\Gamma, x : A \vdash x : A} \text{Var} \quad \frac{\Gamma \vdash A : B \quad \Gamma \vdash C : s}{\Gamma, x : C \vdash A : B} \text{Weak} \\[10pt] \frac{\Gamma \vdash A : * \quad \Gamma, x : A \vdash B : s}{\Gamma \vdash \Pi x : A . B : s} \text{Form} \quad \frac{\Gamma \vdash M : \Pi x : A . B \quad \Gamma \vdash N : A}{\Gamma \vdash M N : B [x := N]} \text{App} \\[10pt] \frac{\Gamma, x : A \vdash M : B \quad \Gamma \vdash \Pi x : A . B : s}{\Gamma \vdash \lambda x : A . M : \Pi x : A . B} \text{Abst} \\[10pt] \frac{\Gamma \vdash A : B \quad B \stackrel{\beta}{=} B' \quad \Gamma \vdash B' : s}{\Gamma \vdash A : B'} \text{Conv} \end{array}$$

Problem

(5.1) Give a diagram of the tree corresponding to the complete tree derivation of line 18 of Section 5.3 (P 107)

Solution.



Problem

(5.2) Give a complete λP derivation of

$$S : * \vdash S \rightarrow S \rightarrow * : \square$$

In tree format and flag format.

Solution.

Tree Derivation.

$$(7) \frac{(3) S : * \vdash S : * \quad (4) \frac{\vdash * : \square \quad \vdash * : \square}{(6) \frac{S : * \vdash * : \square}{S : *, x : S \vdash * : \square} \text{ Weak}} \text{ Weak}}{S : * \vdash S \rightarrow * : \square}$$

$$(3) \frac{\vdash * : \square}{S : * \vdash S : *} \text{Var} \quad (9) \frac{(7) S : * \vdash S \rightarrow * : \square \quad (3) S : * \vdash S : *}{S : *, x : S \vdash S \rightarrow * : \square} \text{Weak}$$

$$\frac{}{S : * \vdash S \rightarrow S \rightarrow * : \square} \text{Form}$$

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Flag Derivation.

1. $* : \square$	Sort
2. $S : *$	
3. $\boxed{S : *}$	1 Var
4. $\boxed{* : \square}$	1,1 Weak
5. $x : S$	
6. $\boxed{x : S}$	4,3 Weak
7. $\boxed{S \rightarrow * : \square}$	3,6 Form
8. $x : S$	
9. $\boxed{x : S}$	7,3 Weak
10. $\boxed{S \rightarrow S \rightarrow * : \square}$	3,9 Form

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