

EXERCISES

CHAPTER 9

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1. Redacted

Definition Extended Rules for λD_0

$$\frac{\Delta; \Gamma \vdash K : L \quad \Delta; \bar{a} : \bar{M} \vdash M : N}{\Delta, (\bar{x} : \bar{A} \triangleright a(\bar{x}) := M : N); \Gamma \vdash K : L} \text{ def}$$

$$\frac{\Delta, \mathcal{D}; \Gamma \vdash * : \square \quad \Delta, \mathcal{D}; \Gamma \vdash \bar{U} : \bar{A} [\bar{x} := \bar{U}] \quad \mathcal{D} \equiv \bar{x} : \bar{A} \triangleright a(\bar{x}) := M : N}{\Delta, \mathcal{D}; \Gamma \vdash a(\bar{U}) : N [\bar{x} := \bar{U}]} \text{ inst}$$

$$\frac{\Delta; \Gamma \vdash x : A \quad \Delta; \Gamma \vdash A : s \quad A \stackrel{\Delta, \beta}{=} B}{\Delta; \Gamma \vdash x : B} \text{ conv}$$

Lemma 1. Given $\mathcal{D} \equiv \bar{x} : \bar{A} \triangleright a(\bar{x}) := M : N$ and $a \notin \Delta$

$$\frac{\Delta; \bar{x} : \bar{A} \vdash M : N}{\Delta, \mathcal{D}; \bar{x} : \bar{A} \vdash a(\bar{x}) : N} \text{ par}$$

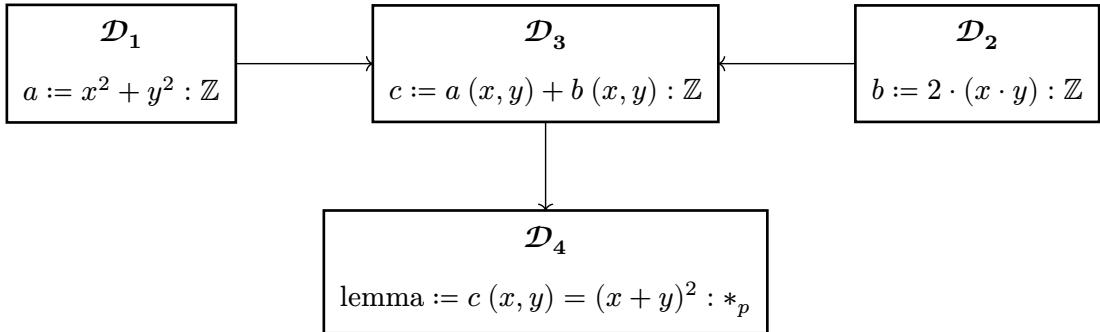
Problem

(9.1) Given

$$\begin{aligned}
 (\mathcal{D}_1) \quad & x : \mathbb{Z}, y : \mathbb{Z} \triangleright a(x, y) := x^2 + y^2 : \mathbb{Z} \\
 (\mathcal{D}_2) \quad & x : \mathbb{Z}, y : \mathbb{Z} \triangleright b(x, y) := 2 \cdot (x \cdot y) : \mathbb{Z} \\
 (\mathcal{D}_3) \quad & x : \mathbb{Z}, y : \mathbb{Z} \triangleright c(x, y) := a(x, y) + b(x, y) : \mathbb{Z} \\
 (\mathcal{D}_4) \quad & x : \mathbb{Z}, y : \mathbb{Z} \triangleright \text{lemma}(x, y) := c(x, y) = (x + y)^2 : *_p
 \end{aligned}$$

Consider $\Delta \equiv \mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3, \mathcal{D}_4$. Describe the dependencies between the four definitions and give all possible linearizations of the corresponding partial order.

Solution. Hasse diagram given below



The only incomparable pair is $(\mathcal{D}_1, \mathcal{D}_2)$. Therefore there are two possible linearizations:

- (1) $\mathcal{D}_1 \leq \mathcal{D}_2 \leq \mathcal{D}_3 \leq \mathcal{D}_4$
- (2) $\mathcal{D}_2 \leq \mathcal{D}_1 \leq \mathcal{D}_3 \leq \mathcal{D}_4$

Problem

(9.2) Consider

$$\begin{aligned}
 \mathcal{D}_i &\equiv \bar{x} : \bar{A} \triangleright a(\bar{x}) := K : L \\
 \mathcal{D}_j &\equiv \bar{y} : \bar{B} \triangleright b(\bar{y}) := M : N
 \end{aligned}$$

Let $\Delta; \Gamma \vdash U : V$ and assume \mathcal{D}_i and \mathcal{D}_j are elements of list Δ , where \mathcal{D}_i precedes \mathcal{D}_j . Describe precisely where constant a may occur in \mathcal{D}_i and \mathcal{D}_j and where constant b may occur in Δ .

Solution. In order for \mathcal{D}_i to be a valid definition, $\bar{x} : \bar{A} \vdash K : L$ must be legal. Therefore by the free variable lemma any free variables in K and L must be in $\bar{x} : \bar{A}$, which by the time, does not yet contain a 's definition. Therefore, a could only appear in \mathcal{D}_j .

By similar reasoning b could only have appeared in definitions after \mathcal{D}_j . Assuming the list sorted by the suffix, then b could only have been in any \mathcal{D}_k where $k > j$.