Excercises

Chapter 4

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1. Reducted

Definition Some rules for reference.

$$\frac{}{\emptyset \vdash * : \square}\ \text{Sort} \qquad \frac{\Gamma \vdash A : s \quad x \notin \text{dom}\ \Gamma}{\Gamma, x : A \vdash x : A}\ \text{Var}$$

$$\frac{\Gamma \vdash A : B \quad \Gamma \vdash C : s \quad x \notin \text{dom}\ \Gamma}{\Gamma, x : C \vdash A : B}\ \text{Weak} \qquad \frac{\Gamma \vdash A : s \quad \Gamma \vdash B : s}{\Gamma \vdash A \to B : s}\ \text{Form}$$

$$\frac{\Gamma \vdash M : A \to B \quad \Gamma \vdash N : A}{\Gamma \vdash M\ N : B}\ \text{App}$$

$$\frac{\Gamma, x : A \vdash M : B \quad \Gamma \vdash A \to B : s}{\Gamma \vdash \lambda x : A\ .\ M : A \to B}\ \text{Abst}$$

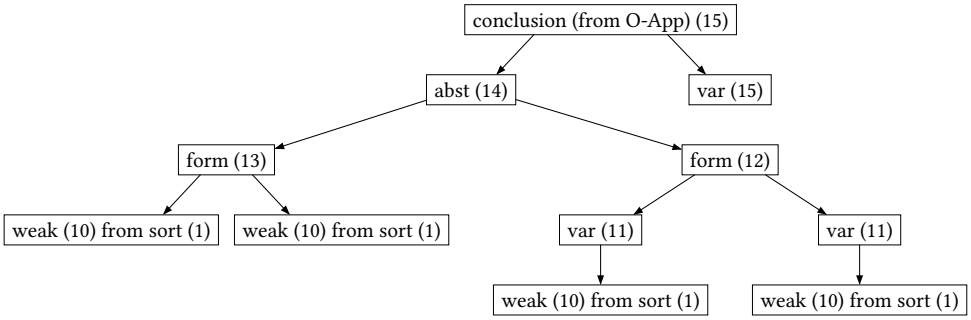
$$\frac{\Gamma \vdash A : B \quad \Gamma \vdash B' : s \quad B \overset{\beta}{=} B'}{\Gamma \vdash A : B'}\ \text{Conv}$$

Previously an alternative version of the flag derivation was used, only putting up a flag for a local premise (abstraction unwrapping) to save horizontal space. Currently, the standard flag derivation format will be used since now single lines will not be as long.

Problem

(4.1) Give a complete tree diagram of the derivation in section 4.5 (95)

Solution.



Problem

(4.2 a) Give a complete $\lambda\underline{\omega}$ derivation in flag format of

$$\emptyset \vdash (* \to *) \to * : \Box$$

Solution.

1. $* : \Box$ **Sort**
2. $* \to * : \Box$ **1,1 Form**
3. $(* \to *) \to * : \Box$ **2,1 Form**

Problem

(4.2 b) Give a complete $\lambda\underline{\omega}$ derivation in flag format of

$$\alpha : *, \beta : * \vdash (\alpha \to \beta) \to \alpha : *$$

Solution.

1.	$\emptyset \vdash * : \square$	**Sort**
2.	$\alpha : *$	
3.	$\quad \alpha : *$	**1 Var**
4.	$\quad * : \square$	**1,1 Weak**
5.	$\quad \beta : *$	
6.	$\quad\quad \alpha : *$	**3,4 Weak**
7.	$\quad\quad \beta : *$	**4 Var**
8.	$\quad\quad \alpha \to \beta : *$	**6,7 Form**
9.	$\quad\quad (\alpha \to \beta) \to \alpha : *$	**8,6 Form**

Problem

(4.3 a) Give a complete $\lambda\underline{\omega}$ derivation in flag format of

$$\alpha, \beta : *, x : \alpha, y : \alpha \to \beta \vdash y\ x : \beta$$

Solution.

1.	$* : \square$	**Sort**
2.	$\alpha : *$	
3.	$\quad \alpha : *$	**1 Var**
4.	$\quad * : \square$	**1,1 Weak**
5.	$\quad \beta : *$	
6.	$\quad\quad \beta : *$	**4 Var**
7.	$\quad\quad \alpha : *$	**3,4 Weak**
8.	$\quad\quad * : \square$	**4,4 Weak**
9.	$\quad\quad x : \alpha$	
10.	$\quad\quad\quad x : \alpha$	**7 Var**
11.	$\quad\quad\quad \alpha : *$	**7,7 Weak**
12.	$\quad\quad\quad \beta : *$	**6,7 Weak**
13.	$\quad\quad\quad \alpha \to \beta : *$	**11,12 Form**
14.	$\quad\quad\quad y : \alpha \to \beta$	
15.	$\quad\quad\quad\quad y : \alpha \to \beta$	**13 Var**
16.	$\quad\quad\quad\quad x : \alpha$	**10,13 Weak**
17.	$\quad\quad\quad\quad y\ x : \beta$	**15,16 App**

Problem

(4.3 b) Give a shortened $\lambda\underline{\omega}$ derivation in flag format of

$$\alpha, \beta : *, x : \alpha, y : \alpha \to \beta, z : \beta \to \alpha \vdash z\ (y\ x) : \alpha$$

Solution.

1.	$\alpha : *$	
2.	$\beta : *$	
3.	$x : \alpha$	
4.	$y : \alpha \to \beta$	
5.	$x : \alpha$	**3 Weak**
6.	$z : \beta \to \alpha$	
7.	$x : \alpha$	**5 Weak**
8.	$y : \alpha \to \beta$	**4 Weak**
9.	$y\ x : \beta$	**8,7 App**
10.	$z\ (y\ x) : \alpha$	**6,9 App**