Module 0.3: Bloch Vector

This notebook is part of the Qiskit v2.x Study Guide. It introduces Bloch vectors, shows how to reason about rotations, and demonstrates how to plot Bloch vectors in Qiskit. The material aligns with the exam objectives for visualizing quantum states. See Module 2.3 for more state-visualization options.

Topics covered

- Review of rotation matrices about x, y, and z axes.
- Bloch vector basics and coordinate conventions.
- Plotting Bloch vectors with Qiskit.
- Predicting results of tensor-product operations using Bloch-sphere reasoning.
- Implementing the examples in Qiskit and verifying with plots.

Documentation and resources

- Qiskit API: `plot_bloch_vector`
  https://quantum.cloud.ibm.com/docs/en/api/qiskit/qiskit.visualization.plot_bloch_vector
- Interactive Bloch sphere (helpful intuition):
  https://javafxpert.github.io/grok-bloch/
- More state visualization tools appear in **Module 2.3**.

In [1]:
```python
# Metadata cell: versions and minimal imports
import sys, platform
print("Python:", sys.version.split()[0])
try:
    import qiskit
    print("Qiskit:", qiskit.__version__)
except Exception as exc:
    print("Qiskit: <not installed in this environment>")
    print("Note: Run this notebook where Qiskit v2.x is available.")

# Core imports (kept concise)
from qiskit import transpile
from qiskit.circuit import QuantumCircuit
from qiskit.quantum_info import Statevector, DensityMatrix, partial_trace
from qiskit.visualization import plot_bloch_vector
import numpy as np

# Fake backend (no real QPUs used)
try:
    from qiskit_ibm_runtime.fake_provider import FakeSherbrooke
    _fake_backend = FakeSherbrooke()
    print("Fake backend:", _fake_backend.name)
except Exception as exc:
    _fake_backend = None
    print("Fake backend unavailable in this environment.")
```

```
Python: 3.12.9
Qiskit: 2.2.1
Fake backend: fake_sherbrooke
```

Bloch sphere diagram

Section 0.3.0 Review: Rotation Matrices

A single-qubit pure state can be represented by a point on the Bloch sphere. Unitary rotations move the Bloch vector on the sphere without changing its norm.

The standard rotation gates about principal axes are:

$$R_x(\theta) = e^{-i\,\theta X/2} = \begin{pmatrix} \cos(\frac{\theta}{2}) & -i\sin(\frac{\theta}{2}) \\ -i\sin(\frac{\theta}{2}) & \cos(\frac{\theta}{2}) \end{pmatrix}$$

$$R_y(\theta) = e^{-i\,\theta Y/2} = \begin{pmatrix} \cos(\frac{\theta}{2}) & -\sin(\frac{\theta}{2}) \\ \sin(\frac{\theta}{2}) & \cos(\frac{\theta}{2}) \end{pmatrix},$$

$$R_z(\theta) = e^{-i\,\theta Z/2} = \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}.$$

On the Bloch vector $\vec{r} = (x, y, z)$, a rotation $R_a(\theta)$ about axis $a \in \{x, y, z\}$ by angle θ transforms $\vec{r}$ by the corresponding

SO(3) rotation. For example, a z-rotation acts as
$x' = x\cos\theta - y\sin\theta$, $y' = x\sin\theta + y\cos\theta$,
$z' = z$.

Conventions used here: $|0\rangle$ maps to $(0,0,1)$, $|1\rangle$ to $(0,0,-1)$, $|+\rangle$ to $(1,0,0)$, $|-\rangle$ to $(-1,0,0)$, $|+i\rangle$ to $(0,1,0)$, and $|-i\rangle$ to $(0,-1,0)$.

Section 0.3.1 Bloch vector concepts

Goals. Understand how (x,y,z) relate to angles θ and ϕ, how the right-hand rule sets axes and rotation sign, and how to write $|\psi\rangle$ in the computational basis.

Bloch coordinates. For a pure one-qubit state described by polar angles $\theta \in [0,\pi]$ and $\phi \in [0,2\pi)$ on the Bloch sphere, the Cartesian components are
$x = \sin\theta\,\cos\phi, \quad y = \sin\theta\,\sin\phi, \quad z = \cos\theta.$

These satisfy $x^2 + y^2 + z^2 = 1$ for pure states. Which means that pure states are on the surface of the sphere. (We will later learn what pure states are, but just keep that in mind for now).

Right-hand rule. The Bloch sphere uses a right-handed coordinate system. Point your right-hand thumb along $+\hat{z}$; curling fingers from $+\hat{x}$ toward $+\hat{y}$ shows the positive rotation sense about $+\hat{z}$. By convention, $|0\rangle$ is at the north pole $(0,0,1)$ and $|1\rangle$ at the south pole $(0,0,-1)$.

Computational-basis form. Any normalized single-qubit pure state can be written as
$|\psi(\theta,\phi)\rangle = \cos\frac{\theta}{2}\,|0\rangle \;+\; e^{i\phi}\,\sin\frac{\theta}{2}\,|1\rangle,$

with global phase omitted. This parameterization maps to (x,y,z) via the formulas above.

Quick check. The code below computes (x,y,z) from (θ,ϕ) and verifies it by extracting the Bloch components from the corresponding density matrix.

In [2]:
```
import numpy as np
from qiskit.quantum_info import Statevector, DensityMatrix

theta = np.pi/2   # 90 degrees
phi   = -np.pi/2  # -90 degrees

state = np.array([np.cos(theta/2), np.exp(1j*phi)*np.sin(theta/2)], dtype=cc
```

```python
sv = Statevector(state)

rho = DensityMatrix(sv).data
pauli_x = np.array([[0,1],[1,0]], dtype=complex)
pauli_y = np.array([[0,-1j],[1j,0]], dtype=complex)
pauli_z = np.array([[1,0],[0,-1]], dtype=complex)

r = np.array([
    np.trace(rho @ pauli_x).real,
    np.trace(rho @ pauli_y).real,
    np.trace(rho @ pauli_z).real
])

print("Statevector:", sv.data)       # ~ [0.70710678+0.j  0.-0.70710678j]
print("Bloch (x,y,z):", r)           # ~ [ 0. -1.  0.]
```

```
Statevector: [7.07106781e-01+0.j         4.32978028e-17-0.70710678j]
Bloch (x,y,z): [ 6.12323400e-17 -1.00000000e+00  2.22044605e-16]
```

Section 0.3.2 Bloch Vector in Qiskit

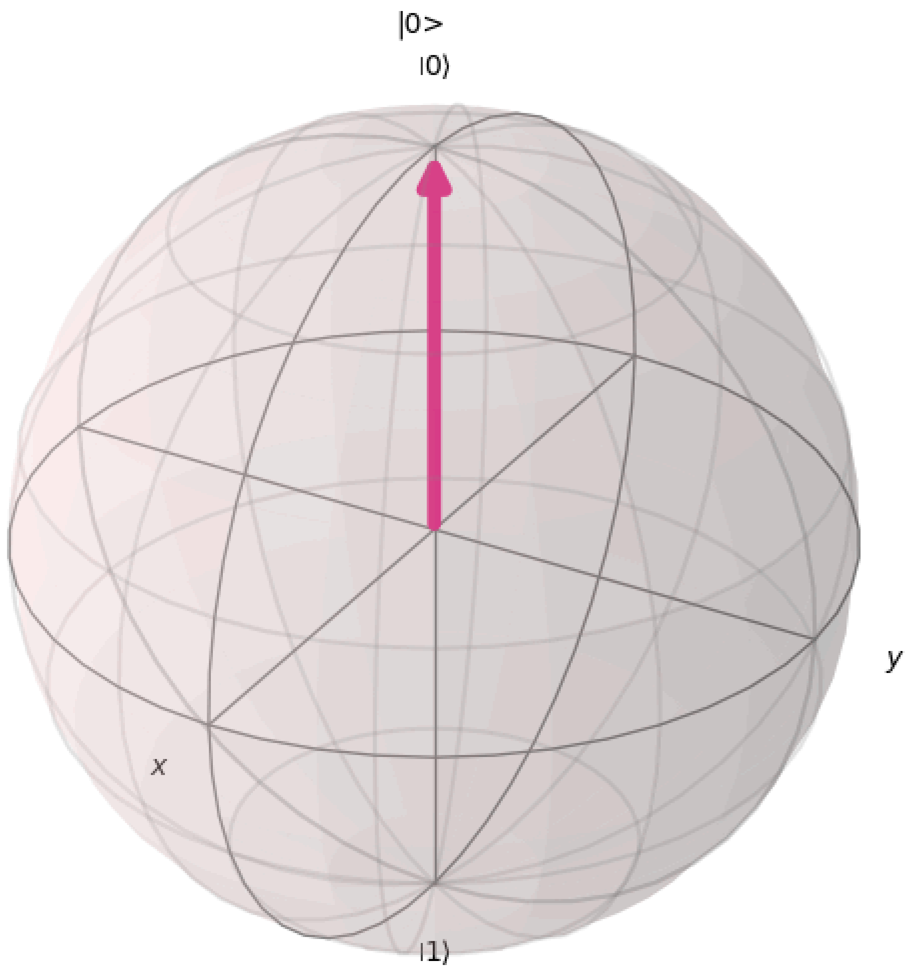
We use `qiskit.visualization.plot_bloch_vector` to plot a vector $\vec{r} = [x, y, z]$ with $\|\vec{r}\| \leq 1$. This is convenient for visualizing ideal pure states and mixed states (reduced single-qubit states).

Below are a few basic examples that reproduce known axes states.

In [3]:
```python
# 0.3.1 — Basic Bloch vectors
from qiskit.visualization import plot_bloch_vector

# |0>  -> (0, 0, 1)
plot_bloch_vector([0, 0, 1], title="|0>")
```

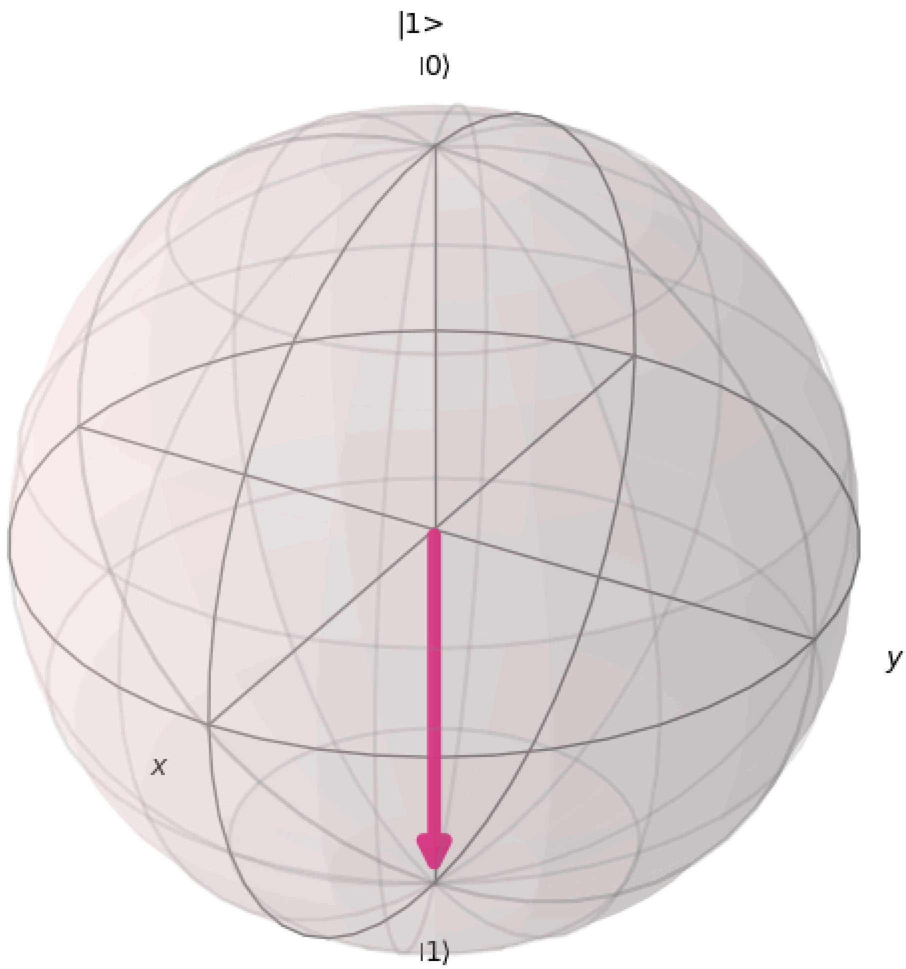
Out[3]:



In [4]:

```
# # |1>  -> (0, 0, -1)
plot_bloch_vector([0, 0, -1], title="|1>")
```

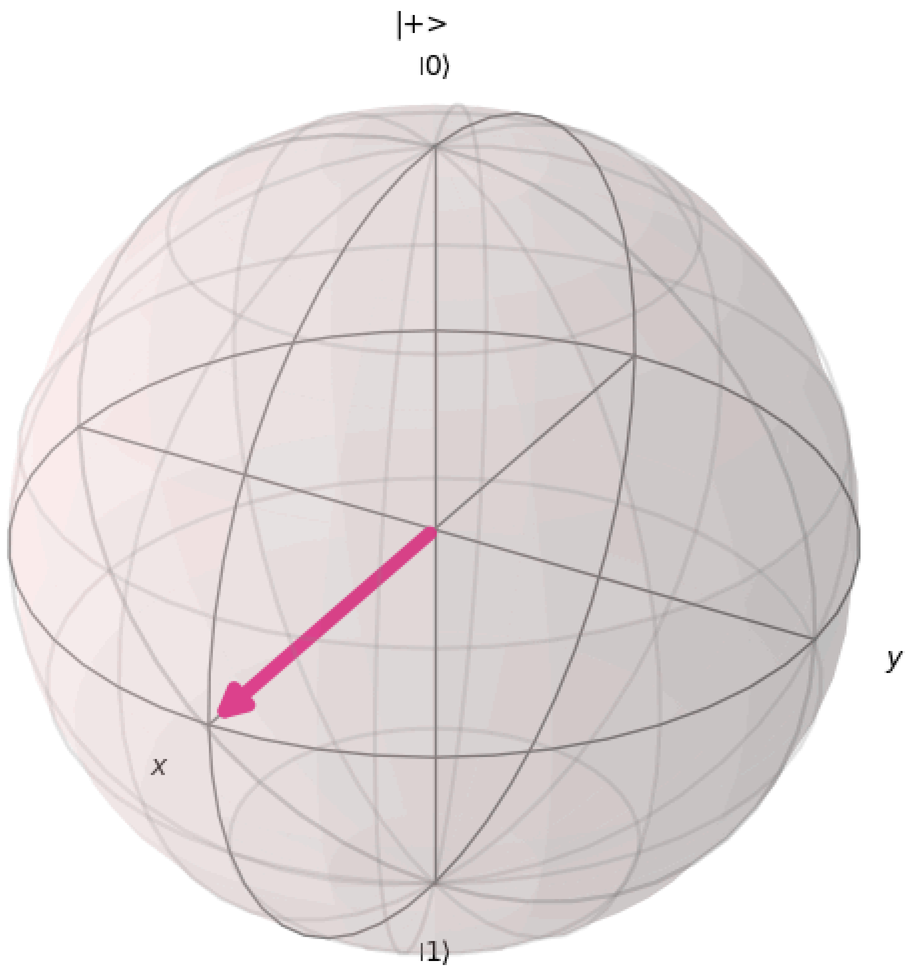
Out[4]:



In [5]:

```
# # |+>  -> (1, 0, 0)
plot_bloch_vector([1, 0, 0], title="|+>")
```

Out [5]:



In [6]:

```
#### Uncomment the rest of these one by one to see where they appear on the
# # |->  -> (-1, 0, 0)
# plot_bloch_vector([-1, 0, 0], title="|->")

# # |+i> -> (0, 1, 0)
# plot_bloch_vector([0, 1, 0], title="|+i>")

# # |-i> -> (0, -1, 0)
# plot_bloch_vector([0, -1, 0], title="|-i>")
```

Common Pitfall: When rotation about an axis doesn't matter

Rotations that *don’t change* a qubit (up to a global phase)

Key idea. If a qubit is **aligned with the axis of rotation** on the Bloch sphere, rotating about that axis leaves the **physical state unchanged** (the Bloch vector stays put). Mathematically, the state picks up only a **global phase**, which has no observable effect.

Example: rotation about the z-axis and $|0\rangle$

- $|0\rangle$ is the $+Z$ **eigenstate** (north pole of the Bloch sphere).
- The Z-rotation is $R_Z(\theta) = e^{-i\theta Z/2}$.
- Since $Z|0\rangle = +|0\rangle$,

$$R_Z(\theta)\,|0\rangle \;=\; e^{-i\theta/2}\,|0\rangle,$$

  which is the **same physical state** (only a global phase $e^{-i\theta/2}$).

Similarly, $R_Z(\theta)|1\rangle = e^{+i\theta/2}|1\rangle$ leaves $|1\rangle$ (the south pole) physically unchanged.

The same pattern for other axes

- **X-axis rotations $R_X(\theta)$:**
  - $|+\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$ is the $+X$ **eigenstate**.
  - $R_X(\theta)|+\rangle = e^{-i\theta/2}|+\rangle$ ⇒ **unchanged (up to phase)**.
  - Likewise for $|-\rangle$ (the $-X$ eigenstate): $R_X(\theta)|-\rangle = e^{+i\theta/2}|-\rangle$.
- **Y-axis rotations $R_Y(\theta)$:**
  - $|+i\rangle = \frac{|0\rangle+i|1\rangle}{\sqrt{2}}$ is the $+Y$ **eigenstate**.
  - $R_Y(\theta)|+i\rangle = e^{-i\theta/2}|+i\rangle$ ⇒ **unchanged (up to phase)**.
  - Likewise for $|-i\rangle$ (the $-Y$ eigenstate): $R_Y(\theta)|-i\rangle = e^{+i\theta/2}|-i\rangle$.

Bloch-sphere intuition

- Rotating around an axis leaves the **points on that axis (the poles)** fixed.
- Therefore, any eigenstate of the corresponding Pauli operator (Z, X, or Y) is **invariant up to global phase** under rotations about that same axis.
- States **not** aligned with the axis **do** move on the sphere, changing their relative phases and measurement statistics.

Section 0.3.3 Predicting tensor-product outputs (Bloch-sphere reasoning)

We now practice predicting the result of tensor-product gates on basis and superposition states, using Bloch-sphere rotations. Throughout, we use Qiskit qubit order where $|q_{n-1}\dots q_1q_0\rangle$ is little-endian, and $|011\rangle$ means $q_2{=}0$, $q_1{=}1$, $q_0{=}1$.

Example A
Question: What is the result of $R_Z(\pi/2)\otimes R_X(\pi)$ on $|--\rangle$?
Reasoning: $|-\rangle$ lies at $(-1,0,0)$. A rotation $R_X(\pi)$ leaves any
point on the x-axis invariant (up to a global phase), so the second qubit stays
at $(-1,0,0)$. For the first qubit, $R_Z(\pi/2)$ rotates $(-1,0,0)$ by $+\pi/2$
around z: $x' = -\cos(\pi/2) = 0$, $y' = -\sin(\pi/2) = -1$, $z' = 0$. Thus the

first qubit becomes $|-i\rangle$. Overall (ignoring any global phase), the state is $|-i\rangle \otimes |-\rangle$.

Example B
Question: What is the result of $I \otimes H \otimes Z$ on $|011\rangle$?
Reasoning: $Z|1\rangle = -|1\rangle$ (a global phase -1).
$H|1\rangle = |-\rangle$. The identity does nothing on $|0\rangle$.
Ignoring the global phase, we get $|0\rangle \otimes |-\rangle \otimes |1\rangle$.

Example C
Question: What is the result of $R_Y(\pi/2) \otimes I$ on $|+\ -\rangle$?
Reasoning: $|+\rangle$ is $(1,0,0)$. A $+\pi/2$ rotation about y maps (x,z) as $x' = x\cos(\frac{\pi}{2}) + z\sin(\frac{\pi}{2})$ and $z' = -x\sin(\frac{\pi}{2}) + z\cos(\frac{\pi}{2})$. Starting from $(1,0,0)$ with $z=0$, we get $x'=0$, $z'=-1$, so the first qubit becomes $|1\rangle$.
The second qubit remains $|-\rangle$. Result: $|1\rangle \otimes |-\rangle$.

Example D
Question: What is the result of $R_Z(\pi) \otimes R_Y(\pi)$ on $|i\,0\rangle$?
Reasoning: $|i\rangle$ is $(0,1,0)$. $R_Z(\pi)$ flips $(x,y) \mapsto (-x,-y)$, so $(0,1,0) \mapsto (0,-1,0) = -|i\rangle$. For $|0\rangle$ at $(0,0,1)$, $R_Y(\pi)$ maps $z \mapsto -z$, giving $(0,0,-1) = |1\rangle$. Result: $-|i\rangle \otimes |1\rangle$.

Section 0.3.4 Implementing the above examples in Qiskit

We now implement each example and verify the per-qubit Bloch vectors by computing single-qubit reduced states and plotting them. We always transpile under a fake backend prior to analysis.

In [7]:
```
# Utility: single-qubit Bloch vectors from an n-qubit circuit/state
def single_qubit_bloch_vectors_from_circuit(qc, backend=_fake_backend):
    tqc = qc
    psi = Statevector.from_instruction(tqc)
    rho = DensityMatrix(psi)
    n = qc.num_qubits
    X = np.array([[0,1],[1,0]], dtype=complex)
    Y = np.array([[0,-1j],[1j,0]], dtype=complex)
    Z = np.array([[1,0],[0,-1]], dtype=complex)
    vecs = []
    for k in range(n):
        traced = partial_trace(rho, [i for i in range(n) if i != k])
        r = traced.data
        vecs.append([np.trace(r @ X).real,
                     np.trace(r @ Y).real,
                     np.trace(r @ Z).real])
```

```
    return vecs, tqc

def plot_per_qubit_bloch(vecs, title_prefix="Qubit"):
    outs = []
    for i, v in enumerate(vecs):
        print(f"{title_prefix} {i} Bloch ≈", np.round(v, 6))
        outs.append(plot_bloch_vector(v, title=f"{title_prefix} {i}"))
    return outs
```

In [8]:
```
# Example A: (Rz(pi/2) ⊗ Rx(pi)) on |-->
qc = QuantumCircuit(2, name="ExA")
# Prepare |--> from |00>
qc.h(0); qc.z(0)        # |-> on q0
qc.h(1); qc.z(1)        # |-> on q1
# Apply gates: first qubit Rz(pi/2), second qubit Rx(pi)
qc.rz(np.pi/2, 0)
qc.rx(np.pi, 1)

vecs, tqc = single_qubit_bloch_vectors_from_circuit(qc)
print(tqc)
_ = plot_per_qubit_bloch(vecs, title_prefix="ExA Q")
```

```
     ┌───┐┌───┐┌─────────┐
q_0: ┤ H ├┤ Z ├┤ Rz(π/2) ├
     ├───┤├───┤└┬───────┬┘
q_1: ┤ H ├┤ Z ├─┤ Rx(π) ├─
     └───┘└───┘ └───────┘
ExA Q 0 Bloch ≈ [-0. -1.  0.]
ExA Q 1 Bloch ≈ [-1.  0.  0.]
```

In [9]:
```
# Example B: (I ⊗ H ⊗ Z) on |011>
qc = QuantumCircuit(3, name="ExB")
# Prepare |011> with convention |q2 q1 q0>
# q2=0 (do nothing), q1=1 (X on q1), q0=1 (X on q0)
qc.x(1); qc.x(0)
# Apply gates
# I on q2 (no-op), H on q1, Z on q0
qc.h(1)
qc.z(0)

vecs, tqc = single_qubit_bloch_vectors_from_circuit(qc)
print(tqc)
_ = plot_per_qubit_bloch(vecs, title_prefix="ExB Q")
```

```
     ┌───┐┌───┐
q_0: ┤ X ├┤ Z ├
     ├───┤├───┤
q_1: ┤ X ├┤ H ├
     └───┘└───┘
q_2: ──────────

ExB Q 0 Bloch ≈ [ 0.  0. -1.]
ExB Q 1 Bloch ≈ [-1.  0.  0.]
ExB Q 2 Bloch ≈ [0. 0. 1.]
```

In [10]:
```
# Example C: (Ry(pi/2) ⊗ I) on |+->
qc = QuantumCircuit(2, name="ExC")
```

```
# Prepare |+->
qc.h(0)                  # |+> on q0
qc.h(1); qc.z(1)         # |-> on q1
# Apply gates
qc.ry(np.pi/2, 0)        # Ry(pi/2) on q0
# I on q1 (no-op)

vecs, tqc = single_qubit_bloch_vectors_from_circuit(qc)
print(tqc)
_ = plot_per_qubit_bloch(vecs, title_prefix="ExC Q")
```

```
     ┌───┐┌─────────┐
q_0: ┤ H ├┤ Ry(π/2) ├
     ├───┤└──┬───┬──┘
q_1: ┤ H ├───┤ Z ├───
     └───┘   └───┘
ExC Q 0 Bloch ≈ [ 0.  0. -1.]
ExC Q 1 Bloch ≈ [-1.  0.  0.]
```

In [11]:
```
# Example D: (Rz(pi) ⊗ Ry(pi)) on |+i, 0>
qc = QuantumCircuit(2, name="ExD")
# Prepare |+i> on q0: S H |0> = Rz(pi/2) H |0>
qc.h(0); qc.s(0)         # H then S gives |+i>
# Prepare |0> on q1 (default)
# Apply gates
qc.rz(np.pi, 0)
qc.ry(np.pi, 1)

vecs, tqc = single_qubit_bloch_vectors_from_circuit(qc)
print(tqc)
_ = plot_per_qubit_bloch(vecs, title_prefix="ExD Q")
```

```
       ┌───┐   ┌───┐┌───────┐
q_0: ──┤ H ├───┤ S ├┤ Rz(π) ├
     ┌─┴───┴─┐ └───┘└───────┘
q_1: ┤ Ry(π) ├───────────────
     └───────┘
ExD Q 0 Bloch ≈ [-0. -1.  0.]
ExD Q 1 Bloch ≈ [ 0.  0. -1.]
```

Section 0.3.5 Multiple Choice Questions

Q1. Which Bloch vector corresponds to the pure state $|+i\rangle$?

A. $(1,0,0)$
B. $(0,1,0)$
C. $(0,0,1)$
D. $(0,-1,0)$

Q2. Consider $R_Z(\pi/2)$ applied to $|-\rangle$. What Bloch vector results?

A. $(0,1,0)$
B. $(0,-1,0)$
C. $(-1,0,0)$
D. $(1,0,0)$

Q3. In Qiskit, which function directly plots a single-qubit Bloch vector $[x, y, z]$?
A. `plot_bloch_multivector`
B. `plot_state_qsphere`
C. `plot_bloch_vector`
D. `plot_histogram`

Q4. For the operation $I \otimes H \otimes Z$ on $|011\rangle$, which of the following is correct after ignoring any global phase?
A. $|0\rangle \otimes |+\rangle \otimes |0\rangle$
B. $|1\rangle \otimes |-\rangle \otimes |1\rangle$
C. $|0\rangle \otimes |-\rangle \otimes |1\rangle$
D. $|1\rangle \otimes |+\rangle \otimes |0\rangle$

Q5: Two-qubit rotations (I).** Start in $|+\rangle \otimes |1\rangle$. Apply, in order:

1. $R_Z(\pi/2)$ on qubit 0;
2. $R_X(\pi)$ on qubit 0;
3. $R_Y(\pi/3)$ on qubit 1.

What Bloch vector results for **qubit 0** (after tracing out qubit 1)?
A. $(0, 1, 0)$
B. $(0, -1, 0)$
C. $(-1, 0, 0)$
D. $(1, 0, 0)$

Q6: Two-qubit rotations (II).** Start in $|0\rangle \otimes |+\rangle$. Apply, in order:

1. $R_X(\pi/2)$ on qubit 0;
2. $R_Z(\pi)$ on qubit 0;
3. $R_Y(\pi/7)$ on qubit 1.

What Bloch vector results for **qubit 0** (after tracing out qubit 1)?
A. $(0, 1, 0)$
B. $(0, -1, 0)$
C. $(1, 0, 0)$
D. $(-1, 0, 0)$

Q7: Single-qubit statevector (I).** Start in $|+\rangle$. Apply, in order:

1. $R_Z(\pi/2)$;
2. X;
3. $R_Y(\pi/2)$.

What is the resulting **single-qubit statevector**, in the basis $\{|0\rangle, |1\rangle\}$, up to a global phase?
A. $\left(\frac{-1+i}{2}, \frac{1+i}{2}\right)$

B. $\left(\frac{1+i}{2},\ \frac{-1+i}{2}\right)$
C. $\left(\frac{1}{\sqrt{2}},\ \frac{i}{\sqrt{2}}\right)$
D. $\left(\frac{i}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right)$

Q8: Single-qubit statevector (II).** Start in $|0\rangle$. Apply, in order:

1. H;
2. $R_Z(\pi)$;
3. X;
4. $R_Y(\pi/2)$.

What is the resulting **single-qubit statevector**, in the basis $\{|0\rangle, |1\rangle\}$, up to a global phase?
A. $(1,\ 0)$
B. $(0,\ 1)$
C. $\left(\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right)$
D. $\left(\frac{1}{\sqrt{2}},\ \frac{-1}{\sqrt{2}}\right)$

▶ **Answer key and explanations**

In [ ]: