Practical Task 8.1

(Credit Task)

Submission deadline: 11:59 pm Sunday, May 16th
Discussion deadline: 11:59 pm Sunday, May 30th

Task Objective

The objective of this task is to study implementation of a *Binary Heap*, a data structure which is seen as a special case of a complete binary tree.

Background

Like a binary tree, a heap consists of a collection of nodes that can be considered as building blocks of the data structure. The tree structure that a binary heap represents is complete; that is, every level, except possibly the last one, is completely filled, and all nodes are as far left as possible. This makes a binary heap with n nodes be always of a $O(\log n)$ height. In addition to the standard binary tree properties, a binary heap must also adhere to the mandatory *Heap Ordering* property. The ordering can be one of the two types:

- The *Min-Heap Property*: the value of each node is greater than or equal to the value of its parent, with the minimum-value element at the root.
- The *Max-Heap Property*: the value of each node is less than or equal to the value of its parent, with the maximum-value element at the root.

Note that a binary heap is not a sorted structure and can be regarded as partially ordered. Indeed, there is no particular relationship among nodes on any given level, even among siblings. From a practical perspective, a binary heap is a very useful data structure when one needs to remove the object with the lowest (or highest, in case of the max-heap ordering) priority.

A binary heap can be uniquely represented by storing its level order traversal in an array or an array-based collection like a list (also known as a vector). Note that the links between nodes are not required. For the convenience of implementation, the first entry of the array with index 0 is skipped; it contains a dummy (default) element. Therefore, the root of a heap is the second item in the array at index 1, and the length of the array is $n + 1$ for a heap with n data elements. This implies that for the k^{th} element of the array the following statements are valid:

- the left child is located at index $2 \cdot k$;
- the right child is located at index $2 \cdot k + 1$;
- the parent is located uniquely at index $\lfloor k/2 \rfloor$.

Insertion of a new element initially appends it to the end of a heap as the last element of the array at index $n + 1$. The *Heap Ordering* property is then repaired by comparing the added element with its parent and moving the added element up a level (swapping positions with the parent). This process is commonly known as "UpHeap", or "Heapify-Up", or "Sift-Up". The comparison is repeated until the parent is larger (or smaller, in case of the max-heap ordering) than or equal to the percolating element. The worst-case runtime of the algorithm is $O(\log n)$, since we need at most one swap on each level of a heap on the path from the inserted node to the root.

The *minimum* (or *maximum*, in case of the max-heap ordering) element can be found at the root, which is the element of the array located at index 1. *Deletion* of the *minimum* element first replaces it with the last element of the array at index n, and then restores the *Heap Ordering* property by following the process known as "DownHeap", or "Heapify-Down", or "Sift-Down". Similar to insertion, the worst-case runtime is $O(\log n)$.

Task Details

In this task, work through each of the following steps to complete the task:

1. Explore the source code attached to this task. Create a new Microsoft Visual Studio project and import the enclosed Heap.cs, IHeapifyable.cs, and Tester.cs files. Your newly built project should compile and work without errors. The objective of the task is to develop the missing functionality of the Heap<K,D> class. The Heap.cs contains a template of the Heap<K,D>. The Tester.cs contains a prepared Main method that should help you to build a fully-functional data structure. It enables a number of tests important for debugging and testing of the Heap<K,D> class and its interfaces for runtime and logical errors.

2. Find the nested Node class presented inside the Heap<K,D> and explore its structure. This is a generic class that represents a node of a binary heap. Think about it as an atomic data structure itself that serves the Heap<K,D> as a building block. It is a data structure consisting of

   - a generic type raw *data* (a *payload*),
   - a generic type *key* necessary to place the node with regard to the order of nodes existing in the Heap<K,D>, and
   - an integer-valued *position* (*index*) that locates the node in the array-based collection of nodes of the Heap<K,D>.

   Because both K and D types are generic, the *key* and the *data* may be of an arbitrary type: a string, an integer, or a user-defined class. Finally, note that the Node class is ready for you to use. It provides the following functionality:

   - **Node(K key, D value, int position)**

     Initializes a new instance of the Node class associated with the specified generic key. The node records the given generic data as well as its own index-based position within the array-based collection of data nodes privately owned by the related Heap<K,D>.

   - **D Data**

     Property. Gets or sets the data of generic type D associated with the Node.

   - **K Key**

     Property. Gets or sets the key of generic type K assigned to the Node.

   - **Position**

     Property. Gets or sets the index-based position of the Node in the array-based collection of nodes constituting the Heap<K,D> .

   - **string ToString()**

     Returns a string that represents the current Node.

   The Node class implements the IHeapifyable<K,D> interface, which is defined in the attached IHeapifyable.cs. Note that this interface is parametrized by the same two generic data types as the Heap<K,D>. The reason for the use of the interface is that the Node class is a data structure internal to the Heap<K,D>, therefore an instance of the Node must not be exposed to a user. It must remain hidden for the user in order to protect the integrity of the whole data structure. Otherwise, manipulating the nodes directly, the user may easily corrupt the structure of a binary heap and violate the important *Heap-Ordering* rule. Nevertheless, because the user needs access to the data that the user owns and stores inside a binary heap, the Node implements the interface that permits reading and modifying the data. Therefore, the primal purpose of the IHeapifyable<K,D> is to record and retrieve the data associated with a particular node and track the position of the node in the array-based collection of nodes of the Heap<K,D>.

Check the IHeapifyable<K,D> interface to see that the only property it allows to change is the Data. The other two properties, the Key and the Position, are read-only. Note that the value of a key is set at the time the node is added to a heap. It then can be changed only via dedicated operations, like *DecreaseKey*. The Heap<K,D> is entirely responsible for *Position*, thus modification of this property by the user is impossible.

3. Proceed with the given template of the Heap<K,D> class and explore the methods that it has implemented for you for the purpose of example, in particular:
   - **Heap(IComparer<K> comparer)**

     Initializes a new instance of the Heap<K,D> class and stores the specified reference to the object that enables comparison of two keys of type K.
   - **Count**

     Property. Gets the number of elements stored by the Heap<K,D>.
   - **IHeapifyable<K, D> Min()**

     Returns the element with the minimum (or maximum) key positioned at the top of the Heap<K,D>, without removing it. The element is casted to the IHeapifyable<K,D>. The method throws InvalidOperationException if the Heap<K,D> is empty.
   - **IHeapifyable<K, D> Insert(K key, D value)**

     Inserts a new node containing the specified key-value pair into the Heap<K,D>. The position of the new element in the binary heap is determined according the Heap-Order policy. It returns the newly created node casted to the IHeapifyable<K,D>.
   - **void Clear()**

     Removes all nodes from the Heap<K,D> and sets the Count to zero.
   - **string ToString()**

     Returns a string representation of the Heap<K,D>.

   Rather than an array, the Heap<K,D> utilizes the native .NET Framework List<T> generic collection as the internal data structure. This collection is dynamic as opposed to an array, which is static. This fact should simplify your work. Furthermore, note that the internal structure of the Heap<K,D> can be explored only implicitly through the positions of the nodes constituting it.

   As you may have noticed, the comparison of nodes is performed by the comparator originally set within the constructor of the Heap<K,D>. Providing different comparator to the constructor will change the behaviour of the Heap<K,D>. When keys are ordered in ascending order, the Heap<K,D> acts as a min-heap. When the comparator orders keys in descending order, the Heap<K,D> behaves as a max-heap.

4. You must complete the Heap<K,D> class and provide the following functionality to the user:
   - **IHeapifyable<K, D> Delete()**

     Deletes and returns the node casted to the IHeapifyable<K,D> positioned at the top of the Heap<K,D>. This method throws InvalidOperationException if the Heap<K,D> is empty.
   - **IHeapifyable<K, D>[] BuildHeap(K[] keys, D[] data)**

     Builds a binary heap following the bottom-up approach. Each new element of the heap is derived by the key-value pair (keys[i],data[i]) specified by the method's parameters. It returns an array of nodes casted to the IHeapifyable<K,D>. Each node at index i must match its key-value pair at index i of the two input arrays. This method throws InvalidOperationException if the Heap<K,D> is not empty.
   - **DecreaseKey(IHeapifyable<K, D> element, K new_key)**

     Decreases the key of the specified element presented in the Heap<K,D>. The method throws InvalidOperationException when the node stored in the Heap<K,D> at the position specified by the element is different to the element. This signals that the given element is inconsistent to the current state of the Heap<K,D>.

Note that you are free in writing your code that is private to the Heap<K,D> unless you respect all the requirements in terms of functionality and signatures of the specified methods.

5. Now you are required to implement some non-standard operations to your Heap H. Firstly, implement a $DeleteElement(X)$ operation that removes element $H[x]$ from the min-heap H consisting of n elements in a logarithmic $O(\log n)$ time. Note that the $H[x]$ does not need to be the minimum-key element in H. Also, naturally, H has to maintain the *Min-Heap Property* after deletion of $H[x]$. Include as a comment in your code a brief analysis of your code's running time as a function of n elements.
6. Min Heap allows us to have a *Min* operation in a constant $O(1)$ time. Now create a new function $KthMinElement(int\ k)$ that finds the k^{th} minimum element in $O(k \log n)$ time, where $1 \leq k \leq n$. Remember that both insertion and removal in a binary min-heap of size n takes a $O(\log n)$ time. Include as a comment in your code a brief analysis of your code's running time as a function of n elements.
7. As you progress with the implementation of the Heap<K,D> class, you should start using the Tester class to thoroughly test the Heap<K,D> aiming on the coverage of all potential logical issues and runtime errors. This (testing) part of the task is as important as writing the Heap<K,D> class. The given version of the testing class covers only some basic cases. Therefore, you should extend it with extra cases to make sure that your data structure is checked against other potential mistakes.

Expected Printout

Appendix A provides an example printout for your program. If you are getting a different printout then your implementation is not ready for submission. Please ensure your program prints out Success for all tests before submission.

Further Notes

- Explore the material of chapter 9.4 of the SIT221 course book "Data structures and algorithms in Java" (2014) by M. Goodrich, R. Tamassia, and M. Goldwasser. You may access the book on-line for free from the reading list application in CloudDeakin available in Resources → Additional Course Resources → Resources on Algorithms and Data Structures → Course Book: Data structures and algorithms in Java. As a supplementary material, to learn more about the theory part and implementation issues of binary heaps, you may refer to the Section 9.2.3 of Chapter 9 of SIT221 Workbook available in CloudDeakin in Resources → Additional Course Resources → Resources on Algorithms and Data Structures → SIT221 Workbook .
- You may find exploring the content of chapter 9.3 of the course book "Data Structures and Algorithms in Java" by Michael T. Goodrich, Roberto Tamassia, and Michael H. Goldwasser (2014) useful to solve these tasks. You may access the book on-line for free from the reading list application in CloudDeakin available in Resources → Additional Course Resources → Resources on Algorithms and Data Structures → Course Book: Data structures and algorithms in Java.
- The lecture notes of week 6 may be the best material to understand the logic behind a binary heap and its array-based implementation.

Marking Process and Discussion

To get your task completed, you must finish the following steps strictly on time.

1. Work on your task either during your allocated lab time or during your own study time.
2. Once the task is complete you should make sure that your program implements all the required functionality, is compliable, and has no runtime errors. Programs causing compilation or runtime errors will not be accepted as a solution. You need to test your program thoroughly before submission. Think about potential errors where your program might fail. Note we can sometime use test cases that are different to those provided so verify you have checked it more thoroughly than just using the test program provided.
3. Submit your solution as an answer to the task via the OnTrack submission system. This first submission must be prior to the submission “S” deadline indicated in the unit guide and in OnTrack.
4. If your task has been assessed as requiring a “Redo” or “Resubmit” then you should prepare a new submission. You will have 1 (7 day) calendar week from the day you receive the assessment from the tutor. This usually will mean you should revise the lecture, the readings indicated, and read the unit discussion list for suggestions. After your submission has been corrected and providing it is still before the due deadline you can resubmit.
5. If your task has been assessed as correct, either after step 3 or 4, you can “discuss” with your tutor. This first discussion must occur prior to the discussion “D”.
6. Meet with your tutor or answer question via the intelligent discussion facility to demonstrate/discuss your submission. Be on time with respect to the specified discussion deadline.
7. The tutor will ask you both theoretical and practical questions. Questions are likely to cover lecture notes, so attending (or watching) lectures should help you with this compulsory interview part. The tutor will tick off the task as complete, only if you provide a satisfactory answer to these questions.
8. If you cannot answer the questions satisfactorily your task will remain on discussion and you will need to study the topic during the week and have a second discussion the following week.
9. Please note, due to the number of students and time constraints tutors will only be expected to mark and/or discuss your task twice. After this it will be marked as a “Exceeded Feedback”.
10. If your task has been marked as “Exceeded Feedback” you are eligible to do the redemption quiz for this task. Go to this unit’s site on Deakin Sync and find the redemption quiz associated with this task. You get three tries at this quiz. Ensure you record your attempt.
    I. Login to Zoom and join a meeting by yourself.
    II. Ensure you have both a camera and microphone working
    III. Start a recording.
    IV. Select Share screen then select “Screen”. This will share your whole desktop. Ensure Zoom is including your camera view of you in the corner.
    V. Bring your browser up and do the quiz.
    VI. Once finished select stop recording.
    VII. After five to ten minutes you should get an email from Zoom providing you with a link to your video. Using the share link, copy this and paste in your chat for this task in OnTrack for your tutor to verify the recording.
11. Note that we will not check your solution after the submission deadline and will not discuss it after the discussion deadline. If you fail one of the deadlines, you fail the task and this reduces the chance to pass the unit. Unless extended for all students, the deadlines are strict to guarantee smooth and on-time work through the unit.
12. Final note, A “Fail” or “Exceeded Feedback” grade on a task does not mean you have failed the unit. It simply means that you have not demonstrated your understanding of that task through OnTrack. Similarly failing the redemption quiz also does not mean you have failed the unit. You can replace a task with a task from a higher grade.

Expected Printout

This section displays the printout produced by the attached Tester class, specifically by its *Main* method. It is based on our solution. The printout is provided here to help with testing your code for potential logical errors. It demonstrates the correct logic rather than an expected printout in terms of text and alignment.

```
Test A: Create a min-heap by calling 'minHeap = new Heap<int, string>(new IntAscendingComparer());'
 :: SUCCESS: min-heap's state []



Test B: Run a sequence of operations:


Insert a node with name Kelly (data) and ID 1 (key).
 :: SUCCESS: min-heap's state [(1,Kelly,1)]


Insert a node with name Cindy (data) and ID 6 (key).
 :: SUCCESS: min-heap's state [(1,Kelly,1),(6,Cindy,2)]


Insert a node with name John (data) and ID 5 (key).
 :: SUCCESS: min-heap's state [(1,Kelly,1),(6,Cindy,2),(5,John,3)]


Insert a node with name Andrew (data) and ID 7 (key).
 :: SUCCESS: min-heap's state [(1,Kelly,1),(6,Cindy,2),(5,John,3),(7,Andrew,4)]


Insert a node with name Richard (data) and ID 8 (key).
 :: SUCCESS: min-heap's state [(1,Kelly,1),(6,Cindy,2),(5,John,3),(7,Andrew,4),(8,Richard,5)]


Insert a node with name Michael (data) and ID 3 (key).
 :: SUCCESS: min-heap's state [(1,Kelly,1),(6,Cindy,2),(3,Michael,3),(7,Andrew,4),(8,Richard,5),(5,John,6)]


Insert a node with name Guy (data) and ID 10 (key).
 :: SUCCESS: min-heap's state [(1,Kelly,1),(6,Cindy,2),(3,Michael,3),(7,Andrew,4),(8,Richard,5),(5,John,6),(10,Guy,7)]


Insert a node with name Elicia (data) and ID 4 (key).
 :: SUCCESS: min-heap's state
[(1,Kelly,1),(4,Elicia,2),(3,Michael,3),(6,Cindy,4),(8,Richard,5),(5,John,6),(10,Guy,7),(7,Andrew,8)]


Insert a node with name Tom (data) and ID 2 (key).
 :: SUCCESS: min-heap's state
[(1,Kelly,1),(2,Tom,2),(3,Michael,3),(4,Elicia,4),(8,Richard,5),(5,John,6),(10,Guy,7),(7,Andrew,8),(6,Cindy,9)]


Insert a node with name Iman (data) and ID 9 (key).
 :: SUCCESS: min-heap's state
[(1,Kelly,1),(2,Tom,2),(3,Michael,3),(4,Elicia,4),(8,Richard,5),(5,John,6),(10,Guy,7),(7,Andrew,8),(6,Cindy,9),(9,Iman,10)]


Insert a node with name Simon (data) and ID 14 (key).
```

:: SUCCESS: min-heap's state
[(1,Kelly,1),(2,Tom,2),(3,Michael,3),(4,Elicia,4),(8,Richard,5),(5,John,6),(10,Guy,7),(7,Andrew,8),(6,Cindy,9),(9,Iman,10),(14,Simon,11)]

Insert a node with name Vicky (data) and ID 12 (key).

:: SUCCESS: min-heap's state
[(1,Kelly,1),(2,Tom,2),(3,Michael,3),(4,Elicia,4),(8,Richard,5),(5,John,6),(10,Guy,7),(7,Andrew,8),(6,Cindy,9),(9,Iman,10),(14,Simon,11),(12,Vicky,12)]

Insert a node with name Kevin (data) and ID 11 (key).

:: SUCCESS: min-heap's state
[(1,Kelly,1),(2,Tom,2),(3,Michael,3),(4,Elicia,4),(8,Richard,5),(5,John,6),(10,Guy,7),(7,Andrew,8),(6,Cindy,9),(9,Iman,10),(14,Simon,11),(12,Vicky,12),(11,Kevin,13)]

Insert a node with name David (data) and ID 13 (key).

:: SUCCESS: min-heap's state
[(1,Kelly,1),(2,Tom,2),(3,Michael,3),(4,Elicia,4),(8,Richard,5),(5,John,6),(10,Guy,7),(7,Andrew,8),(6,Cindy,9),(9,Iman,10),(14,Simon,11),(12,Vicky,12),(11,Kevin,13),(13,David,14)]

Test C: Run a sequence of operations:

Delete the minimum element from the min-heap.

:: SUCCESS: min-heap's state
[(2,Tom,1),(4,Elicia,2),(3,Michael,3),(6,Cindy,4),(8,Richard,5),(5,John,6),(10,Guy,7),(7,Andrew,8),(13,David,9),(9,Iman,10),(14,Simon,11),(12,Vicky,12),(11,Kevin,13)]

Delete the minimum element from the min-heap.

:: SUCCESS: min-heap's state
[(3,Michael,1),(4,Elicia,2),(5,John,3),(6,Cindy,4),(8,Richard,5),(11,Kevin,6),(10,Guy,7),(7,Andrew,8),(13,David,9),(9,Iman,10),(14,Simon,11),(12,Vicky,12)]

Delete the minimum element from the min-heap.

:: SUCCESS: min-heap's state
[(4,Elicia,1),(6,Cindy,2),(5,John,3),(7,Andrew,4),(8,Richard,5),(11,Kevin,6),(10,Guy,7),(12,Vicky,8),(13,David,9),(9,Iman,10),(14,Simon,11)]

Delete the minimum element from the min-heap.

:: SUCCESS: min-heap's state
[(5,John,1),(6,Cindy,2),(10,Guy,3),(7,Andrew,4),(8,Richard,5),(11,Kevin,6),(14,Simon,7),(12,Vicky,8),(13,David,9),(9,Iman,10)]

Delete the minimum element from the min-heap.

:: SUCCESS: min-heap's state
[(6,Cindy,1),(7,Andrew,2),(10,Guy,3),(9,Iman,4),(8,Richard,5),(11,Kevin,6),(14,Simon,7),(12,Vicky,8),(13,David,9)]

Delete the minimum element from the min-heap.

:: SUCCESS: min-heap's state
[(7,Andrew,1),(8,Richard,2),(10,Guy,3),(9,Iman,4),(13,David,5),(11,Kevin,6),(14,Simon,7),(12,Vicky,8)]

Delete the minimum element from the min-heap.

:: SUCCESS: min-heap's state [(8,Richard,1),(9,Iman,2),(10,Guy,3),(12,Vicky,4),(13,David,5),(11,Kevin,6),(14,Simon,7)]

Delete the minimum element from the min-heap.

:: SUCCESS: min-heap's state [(9,Iman,1),(12,Vicky,2),(10,Guy,3),(14,Simon,4),(13,David,5),(11,Kevin,6)]

Delete the minimum element from the min-heap.

:: SUCCESS: min-heap's state [(10,Guy,1),(12,Vicky,2),(11,Kevin,3),(14,Simon,4),(13,David,5)]

Delete the minimum element from the min-heap.

:: SUCCESS: min-heap's state [(11,Kevin,1),(12,Vicky,2),(13,David,3),(14,Simon,4)]

Delete the minimum element from the min-heap.

:: SUCCESS: min-heap's state [(12,Vicky,1),(14,Simon,2),(13,David,3)]

Delete the minimum element from the min-heap.

:: SUCCESS: min-heap's state [(13,David,1),(14,Simon,2)]

Delete the minimum element from the min-heap.

:: SUCCESS: min-heap's state [(14,Simon,1)]

Delete the minimum element from the min-heap.

:: SUCCESS: min-heap's state []

Test D: Delete the minimum element from the min-heap.

:: SUCCESS: InvalidOperationException is thrown because the min-heap is empty

Test E: Run a sequence of operations:

Insert a node with name Kelly (data) and ID 1 (key).

:: SUCCESS: min-heap's state [(1,Kelly,1)]

Build the min-heap for the pair of key-value arrays with
[1, 6, 5, 7, 8, 3, 10, 4, 2, 9, 14, 12, 11, 13] as keys and
[Kelly, Cindy, John, Andrew, Richard, Michael, Guy, Elicia, Tom, Iman, Simon, Vicky, Kevin, David] as data elements

:: SUCCESS: InvalidOperationException is thrown because the min-heap is not empty

Test F: Run a sequence of operations:

Clear the min-heap.

:: SUCCESS: min-heap's state []

Build the min-heap for the pair of key-value arrays with

[1, 6, 5, 7, 8, 3, 10, 4, 2, 9, 14, 12, 11, 13] as keys and

[Kelly, Cindy, John, Andrew, Richard, Michael, Guy, Elicia, Tom, Iman, Simon, Vicky, Kevin, David] as data elements

:: SUCCESS: min-heap's state
[(1,Kelly,1),(2,Tom,2),(3,Michael,3),(4,Elicia,4),(8,Richard,5),(5,John,6),(10,Guy,7),(6,Cindy,8),(7,Andrew,9),(9,Iman,10),(14,Simon,11),(12,Vicky,12),(11,Kevin,13),(13,David,14)]

Test G: Run a sequence of operations:

Delete the minimum element from the min-heap.

:: SUCCESS: min-heap's state
[(2,Tom,1),(4,Elicia,2),(3,Michael,3),(6,Cindy,4),(8,Richard,5),(5,John,6),(10,Guy,7),(13,David,8),(7,Andrew,9),(9,Iman,10),(14,Simon,11),(12,Vicky,12),(11,Kevin,13)]

Delete the minimum element from the min-heap.

:: SUCCESS: min-heap's state
[(3,Michael,1),(4,Elicia,2),(5,John,3),(6,Cindy,4),(8,Richard,5),(11,Kevin,6),(10,Guy,7),(13,David,8),(7,Andrew,9),(9,Iman,10),(14,Simon,11),(12,Vicky,12)]

Run DecreaseKey(node,0) for node (13,David,8) by setting the new value of its key to 0

:: SUCCESS: min-heap's state
[(0,David,1),(3,Michael,2),(5,John,3),(4,Elicia,4),(8,Richard,5),(11,Kevin,6),(10,Guy,7),(6,Cindy,8),(7,Andrew,9),(9,Iman,10),(14,Simon,11),(12,Vicky,12)]

Test H: Run a sequence of operations:

Create a max-heap by calling 'maxHeap = new Heap<int, string>(new IntDescendingComparer());'

:: SUCCESS: max-heap's state []

Build the max-heap for the pair of key-value arrays with

[1, 6, 5, 7, 8, 3, 10, 4, 2, 9, 14, 12, 11, 13] as keys and

[Kelly, Cindy, John, Andrew, Richard, Michael, Guy, Elicia, Tom, Iman, Simon, Vicky, Kevin, David] as data elements

:: SUCCESS: max-heap's state
[(14,Simon,1),(9,Iman,2),(13,David,3),(7,Andrew,4),(8,Richard,5),(12,Vicky,6),(10,Guy,7),(4,Elicia,8),(2,Tom,9),(6,Cindy,10),(1,Kelly,11),(3,Michael,12),(11,Kevin,13),(5,John,14)]

Test I: Run a sequence of operations:

Delete the Richard element from the min-heap.

:: SUCCESS: min-heap's state
[(0,David,1),(3,Michael,2),(5,John,3),(4,Elicia,4),(9,Iman,5),(11,Kevin,6),(10,Guy,7),(6,Cindy,8),(7,Andrew,9),(12,Vicky,10),(14,Simon,11)]

:: SUCCESS: min-heap's state
[(0,David,1),(3,Michael,2),(5,John,3),(4,Elicia,4),(9,Iman,5),(11,Kevin,6),(10,Guy,7),(6,Cindy,8),(7,Andrew,9),(12,Vicky,10),(14,Simon,11)]

```
Test J: Run a sequence of operations:

min-heap's state
[(0,David,1),(3,Michael,2),(5,John,3),(4,Elicia,4),(9,Iman,5),(11,Kevin,6),(10,Guy,7),(6,Cindy,8),(7,Andrew,9),(12,Vicky,10
),(14,Simon,11)]

 :: SUCCESS: 4th Node is (5,John,-1)

 :: SUCCESS: min-heap's state
[(0,David,1),(3,Michael,2),(10,Guy,3),(6,Cindy,4),(4,Elicia,5),(11,Kevin,6),(12,Vicky,7),(14,Simon,8),(7,Andrew,9),(9,Iman
,10),(5,John,11)]



 ------------------- SUMMARY -------------------

Tests passed: ABCDEFGHIJ
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