## Cheat Sheet

May 24, 2022

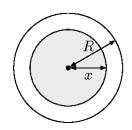
## 1 Hyperbolic Random Graphs

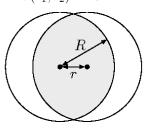
$$p(\theta, r) := \frac{\alpha}{2\pi} \frac{\sinh(\alpha r)}{\cosh(\alpha R) - 1} \tag{1}$$

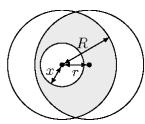
$$R = 2\log n + C \tag{2}$$

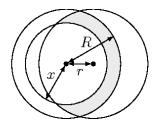
Average degree: 
$$\overline{\delta} = (1 + o(1)) \frac{2\alpha^2 e^{-C/2}}{\pi(\alpha - 1/2)^2}$$
 (3)

$$\theta(r_1, r_2) = 2e^{\frac{R-r_1-r_2}{2}} + \varepsilon \tag{4}$$









$$\mu(B_o(x)) = (1 + o(1))e^{-\alpha(R - x)}$$
(5)

$$\mu(B_r(R) \cap B_o(R)) = \frac{2\alpha e^{-r/2}}{\pi(\alpha - \frac{1}{2})} \left(1 \pm \mathcal{O}(e^{-(\alpha - \frac{1}{2})r} + e^{-r})\right)$$
(6)

$$\mu((B_r(R) \cap B_o(R))/B_o(x)) = \frac{2\alpha e^{-r/2}}{\pi(\alpha - \frac{1}{2})} (1 \pm \mathcal{O}(e^{-(\alpha - \frac{1}{2})r} + e^{-r})) \text{ for } x \le R - r$$
(7)

$$\mu((B_r(R) \cap B_0(R)) \ B_0(x)) =$$

$$\frac{2^{-r/2}}{\pi(\alpha - 1/2)} \left(1 - \left(1 + \frac{\alpha - 1/2}{\alpha + 1/2}e^{-2\alpha x}\right)e^{-(\alpha - 1/2)(R - x)}\right) \left(1 \pm \mathcal{O}(e^{-r} + e^{-r - (R - x)(\alpha - 3/2)})\right) \text{ for } x \ge R - r$$
(8)

Highest degree: 
$$\Delta(\alpha, n) = n^{\frac{1}{2\alpha} + o(1)}$$
 (9)

## 2 Trigonometry

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \tag{10}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \tag{11}$$

$$\cosh(x \pm y) = \cosh(x)\cosh(y) \pm \sinh(x)\sinh(y) \tag{12}$$

$$\cos(\theta) \ge 1 - \frac{\theta^2}{2} \tag{13}$$

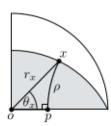
$$\cos(\theta) \le 1 - \frac{\theta^2}{2} + \frac{\theta^4}{4!} \tag{14}$$

$$\cosh(x)\cosh(y) = e^{x+y}(1 + \mathcal{O}(e^{-2x} + e^{-2y})) \tag{15}$$

$$\sinh(x)\sinh(y) = e^{x+y}(1 - \mathcal{O}(e^{-2x} + e^{-2y})) \tag{16}$$

$$\frac{\Delta}{\Delta x}\cosh(x) = \sinh(x) \tag{17}$$

$$sin(\theta_x) = \frac{\sinh(\rho)}{\sinh(r_x)} \tag{18}$$



## 3 Equalities and Inequalities

$$1 + x \le e^x \tag{19}$$

$$\sqrt{1+x} = 1 + \frac{x}{2} + \Theta(x^2) \iff |x| \le 1$$
 (20)

$$\frac{1}{1-x} = 1 + \Theta(x) \iff x \in o(1)$$
 (21)

$$(1-p)^n \le e^{-pn} \tag{22}$$