Cheat Sheet

May 18, 2022

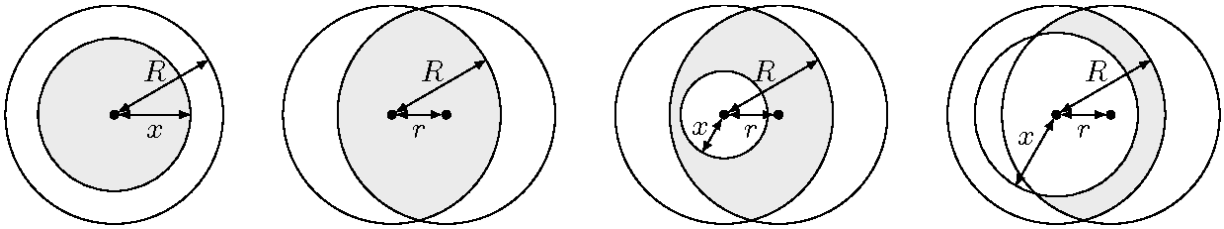
1 Hyperbolic Random Graphs

$$p(\theta, r) := \frac{\alpha}{2\pi} \frac{sinh(\alpha r)}{cosh(\alpha R) - 1} \tag{1}$$

$$R = 2 \log n + C \tag{2}$$

$$\text{Average degree: } \overline{\delta} = (1 + o(1)) \frac{2\alpha^2 e^{-C/2}}{\pi(\alpha - 1/2)^2} \tag{3}$$

$$\theta(r_1, r_2) = 2e^{\frac{R - r_1 - r_2}{2}} + \varepsilon \tag{4}$$



$$\mu(B_o(x)) = (1 + o(1)) e^{-\alpha(R - x)} \tag{5}$$

$$\mu(B_r(R) \cap B_o(R)) = \frac{2\alpha e^{-r/2}}{\pi(\alpha - \frac{1}{2})} (1 \pm \mathcal{O}(e^{-(\alpha - \frac{1}{2})r} + e^{-r})) \tag{6}$$

$$\mu((B_r(R) \cap B_o(R))/B_o(x)) = \frac{2\alpha e^{-r/2}}{\pi(\alpha - \frac{1}{2})} (1 \pm \mathcal{O}(e^{-(\alpha - \frac{1}{2})r} + e^{-r})) \textbf{ for } x \leq R - r \tag{7}$$

$$\mu((B_r(R) \cap B_0(R)) \; B_0(x)) = \frac{2^{-r/2}}{\pi(\alpha - 1/2} (1 - (1 + \frac{\alpha - 1/2}{\alpha + 1/2} e^{-2\alpha x}) e^{-(\alpha - 1/2)(R - x)}) (1 \pm \mathcal{O}(e^{-r} + e^{-r - (R - x)(\alpha - 3/2)})) \textbf{ for } x \geq R - r \tag{8}$$

$$\text{Highest degree: } \Delta(\alpha, n) = n^{\frac{1}{2\alpha} + o(1)} \tag{9}$$

2 Trigonometry

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \tag{10}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \tag{11}$$

$$\cosh(x \pm y) = \cosh(x)\cosh(y) \pm \sinh(x)\sinh(y) \tag{12}$$

$$\cos(\theta) \geq 1 - \frac{\theta^2}{2} \tag{13}$$

$$\cos(\theta) \leq 1 - \frac{\theta^2}{2} + \frac{\theta^4}{4!} \tag{14}$$

$$\cosh(x)\cosh(y) = e^{x+y}(1 + \mathcal{O}(e^{-2x} + e^{-2y})) \tag{15}$$

$$\sinh(x)\sinh(y) = e^{x+y}(1 - \mathcal{O}(e^{-2x} + e^{-2y})) \tag{16}$$

$$\frac{\Delta}{\Delta x}\cosh(x) = \sinh(x) \tag{17}$$

3 Equalities and Inequalities

$$1 + x \leq e^x \tag{18}$$

$$\sqrt{1+x} = 1 + \frac{x}{2} + \Theta(x^2) \iff |x| \leq 1 \tag{19}$$

$$\frac{1}{1-x} = 1 + \Theta(x) \iff x \in o(1) \tag{20}$$

$$(1-p)^n \leq e^{-pn} \tag{21}$$