I implemented a program that takes in a file containing a list of types defined like types in System F. For each type it determines if the type is uninhabited or infinite. If the type is inhabited and finite, then it determines what terms fulfill that type. In order to evaluate the types, I created a dictionary that assigns a variable to each type, then used a function to append to the dictionary types that must be generated using other inputs.

For example, if the input is "VX.VY.VZ.(X->X->Z)->(Y->X)->(Y)->Z". Then the lambda calculus term is: $VX.VY.VZ.\0:(X->X->Z).\1:(Y->X).\2:(Y)$. 1212. With 1212 being the only inhabitant. In order to use the substitute function, I had to append the type dictionary to contain $\{X:12\}$.

I created a test file containing different types in order to test my program.

Explanation of tests in testFile.txt:

1. "VX.VY.(X->X->X)->(Y)->X"

Tests for an uninhabited type.

2. "VX.VY.(X->X->Y)->Y"

Tests for an uninhabited type.

3. "VX.VY.(X->X->Y)->(X)->Y"

Tests for type inhabited by one term

4. "VX.VY.(X->X->Y)->(X)->(X)->Y"

Tests for a type inhabited by four terms.

5. "VX.(X->X->X)->(X)->X"

Test for infinite type.

6. "VX.VY.VZ.(X->X->Z)->(Y->X)->(Y)->Z"

Test for type that relies on variable that must be generated by genTerms. One Term.

7. "VX.VY.VZ.(X->X->Z)->(Y->X)->(Y)->(X)->Z"

Test for a type inhabited by four terms, but one of the substituting terms must be created in genTerms.

8. "VX.VY(X->X->X)->(Y->X)->(Y)->X"

Test for infinite type that has terms created in genTerms.

9. "VX.VY.VZ.VR.(X->X->Z)->(R->X)->(Y->R)->(Y->R)->(Y)->Z"

Test for multiple ways term can be constructed in genTerms. Four Terms

10. "VX.VY.VR.VQ.(X->X->Q)->(R->X)->(Y->R)->(Y)->(Y)->Q"

Test for multiple ways term can be constructed in genTerms. Four terms.

11. "VX.VY.VR.VQ.(X->X->Q)->(R->X)->(Y->R)->(Y->R)->(Y)->(Y)->Q"

Test for multiple ways term can be constructed in genTerms. Sixteen terms.

12. "VX.VY.VZ.(X->X->X)->(X)->Z"

Test for type uninhabited because first term does not result in final term.

Note about inputs:

- Type must be wrapped in quotes, no spaces
- All types except for the final type must be wrapped in parentheses
- No parentheses can be contained in types
- Final type should only be one term (no arrows)
- Type variables cannot be numbers

Typing in System F: https://www3.nd.edu/~dchiang/teaching/pl/2022/f.html