

VE477

Introduction to Algorithms

Homework 3

Manuel — UM-JI (Fall 2019)

Reminders

- Write in a neat and legible handwriting or use \LaTeX
- Clearly explain the reasoning process
- Write in a complete style (subject, verb, and object)
- Be critical on your results

*Questions preceded by a * are optional. Although they can be skipped without any deduction, it is important to know and understand the results they contain.*

Ex. 1 — Hamiltonian path

- * 1. Explain and present Depth-First Search (DFS).
- * 2. Explain and present topological sorting.
- 3. Write the pseudo-code of a polynomial time algorithm which decides if a directed acyclic graph contains a Hamiltonian path.
- 4. Prove its complexity.
- 5. To what complexity class does the Hamiltonian path problem belong?

Ex. 2 — Critical thinking

- 1. Is the function $\lceil \log n \rceil!$ bounded by a polynomial?
- 2. Is the function $\log^* \log n$ asymptotically larger than $\log \log^* n$?
- 3. Given eight balls of similar size but where one is lighter, detect which one it is, while minimizing the number of weighting. Provide the pseudocode.

Ex. 3 — Rubik's Cube

In about half a page explain the game and at least two algorithms to solve it. Provide references.

Ex. 4 — The \mathcal{NP} classe

Prove that the following problems are in \mathcal{NP} .

- 1. Does a given graph have a simple path?
- * 2. Is a given integer composite?
- 3. Does a given graph have a vertex cover of size k , for some integer k ?

Ex. 5 — PRIMES is in \mathcal{P}

The PRIMES problem consists in deciding if a given integer n is prime. A simple algorithm to solve PRIMES is trial division which runs in time $\mathcal{O}(n)$. Is it sufficient to prove that PRIMES is in \mathcal{P} ? Explain.

Hint: use the Prime Number Theorem.