VE477 HW 7

Wu Jiayao 517370910257

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1 Karger-Stein's Algorithm

1.1 Proof

$$P = \frac{\binom{t}{2}}{\binom{|V|}{2}} \geq \frac{1}{\binom{|V|}{2}} > \frac{1}{\binom{6}{2}} = \frac{1}{15}$$

1.2 Show

$$p_0 = 1/15$$

$$\begin{aligned} P(t) &= 1 - \left(1 - \frac{1}{2} P\left(\frac{t}{\sqrt{2}}\right)\right)^2 \\ &= P(\frac{t}{\sqrt{2}}) - \frac{1}{4} P(\frac{t}{\sqrt{2}})^2 \end{aligned}$$

$p_0 = 1/15$, $p_k = P(\frac{t}{\sqrt{2}})$, $p_{k+1} = P(t)$.

1.3

$$z_0 = 4/p_k - 1 = 60 - 1 = 59$$

$$\begin{aligned} z_{k+1} &= 4/p_{k+1} - 1 \\ &= \frac{4}{p_k - \frac{1}{4}p_k^2} - 1 \\ &= \frac{16 - 2(4-p_k)p_k}{(4-p_k)p_k} + 1 \\ &= \frac{(4-p_k)^2 + p_k^2}{(4-p_k)p_k} + 1 \\ &= \frac{4-p_k}{p_k} + \frac{p_k}{4-p_k} + 1 \\ &= z_k + \frac{1}{z_k} + 1 \end{aligned}$$

1.4

Omit

1.5

We can conclude that $k \leq 2\log_2 n + \mathcal{O}(1)$

$$\begin{gathered} k < z_k < 59 + 2k \\ \log_2 n + 1 < \frac{4}{p_k} - 1 < 59 + 4\log_2 n + \mathcal{O}(1) \\ \frac{1}{\log_2 n + \mathcal{O}(1)} < p_k < \frac{4}{\log_2 n + \mathcal{O}(1)} \end{gathered}$$

Hence, $p_k = \Omega(1/\log n)$

2 Simplex Problem

2.1 Tableaux

	z	x_1	x_2	x_3	s_1	s_2	s_3	
R_0	1	-3	-1	-2	0	0	0	0
R_1	0	1	1	3	1	0	0	30
R_2	0	2	2	5	0	1	0	24
R_3	0	4	1	2	0	0	1	36

Table 1: Initialization

	z	x_1	x_2	x_3	s_1	s_2	s_3	
$4 \times R_0 + 3 \times R_3$	4	0	-1	-2	0	0	3	108
$4 \times R_1 - R_3$	0	0	3	10	4	0	-1	84
$2 \times R_2 - R_3$	0	0	3	8	0	2	-1	12
R_3	0	4	1	2	0	0	1	36

Table 2: Follow the rule of simplex method to remove the biggest restriction x_1

R_2 is the tightest with the maximum A_{i3}/s_i

	z	x_1	x_2	x_3	s_1	s_2	s_3	
$4 \times R_0 + R_2$	16	0	-1	0	4	0	11	444
$4 \times R_1 - 5 \times R_2$	0	0	-3	0	4	-10	1	276
$5 \times R_2 - 4 \times R_1$	0	0	3	8	0	2	-1	12
$4 \times R_3 - R_2$	0	16	1	0	0	-2	5	132

Table 3: Follow the rule of simplex method to remove the biggest restriction x_3

R_2 is the tightest with the maximum A_{i2}/s_i.

	z	x_1	x_2	x_3	s_1	s_2	s_3	
$3 \times R_0 + 2 \times R_2$	48	0	0	8	0	8	32	1344
$R_1 + R_2$	0	0	0	8	4	-8	0	720
R_2	0	0	3	8	0	2	-1	12
$3 \times R_3 - R_2$	0	48	0	-8	0	-8	16	384

Table 4: Follow the rule of simplex method to remove the biggest restriction x_2

The optimal solution for z is

$$z = \frac{1344}{16} = 28$$

2.2 Geometric

It means to find the optimized vertex in the convex polyhedron bounded by given constraints in an n-dimension space. The simplex method indicates how to find the next optimized result and when to halt when the best solution is found.

3 Critical Thinking

It works like this.

There are two arrays, one to store numbers, called **st**, one to store the location of the minimum element, called **min_loc**.

When **top**, return the last element of **st**.

When **get_min**, return $st[min\_loc.back]$.

When **push**, push the element x to the back of **st**. If x is smaller than the minimum element or x is the first element in the stack, push $st.size - 1$ to the back of **min_loc**.

When **pop**, pop the last element from **st**. After pop, if the last element of **min_loc** equals the $st.size$, pop the last element of **min_Loc**.

C++ code is like.

```
vector<int> st;
vector<int> min_loc;
int min;
void push(int x) {
    st.push_back(x);
    if(min_loc.empty() || x < st[min_loc.back()])
    {
        min_loc.push_back(st.size()-1);
    }
}

void pop() {
    st.pop_back();
    if(min_loc.back() == st.size())
    {
        min_loc.pop_back();
    }
}

int top() {
    return st.back();
}
```

```
    int getMin() {
        return st[min_loc.back()];
    }
```

4 Farka's lemma

1. If $Mx \leq 0$ and $V^T x > 0$, suppose that $M^T y = V \ and \ y \geq 0$. Then,

$$\begin{aligned} V^T x &= (M^T y)^T x \\ &= y^T M x \\ &= y^T (Mx) \leq 0 \end{aligned}$$

   Since Mx is a m-vector, $y^T \geq 0 \equiv y \geq 0$, this leads to a contradiction. Hence ,when $Mx \leq 0$ and $V^T x > 0$, $M^T y = V \ and \ y \geq 0$ is false

2. If $M^T y = V \ and \ y \geq 0$, suppose that $Mx \leq 0$ and $V^T x > 0$. Then,

$$\begin{aligned} V^T x &= (M^T y)^T x \\ &= y^T M x \\ &= y^T (Mx) > 0 \end{aligned}$$

   Since Mx is a m-vector, $y^T \geq 0 \equiv y \geq 0$, we can conclude that $Mx > 0$, which leads to a contradiction. Hence ,when $M^T y = V \ and \ y \geq 0$, $Mx \leq 0$ and $V^T x > 0$ is false.