Announcements

- Mid-term exam: June 22, 4pm-5:40pm
  - Open book, open notes
  - No communication
- HW5 on CSP
  - Released today
  - Due June 24 at 11:59pm
- P3 on MDP and RL
  - Early release
  - Due July 3 at 11:59pm

Ve492: Introduction to Artificial Intelligence

Mid-term Review

Paul Weng

UM-SJTU Joint Institute

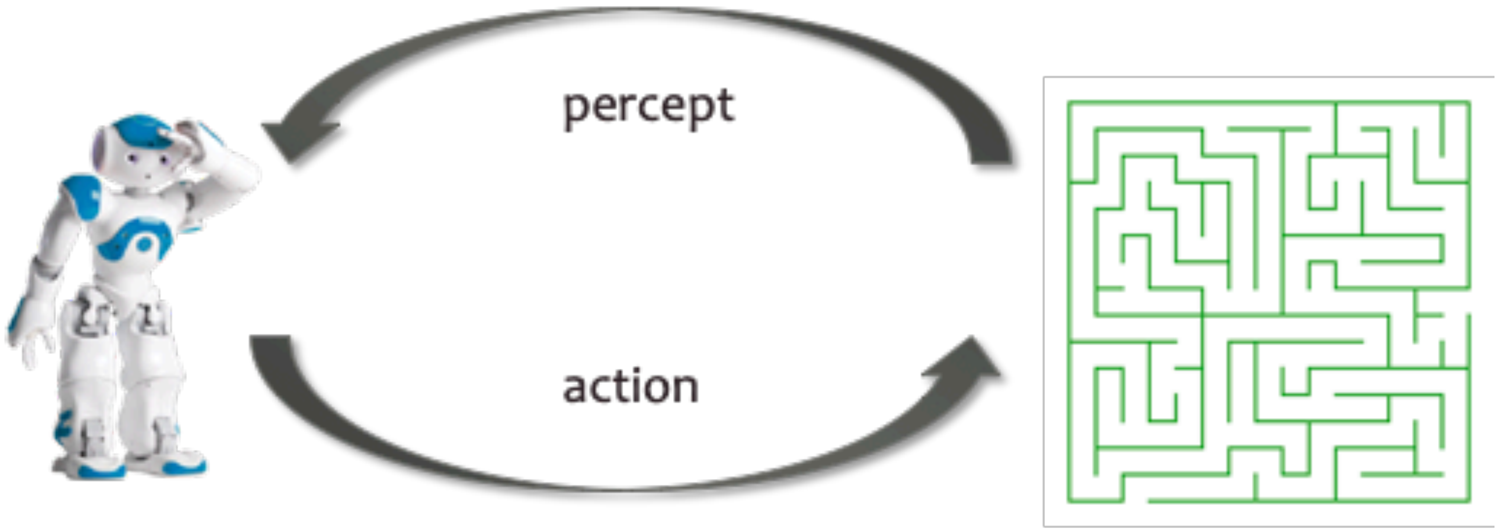
Slides adapted from http://ai.berkeley.edu, AIMA, UM, CMU

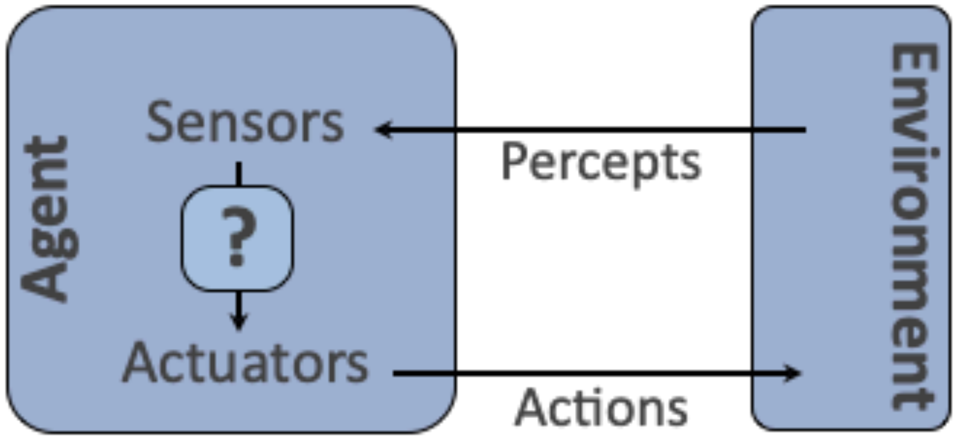
What have we learned so far?

- Search and planning
  - Define a state space, goal test; Find path from start to goal
- Game trees
  - Define utilities; Find path from start that maximizes utility
- Decision theory and game theory
  - Foundation for MEU; Basic concepts in game theory
- MDPs
  - Define rewards, utility = (discounted) sum of rewards
  - Find policy that maximizes utility
- Reinforcement learning
  - Just like MDPs, only T and/or R are not known in advance
- Constraint satisfaction
  - Find solution that satisfies constraints; Not just for finding a sequential plan

High-Level Framework

- How to build AI system?





Search

- Environment: single-agent, fully-observable state, deterministic transition, sequential, model known
- Search problem
  - States, transition model, goal test, initial state — graph
  - Search tree
- Algorithms
  - Uninformed search
    - BFS, DFS, UCS
  - Informed search — heuristic fcn.
    - Greedy search, A*
- Properties
  - Complete, optimal
  - Space and computational complexities

tree / graph search

Search in Games

- Environment: multi-agent, fully-observable state, deterministic or stochastic transition, turn-taking , model known
- Multi-agent search problems as games
  - States, players, transition model, terminal test/values, initial state
  - Game tree
- Algorithm for adversarial agent (zero-sum game)
  - Minimax search algorithm
  - Alpha-beta pruning
  - Depth-limited search, iterative deepening
- Algorithm for random agent
  - Expectimax
- Algorithm for multi-agent search
  - Expectiminimax

Decision Theory and Game Theory

- Axiomatization of Expected Utility
    - Completeness, Transitivity, Independence, Continuity
    - Unicity of utility function up to positive affine transformation
    - Preference elicitation
- Game theory
    - Extensive form vs normal form
    - Best response, dominant/dominated strategies
    - Nash equilibrium (pure or mixed)
    - Pareto optimal, correlated equilibrium

Markov Decision Process

- Environment: single-agent, fully-observable state, stochastic transition, sequential, model known
- Model
  - States, actions, transition function, reward function
- Algorithms
  - Policy evaluation
  - Policy extraction
  - Value iteration
  - Policy iteration

$\pi : S \to A$

$V : S \to \mathbb{R}$

Reinforcement Learning

- Environment: single-agent, fully-observable state, stochastic transition, sequential, model unknown
- MDP Model, but unknown!
  - States, actions, transition function, reward function
- Algorithms
  - Policy evaluation with TD learning
  - Policy learning with Q-learning
  - Approximate Q-learning — $\sum_{i=1}^{n} \overline{\phi_i(s,a)} \times w_i = \hat{Q}(s,a)$
  - Action selection with ϵ-greedy or exploration function

Constraint Satisfaction

- CSP
  - Set of variables, set of domains, set of constraints $(x_1, \ldots, x_n)$
  - Find assignments to variables such that all constraints are satisfied
- Algorithms
  - Backtracking search
    - Filtering, forward-checking, arc consistency, k-consistency
    - Ordering of variables and values
  - Structure of constraint graph
    - Two-pass algorithm for tree-structured constraint graph
    - Cutset conditioning
  - Iterative~~ment~~ improvement
- Local search

Quiz: Search

- Consider a graph search problem where for every action, the cost is at least ϵ, with ϵ>0. Assume the used heuristic is consistent.
  - Greedy graph search is guaranteed to return an optimal solution. F
  - A* graph search is guaranteed to return an optimal solution. T
  - A* graph search is guaranteed to expand no more nodes than depth-first graph search. F
  - A* graph search is guaranteed to expand no more nodes than uniform-cost graph search. T

$g(n) + h(n)$

Quiz: A* Heuristics

- Let H_1 and H_2 both be admissible heuristics.
  - $max(H_1, H_2)$ is necessarily admissible T
  - $min(H_1, H_2)$ is necessarily admissible T
  - $(H_1 + H_2)/2$ is necessarily admissible T
  - $max(H_1, H_2)$ is necessarily consistent F

$\forall n,\ h(n) \leq h^*(n)$

$\forall n,\ h(n) \leq c(n, n') + h(n')$

Quiz: Search under Uncertainty

- You are given a game tree for which you are the maximizer, and in the nodes in which you don't get to make a decision an action is chosen uniformly at random amongst the available options. Your objective is to maximize the probability you win $10 or more (rather than the usual objective to maximize your expected value).
    - Running expectimax will result in finding the optimal strategy to maximize the probability of winning $10 or more. F
    - Running minimax, where chance nodes are considered minimizers, will result in finding the optimal strategy to maximize the probability of winning $10 or more. F
    - Running expectimax in a modified game tree where every pay-off of $10 or more is given a value of 1, and every pay-off lower than $10 is given a value of 0 will result in finding the optimal strategy to maximize the probability of winning $10 or more. T
    - Running minimax in a modified game tree where every pay-off of $10 or more is given a value of 1, and every pay-off lower than $10 is given a value of 0 will result in finding the optimal strategy to maximize the probability of winning $10 or more. F

Quiz: Adversarial Search

- In the context of adversarial search, α-β pruning
  - can reduce computation time by pruning portions of the game tree T
  - is generally faster than minimax, but loses the guarantee of optimality F
  - always returns the same value as minimax for the root of the tree T
  - always returns the same value as minimax for all nodes of the tree F

Game Theory: Zero-Sum Game

- Two players choose simultaneously a coin of 10 cents, 50 cents or 1 dollar, which they show to each other.
- If they chose the same coin, player I wins. Otherwise, player II wins.
- Write this game in normal form. Is there any pure NE?
- Express a system of inequalities to find a mixed NE.

$[(p_I, q_I, 1-p_I-q_I), (p_{II}, q_{II}, 1-p_{II}-q_{II})]$

	p_{II}: 10	q_{II}: 50	$(1-p-q)_{II}$: 100
p_I: 10	10	-10	-10
q_I: 50	-50	50	-50
$1-p_I-q_I$: 100	-100	-100	100

$$\begin{cases} 10p - 10q - 10(1-p-q) \geq v \\ -50p + 50q - 50(1-p-q) \geq v \\ -100p - 100q + 100(1-p-q) \geq v \\ 0 \leq p \leq 1 \quad 0 \leq q \leq 1 \end{cases}$$

$\Downarrow$ value of game

Fourier Elimination

$20p \geq v + 10$

$100q \geq v + 50$

$-200p \geq v + 200q - 100$

$0 \leq p$ | $p \leq 1$

$\frac{1}{20}v + \frac{1}{2} \leq p$ | $p \leq -\frac{1}{200}v - q + \frac{1}{2}$

$0 \leq q \leq 1$ | $q \geq \frac{1}{100}v + \frac{1}{2}$

$$\max\left(0, \frac{1}{20}v + \frac{1}{2}\right) \le p \le \min\left(1, -\frac{1}{200}v - q + \frac{1}{2}\right)$$

$$\begin{cases} \cancel{0 \le t} \\ 0 \le -\frac{1}{200}v - q + \frac{1}{2} \\ \frac{1}{20}v + \frac{1}{2} \le 1 \\ \frac{1}{20}v + \cancel{\frac{1}{2}} \le -\frac{1}{200}v - q + \cancel{\frac{1}{2}} \\ 0 \le q \le 1 \end{cases}$$

$$q \ge \frac{1}{100}v + \frac{1}{2}$$

$$\begin{cases} 0 \le q \\ \frac{1}{100}v + \frac{1}{2} \le q \\ q \le -\frac{1}{200}v + \frac{1}{2} \\ q \le -\frac{11}{200}v \\ q \le 1 \\ \frac{1}{20}v + \frac{1}{2} \le 1 \end{cases}$$

$v \le -\frac{100}{13}$

$0 \le -\frac{1}{200}v + \frac{1}{2}$ $\quad v \le 100$

$0 \le -\frac{11}{200}v$ $\quad v \le 0$

$\frac{1}{100}v + \cancel{\frac{1}{2}} \le -\frac{1}{200}v + \cancel{\frac{1}{2}}$

$\cancel{\frac{3}{200}}v \le 0$

$\frac{1}{100}v + \frac{1}{2} \le -\frac{11}{200}v$ $\quad \frac{13}{200}v \le -\frac{1}{2}$

$\frac{1}{100}v + \frac{1}{2} \le 1$ $\quad v \le 50$

$\frac{1}{20}v + \frac{1}{2} \le 1$ $\quad v \le 10$

probabilities

$$\max_{\sigma} \min_{\sigma'} \sum_{i,j} \sigma_i \sigma'_j u_{ij} = v$$

Quiz: MDP

- For Markov Decisions Processes (MDPs), we have that:
  - T A small discount (close to 0) encourages shortsighted, greedy behavior.
  - T A large, negative living reward ($\ll 0$) encourages shortsighted, greedy behavior.
  - F A negative living reward can always be expressed using a discount<1.
  - F A discount<1 can always be expressed as a negative living reward.

Quiz: MDP

F ❖ Value iteration can converge only if the discount factor (γ) satisfies $0 < \gamma < 1$.

F ❖ Policies found by value iteration may be superior to policies found by policy iteration.

F ❖ Policies found by policy iteration may be superior to policies found by value iteration.

T ❖ In some problems, value iteration can converge even though policy iteration may not. $\gamma = 1$

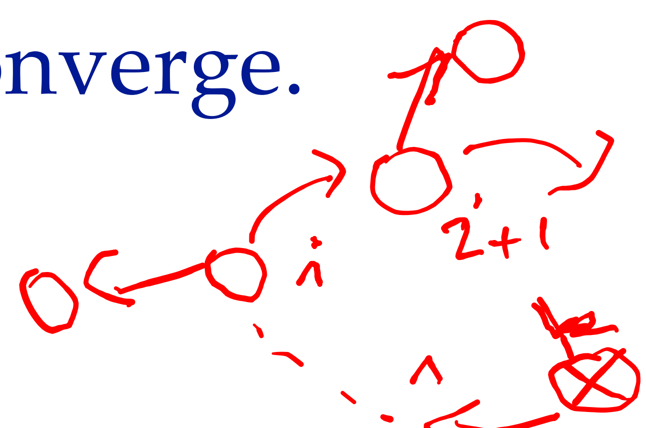
Quiz: Reinforcement Learning

- Assume that the agent observes the true reward with some
T Gaussian noise $\mathcal{N}(0,1)$, Q-learning would still converge

- off-policy
Q-learning can learn the optimal Q-function Q^* without
T ever executing the optimal policy.

- If an MDP has a transition model T that assigns non-zero
F probability for all triples T (s, a, s′) then Q-learning will fail.

- In Q-learning, we decide to explore every k steps, i.e., if t = 0 [k] we choose a random action with a uniform
F distribution, otherwise we choose the greedy action. This version would still converge.



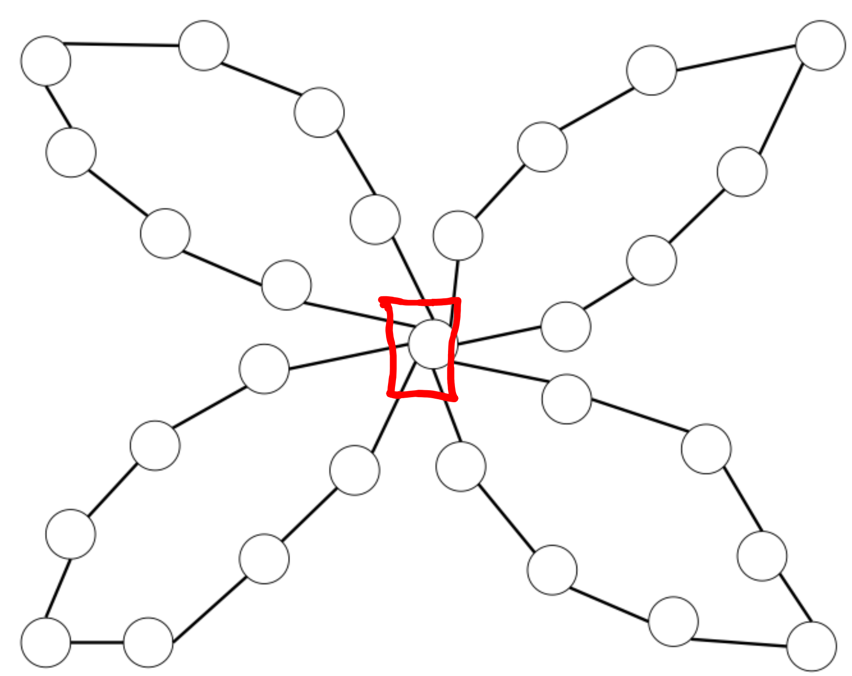
Quiz: CSP

- Assume given a CSP whose constraint graph is given below and that all the variables have the same domain.
- What is the complexity of solving it with a direct application of backtracking search? $O(d^{29})$
- Which efficient strategy could you apply to solve it? What would be the complexity?

29 variables

d size of domain

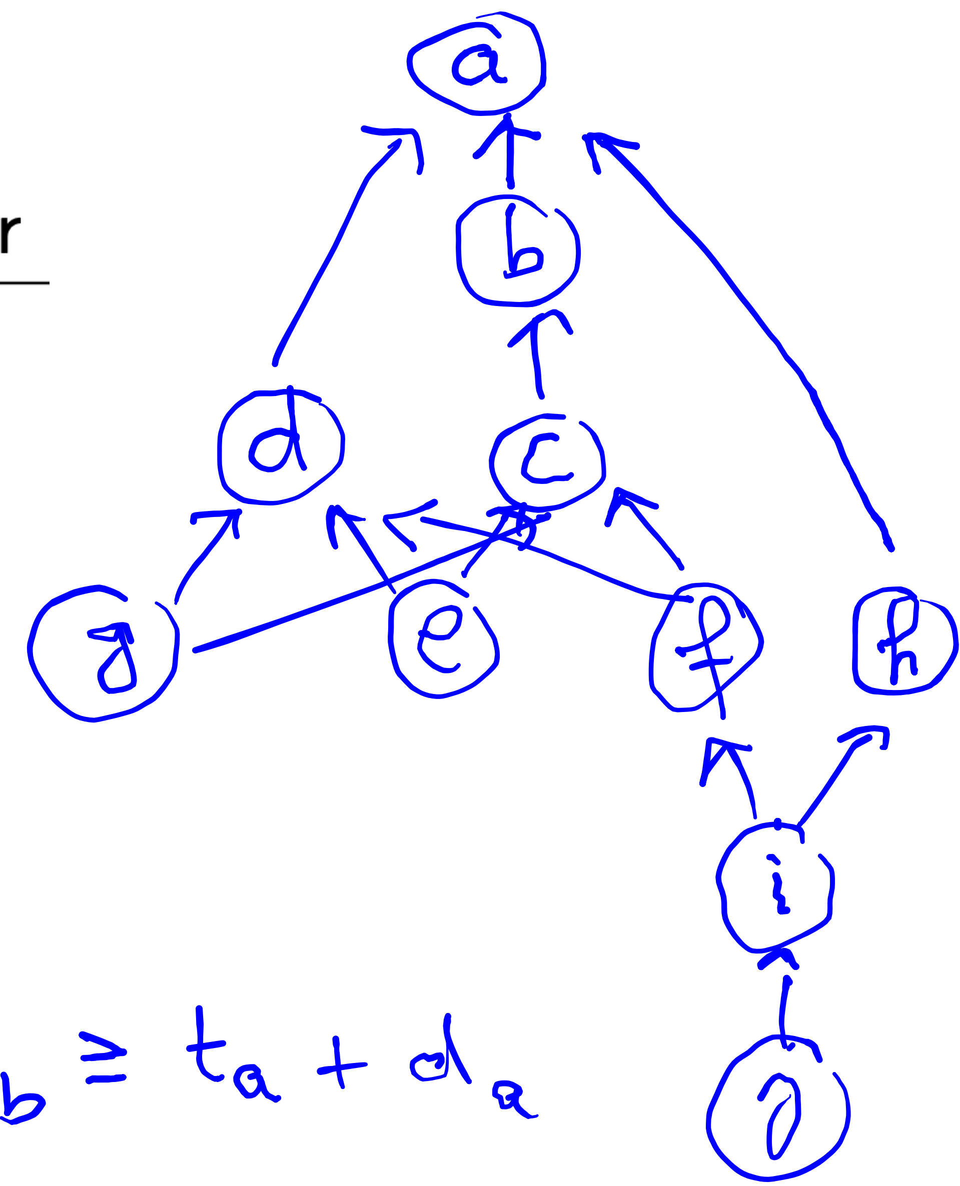
$O(d \times d^2 \times \cancel{28})$



CSP Problem: Job Scheduling

- When can I move in?

Task	Description	Duration	Predecessor
a	Erecting walls	7	none
b	Carpentry for roof	3	a
c	Roof	1	b
d	Installations	8	a
e	Facade painting	2	c & d
f	Windows	1	c & d
g	Garden	1	c & d
h	Ceilings	3	a
i	Painting	2	f & h
j	Moving in	1	i



$t_b \geq t_a + d_a$