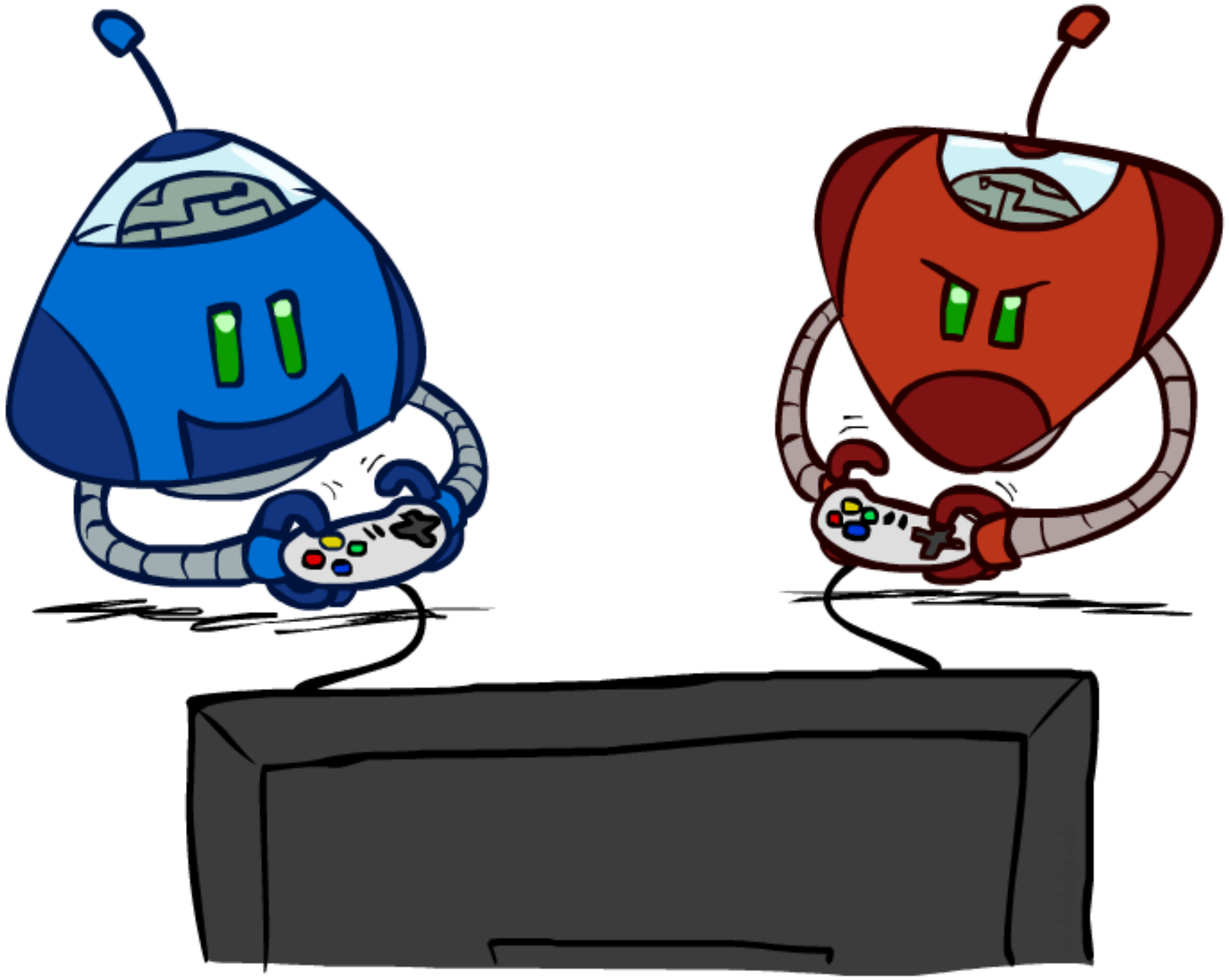
Announcements

- Project 1: Search
  - Due Wed. June 3 at 11:59pm.
  - Solo or in group of two. For groups of two, both of you need to submit your code into JOJ!
- Homework 1: Single-agent search
  - Due Wed. May 27 at 11:59pm.
- Homework 2: Multi-agent search
  - Release Wed., May 27, due Wed, June 3 at 11:59pm.
- Project 2: Multi-agent search
  - Release Wed. June 3, due Wed June 17 at 11:59pm

Ve492: Introduction to Artificial Intelligence
Games with Chance; Decision Theory



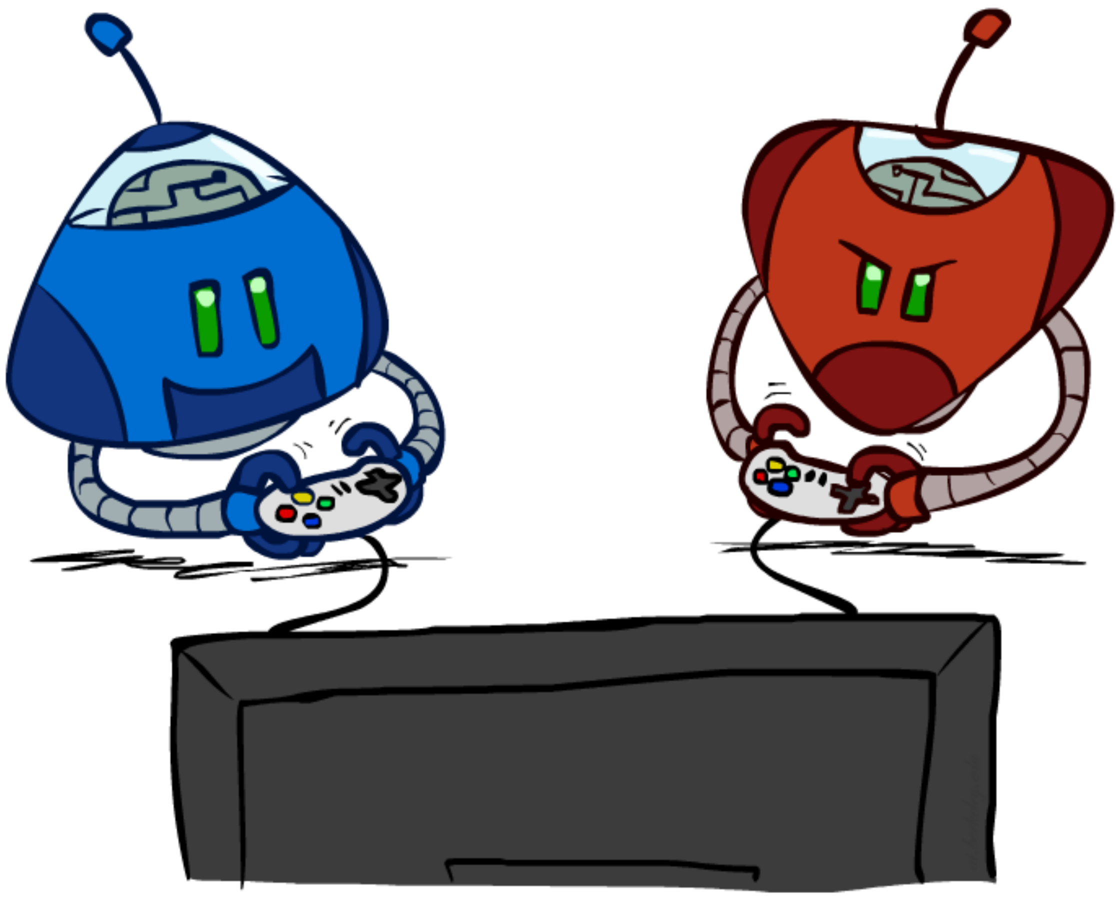
Paul Weng

UM-SJTU Joint Institute

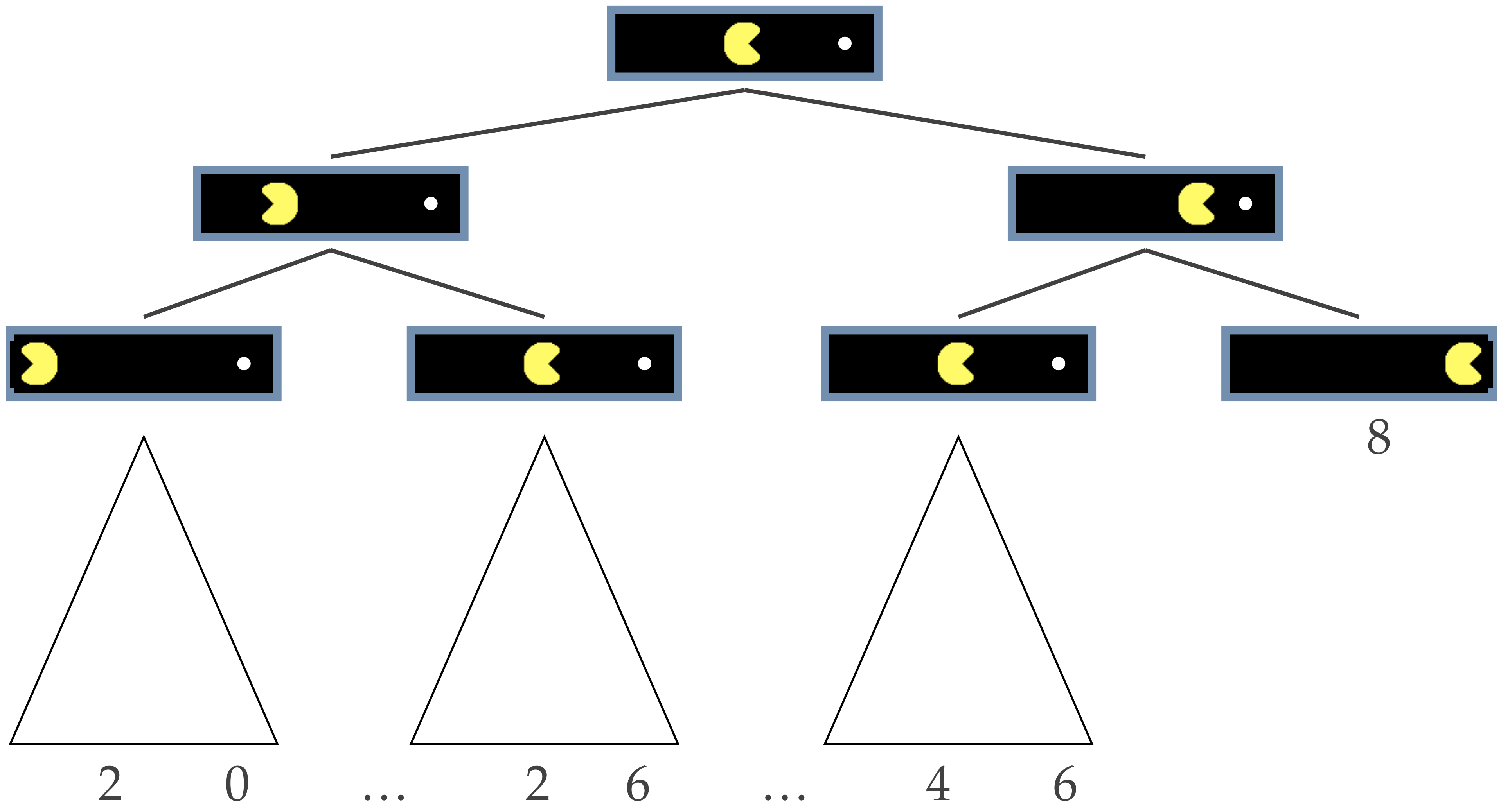
Slides adapted from http://ai.berkeley.edu, AIMA, UM, CMU

Outline

- Multi-agent search
- Games with chance
- Decision Theory

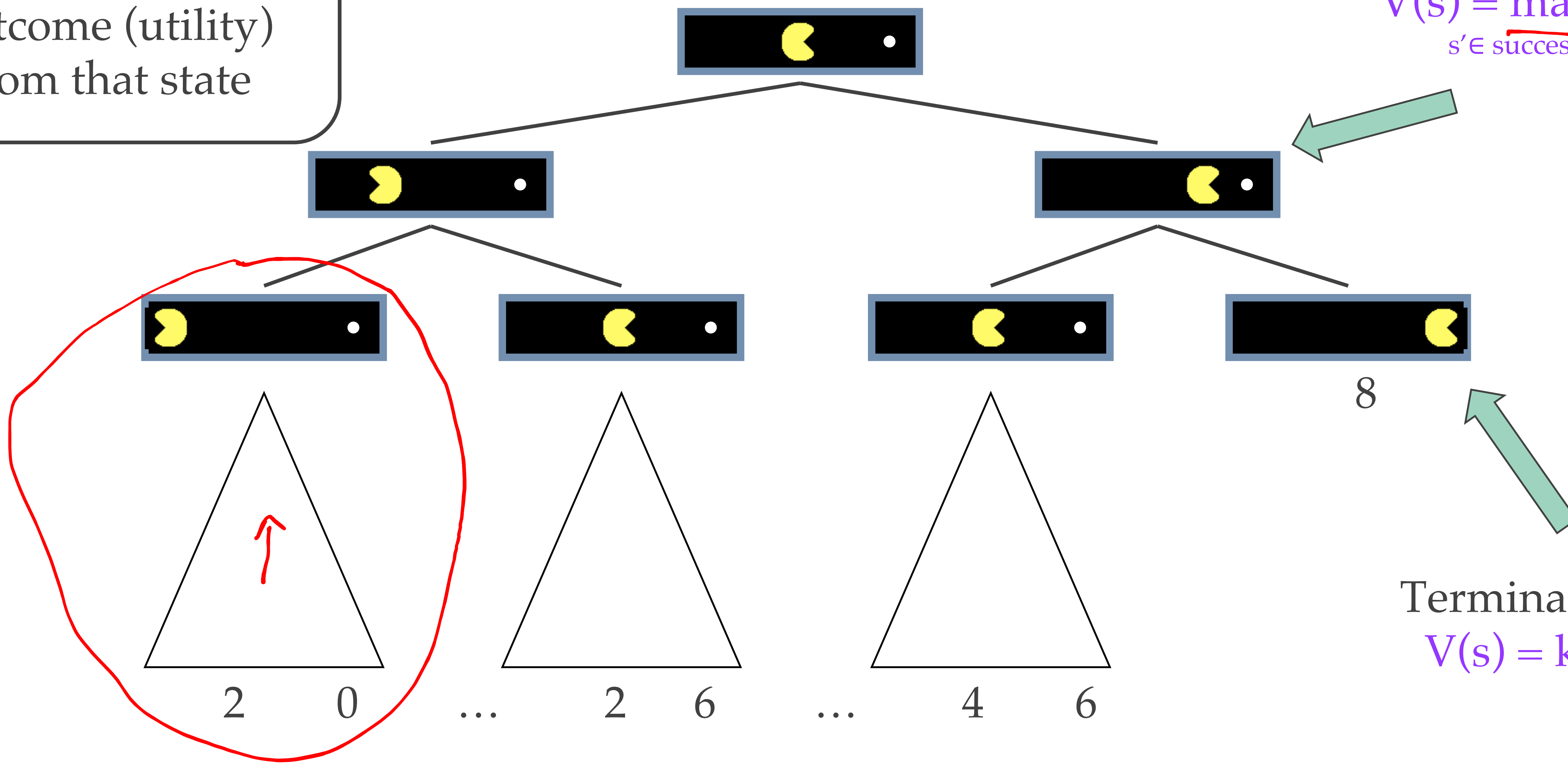


Single-Agent Trees



Value of a State

Value of a state: The best achievable outcome (utility) from that state



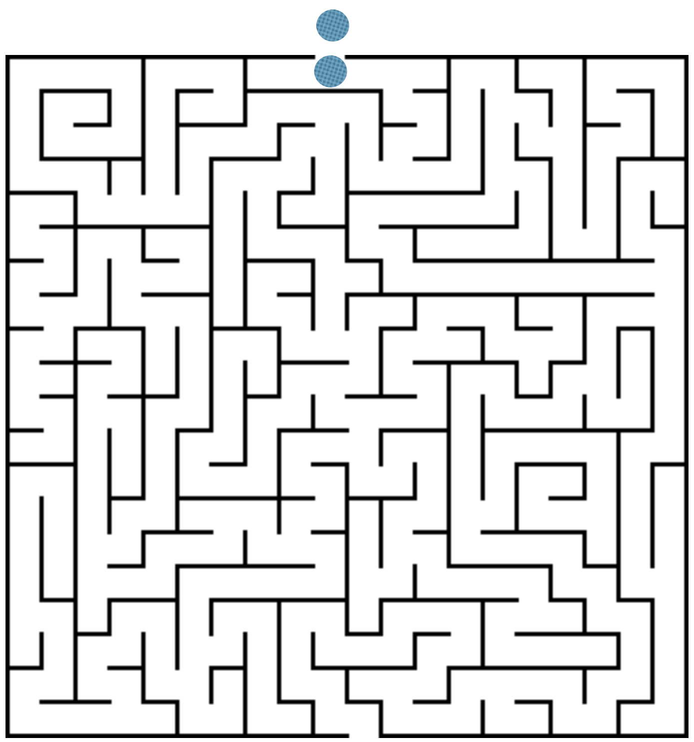
Non-Terminal States:

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

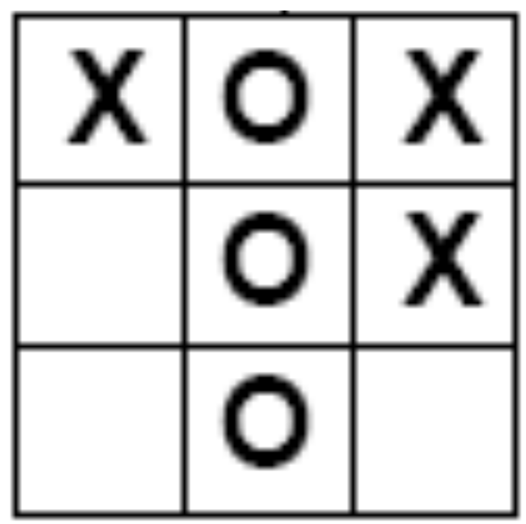
Terminal States:

$V(s) = \text{known}$

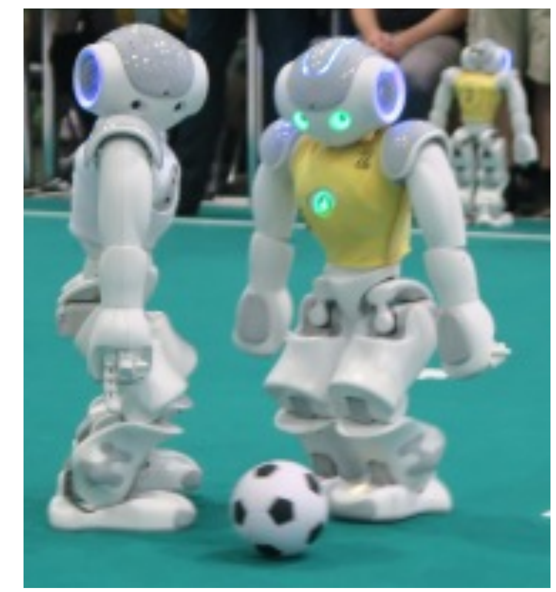
Multi-Agent Applications



Collaborative Maze Solving



Adversarial



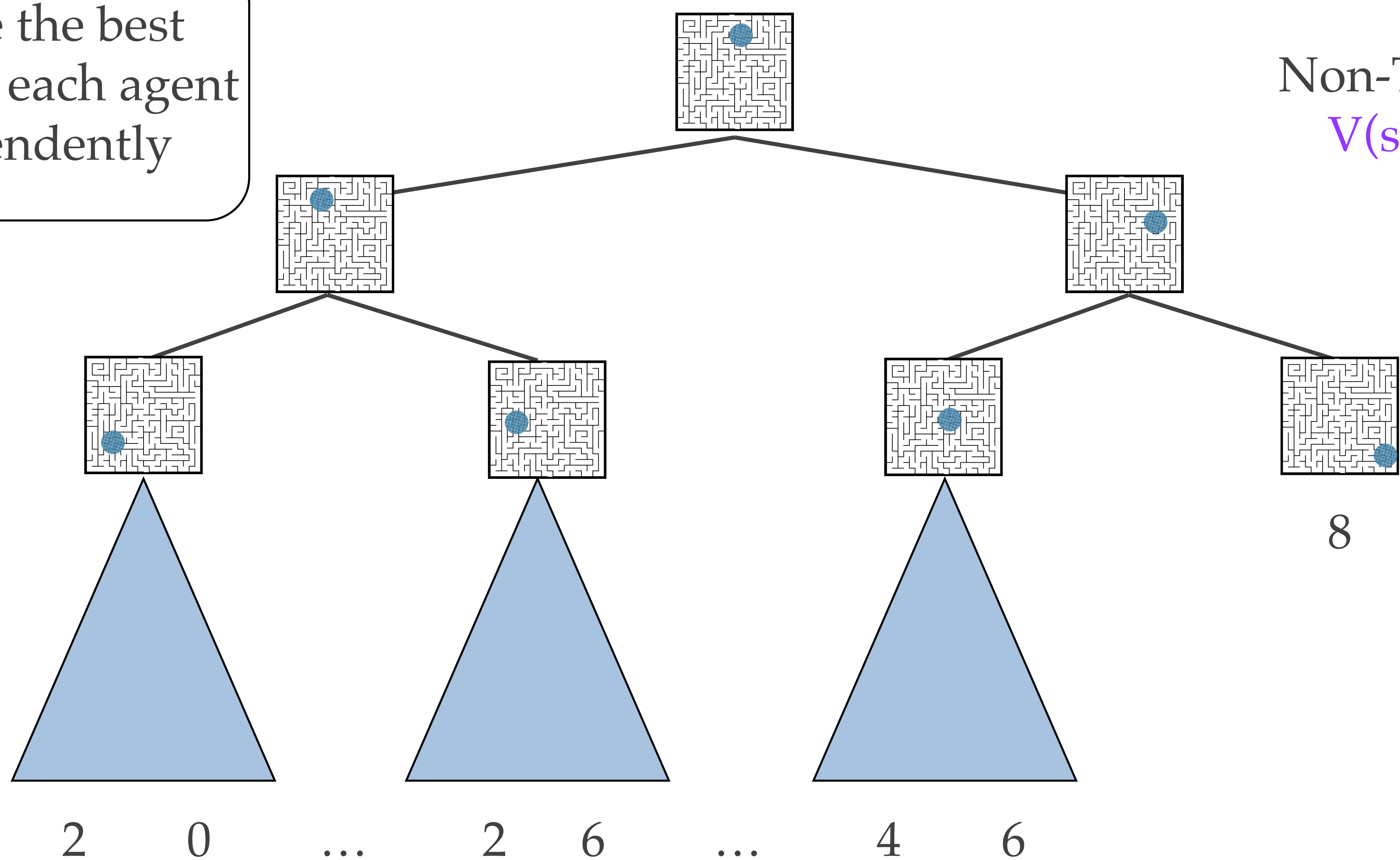
Team: Collaborative
Competition: Adversarial

- How could we model multi-agent problems?
    - Depends on problem assumptions

Idea 1: Independent Decision-making

- Each agent plans their own actions separately from others => Many single-agent trees

Choose the best action for each agent independently



Non-Terminal States:

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

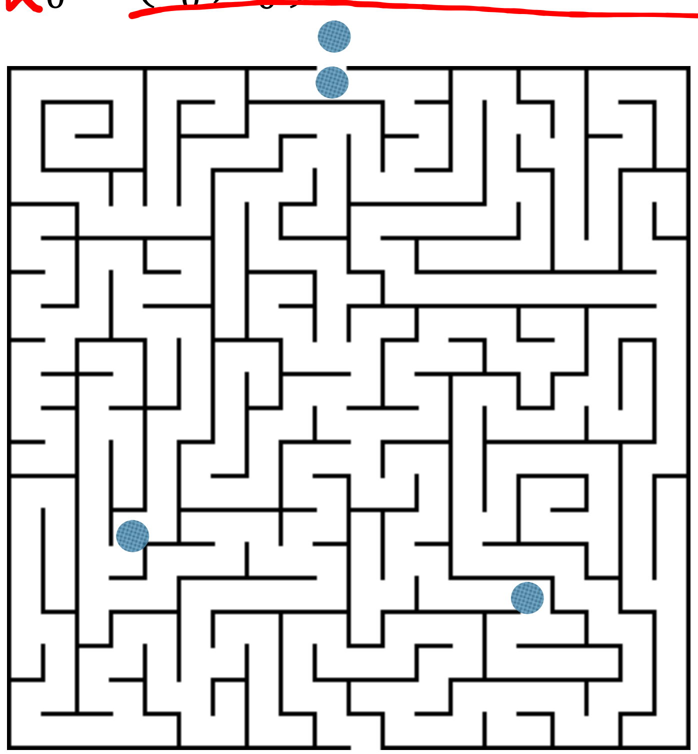
Idea 2: Joint State/Action Spaces

- Combine the states and actions of the N agents
- Search looks through all combinations of all agents' states and actions
- Think of one brain controlling many agents

S : state space of one agent

A = action

$S_0 = (S_0^A, S_0^B)$



$S_K = (S_K^A, S_K^B)$

What is the size of the state space? $|S|^N$

What is the size of the action space? $|A|^N$

What is the size of the search tree? $(|A|^N)^h$

Idea 3: Coordinated Decision Making

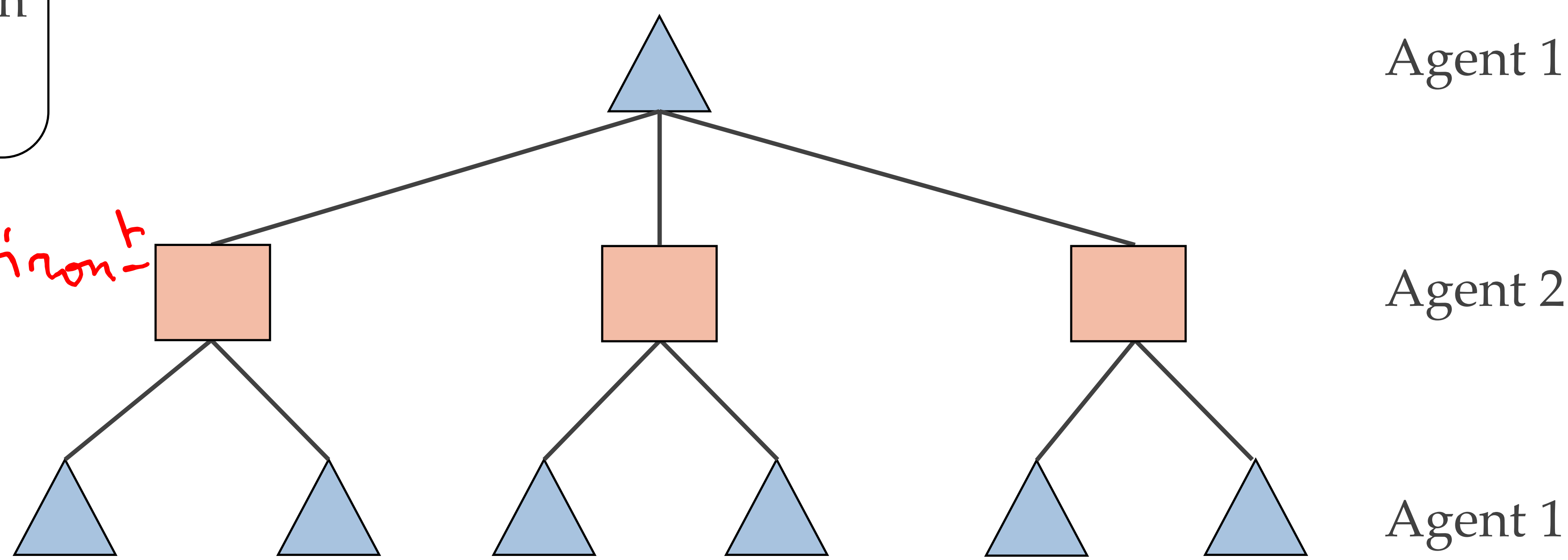
- Each agent proposes their actions and computer confirms the joint plan
- Example: [Autonomous driving through intersections]

Idea 4: Alternate Searching One Agent at a Time

- Search one agent's actions from a state, search the next agent's actions from those resulting states , etc…

Choose the best cascading combination of actions

S state of environ^t



What is the size of the state space?

$|S|$

What is the size of the action space?

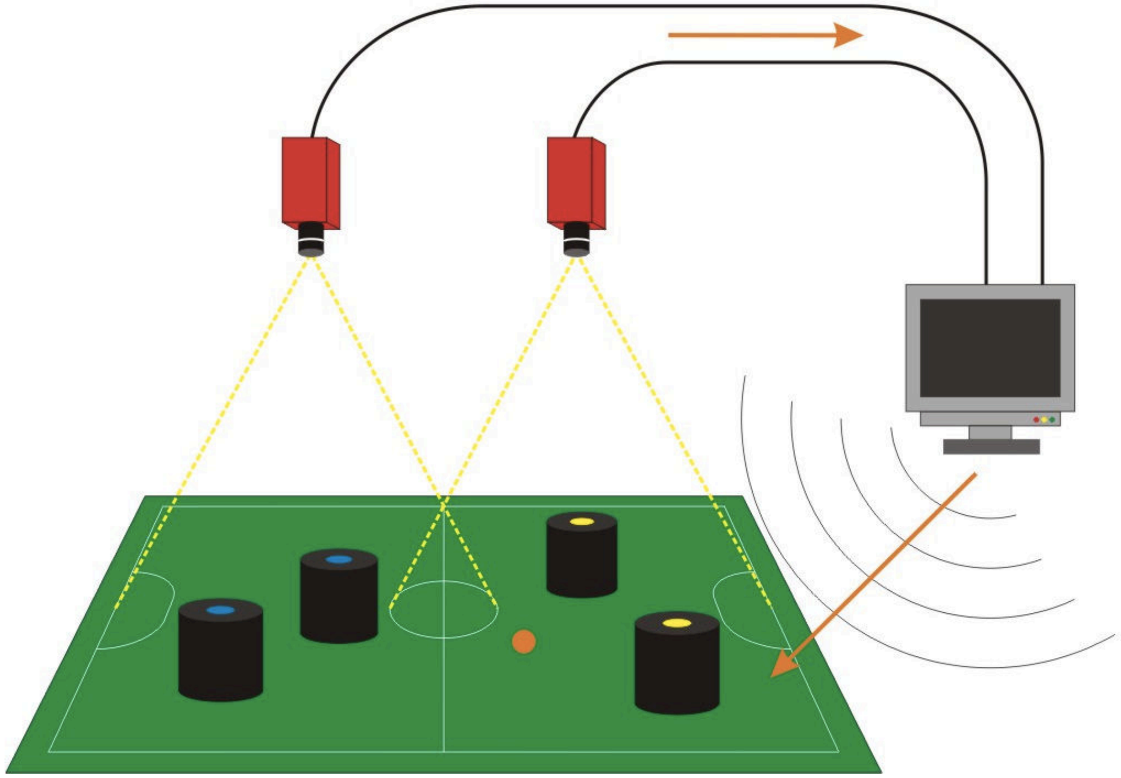
$|A|$

What is the size of the search tree?

$|A|^h$ ← factor that depends on N.

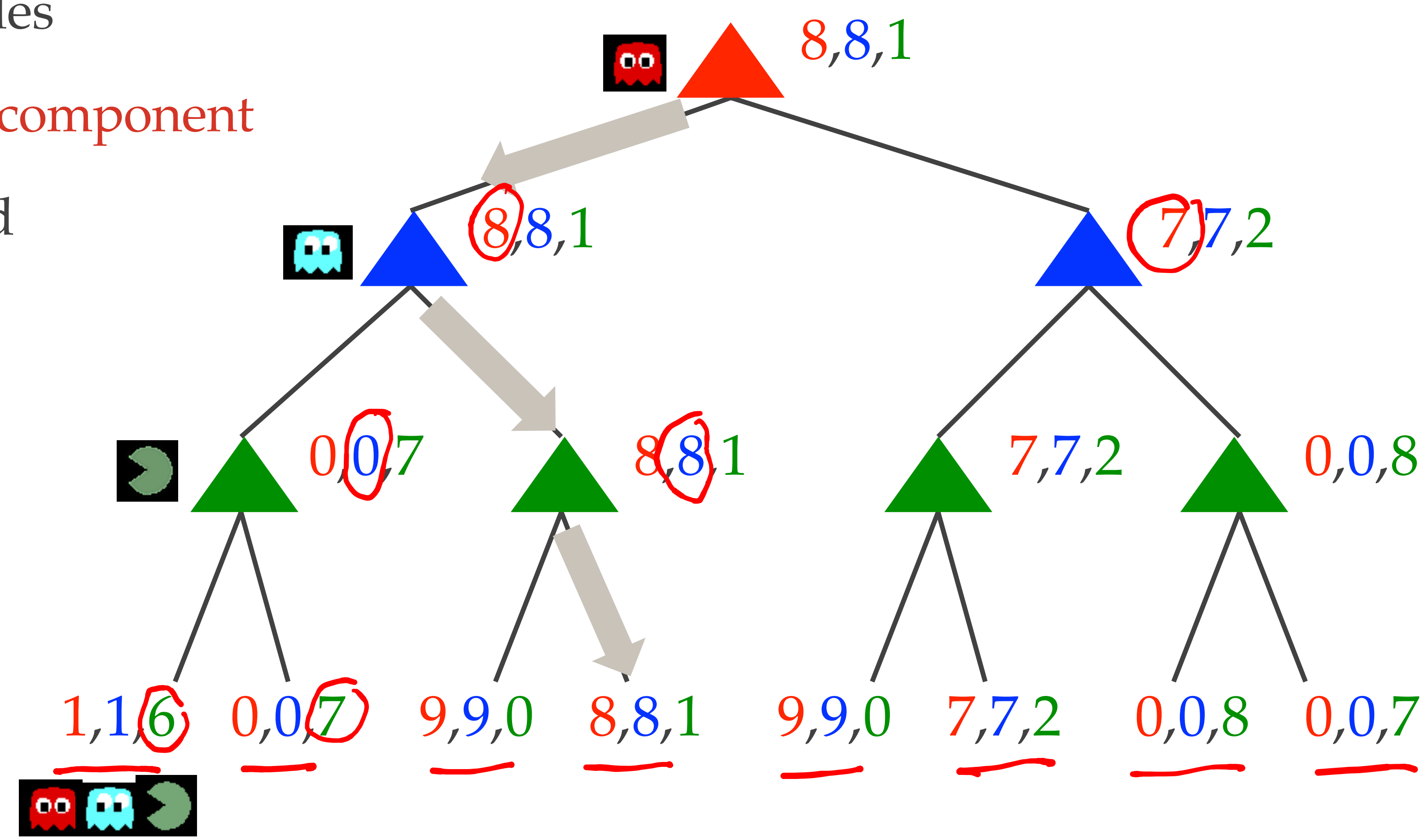
Minimax Search with Two Teams

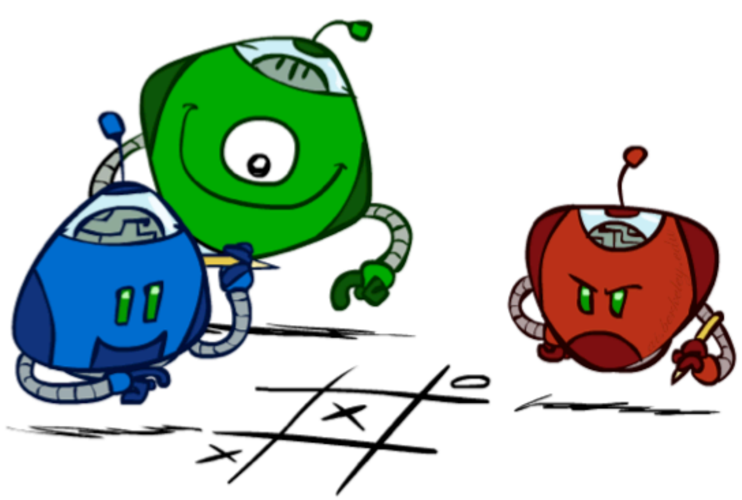
- Joint State / Action space and search for our team
- Adversarial search to predict the opponent team
- Example: Small Size Robot Soccer



Generalized minimax

- What if the game is not zero-sum, or has multiple players?
- Generalization of minimax:
  - Terminals have utility tuples
  - Node values are also utility tuples
  - Each player maximizes its own component
  - Can give rise to cooperation and competition dynamically…



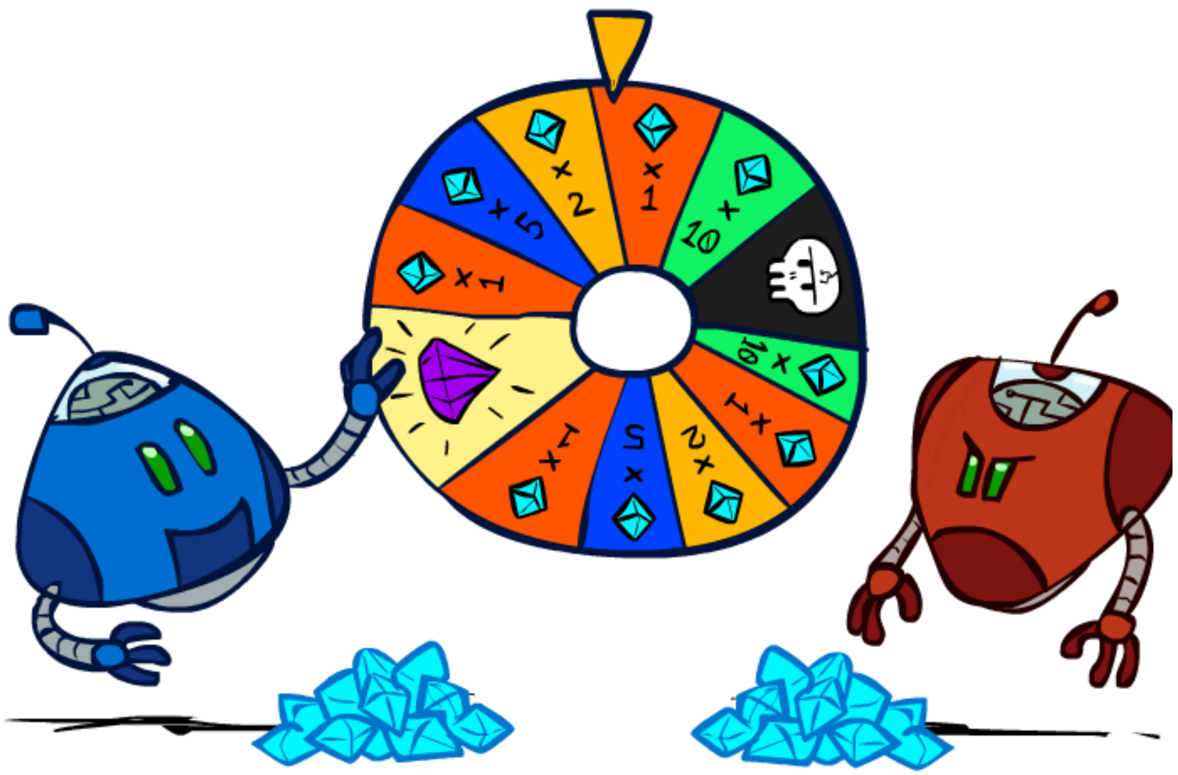


Three-Person Chess

From Wikipedia

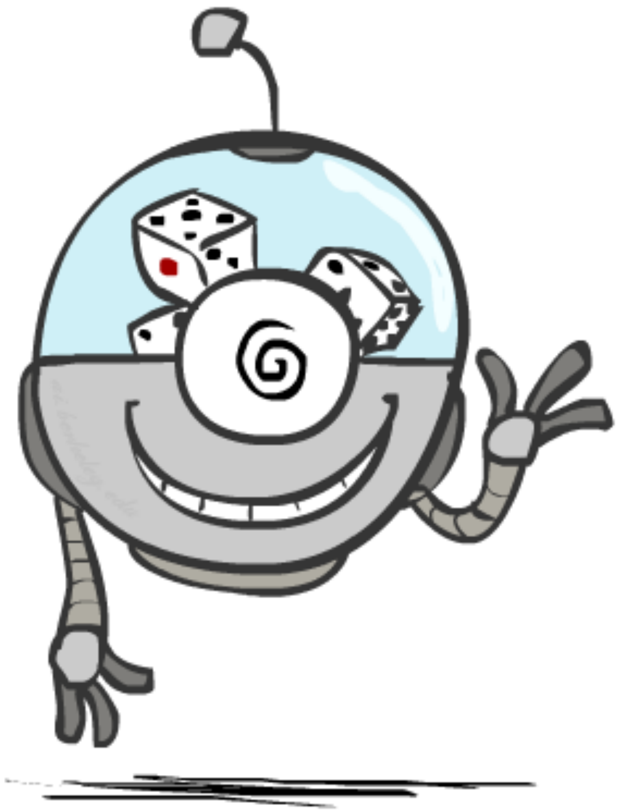
Games with Chance

Search with Random Outcomes



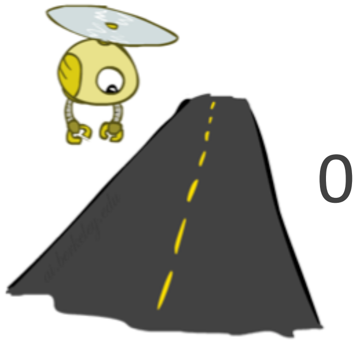
Games with Chance

Probabilities



Reminder: Probabilities

- A random variable represents an event whose outcome is unknown
- A probability distribution is an assignment of weights to outcomes
- Example: Traffic on freeway
  - Random variable: T = whether there's traffic
  - Outcomes: T in {none, light, heavy}
  - Distribution: P(T=none) = 0.25, P(T=light) = 0.50, P(T=heavy) = 0.25
- Some laws of probability (more later):
  - Probabilities are always non-negative
  - Probabilities over all possible outcomes sum to one
- As we get more evidence, probabilities may change:
  - P(T=heavy) = 0.25, P(T=heavy | Hour=8am) = 0.60
  - We'll talk about methods for reasoning and updating probabilities later



0.25

0.50

0.25

Reminder: Expectations

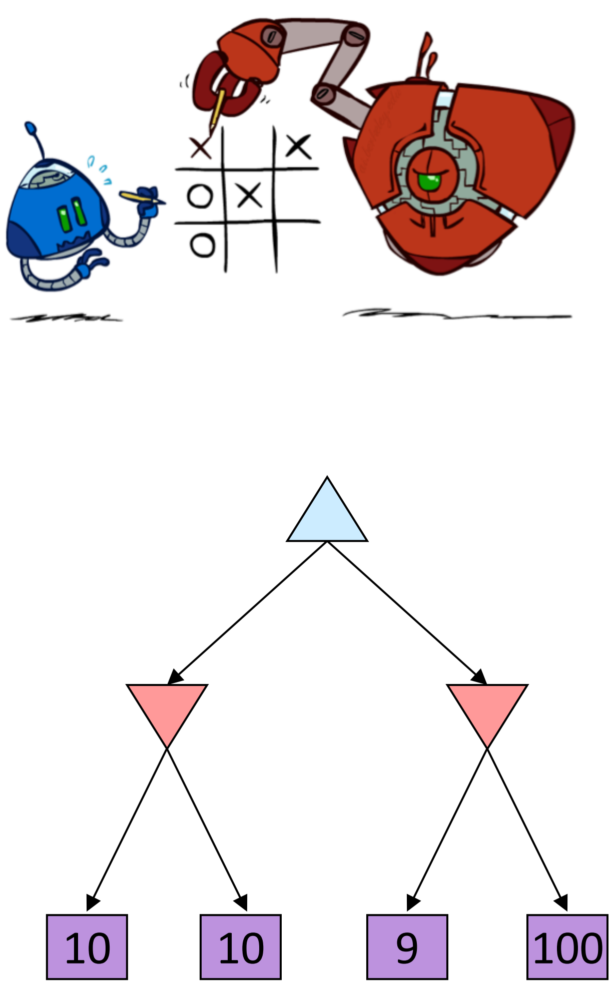
- The expected value of a random variable is the average, weighted by the probability distribution over outcomes

- Example: How long to get to the airport?

Time:	20 min	+	30 min	+	60 min		
	x		x		x	⇒	35 min
Probability:	0.25		0.50		0.25		

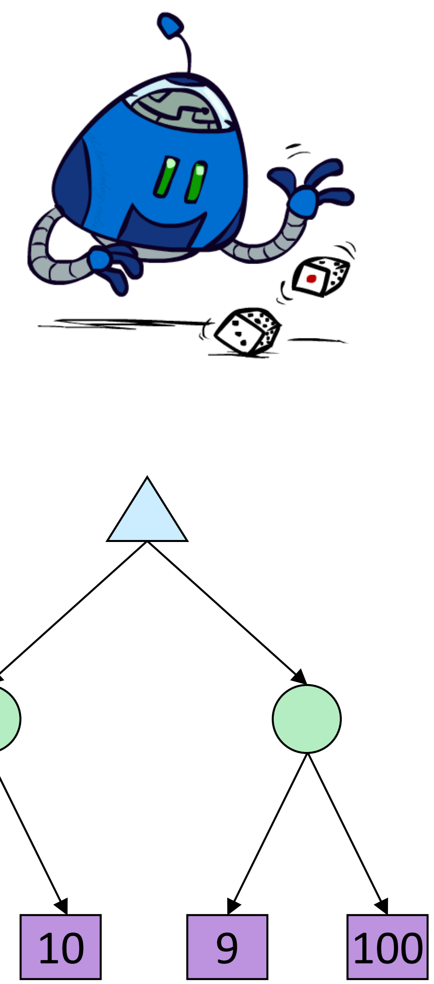
  

Different Game Trees



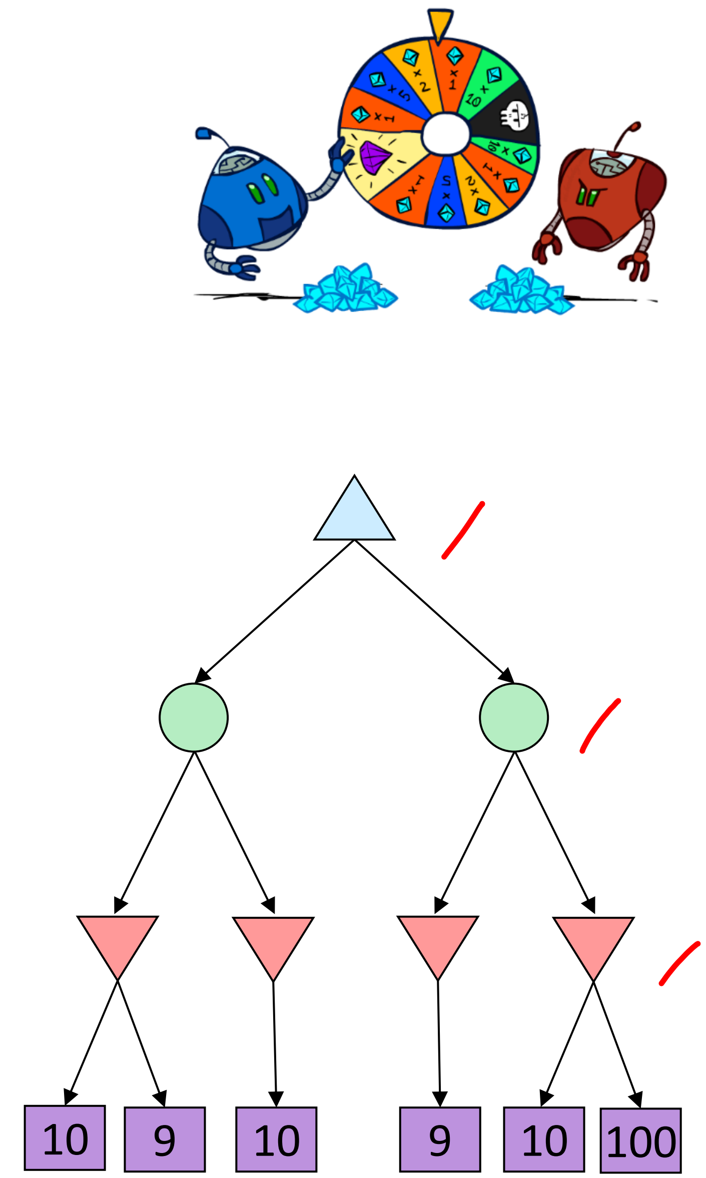
Tictactoe, chess

Minimax



Tetris, investing

Expectimax



Backgammon, Monopoly

Expectiminimax

Minimax

```
function decision(s) returns an action
    return the action a in Actions(s) with the highest
     value(Succ(s,a))
```

```
function value(s) returns a value
      if Terminal-Test(s) then return Utility(s)
      if Player(s) = MAX    then return max_{a in Actions(s)} value(Succ(s,a))
      if Player(s) = MIN    then return min_{a in Actions(s)} value(Succ(s,a))
```

Expectimax

function decision(s) returns an action

return the action a in Actions(s) with the highest value(Succ(s,a))

function value(s) returns a value

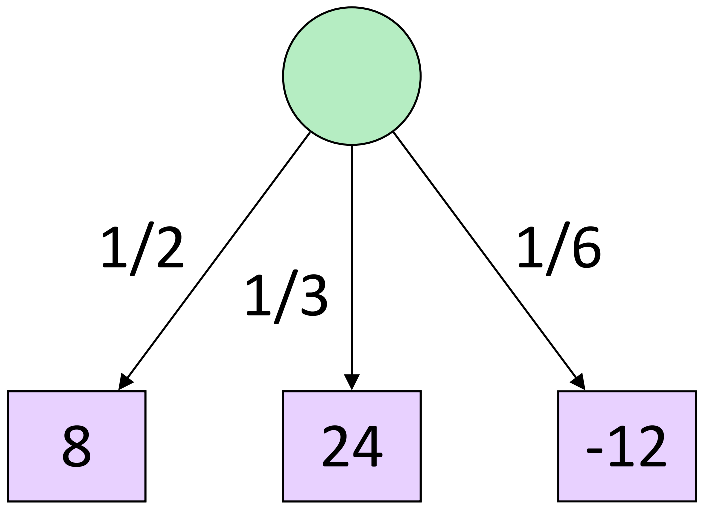
if Terminal-Test(s) then return Utility(s)

if Player(s) = MAX then return $\max_{a \in Actions(s)}$ value(Succ(s,a))

if Player(s) = CHANCE then return $\text{sum}_{a \in Actions(s)}$ Pr(a) * value(Succ(s,a))

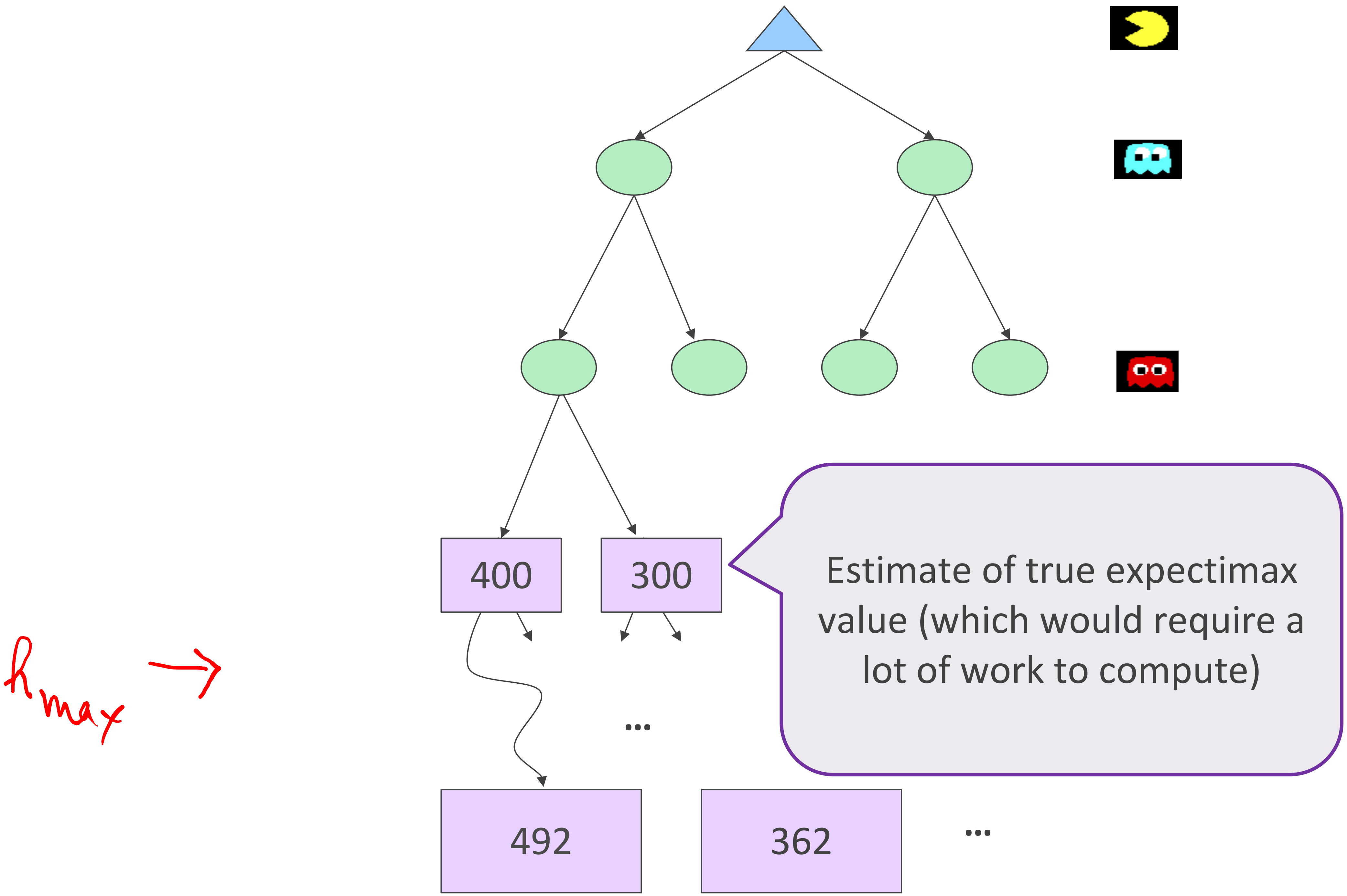
Expectimax Pseudocode

$\text{sum}_{a \text{ in Outcome}(s)} \text{Pr}(a) * \text{value}(\text{Succ}(s,a))$

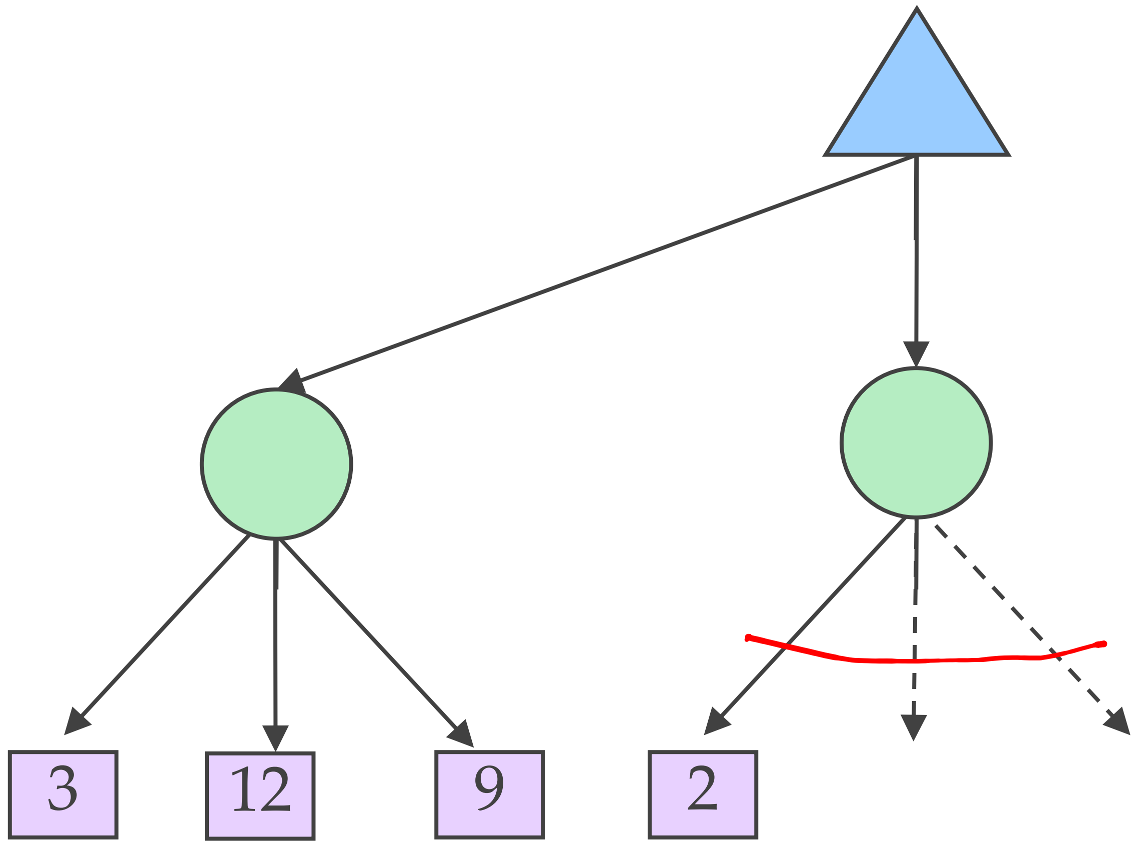


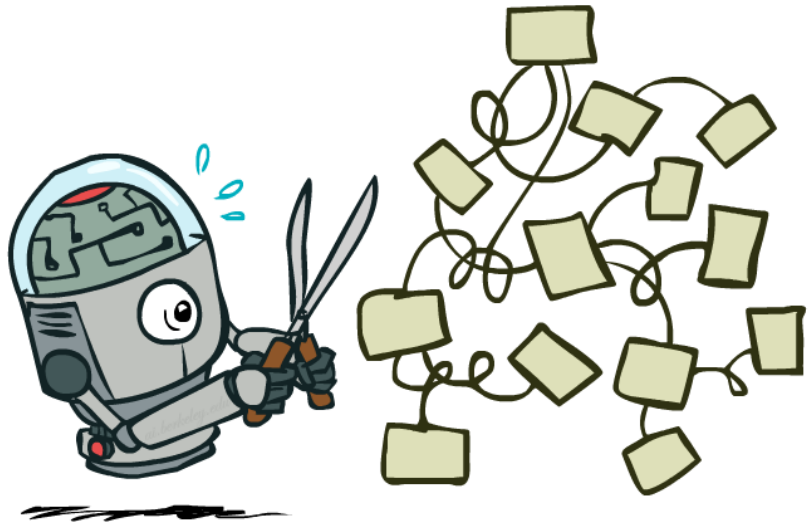
v = (1/2) (8) + (1/3) (24) + (1/6) (-12) = 10

Depth-Limited Expectimax



Expectimax Pruning?





Expectiminimax

function decision(s) returns an action

return the action a in Actions(s) with the highest value(Succ(s,a))

function value(s) returns a value

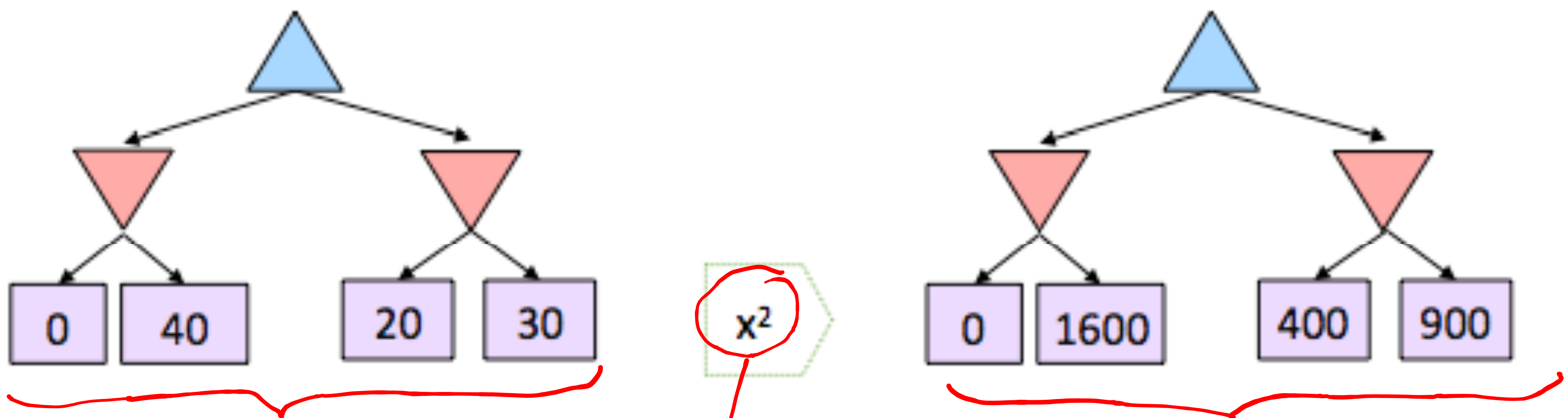
if Terminal-Test(s) then return Utility(s)

if Player(s) = MAX then return $\max_{a \text{ in Actions}(s)}$ value(Succ(s,a))

if Player(s) = MIN then return $\min_{a \text{ in Actions}(s)}$ value(Succ(s,a))

if Player(s) = CHANCE then return $\text{sum}_{a \text{ in Actions}(s)}$ Pr(a) * value(Succ(s,a))

What Values to Use?



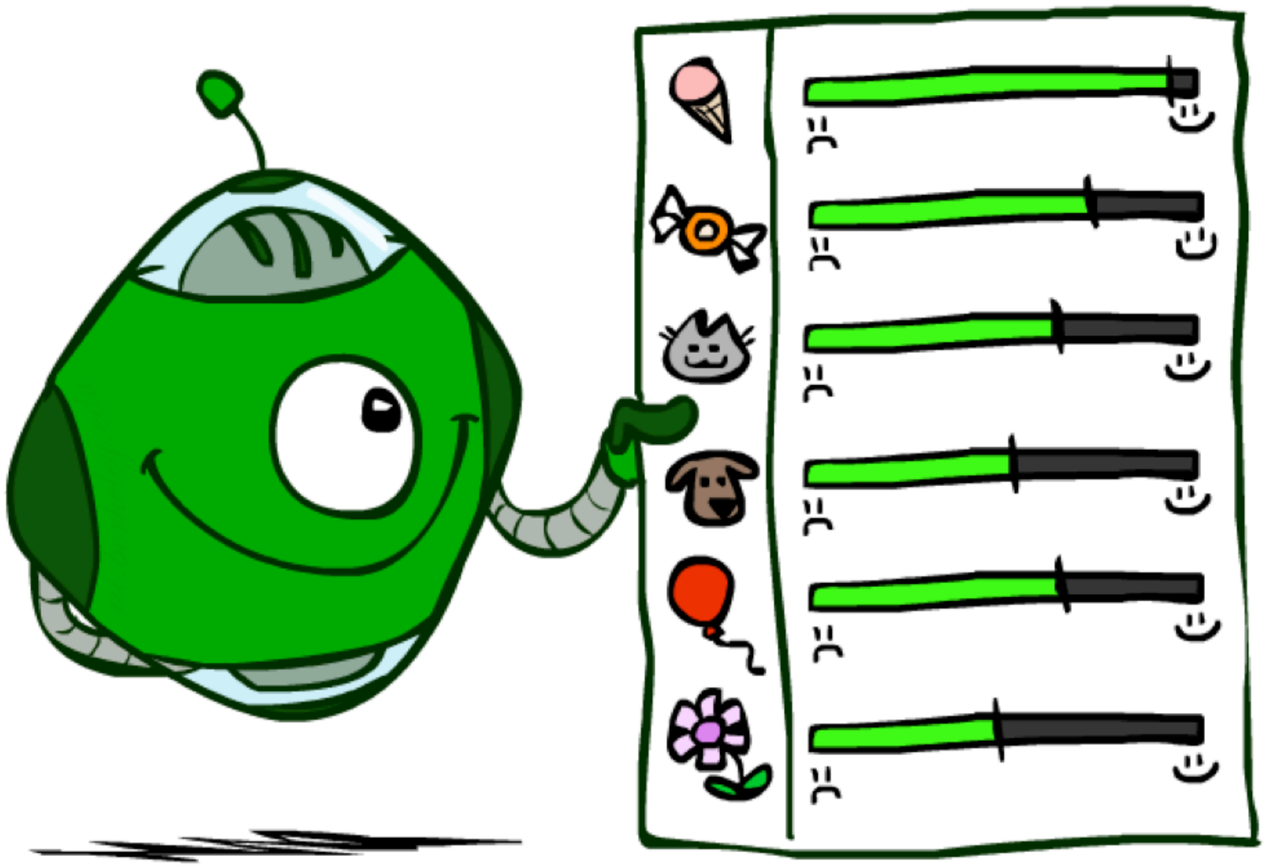
x>y => f(x)>f(y)

f(x) = Ax+B where A>0

- For worst-case minimax reasoning, evaluation function scale doesn't matter
  - We just want better states to have higher evaluations (get the ordering right)
  - Minimax decisions are ***invariant with respect to monotonic transformations on values***

- Expectiminimax decisions are ***invariant with respect to positive affine transformations***
- Expectiminimax evaluation functions have to be aligned with actual win probabilities!

Decision Theory



Decision Theory

- Decision problem:
  - Choose $a \in A$ assuming given preference relation $\succsim$ over A
- Often, choice has uncertain outcomes
  - Probability distribution over outcomes
- Here, we assume single-agent decision-making
- Which decision criterion should we choose?
  - Descriptive
  - Normative

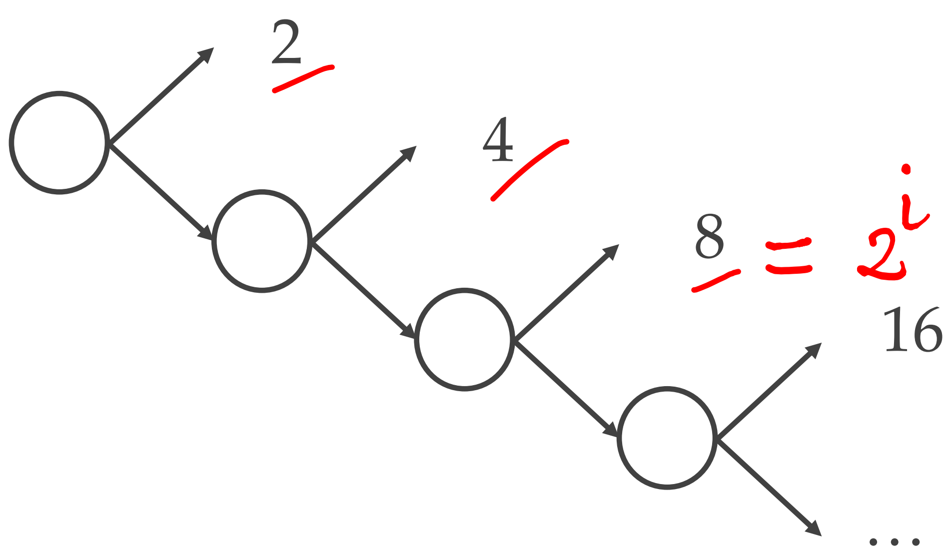
Maximum Expected Utility

- MEU principle:
  - $\max_{p} \sum_{o} p(o) \times U(o)$

- Why is the MEU principle considered rational?
- Where do the utilities come from?
- Where do the probabilities come from?

St Petersburg Paradox

- Game:



- How much would you pay to play this game? 3, 4, . . .
- Expectation:
  - $\frac{1}{2} \cdot 2 + \frac{1}{4} \cdot 4 + \frac{1}{8} \cdot 8 + \frac{1}{16} \cdot 16 + \ldots = 1 + 1 + 1 + 1 + \ldots = + \infty$
- $\mathrm{EU(L)} = \sum_{o \in O} p(o) \log(o)$

U

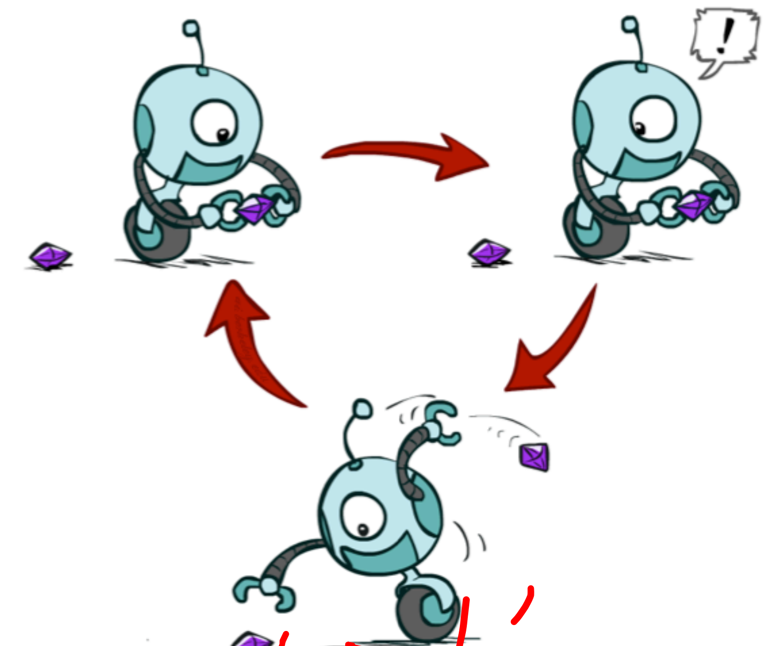
Axiomatization of MEU

- Decision under uncertainty
  - Outcomes: any consequences from a choice
  - Lotteries: distributions over outcomes
  - Preference relation over lotteries: $\gtrsim$
- Two decision models seen so far: EU and minimax
- Axiomatization of decision model C:
  - If set of conditions on $>$ are satisfied, $L \gtrsim L' \Leftrightarrow C(L) \geq C(L')$

Transitivity

- For any three lotteries, L, L′, and L″:
  - $(L \succsim L') \text{ and } (L' \succsim L'') \Rightarrow (L \succsim L'')$

$L'' \succsim L$



- Is it a reasonable axiom?

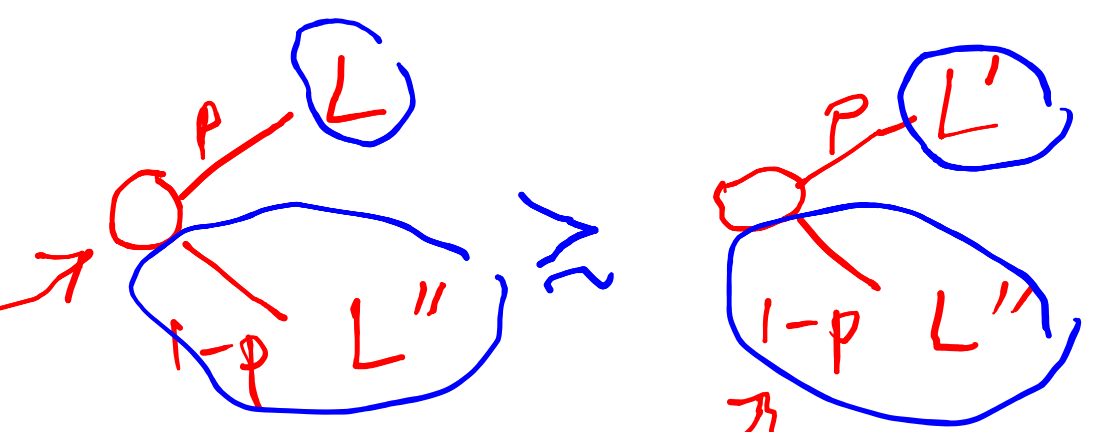
- Money pump argument:

$L'' + 1€ \Leftrightarrow L'$

  - If L′ > L″, then an agent with L″ would pay (say) 1 cent to get L′
  - If L > L′, then an agent with L′ would pay (say) 1 cent to get L
  - If L″ > L, then an agent with L would pay (say) 1 cent to get L″

Axioms of MEU

- Completeness
  - $L \succsim L'$ or $L' \succsim L$
- Transitivity
  - $(L \succsim L')$ and $(L' \succsim L'') \Rightarrow (L \succsim L'')$
- Independence
  - $(L \succsim L') \Rightarrow [p, L; 1\text{-}p, L''] \succsim [p, L'; 1\text{-}p, L'']$



- Continuity
  - $(L \succsim L' \succsim L'') \Rightarrow \exists p, [p, L; 1\text{-}p, L''] \sim L'$

Characterization of MEU

- Theorem [Ramsey, 1931; von Neumann & Morgenstern, 1944; Machin~~e~~a, 1988]
  - If $\succsim$ satisfies the 4 previous axioms, there exists a utility function $U : O \rightarrow \mathbb{R}$ such that
    - $L \succsim L' \Leftrightarrow EU(L) \geq EU(L')$
    - $EU(L) = \sum_{o \in O} p_L(o) U(o)$
- If we agree with the 4 axioms, we should apply MEU
- However, most axioms are debatable
- More general axiomatization where probabilities are not assumed to be given (Savage, 1954)
- Decision theory moved to more general notion of rationality

Risk-Sensitive Decision-Making

- Certainty equivalent of lottery L
  - Outcome o_L such that $\mathrm{U}(o_L) = \mathrm{EU}(\mathrm{L})$
- Risk-neutral decision-making
  - $o_L = \sum_{o \in O} p_L(o) \times o$
  - This is the case if U linear
- Risk-averse decision-making
  - $o_L < \sum_{o \in O} p_L(o) \times o$
  - This is the case if U concave
- Risk-seeking decision-making
  - $o_L > \sum_{o \in O} p_L(o) \times o$
  - This is the case if U convex

Preference Elicitation

- Utility function is unique up to a positive affine transformation
- How to specify a utility function for a given decision problem?
    - Assume U is normalized $U(o^+)=1$ and $U(o^-)=0$
    - Compare any outcome with binary lotteries:
        - For which p, is this true: $o \sim [p, o^+; 1\text{-}p, o^-]$?
        - The answer gives $U(o) = p$
    - Extend to a all lotteries

Uncertainty Elicitation

- How to specify a probability distribution for a given decision problem, if unknown?
  - For which o, is this true: $[E, o^{+}; E^{c}, o^{-}] \sim [1, o; 0, o^{-}]$
  - The answer gives P(E) = U(o)

Allais Paradox (1953)

- What do you prefer?
  - A: [0.8, $4k; 0.2; $0]
  - B: [1.0, $3k; 0.0; $0]
- What do you prefer?
  - C: [0.2, $4k; 0.8; $0]
  - D: [0.25, $3k; 0.75; $0]
- Usually, B > A and C > D
- However, incompatible with MEU! Assuming U($0)=0:
  - B > A => U($3k) > 0.8 U($4k)
  - C > D => U($4k) > U($3k)

Ellsberg Paradox

- Urn with 30 red balls and 60 other balls, which are either black or yellow.
- What do you prefer?
  - A: [R, $100; B or Y; $0]
  - B: [B, $100; R or Y; $0]
- What do you prefer?
  - C: [R or Y, $100; B; $0]
  - D: [B or Y, $100; R; $0]
- Usually, A > B and D > C
- However, incompatible with MEU!

Summary

- Multi-agent problems can require more space or deeper trees to search
  - Bounded-depth search and approximate evaluation functions
  - Alpha-beta pruning
- Game playing has produced important research ideas
  - Reinforcement learning (checkers)
  - Iterative deepening (chess)
  - Monte Carlo tree search (Go)
  - Solution methods for partial-information games in economics (poker)
- Video games present much greater challenges – lots to do!
  - $b = 10^{500}$, $|S| = 10^{4000}$, $m = 10{,}000$
- Basics of decision theory