MTRE 4200
Robotics Analysis and Synthesis
(Part #2)

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Section 3.2 The D-H Convention

(1) What is the D-H convention?

A systematic procedure to assign a coordinate frame for each link and calculate the forward kinematics.

(2) D-H conditions

Consider two frames $o_{i-1}x_{i-1}y_{i-1}z_{i-1}$ and $o_i x_i y_i z_i$

If:

- The axis x_i is perpendicular to the axis z_{i-1} (DH1)
- The axis x_i intersects the axis z_{i-1} (DH2)

Then the frames meet the D-H conditions

(3) An example about the D-H conditions

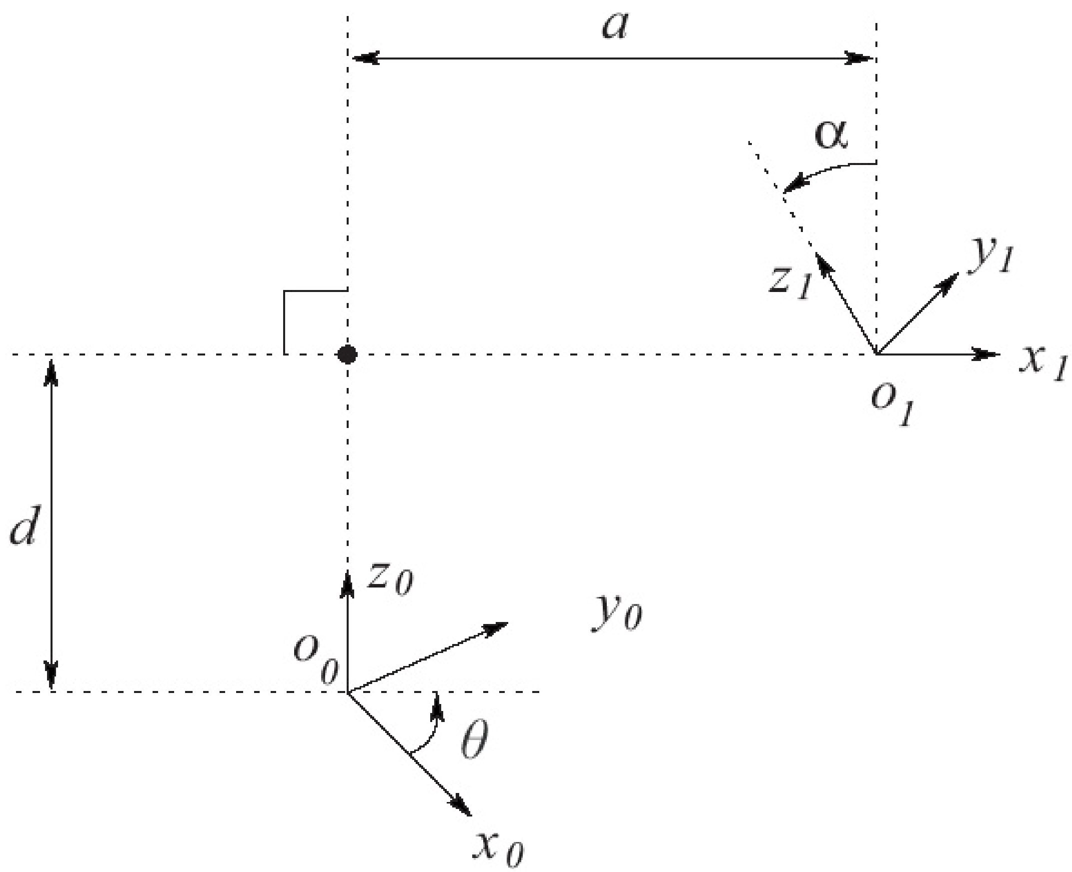


Figure 3.2: Coordinate frames satisfying assumptions DH1 and DH2.

(4) An important conclusion

If the frame $o_{i-1}x_{i-1}y_{i-1}z_{i-1}$ and $o_i x_i y_i z_i$ meet the D-H conditions, then there exist four unique parameters a, d, θ, α such that:

$$H_i^{i-1} = Rot_{z,\theta} \bullet Trans_{z,d} \bullet Trans_{x,a} \bullet Rot_{x,\alpha}$$

Where: ***Rot*** and ***Trans*** are one of six basic homogeneous matrices.

- If you know the four parameters a, d, θ, α, then you can directly get the H matrix with the above equation.

(5) Physical explanation of the four parameters

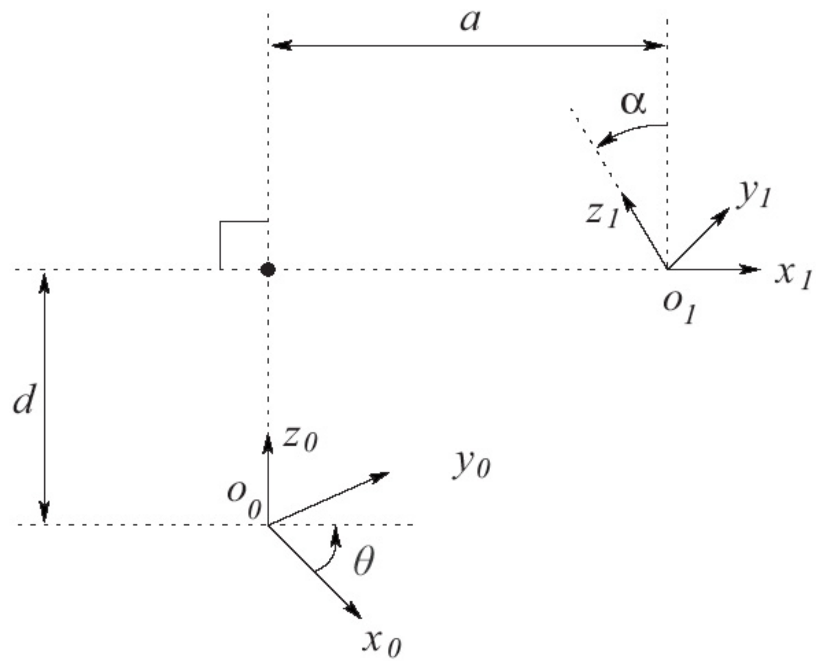


Figure 3.2: Coordinate frames satisfying assumptions DH1 and DH2.

a: the distance between the axis z_{i-1} and z_i measured along the axis x_i .

d: the distance from the origin o_{i-1} to the intersection of axis x_i with z_{i-1} along axis z_{i-1}.

α: The angle between the axis z_{i-1} and z_i measured in a plane normal to x_i.

θ: The angle between the axis x_{i-1} and x_i measured in a plane normal to z_{i-1}.

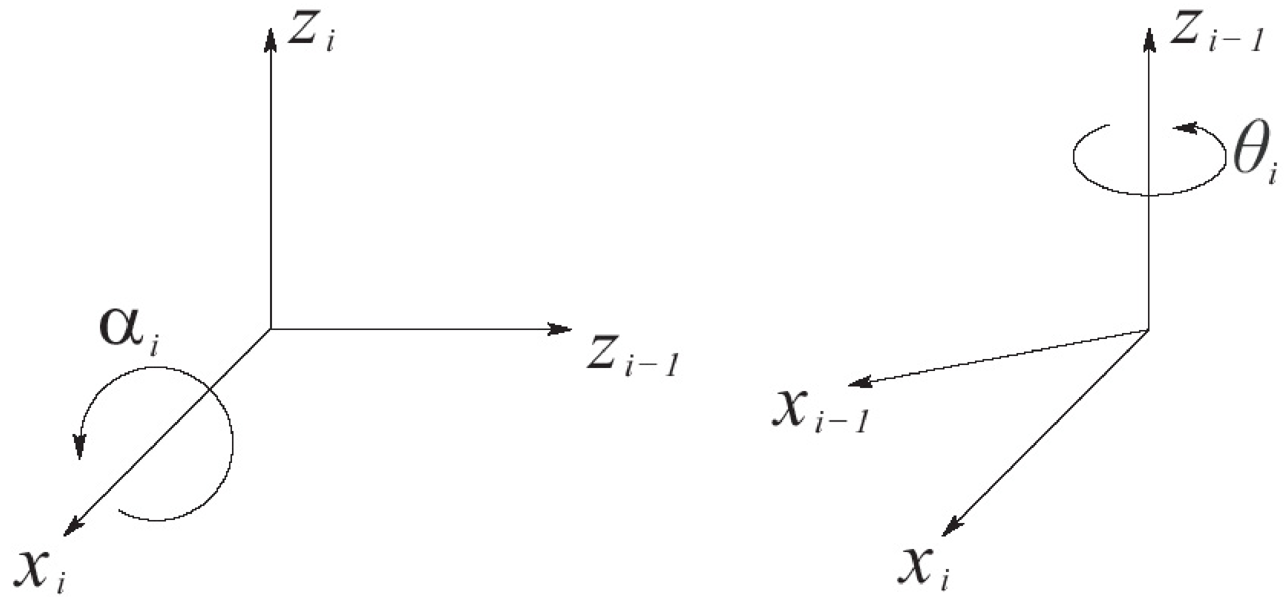


Figure 3.3: Positive sense for α_i and θ_i.

Youtube Video for D-H Covention

Youtube Video:

http://www.youtube.com/watch?v=rA9tm0gTln8

(6) Find forward kinematics using D-H convention

Step #1: Establish the base frame

The base frame is attached to Link #0

Step #2: Assign a frame for each link

Step #3: Establish the end-effector frame

Please note : In Step #1 - #3, make sure any two adjacent frames meet the D-H condition. In addition, each frame is **attached to** a link rather than a joint.

Step #4: Find the H matrix (forward kinematics)

- Determine the four parameters a_i, d_i, θ_i, α_i for two adjacent frames.
- Calculate

$$H_i^{i-1} = Rot_{z,\theta} \bullet Trans_{z,d} \bullet Trans_{x,a} \bullet Rot_{x,\alpha}$$

- Calcualte:

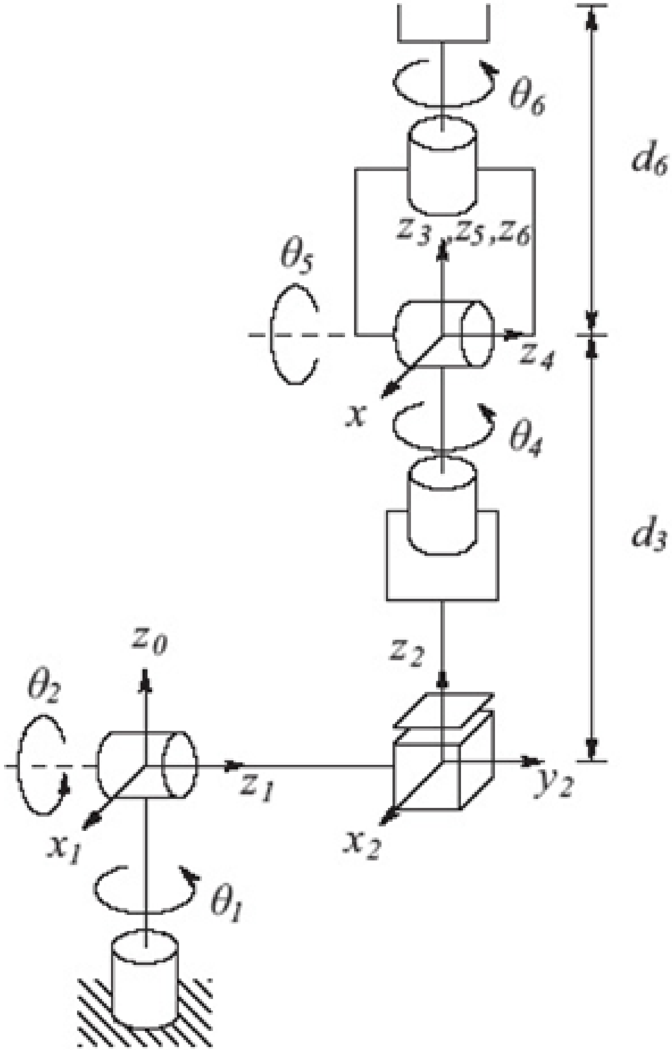
$$H_n^0 = H_1^0 H_2^1 \cdots H_n^{n-1}$$

Some important facts:

- **Two adjacent frames must meet the D-H conditions.**
- If the current joint is a revolute joint, then one of θ_i or α_i is a variable, and the other three parameters are the constants.
- If the current joint is a prismatic joint, then one of a_i or d_i is a variable, and the other three parameters are the constants.

Section 3.3 Inverse Kinematics

Example 3.7 Stanford Manipulator



Question:

If we know:

$$H_6^0 = \begin{bmatrix} 0 & 1 & 0 & -0.154 \\ 0 & 0 & 1 & 0.763 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

how much are $\theta_1, \theta_2, d_3, \theta_4, \theta_5, \theta_6$?

Section 3.3 Inverse Kinematics

Solution:

Step #1: Derive the forward kinematics of Stanford Manipulator

$$H_6^0 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned}
r_{11} &= c_1[c_2(c_4c_5c_6 - s_4s_6) - s_2s_5c_6] - d_2(s_4c_5c_6 + c_4s_6) \\
r_{21} &= s_1[c_2(c_4c_5c_6 - s_4s_6) - s_2s_5c_6] + c_1(s_4c_5c_6 + c_4s_6) \\
r_{31} &= -s_2(c_4c_5c_6 - s_4s_6) - c_2s_5c_6 \\
r_{12} &= c_1[-c_2(c_4c_5s_6 + s_4c_6) + s_2s_5s_6] - s_1(-s_4c_5s_6 + c_4c_6) \\
r_{22} &= -s_1[-c_2(c_4c_5s_6 + s_4c_6) + s_2s_5s_6] + c_1(-s_4c_5s_6 + c_4c_6) \\
r_{32} &= s_2(c_4c_5s_6 + s_4c_6) + c_2s_5s_6 \\
r_{13} &= c_1(c_2c_4s_5 + s_2c_5) - s_1s_4s_5 \\
r_{23} &= s_1(c_2c_4s_5 + s_2c_5) + c_1s_4s_5 \\
r_{33} &= -s_2c_4s_5 + c_2c_5 \\
d_x &= c_1s_2d_3 - s_1d_2 + +d_6(c_1c_2c_4s_5 + c_1c_5s_2 - s_1s_4s_5) \\
d_y &= s_1s_2d_3 + c_1d_2 + d_6(c_1s_4s_5 + c_2c_4s_1s_5 + c_5s_1s_2) \\
d_z &= c_2d_3 + d_6(c_2c_5 - c_4s_2s_5)
\end{aligned}$$

Step #2: List the 12 non-linear equations

$$
H^0_6 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -0.154 \\ 0 & 0 & 1 & 0.763 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\begin{aligned}
c_1[c_2(c_4c_5c_6 - s_4s_6) - s_2s_5c_6] - s_1(s_4c_5c_6 + c_4s_6) &= 0 \\
s_1[c_2(c_4c_5c_6 - s_4s_6) - s_2s_5c_6] + c_1(s_4c_5c_6 + c_4s_6) &= 0 \\
-s_2(c_4c_5c_6 - s_4s_6) - c_2s_5c_6 &= 1 \\
c_1[-c_2(c_4c_5s_6 + s_4c_6) + s_2s_5s_6] - s_1(-s_4c_5s_6 + c_4c_6) &= 1 \\
s_1[-c_2(c_4c_5s_6 + s_4c_6) + s_2s_5s_6] + c_1(-s_4c_5s_6 + c_4c_6) &= 0 \\
s_2(c_4c_5s_6 + s_4c_6) + c_2s_5s_6 &= 0 \\
c_1(c_2c_4s_5 + s_2c_5) - s_1s_4s_5 &= 0 \\
s_1(c_2c_4s_5 + s_2c_5) + c_1s_4s_5 &= 1 \\
-s_2c_4s_5 + c_2c_5 &= 0 \\
c_1s_2d_3 - s_1d_2 + d_6(c_1c_2c_4s_5 + c_1c_5s_2 - s_1s_4s_5) &= -0.154 \\
s_1s_2d_3 + c_1d_2 + d_6(c_1s_4s_5 + c_2c_4s_1s_5 + c_5s_1s_2) &= 0.763 \\
c_2d_3 + d_6(c_2c_5 - c_4s_2s_5) &= 0
\end{aligned}
$$

Step #3: Solve the 12 non-linear equations, we get the joint variable:

$$\theta_1, \theta_2, d_3, \theta_4, \theta_5, \theta_6$$

Challenges in Finding the Inverse Kinematics of a Robotic Arm

- Very difficult to solve the 12 nonlinear equations.
- The equations may have no solutions or infinite solutions, depending on the number of joints.

An Engineering Solution

- Design a robot with simple kinematics. (A simple robot is a good robot!)
- Solve three nonlinear equations only every time.