VAR Baseline for Cambridge UK Weather Forecasting

Vector autoregression (VAR) models generalise the single-variable autoregressive model by capturing the relationships between multivariate time series. This notebook updates the earlier VAR model from the baselines notebook with more data, cleaner data and more extensively tuned parameters.

This notebook is being developed on Google Colab, using statsmodels and the Darts time series package. Initially I was most interested in short term temperature forecasts (less than 2 hours) but now mostly produce results up to 24 hours in the future for comparison with earlier baselines.

See my previous notebooks, web apps etc:

- Cambridge UK temperature forecast python notebooks
- Cambridge UK temperature forecast R models
- Bayesian optimisation of prophet temperature model
- Cambridge University Computer Laboratory weather station R shiny web app

The linked notebooks, web apps etc contain further details including:

- data description
- data cleaning and preparation
- data exploration

In particular, see the notebooks:

- cammet_baselines_2021 including persistent, simple exponential smoothing, Holt Winter's exponential smoothing and vector autoregression
- gradient_boosting notebook uses the results of the updated VAR model as the main baseline for model comparison
- keras_mlp_fcn_resnet_time_series, which uses a streamlined version of data preparation from Tensorflow time series forecasting tutorial
- lstm_time_series with stacked LSTMs, bidirectional LSTMs and ConvLSTM1D networks
- cnn_time_series with Conv1D, multi-head Conv1D, Conv2D and Inception-style models
- encoder_decoder which includes autoencoder with attention, encoder decoder with teacher forcing, transformer with teacher forcing and padding, encoder only with MultiHeadAttention
- feature_engineering solar-based and meteorology-based calculated features, rolling stats, tsfeatures, catch22, bivariate features and more
- tsfresh_feature_engineering automated feature engineering and selection for time series analysis of Cambridge UK weather measurements

Most of the above repositories, notebooks, web apps etc were built on both less data and less thoroughly cleaned data.

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TODO Add internal links before "final" commits

Some sections may get added/deleted during development.

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Code Setup

Install darts Library

Install darts because it is currently not available on google colab. The darts `TimeSeries` class is used in the `get_var_backtest` function and dependent functions like `get_historic_comparison`.

WARNING: You may need to restart the google colab runtime after this install.

```
!pip install "u8darts[notorch]"
```

```
Requirement already satisfied: pytz>=2020.1 in /usr/local/lib/python3.10/dist-packages (from pandas>=1.0.5->u8darts[notor
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Requirement already satisfied: urllib3 in /usr/local/lib/python3.10/dist-packages (from pmdarima>=1.8.0->u8darts[notorch]
Requirement already satisfied: setuptools!=50.0.0,>=38.6.0 in /usr/local/lib/python3.10/dist-packages (from pmdarima>=1.8
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Requirement already satisfied: idna<4,>=2.5 in /usr/local/lib/python3.10/dist-packages (from requests>=2.22.0->u8darts[no
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Requirement already satisfied: threadpoolctl>=2.0.0 in /usr/local/lib/python3.10/dist-packages (from scikit-learn>=1.0.1-
Collecting slicer==0.0.7 (from shap>=0.40.0->u8darts[notorch])
  Downloading slicer-0.0.7-py3-none-any.whl (14 kB)
Requirement already satisfied: cloudpickle in /usr/local/lib/python3.10/dist-packages (from shap>=0.40.0->u8darts[notorch
Collecting fugue>=0.8.1 (from statsforecast>=1.4->u8darts[notorch])
  Downloading fugue-0.8.7-py3-none-any.whl (279 kB)
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  Downloading utilsforecast-0.1.2-py3-none-any.whl (40 kB)
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Requirement already satisfied: patsy>=0.5.4 in /usr/local/lib/python3.10/dist-packages (from statsmodels>=0.14.0->u8darts
Requirement already satisfied: stanio~=0.3.0 in /usr/local/lib/python3.10/dist-packages (from cmdstanpy>=1.0.4->prophet>=
Collecting triad>=0.9.3 (from fugue>=0.8.1->statsforecast>=1.4->u8darts[notorch])
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Collecting adagio>=0.2.4 (from fugue>=0.8.1->statsforecast>=1.4->u8darts[notorch])
  Downloading adagio-0.2.4-py3-none-any.whl (26 kB)
Collecting qpd>=0.4.4 (from fugue>=0.8.1->statsforecast>=1.4->u8darts[notorch])
  Downloading qpd-0.4.4-py3-none-any.whl (169 kB)
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Collecting fugue-sql-antlr>=0.1.6 (from fugue>=0.8.1->statsforecast>=1.4->u8darts[notorch])
  Downloading fugue-sql-antlr-0.2.0.tar.gz (154 kB)
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  Preparing metadata (setup.py) ... done
Requirement already satisfied: sqlglot in /usr/local/lib/python3.10/dist-packages (from fugue>=0.8.1->statsforecast>=1.4-
Requirement already satisfied: jinja2 in /usr/local/lib/python3.10/dist-packages (from fugue>=0.8.1->statsforecast>=1.4->
Requirement already satisfied: llvmlite<0.42,>=0.41.0dev0 in /usr/local/lib/python3.10/dist-packages (from numba>=0.51->p
Requirement already satisfied: tenacity>=6.2.0 in /usr/local/lib/python3.10/dist-packages (from plotly->catboost>=1.0.6->
Collecting antlr4-python3-runtime<4.12 (from fugue-sql-antlr>=0.1.6->fugue>=0.8.1->statsforecast>=1.4->u8darts[notorch])
  Downloading antlr4_python3_runtime-4.11.1-py3-none-any.whl (144 kB)
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Requirement already satisfied: pyarrow>=6.0.1 in /usr/local/lib/python3.10/dist-packages (from triad>=0.9.3->fugue>=0.8.1
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Collecting fs (from triad>=0.9.3->fugue>=0.8.1->statsforecast>=1.4->u8darts[notorch])
  Downloading fs-2.4.16-py2.py3-none-any.whl (135 kB)
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Requirement already satisfied: MarkupSafe>=2.0 in /usr/local/lib/python3.10/dist-packages (from jinja2->fugue>=0.8.1->sta
Requirement already satisfied: appdirs~=1.4.3 in /usr/local/lib/python3.10/dist-packages (from fs->triad>=0.9.3->fugue>=0
Building wheels for collected packages: pyod, fugue-sql-antlr
  Building wheel for pyod (setup.py) ... done
  Created wheel for pyod: filename=pyod-1.1.3-py3-none-any.whl size=190250 sha256=bcb832e7100ea0c9de7f6e4b90e74a4384393d0
  Stored in directory: /root/.cache/pip/wheels/05/f8/db/124d43bec122d6ec0ab3713fadfe25ebed8af52ec561682b4e
  Building wheel for fugue-sql-antlr (setup.py) ... done
  Created wheel for fugue-sql-antlr: filename=fugue_sql_antlr-0.2.0-py3-none-any.whl size=158196 sha256=af49fd7e057bf3321
  Stored in directory: /root/.cache/pip/wheels/5a/b5/4e/216953a1c711da55de29ed7ecf158b4a5bf32ef93d69ad66dd
Successfully built pyod fugue-sql-antlr
Installing collected packages: antlr4-python3-runtime, slicer, fs, utilsforecast, triad, shap, pyod, nfoursid, catboost,
Successfully installed adagio-0.2.4 antlr4-python3-runtime-4.11.1 catboost-1.2.3 fs-2.4.16 fugue-0.8.7 fugue-sql-antlr-0.
```

Load Libraries

Load the required packages:

```
import re
import sys
import math
import timeit
import warnings
import datetime
import itertools
import subprocess
```

```
import pkg_resources

import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import matplotlib.dates as mdates
from scipy import stats
from itertools import product
from statsmodels.tsa.api import VAR
from statsmodels.tools.eval_measures import rmse, medianabs
from statsmodels.nonparametric.smoothers_lowess import lowess
from sklearn.utils import check_X_y
from sklearn.metrics import r2_score
from sklearn.preprocessing import MinMaxScaler

from darts import TimeSeries


# Reduces variance in results but won't eliminate it :-(
%env PYTHONHASHSEED=0
import random

# set seed to make all processes deterministic
seed = 0
random.seed(seed)
np.random.seed(seed)


%matplotlib inline
```

```
env: PYTHONHASHSEED=0
```

Environment Variables

Set some environment variables:

```
HORIZON = 48
Y_COL   = 'y_des' # 'y_des_fft' 'y_res' 'y'
CORE_FEATS = [Y_COL, 'dew.point_des', 'humidity', 'pressure']
FUT_FEATS  = ['irradiance', 'za_rad', 'azimuth_cos']

DAY  = 24 * 60 * 60
YEAR = 365.2425 * DAY

DAILY_OBS  = 48
YEARLY_OBS = int(365.2425 * DAILY_OBS)  # annual observations
DAY_SECS_STEP = int(DAY / DAILY_OBS)

VALID_YEAR = 2021
TEST_YEAR  = 2022
```

Custom Functions

Next, define some utility functions:

- `_check_obs_preds_lens_eq`
- `rmse_`
- `mse_`
- `mae_`
- `load_features_file`
- `load_train_valid_test_features`
- `drop_correlated_cols`
- `plot_observation_examples`
- `sanity_check_df_rows_cols_labels`
- `sanity_check_before_after_dfs`
- `compare_train_valid_test_sanity_dfs`
- `sanity_check_train_valid_test`
- `print_train_valid_test_shapes`
- `plot_feature_history_single_df`
- `plot_feature_history`
- `plot_feature_history_separately`
- `check_high_low_thresholds`

- print_df_summary
- get_approx_overlap
- expand_grid
- keep_key
- get_historic_comparison
- summarise_historic_comparison
- print_rmse_mae
- _plot_xy_for_label
- plot_multistep_obs_vs_preds
- plot_multistep_obs_vs_mean_preds_by_step
- plot_multistep_obs_preds_dists
- plot_multistep_residuals
- plot_multistep_residuals_dist
- plot_multistep_residuals_vs_predicted
- se_
- metric_ci_vals
- plot_horizon_metrics
- plot_horizon_metrics_boxplots
- plot_multistep_diagnostics

```
def _check_obs_preds_lens_eq(obs, preds):
    obs_preds_lens_eq = 1

    if len(obs) != len(preds):
        print("obs:  ", len(obs))
        print("preds:", len(preds))
        obs_preds_lens_eq = 0

    return obs_preds_lens_eq


def rmse_(obs, preds):
    if _check_obs_preds_lens_eq(obs, preds) == 0:
        stop()
    else:
        return np.sqrt(np.mean((obs - preds) ** 2))


def mse_(obs, preds):
    if _check_obs_preds_lens_eq(obs, preds) == 0:
        stop()
    else:
        return np.mean((obs - preds) ** 2)


def mae_(obs, preds):
    "mean absolute error - equivalent to the keras loss function"
    if _check_obs_preds_lens_eq(obs, preds) == 0:
        stop()
    else:
        return np.mean(np.abs(obs - preds))      # keras loss
        # return np.median(np.abs(obs - preds))  # earlier baselines


def load_features_file(feature_set,
                       data_set,
                       location = 'gdrive',
                       date_str = '.2022.09.20',
                       filex    = '.parquet'):

  if location == 'github':
    base_url = 'https://github.com/makeyourownmaker/CambridgeTemperatureNotebooks/blob/main/data/features/'
    filex += '?raw=true'
  elif location == 'gdrive':
    base_url = '/content/drive/MyDrive/data/CambridgeTemperatureNotebooks/features/'
  else:
    print("Unsupported 'location' in load_features_file function:")
    print('  location =', location)

  file_str = feature_set + '_' + data_set + date_str + filex
  data_url = base_url + file_str

  df = pd.read_parquet(data_url)
```

```
  df.set_index('ds', drop=False, inplace=True)
  df = df[~df.index.duplicated(keep='first')]
  df = df.asfreq(freq='30min')

  return df


def load_train_valid_test_features(feature_set, location='gdrive'):
  train = load_features_file(feature_set, 'train', location)
  valid = load_features_file(feature_set, 'valid', location)
  test  = load_features_file(feature_set, 'test',  location)

  sanity_check_train_valid_test(train, valid, test)

  check_high_low_thresholds(train, 'train '+feature_set)
  check_high_low_thresholds(valid, 'valid '+feature_set)
  check_high_low_thresholds(test,   'test '+feature_set)

  return train, valid, test



def drop_correlated_cols(dataset, threshold=0.95):
  '''Adapted from https://stackoverflow.com/a/44674459/100129'''

  col_corr = set()  # Set of all the names of deleted columns
  corr_matrix = dataset.corr(numeric_only=True).abs()

  for i in range(len(corr_matrix.columns)):
    for j in range(i):
      if (corr_matrix.iloc[i, j] >= threshold) and (corr_matrix.columns[j] not in col_corr):
        colname = corr_matrix.columns[i]
        col_corr.add(colname)
        if colname in dataset.columns:
          del dataset[colname]

  return dataset


def plot_observation_examples(df, cols, num_plots = 9):
    """Plot 9 sets of observations in 3 * 3 matrix"""

    num_plots_sqrt = int(np.sqrt(num_plots))
    assert num_plots_sqrt ** 2 == num_plots

    days = df.ds.dt.date.sample(n = num_plots).sort_values()
    p_data = [df[df.ds.dt.date.eq(days[i])] for i in range(num_plots)]

    fig, axs = plt.subplots(num_plots_sqrt, num_plots_sqrt, figsize = (15, 10))
    axs = axs.ravel()  # apl for the win :-)

    for i in range(num_plots):
        for col in cols:
            axs[i].plot(p_data[i]['ds'], p_data[i][col])
            axs[i].xaxis.set_tick_params(rotation = 20, labelsize = 10)

    fig.suptitle('Observation examples')
    fig.legend(cols, loc = 'lower center',  ncol = len(cols))

    return None


# TODO Change to operate on single dataframe - More useful function :-)
#      Change as far as possible - merge(), common_cols etc
#      Then write a wrapper to operate on before and after dataframes
#      combine results and calculate differences
def sanity_check_df_rows_cols_labels(before, after,
                                     row_var_cutoff=0.005, col_var_cutoff=0.05,
                                     col_corr_cutoff=0.,
                                     fast=True, verbose=False):
  '''Sanity check dataframes before and after modifications

  WARN: default row_var_cutoff, col_var_cutoff, col_corr_cutoff are fairly arbitrary
        there is some redundancy between these tests

  '''

  print_v = print if verbose else lambda *a, **k: None

  df = pd.DataFrame(columns = ['before', 'after', 'diff'])
  df_labels = []
```

```
label = 'rows'
# start_time = timeit.default_timer()
i = 0
df.loc[len(df), df.columns] = before.shape[i], after.shape[i], 0
df_labels.append(label)
# print('\t', label, round(timeit.default_timer() - start_time, 2))

label = 'cols'
# start_time = timeit.default_timer()
i = 1
df.loc[len(df), df.columns] = before.shape[i], after.shape[i], 0
df_labels.append(label)
# print('\t', label, round(timeit.default_timer() - start_time, 2))

label = 'missing_rows'
# start_time = timeit.default_timer()
i = 0
before_after = pd.merge(before, after, left_index=True, right_index=True, how='outer', indicator=True)
missing_rows = before_after.loc[before_after['_merge'] == 'left_only', :]
df.loc[len(df), df.columns] = 0, missing_rows.shape[i], 0
if missing_rows.shape[i] > 0:
  print_v('\n', label, ':')
  print_v(missing_rows)
df_labels.append(label)
# print('\t', label, round(timeit.default_timer() - start_time, 2))

label = 'missing_cols'
# start_time = timeit.default_timer()
i = 1
common_cols = before.columns.intersection(after.columns)
missing_cols = before.shape[i] - len(common_cols)
df.loc[len(df), df.columns] = 0, missing_cols, 0
if missing_cols > 0:
  print_v('\n', label, ':')
  print_v(set(before.columns) - set(common_cols))
df_labels.append(label)
# print('\t', label, round(timeit.default_timer() - start_time, 2))

label = 'total_nas'
# start_time = timeit.default_timer()
df.loc[len(df), df.columns] = before.isna().sum().sum(), \
                              after.isna().sum().sum(), 0
df_labels.append(label)
# print('\t', label, round(timeit.default_timer() - start_time, 2))

label = 'rows_with_nas'
# start_time = timeit.default_timer()
before_rows_nas = before.isnull().any(axis=1).sum()
after_rows_nas  = after.isnull().any(axis=1).sum()
df.loc[len(df), df.columns] = before_rows_nas, after_rows_nas, 0
if before_rows_nas != after_rows_nas:
  print_v('\n', label, ':')
  print_v(before[before.isnull().any(axis=1)])
  print_v(after[after.isnull().any(axis=1)])
df_labels.append(label)
# print('\t', label, round(timeit.default_timer() - start_time, 2))

label = 'cols_with_nas'
# start_time = timeit.default_timer()
before_cols_nas = before.isnull().any().sum()
after_cols_nas  = after.isnull().any().sum()
df.loc[len(df), df.columns] = before_cols_nas, after_cols_nas, 0
if before_cols_nas != after_cols_nas:
  print_v('\n', label, ':')
  print_v(before.isnull().any().index.values)
  print_v(after.isnull().any().index.values)
df_labels.append(label)
# print('\t', label, round(timeit.default_timer() - start_time, 2))

label = 'single_value_rows'
if not fast:
  # start_time = timeit.default_timer()
  before_single_value_rows = np.sum(before.nunique(axis=1) <= 1)
  after_single_value_rows  = np.sum(after.nunique(axis=1) <= 1)
  df.loc[len(df), df.columns] = before_single_value_rows, \
                                after_single_value_rows, 0
  if before_single_value_rows != after_single_value_rows:
    print_v('\n', label, ':')
    print_v(before[before.nunique(axis=1) <= 1])
    print_v(after[after.nunique(axis=1) <= 1])
  df_labels.append(label)
```

```
  # print('\t', label, round(timeit.default_timer() - start_time, 2))

label = 'single_value_cols'
# start_time = timeit.default_timer()
before_single_value_cols = np.sum(before.nunique() <= 1)
after_single_value_cols  = np.sum(after.nunique() <= 1)
df.loc[len(df), df.columns] = before_single_value_cols, \
                              after_single_value_cols, 0
if before_single_value_cols != after_single_value_cols:
  print_v('\n', label, ':')
  print_v(before.columns[before.nunique() <= 1].values)
  print_v(after.columns[after.nunique() <= 1].values)
df_labels.append(label)
# print('\t', label, round(timeit.default_timer() - start_time, 2))

# warnings.resetwarnings()

with warnings.catch_warnings():
  warnings.simplefilter('ignore')

  label = 'low_var_rows'
  # start_time = timeit.default_timer()
  before_low_var_rows = (before.select_dtypes(include=[np.number]).std(axis=1) <= row_var_cutoff).sum()
  after_low_var_rows  = (after.select_dtypes(include=[np.number]).std(axis=1) <= row_var_cutoff).sum()
  df.loc[len(df), df.columns] = before_low_var_rows, after_low_var_rows, 0
  if before_low_var_rows != after_low_var_rows:
    print_v('\n', label, ':')
    print_v(before.select_dtypes(include=[np.number]).std(axis=1) <= row_var_cutoff)
    print_v(after.select_dtypes(include=[np.number]).std(axis=1)  <= row_var_cutoff)
  df_labels.append(label)
  # print('\t', label, round(timeit.default_timer() - start_time, 2))

  label = 'low_var_cols'
  # start_time = timeit.default_timer()
  before_low_var_cols = (before.select_dtypes(include=[np.number]).std() <= col_var_cutoff).sum()
  after_low_var_cols  = (after.select_dtypes(include=[np.number]).std() <= col_var_cutoff).sum()
  df.loc[len(df), df.columns] = before_low_var_cols, after_low_var_cols, 0
  if before_low_var_cols != after_low_var_cols:
    print_v('\n', label, ':')
    s = before.select_dtypes(include=[np.number]).std() <= col_var_cutoff
    t = after.select_dtypes(include=[np.number]).std()  <= col_var_cutoff
    print_v(s[s].index.values)
    print_v(t[t].index.values)
  df_labels.append(label)
  # print('\t', label, round(timeit.default_timer() - start_time, 2))

label = 'duplicate_rows'
# start_time = timeit.default_timer()
before_dup_rows = before.shape[0] - before.drop_duplicates().shape[0]
after_dup_rows  = after.shape[0]  - after.drop_duplicates().shape[0]
df.loc[len(df), df.columns] = before_dup_rows, after_dup_rows, 0
if before_dup_rows != after_dup_rows:
  print_v('\n', label, ':')
  print_v(before[before.duplicated(keep=False)])
  print_v(after[after.duplicated(keep=False)])
df_labels.append(label)
# print('\t', label, round(timeit.default_timer() - start_time, 2))

label = 'highly_correlated_cols'
# .copy() so we don't modify the original dataframe
if not fast:
  # start_time = timeit.default_timer()
  before_high_corr_cols = before.shape[1] - drop_correlated_cols(before.copy(), col_corr_cutoff).shape[1]
  after_high_corr_cols  = after.shape[1]  - drop_correlated_cols(after.copy(), col_corr_cutoff).shape[1]
  df.loc[len(df), df.columns] = before_high_corr_cols, after_high_corr_cols, 0
  if before_high_corr_cols != after_high_corr_cols:
    print_v('\n', label, ':')
    print_v(set(before.columns) - set(drop_correlated_cols(before.copy(), col_corr_cutoff).columns))
    print_v(set(after.columns) - set(drop_correlated_cols(after.copy(), col_corr_cutoff).columns))
  df_labels.append(label)
  # print('\t', label, round(timeit.default_timer() - start_time, 2))

label = 'duplicate_index_labels'
# start_time = timeit.default_timer()
before_idx_labels = before.index.duplicated().sum()
after_idx_labels  = after.index.duplicated().sum()
df.loc[len(df), df.columns] = before_idx_labels, after_idx_labels, 0
if before_idx_labels != after_idx_labels:
  print_v('\n', label, ':')
  print_v(before.index.duplicated())
  print_v(after.index.duplicated())
df_labels.append(label)
```

```
  # print('\t', label, round(timeit.default_timer() - start_time, 2))

  label = 'duplicate_col_labels'
  # start_time = timeit.default_timer()
  before_dup_col_labels = before.columns.duplicated().sum()
  after_dup_col_labels  = after.columns.duplicated().sum()
  df.loc[len(df), df.columns] = before_dup_col_labels, after_dup_col_labels, 0
  if before_dup_col_labels != after_dup_col_labels:
    print_v('\n', label, ':')
    print_v(before.columns.duplicated())
    print_v(after.columns.duplicated())
  df_labels.append(label)
  # print('\t', label, round(timeit.default_timer() - start_time, 2))

  # TODO Find renamed columns from before in after?


  df['diff'] = df['after'] - df['before']
  df.index = df_labels

  return df


def sanity_check_before_after_dfs(before_, after_, ds_name, fast=True, verbose=False):

  print('\n', ds_name, sep='')

  # Reasons I HATE pandas number Inf a neverending series:
  # PerformanceWarning: DataFrame is highly fragmented.  This is usually the
  # result of calling `frame.insert` many times, which has poor performance.
  # Consider joining all columns at once using pd.concat(axis=1) instead. To
  # get a de-fragmented frame, use `newframe = frame.copy()`
  before = before_.copy()
  after  = after_.copy()

  # start_time = timeit.default_timer()
  sanity_df = sanity_check_df_rows_cols_labels(before, after, fast=fast, verbose=verbose)
  # print('\t sanity_check_df_rows_cols_labels', round(timeit.default_timer() - start_time, 2))


  # start_time = timeit.default_timer()
  print('before.index.equals(after.index):', before.index.equals(after.index))

  # check index freq is set and are equal
  print('before.index.freq == after.index.freq:', before.index.freq == after.index.freq)
  if verbose:
    print('before.index.freq:', before.index.freq)
    print('after.index.freq:',  after.index.freq)


  # check if common column dtypes have changed
  common_cols = before.columns.intersection(after.columns)
  print('before[common_cols].dtypes == after[common_cols].dtypes:',
        (before[common_cols].dtypes == after[common_cols].dtypes).all())
  if verbose:
    print('before[common_cols].dtypes:', before[common_cols].dtypes)
    print('after[common_cols].dtypes:',  after[common_cols].dtypes)

  # check if describe() summaries are equal
  print('before[common_cols].describe() == after[common_cols].describe():',
        (before[common_cols].describe() == after[common_cols].describe()).all().all())
  if verbose:
    print(before[common_cols].describe() == after[common_cols].describe())

  # check after subsetted by before equals before
  print('\nbefore[common_cols].equals(after[common_cols]):',
  before[common_cols].dropna().drop_duplicates().equals(after[common_cols].dropna().drop_duplicates())
  )
  if verbose:
    print('before.isin(after):',
    before[common_cols].dropna().drop_duplicates().isin(after[common_cols].dropna().drop_duplicates()).all().all()
    )
    print(before.dropna().drop_duplicates().isin(after.dropna().drop_duplicates()).all())
    print(before.dropna().drop_duplicates().isin(after.dropna().drop_duplicates()))


  # Reasons I HATE pandas number Inf a neverending series:
  # PerformanceWarning: DataFrame is highly fragmented.  This is usually the
  # result of calling `frame.insert` many times, which has poor performance.
  # Consider joining all columns at once using pd.concat(axis=1) instead. To
  # get a de-fragmented frame, use `newframe = frame.copy()`
  # calculate duplicate row counts then find mean duplicate count
```

```
  # for each column and finally find mean of means aka redundancy
  # warnings.resetwarnings()
  with warnings.catch_warnings():
    warnings.simplefilter('ignore')
    before_red = before.dropna().groupby(before.select_dtypes(include=np.number).columns.tolist(), as_index=False).size().mea
    after_red  = after.dropna().groupby(after.select_dtypes(include=np.number).columns.tolist(), as_index=False).size().mean
    print('redundancy before > after:', before_red > after_red)
    print('mean before feature redundancy:', round(before_red, 3))
    print('mean after feature redundancy: ', round(after_red,  3))

  # Check all data is numeric, finite (but allow NAs) and reasonably shaped
  # If any problems then this will error out
  # Only checking 'after' dataframe
  # https://scikit-learn.org/stable/modules/generated/sklearn.utils.check_X_y.html
  if Y_COL in after.columns:
    _, _ = check_X_y(after.drop(columns=[Y_COL, 'ds']),
                     after[Y_COL],
                     y_numeric = True,
                     force_all_finite = 'allow-nan')

  print()
  # print('\t end sanity_check_before_after_dfs', round(timeit.default_timer() - start_time, 2))

  display(sanity_df)

  return sanity_df


def compare_train_valid_test_sanity_dfs(train_sanity, valid_sanity, test_sanity, ex_labels=None):
  '''...'''

  if ex_labels is None:
    ex_labels = ['rows']

  train_sanity = train_sanity.loc[~train_sanity.index.isin(ex_labels)]
  valid_sanity = valid_sanity.loc[~valid_sanity.index.isin(ex_labels)]
  test_sanity  =  test_sanity.loc[~test_sanity.index.isin(ex_labels)]

  if not train_sanity.equals(valid_sanity):
    print('WARN: train_sanity != valid_sanity')
    display(pd.concat([train_sanity, valid_sanity]).drop_duplicates(keep=False))

  if not train_sanity.equals(test_sanity):
    print('WARN: train_sanity != test_sanity')
    display(pd.concat([train_sanity, test_sanity]).drop_duplicates(keep=False))

  if not test_sanity.equals(valid_sanity):
    print('WARN: test_sanity != valid_sanity')
    display(pd.concat([test_sanity, valid_sanity]).drop_duplicates(keep=False))

  return None


# TODO Remove some of the code duplication
def sanity_check_train_valid_test(train_df, valid_df, test_df,
                                  over_cols = ['y_des', 'dew.point_des', 'humidity', 'pressure'],
                                  dp = 2):

  # Check number of columns is equal
  if (train_df.shape[1] != valid_df.shape[1]) or \
     (train_df.shape[1] != test_df.shape[1])  or \
     (valid_df.shape[1] != test_df.shape[1]):
    print('ERROR: Inconsistent number of columns!')
    print('train_df.shape[1]:', train_df.shape[1])
    print('valid_df.shape[1]:', valid_df.shape[1])
    print('test_df.shape[1]:',  test_df.shape[1])


  # Check column names are equal
  if not (train_df.columns == valid_df.columns).all():
    print('ERROR: Inconsistent train_df, valid_df column names!')
    print('train_df.columns:', train_df.columns)
    print('valid_df.columns:', valid_df.columns)

  if not (train_df.columns == test_df.columns).all():
    print('ERROR: Inconsistent train_df, test_df column names!')
    print('train_df.columns:', train_df.columns)
    print('test_df.columns:',  test_df.columns)

  if not (valid_df.columns == test_df.columns).all():
    print('ERROR: Inconsistent valid_df, test_df column names!')
    print('valid_df.columns:', valid_df.columns)
```

```
  print('test_df.columns:',  test_df.columns)


# Check column dtypes are equal
if not (train_df.dtypes == valid_df.dtypes).all():
  print('ERROR: Inconsistent train_df, valid_df dtypes!')
  print('train_df.dtypes:', train_df.dtypes)
  print('valid_df.dtypes:', valid_df.dtypes)

if not (train_df.dtypes == test_df.dtypes).all():
  print('ERROR: Inconsistent train_df, test_df dtypes!')
  print('train_df.dtypes:', train_df.dtypes)
  print('test_df.dtypes:',  test_df.dtypes)

if not (valid_df.dtypes == test_df.dtypes).all():
  print('ERROR: Inconsistent valid_df, test_df dtypes!')
  print('valid_df.dtypes:', valid_df.dtypes)
  print('test_df.dtypes:',  test_df.dtypes)


# Check index freqs are equal
if train_df.index.freq != valid_df.index.freq:
  print('ERROR: Inconsistent train_df, valid_df index frequencies!')
  print('train_df.index.freq:', train_df.index.freq)
  print('valid_df.index.freq:', valid_df.index.freq)

if train_df.index.freq != test_df.index.freq:
  print('ERROR: Inconsistent train_df, test_df index frequencies!')
  print('train_df.index.freq:', train_df.index.freq)
  print('test_df.index.freq:',   test_df.index.freq)

if valid_df.index.freq != test_df.index.freq:
  print('ERROR: Inconsistent valid_df, test_df index frequencies!')
  print('valid_df.index.freq:', valid_df.index.freq)
  print('test_df.index.freq:',   test_df.index.freq)


# Verify dataframes are different!
if train_df.equals(valid_df):
  print('ERROR: train_df == valid_df!')

if train_df.equals(test_df):
  print('ERROR: train_df == test_df!')

if valid_df.equals(test_df):
  print('ERROR: valid_df == test_df!')


# Check no overlap between train_df.index and valid_df.index
# train_df.index strictly before valid_df.index and test_df.index
if max(train_df.index) >= min(valid_df.index):
  print('ERROR: Overlap between train_df, valid_df indices!')
  print('max(train_df.index):', max(train_df.index))
  print('min(valid_df.index):', max(valid_df.index))

# Check no overlap between train_df.index and test_df.index
# train_df.index strictly before valid_df.index and test_df.index
if max(train_df.index) >= min(test_df.index):
  print('ERROR: Overlap between train_df, test_df indices!')
  print('max(train_df.index):', max(train_df.index))
  print('min(test_df.index):',  max(test_df.index))


# Check no overlap between valid_df.index and test_df.index
# valid_df.index can be before or after test_df.index
if (max(valid_df.index) >= min(test_df.index)) and \
   (max(valid_df.index) <= max(test_df.index)):
  print('ERROR: Overlap between valid_df, test_df indices!')
  print('valid_df.index:', max(valid_df.index), '-', max(valid_df.index))
  print('test_df.index:',  max(test_df.index),  '-', max(test_df.index))

if (min(valid_df.index) >= min(test_df.index)) and \
   (min(valid_df.index) <= max(test_df.index)):
  print('ERROR: Overlap between valid_df, test_df indices!')
  print('valid_df.index:', max(valid_df.index), '-', max(valid_df.index))
  print('test_df.index:',  max(test_df.index),  '-', max(test_df.index))


# TODO: Consider enforcing a gap of 1 day to 1 week between
#       train_df.index and {valid_df,test_df}.index to avoid data leakage?
```

```
  # Check train_df has more observations than valid_df and test_df
  if valid_df.shape[0] > train_df.shape[0]:
    print('ERROR: valid_df more observations than train_df!')
    print('train_df observations:', train_df.shape[0])
    print('valid_df observations:', valid_df.shape[0])

  if test_df.shape[0] > train_df.shape[0]:
    print('ERROR: test_df more observations than train_df!')
    print('train_df observations:', train_df.shape[0])
    print('test_df observations:',  test_df.shape[0])


  # Check valid_df and test_df have equal number of observations
  # valid_df and test_df may be different sizes but
  # large size difference may indicate an issue
  # TODO: Use calendar.isleap() to check if leap year
  if valid_df.shape[0] != test_df.shape[0]:
    print('WARN: Inconsistent number of valid_df, test_df rows.  Leap year?')


  # Check valid_df and test_df are each 1 year long
  YEAR_OBS_MIN = 48 * 365
  YEAR_OBS_MAX = 48 * 366
  if (valid_df.shape[0] < YEAR_OBS_MIN) or \
     (valid_df.shape[0] > YEAR_OBS_MAX):
    print('ERROR: valid_df should be 1 year long [',
          YEAR_OBS_MIN, ',', YEAR_OBS_MAX, ']!')
    print('valid_df observations:', valid_df.shape[0])

  if (test_df.shape[0] < YEAR_OBS_MIN) or \
     (test_df.shape[0] > YEAR_OBS_MAX):
    print('ERROR: test_df should be 1 year long [',
          YEAR_OBS_MIN, ',', YEAR_OBS_MAX, ']!')
    print('test_df observations:', test_df.shape[0])

  # Check approx number of overlapping rows between train_df and valid_df
  dups_pc_lim = 15.0
  n_dups, dups_pc = get_approx_overlap(train_df, valid_df, over_cols, decs=dp)
  if dups_pc > dups_pc_lim:
    print('WARN: high overlap between train_df and valid_df rows!')
    print(f"Number of shared rows: {n_dups}")
    print(f'Approximate overlap: {dups_pc} %\n')
    # print(f'Decimal places: {dp}')
    # print('Overlap features:', over_cols)

  # Check approx number of overlapping rows between train_df and test_df
  n_dups, dups_pc = get_approx_overlap(train_df, test_df, over_cols, decs=dp)
  if dups_pc > dups_pc_lim:
    print('WARN: high overlap between train_df and test_df rows!')
    print(f"Number of shared rows: {n_dups}")
    print(f'Approximate overlap: {dups_pc} %\n')
    # print(f'Decimal places: {dp}')
    # print('Overlap features:', over_cols)

  return None


def print_train_valid_test_shapes(df, train_df, valid_df, test_df):
  print("df shape: ",             df.shape)
  print("train shape:   ", train_df.shape)
  print("valid shape:   ", valid_df.shape)
  print("test shape:    ",  test_df.shape)

  return None


def plot_feature_history_single_df(data, var, missing=False):
    plt.figure(figsize = (12, 6))
    plt.scatter(data.index, data[var],
                label='train', color='black', s=3)
    if missing:
      label = 'missing'
      x_lab = data.loc[data[label] == 1.0, 'ds']
      y_lab = data.loc[data[label] == 1.0, var]
      plt.scatter(x_lab, y_lab, color='red', label=label, s=3)

    plt.title(var)
    plt.show()


def plot_feature_history(train, valid, test, var, missing=False):
    label = 'missing'
```

```
    plt.figure(figsize = (12, 6))
    plt.scatter(train.index, train[var],
                label='train', color='black', s=3)
    if missing:
      x_lab = train.loc[train[label] == 1.0, 'ds']
      y_lab = train.loc[train[label] == 1.0, var]
      plt.scatter(x_lab, y_lab, color='red', label=label, s=3)

    plt.scatter(valid.index, valid[var],
                label='valid', color='blue', s=3)
    if missing:
      x_lab = valid.loc[valid[label] == 1.0, 'ds']
      y_lab = valid.loc[valid[label] == 1.0, var]
      plt.scatter(x_lab, y_lab, color='red', label=label, s=3)

    plt.scatter(test.index,  test[var],
                label='test', color='purple', s=3)
    if missing:
      x_lab = test.loc[test[label] == 1.0, 'ds']
      y_lab = test.loc[test[label] == 1.0, var]
      plt.scatter(x_lab, y_lab, color='red', label=label, s=3)

    plt.title(var)
    plt.show()


def plot_feature_history_separately(train, valid, test, var):
    fig, axs = plt.subplots(1, 3, figsize = (14, 7))

    axs[0].plot(train.index, train[var])
    axs[0].set_title('train')

    axs[1].plot(valid.index, valid[var])
    axs[1].set_title('valid')
    axs[1].set_xticks(axs[1].get_xticks(), axs[1].get_xticklabels(), rotation=45, ha='right')

    axs[2].plot(test.index,  test[var])
    axs[2].set_title('test')
    axs[2].set_xticks(axs[2].get_xticks(), axs[2].get_xticklabels(), rotation=45, ha='right')

    fig.suptitle(var)
    plt.show()


def check_high_low_thresholds(df, ds=None):
  '''Check main features from dataframe are within reasonable thresholds'''

  all_ok = True
  feats = ['y', 'dew.point', 'humidity', 'pressure',
           'wind.speed.mean', 'wind.speed.max']
  highs = [ 45,  25, 100, 1060, 35, 70]
  lows  = [-20, -20,   5,  950,  0,  0]

  thresh = pd.DataFrame({'feat': feats,
                         'high': highs,
                         'low':  lows,})
  thresh.index = feats

  for feat in feats:
    feat_high = thresh.loc[feat, 'high']
    feat_low  = thresh.loc[feat, 'low']

    if not df[feat].between(feat_low, feat_high).all():
      all_ok = False
      print('%15s [%3d, %3d] - % 7.3f, % 7.3f' %
            (feat, feat_low, feat_high,
            round(min(df[feat]), 3), round(max(df[feat]), 3)))

  # check if dew.point ever greater than temperature
  if df.loc[df['dew.point'] > df['y'], ['y', 'dew.point']].shape[0] != 0:
    all_ok = False
    print('dew.point > y:')
    display(df.loc[df['dew.point'] > df['y'], ['y', 'dew.point']])

  if all_ok is False:
    print(' ... from', ds)

  return None


def print_df_summary(df):
```

```
  print("Shape:")
  display(df.shape)

  total_nas = df.isna().sum().sum()
  rows_nas  = df.isnull().any(axis=1).sum()
  cols_nas  = df.isnull().any().sum()
  print('\nTotal NAs:', total_nas)
  print('Rows with NAs:', rows_nas)
  print('Cols with NAs:', cols_nas)

  print("\nInfo:")
  display(df.info())

  print("\nSummary stats:")
  display(df.describe())

  print("\nRaw data:")
  display(df)
  print("\n")


def get_approx_overlap(X1, X2, over_cols, decs=2, verbose=False):
  '''Calculate approximate overlap between 2 dataframes of different sizes.

  If exact values are used then overlap is probably too low,
  so use np.round() to reduce precision.
  Use MinMaxScaler so single decimals parameter is applicable to all columns.
  Assumes X1 is train and X2 is valid/test.
  Duplicates dropped from X1 & X2 before calculating overlap.
  Percent overlap can be greater than 100 if decs is too low.

  Based on https://stackoverflow.com/a/71002234/100129
  '''

  assert X1.shape[0] >= X2.shape[0]

  X1 = X1[over_cols].drop_duplicates()
  X2 = X2[over_cols].drop_duplicates()

  Xcomb = pd.concat((X1, X2), axis=0, ignore_index=True)

  # scale
  scaler = MinMaxScaler()
  Xscl = scaler.fit_transform(Xcomb)

  # round
  # df_scl = pd.DataFrame(np.round(Xcomb, decimals=decs), columns=over_cols)
  df_scl = pd.DataFrame(np.round(Xscl, decimals=decs), columns=over_cols)

  # count overlaps
  n_uniq = df_scl.drop_duplicates().shape[0]
  n_dup = X1.shape[0] + X2.shape[0] - n_uniq
  dup_pc = round(n_dup * 100 / X2.shape[0], 2)

  if verbose:
    print(f"Number of shared rows: {n_dup}")
    print(f'Approximate overlap: {dup_pc} %\n')

  if dup_pc > 100.0:
    print('Approx. overlap over 100 %!')
    print('Increase decs argument')
    print(f"decs = {decs}\n")

  return n_dup, dup_pc


def expand_grid(dictionary):
  return pd.DataFrame([row for row in product(*dictionary.values())],
                      columns = dictionary.keys())


def keep_key(d, k):
  """ models = keep_key(models, 'datasets') """
  return {k: d[k]}


def get_historic_comparison(backtest, df, y_col = Y_COL, horizon = HORIZON):
    if horizon > 1:
      assert len(backtest[0]) > 1

    if y_col == 'y_des':
      # cols = ['y_des', 'y_seasonal']
```

```
    cols = ['y_des', 'y_yearly', 'y_daily', 'y_trend']
  elif y_col == 'y_des_fft':
    cols = ['y_des_fft', 'y_fft']
  elif y_col == 'y_res':
    cols = ['y_res', 'y_yearly', 'y_daily']
  elif y_col == 'y':
    cols = ['y']

  # cols.extend(['missing', 'mi_filled', 'isd_outlier', 'hist_average'])
  cols.extend(['missing', 'isd_outlier'])

  preds_df = pd.concat([backtest[i].pd_dataframe() for i in range(len(backtest))], axis=0)
  trues_df = df.loc[preds_df.index, cols]

  hist_comp = pd.concat([trues_df, preds_df[y_col]], axis = 1)
  cols.append('pred')
  hist_comp.columns = cols

  # re-seasonalise
  if y_col == 'y_des':
    hist_comp['y_des'] += hist_comp['y_yearly'] + hist_comp['y_daily'] + hist_comp['y_trend']
    hist_comp['pred']  += hist_comp['y_yearly'] + hist_comp['y_daily'] + hist_comp['y_trend']
  elif y_col == 'y_des_fft':
    hist_comp['y_des_fft'] += hist_comp['y_fft']
    hist_comp['pred']      += hist_comp['y_fft']
  elif y_col == 'y_res':
    hist_comp['y_res'] += hist_comp['y_yearly'] + hist_comp['y_daily']
    hist_comp['pred']  += hist_comp['y_yearly'] + hist_comp['y_daily']


  hist_comp['res']    = hist_comp[y_col] - hist_comp['pred']
  hist_comp['res^2']  = hist_comp['res'] * hist_comp['res']
  hist_comp['res_sign']  = np.sign(hist_comp['res'])
  hist_comp['missing']   = hist_comp['missing']#.astype(int)
  # hist_comp['mi_filled'] = hist_comp['mi_filled']#.astype(int)
  # hist_comp['hist_average'] = hist_comp['hist_average']#.astype(int)

  list_int = [i for i in range(1, horizon+1)]
  reps = len(hist_comp) // len(list_int)
  hist_comp['step'] = np.tile(list_int, reps)
  hist_comp['id']   = np.repeat([i for i in range(reps)], horizon)
  hist_comp['date'] = hist_comp.index.values

  return hist_comp


def summarise_historic_comparison(hc, df, horizon = HORIZON,
                              digits = 6,
                              y_col = Y_COL,
                              df_name = 'valid_df'):

  print('\n')
  print_rmse_mae(hc[y_col], hc['pred'], 'all')

  obs   = hc.loc[hc['step'] == horizon, y_col]
  preds = hc.loc[hc['step'] == horizon, 'pred']
  if horizon == 1:
    post_str = '1st'
  elif horizon == 2:
    post_str = '2nd'
  elif horizon == 3:
    post_str = '3rd'
  else:
    post_str = str(horizon) + 'th'
  print_rmse_mae(obs, preds, post_str, '# ')

  obs   = hc.loc[hc['missing'] == 0.0, y_col]
  preds = hc.loc[hc['missing'] == 0.0, 'pred']
  print_rmse_mae(obs, preds, 'miss==0')

  obs   = hc.loc[hc['missing'] == 1.0, y_col]
  preds = hc.loc[hc['missing'] == 1.0, 'pred']
  print_rmse_mae(obs, preds, 'miss==1')

  if y_col == 'y_des':
    # preds = hc['pred'] - hc['y_seasonal']
    preds = hc['pred'] - hc['y_yearly'] - hc['y_daily'] - hc['y_trend']
  elif y_col == 'y_des_fft':
    preds = hc['pred'] - hc['y_fft']
  elif y_col == 'y':
    preds = hc['pred']
  elif y_col == 'y_res':
```

```
      preds = hc['pred'] - hc['y_yearly'] - hc['y_daily']

    preds.dropna(inplace=True)
    lasttest_stats = stats.describe(preds)
    print("\nbacktest['", y_col, "']:", sep='')
    print("count\t", len(preds))
    print("mean\t",  round(lasttest_stats[2], digits))
    print("std\t",   round(np.sqrt(lasttest_stats[3]), digits))
    print("min\t",   round(np.min(lasttest_stats[1]), digits))
    print("25%\t",   round(np.percentile(preds, 25), digits))
    print("50%\t",   round(np.median(preds), digits))
    print("75%\t",   round(np.percentile(preds, 75), digits))
    print("max\t",   round(np.max(lasttest_stats[1]), digits))

    print("\n", df_name, "['", y_col, "']:\n", df[y_col].describe(), '\n', sep='')


def print_rmse_mae(obs, preds, postfix_str, prefix_str = '', digits = 6):
    print(prefix_str, "Backtest RMSE ", postfix_str, ": ",
          round(rmse_(obs, preds), digits),
          sep='')
    print(prefix_str, "Backtest MAE ",  postfix_str, ":  ",
          round( mae_(obs, preds), digits),
          sep='')
    print()


def _plot_xy_for_label(data, label, x_feat, y_feat, color):
    x = data.loc[data[label] == 1.0, x_feat]
    y = data.loc[data[label] == 1.0, y_feat]

    if len(x) > 0:
        plt.scatter(x = x, y = y, color=color, alpha=0.5, label=label)


def plot_multistep_obs_vs_preds(hist, title, y_col=Y_COL):
    plt.figure(figsize = (12, 16))
    plt.subplot(5, 1, 1)
    plt.scatter(x = hist[y_col], y = hist['pred'])
    _plot_xy_for_label(hist, 'missing', y_col, 'pred', 'red')
    plt.axline((0, 0), slope=1.0, color="grey")
    plt.xlabel('Observations')
    plt.ylabel('Predictions')
    plt.legend(loc='lower right')
    obs   = hist.loc[hist[[y_col, 'pred']].notnull().all(1), y_col]
    preds = hist.loc[hist[[y_col, 'pred']].notnull().all(1), 'pred']
    r2score = r2_score(obs, preds)
    plt.annotate("$R^2$ = {:.3f}".format(r2score), (-9, 31))
    plt.title(title)
    plt.xlim((-10, 35))
    plt.ylim((-10, 35))
    plt.show()


def plot_multistep_obs_vs_mean_preds_by_step(hist, title, y_col = Y_COL,
                                             step_ = HORIZON, ci = False):
    '''For specific step, plot mean prediction for each observation

    A 95 % confidence interval is plotted, but can be disabled
    '''

    mean_preds = hist.loc[hist['step'] == step_, [y_col, 'pred']].groupby(y_col).mean('pred')
    obs   = mean_preds.index.values
    preds = mean_preds['pred'].values

    plt.figure(figsize = (12, 16))
    ax = plt.subplot(5, 1, 2)
    plt.plot(obs, preds)

    if ci is True:
      ci = 1.96 * np.std(preds) / np.sqrt(len(obs))
      ax.fill_between(obs, (preds - ci), (preds + ci), color='b', alpha=.1)

    plt.axline((0, 0), slope=1.0, color="grey")
    r2score = r2_score(obs, preds)
    plt.annotate("$R^2$ = {:.3f} - step = {}".format(r2score, step_), (-9, 31))
    plt.title(title + ' step = ' + str(step_))
    plt.xlabel('Temperature')
    plt.ylabel('Mean prediction')
    plt.xlim((-10, 35))
    plt.ylim((-10, 35))
    plt.show()
```

```
def plot_multistep_obs_preds_dists(hist, title, y_col=Y_COL):
    obs   = hist.loc[hist[[y_col, 'pred']].notnull().all(1), y_col]
    preds = hist.loc[hist[[y_col, 'pred']].notnull().all(1), 'pred']
    r2score = r2_score(obs, preds)
    plt.figure(figsize = (12, 16))
    plt.subplot(5, 1, 3)
    pd.Series(obs).plot(kind = 'density', label='observations')
    pd.Series(preds).plot(kind = 'density', label='predictions')
    plt.xlim(-10, 40)
    plt.title(title)
    plt.legend()
    plt.annotate("$R^2$ = {:.3f}".format(r2score), (-7.5, 0.055))
    # plt.tight_layout()
    plt.show()


def plot_multistep_residuals(hist, title):
    plt.figure(figsize = (12, 16))
    plt.subplot(5, 1, 4)
    plt.scatter(x = range(len(hist)), y = hist['res'])
    hist['id.2'] = range(len(hist))
    _plot_xy_for_label(hist, 'missing', 'id.2', 'res', 'red')
    plt.axhline(y = 0, color = 'grey')
    plt.xlabel('Index position')
    plt.ylabel('Residuals')
    plt.legend(loc='lower right')
    plt.title(title)
    plt.show()


def plot_multistep_residuals_dist(hist, title):
    plt.figure(figsize = (12, 16))
    plt.subplot(5, 1, 5)
    pd.Series(hist['res']).plot(kind = 'density', label='residuals')
    plt.xlim(-10, 10)
    plt.title(title)
    plt.show()


# Unused?
# TODO Diagonal structure of these plots might need further consideration
#      Add lowess fit to check for problems
def plot_multistep_residuals_vs_predicted(hist, title):
    plt.subplot(5, 1, 5)
    plt.scatter(x = hist['pred'], y = hist['res'])
    _plot_xy_for_label(hist, 'missing', 'pred', 'res', 'red')
    plt.axhline(y = 0, color = 'grey')

    n = 24  # slow to run all points :-(
            # 12 takes approx 2 mins to run
            #  8 takes approx 4 mins to run
    xy = hist.iloc[::n, :]
    # x = hist.iloc[::n, :]
    y_l = lowess(xy['res'], xy['pred'])
    plt.plot(y_l[:, 0], y_l[:, 1], 'green', label='lowess fit')

    plt.xlabel('Predictions')
    plt.ylabel('Residuals')
    plt.legend(loc='upper right')
    plt.title(title);


def se_(obs, preds, metric):
    '''Standard error of sum of squared residuals or sum of absolute residuals'''

    if _check_obs_preds_lens_eq(obs, preds) == 0:
        stop()

    if metric == 'rmse':
        se = np.sqrt(np.sum((obs - preds) ** 2) / len(obs))
    elif metric == 'mae':
        se = np.sqrt(np.sum(np.abs(obs - preds)) / len(obs))
    else:
        print('Unrecognised metric:', metric)
        print("metric should be 'rmse' or 'mae'")
        stop()

    return se
```

```
def metric_ci_vals(test_val, se, z_val = 1.95996):
    cil = z_val * se
    metric_cil = test_val - cil
    metric_ciu = test_val + cil

    return metric_cil, metric_ciu


# TODO: Remove unused confidence intervals
# NOTE: VAR baseline metrics cvar_rmse and cvar_mae hardcoded to 48 steps
def plot_horizon_metrics(hist, title, y_col=Y_COL, horizon = HORIZON, ci=False):
    steps = [i for i in range(1, horizon+1)]

    # calculate metrics
    z_val_95 = 1.95996
    z_val_50 = 0.674
    rmse_h,   mae_h    = np.zeros(horizon), np.zeros(horizon)
    res_se_h, abs_se_h = np.zeros(horizon), np.zeros(horizon)
    rmse_ciu, rmse_cil = np.zeros(horizon), np.zeros(horizon)
    mae_ciu,  mae_cil  = np.zeros(horizon), np.zeros(horizon)

    for i in range(1, horizon+1):
      obs   = hist.loc[hist['step'] == i, y_col]
      preds = hist.loc[hist['step'] == i, 'pred']
      rmse_h[i-1] = rmse_(obs, preds)
      mae_h[i-1]  =  mae_(obs, preds)
      res_se_h[i-1] = se_(obs, preds, 'rmse')
      abs_se_h[i-1] = se_(obs, preds, 'mae')
      rmse_cil[i-1], rmse_ciu[i-1] = metric_ci_vals(rmse_h[i-1], res_se_h[i-1], z_val_50)
      mae_cil[i-1],  mae_ciu[i-1]  = metric_ci_vals(mae_h[i-1],  abs_se_h[i-1], z_val_50)

    # plot metrics for horizons
    fig, axs = plt.subplots(1, 2, figsize = (14, 7))
    fig.suptitle(title + ' forecast horizon errors')
    axs = axs.ravel()


    mean_val_lab = title + ' mean value'
    axs[0].plot(steps, rmse_h, color='green', label=title)

    if ci is True:
      axs[0].fill_between(steps, rmse_cil, rmse_ciu, color='green', alpha=0.25)

    # i - initial, u - updated, c - corrected
    #ivar_rmse = np.array([0.39, 0.52, 0.64, 0.75, 0.86, 0.96, 1.06, 1.15, 1.23,
    #                     1.31, 1.38, 1.45, 1.51, 1.57, 1.63, 1.68, 1.73, 1.77,
    #                     1.81, 1.85, 1.89, 1.92, 1.96, 1.99, 2.02, 2.05, 2.08,
    #                     2.1 , 2.13, 2.15, 2.18, 2.2 , 2.22, 2.24, 2.26, 2.28,
    #                     2.3 , 2.31, 2.33, 2.35, 2.36, 2.38, 2.39, 2.4 , 2.42,
    #                     2.43, 2.44, 2.45])
    # NOTE: uvar_rmse tested on test_df
    #uvar_rmse = np.array([0.36, 0.49, 0.6, 0.7, 0.8, 0.89, 0.98, 1.06, 1.14,
    #                      1.21, 1.28, 1.35, 1.41, 1.47, 1.52, 1.57, 1.62, 1.66,
    #                      1.7, 1.74, 1.78, 1.81, 1.84, 1.87, 1.9, 1.93, 1.96,
    #                      1.99, 2.01, 2.03, 2.06, 2.08, 2.1, 2.12, 2.14, 2.16,
    #                      2.18,  2.19, 2.21, 2.23, 2.24, 2.26, 2.27, 2.29, 2.3,
    #                      2.31, 2.33, 2.34])
    cvar_rmse = np.array([0.49318888, 0.70222546, 0.88570688, 1.05495349,
    1.21081157, 1.34945832, 1.46844034, 1.57779714, 1.67754323, 1.7665827,
    1.84567039, 1.91561743, 1.97899766, 2.03616174, 2.08661944, 2.13396441,
    2.17809725, 2.21946156, 2.25780078, 2.29370568, 2.3272055,  2.35760153,
    2.38520845, 2.41076185, 2.43404716, 2.45466806, 2.47361784, 2.49117761,
    2.50625606, 2.52023589, 2.53319205, 2.54566125, 2.55764924, 2.56870554,
    2.57976955, 2.59102429, 2.6018822, 2.61242356, 2.62280045, 2.63353767,
    2.64410312, 2.65458709, 2.66532837, 2.67609086, 2.68675178, 2.69745108,
    2.71002892, 2.72445726])
    # axs[0].plot(steps, ivar_rmse, color='black', label='Initial VAR')
    axs[0].plot(steps, cvar_rmse, color='blue', label='Updated VAR')
    axs[0].hlines(np.mean(rmse_h), xmin=1, xmax=horizon,
                  color='green', linestyles='dotted', label=mean_val_lab)
    axs[0].hlines(np.mean(cvar_rmse), xmin=1, xmax=horizon,
                  color='blue', linestyles='dotted', label='Updated VAR mean value')
    axs[0].set_xlabel("horizon - half hour steps")
    axs[0].set_ylabel("rmse")


    axs[1].plot(steps, mae_h, color='green', label=title)

    if ci is True:
      axs[1].fill_between(steps, mae_cil, mae_ciu, color='green', alpha=0.25)

    # NOTE: ivar_mae tested on test_df
```

```
    #ivar_mae = np.array([0.39, 0.49, 0.57, 0.66, 0.74, 0.83, 0.91, 0.98, 1.05,
    #                    1.12, 1.18, 1.24, 1.29, 1.34, 1.39, 1.43, 1.47, 1.5 ,
    #                    1.53, 1.56, 1.59, 1.62, 1.64, 1.66, 1.68, 1.7 , 1.72,
    #                    1.73, 1.75, 1.76, 1.77, 1.78, 1.8 , 1.81, 1.82, 1.83,
    #                    1.83, 1.84, 1.85, 1.85, 1.86, 1.86, 1.87, 1.87, 1.88,
    #                    1.88, 1.89, 1.89])
    #uvar_mae = np.array([0.36, 0.45, 0.53, 0.61, 0.69, 0.76, 0.83, 0.9, 0.97,
    #                     1.03, 1.09, 1.14, 1.19, 1.24, 1.28, 1.32, 1.36, 1.4,
    #                     1.43, 1.46, 1.49, 1.52, 1.54, 1.56, 1.58, 1.6, 1.62,
    #                     1.63, 1.65, 1.66, 1.68, 1.69, 1.7, 1.71, 1.72, 1.73,
    #                     1.74, 1.74, 1.75, 1.75, 1.76, 1.76, 1.77, 1.77, 1.78,
    #                     1.78, 1.78, 1.78])
    cvar_mae = np.array([0.34694645, 0.50765333, 0.65132003, 0.78584432,
    0.9077075,  1.01705088, 1.11113622, 1.19759807, 1.27696634, 1.34941444,
    1.4134705,  1.47180058, 1.52304802, 1.56961154, 1.60903759, 1.64763418,
    1.68391297, 1.71690735, 1.74787094, 1.77721642, 1.80442554, 1.82951782,
    1.85358226, 1.87488643, 1.89346337, 1.91069565, 1.92613218, 1.94071845,
    1.95245349, 1.96323923, 1.9736734,  1.98370815, 1.99367508, 2.00204077,
    2.00992601, 2.01796976, 2.02747736, 2.03477489, 2.04173317, 2.04985428,
    2.05843847, 2.06731348, 2.07606609, 2.08533656, 2.09560914, 2.10668272,
    2.1183637,  2.13164371])
    # axs[1].plot(steps, ivar_mae, color='black', label='Initial VAR')
    axs[1].plot(steps, cvar_mae, color='blue', label='Updated VAR')
    axs[1].hlines(np.mean(mae_h), xmin=1, xmax=horizon,
                  color='green', linestyles='dotted', label=mean_val_lab)
    axs[1].hlines(np.mean(cvar_mae), xmin=1, xmax=horizon,
                  color='blue', linestyles='dotted', label='Updated VAR mean value')
    axs[1].set_xlabel("horizon - half hour steps")
    axs[1].set_ylabel("mae")

    plt.legend(bbox_to_anchor=(1.04, 0.5), loc="center left", borderaxespad=0)
    plt.show()


def plot_horizon_metrics_boxplots(hist, title):

  hist['abs_err'] = np.abs(hist['res'])
  hist[['abs_err', 'step']].boxplot(by='step',
                                    meanline=False,
                                    showmeans=True,
                                    showcaps=True,
                                    showbox=True,
                                    showfliers=False,
                                    )
  plt.title(title + '\nboxplots with mean and median')
  plt.suptitle('')
  plt.xlabel("horizon - half hour steps")
  plt.ylabel("absolute error")
  x_step = 10.0
  x_max  = np.ceil(np.max(hist.step) / x_step) * int(x_step)
  plt.xticks(np.arange(0, x_max, int(x_step)))
  plt.show()


def plot_multistep_diagnostics(hist, title, y_col=Y_COL):
  title = 'Multi-step ' + title
  plot_multistep_obs_vs_preds(hist, title, y_col)
  plot_multistep_obs_vs_mean_preds_by_step(hist, title, y_col)
  plot_multistep_obs_preds_dists(hist, title, y_col)
  plot_multistep_residuals(hist, title + ' residuals')
  plot_multistep_residuals_dist(hist, title + ' residuals density')
  plot_horizon_metrics(hist, title, y_col)
  plot_horizon_metrics_boxplots(hist, title)
  # plot_multistep_forecast_examples(hist, title + ' forecast examples')


def plot_baseline_metrics(metrics, main_title):
  fig, axs = plt.subplots(1, 2, figsize = (14, 7))
  fig.suptitle(main_title)
  axs = axs.ravel()  # APL ftw!

  methods = metrics.method.unique()

  for method in methods:
    met_df = metrics.query('metric == "rmse" & method == "%s"' % method)
    axs[0].plot(met_df.horizon, met_df.value, color='blue', label='Updated VAR')

  ivar_rmse = np.array([0.39, 0.52, 0.64, 0.75, 0.86, 0.96, 1.06, 1.15, 1.23,
                        1.31, 1.38, 1.45, 1.51, 1.57, 1.63, 1.68, 1.73, 1.77,
                        1.81, 1.85, 1.89, 1.92, 1.96, 1.99, 2.02, 2.05, 2.08,
                        2.1 , 2.13, 2.15, 2.18, 2.2 , 2.22, 2.24, 2.26, 2.28,
                        2.3 , 2.31, 2.33, 2.35, 2.36, 2.38, 2.39, 2.4 , 2.42,
```

```
                          2.43, 2.44, 2.45])
  steps = [i for i in range(1, len(ivar_rmse)+1)]
  axs[0].plot(steps, ivar_rmse, color='black', label='Initial VAR')

  axs[0].set_xlabel("horizon - half hour steps")
  axs[0].set_ylabel("rmse")


  for method in methods:
    met_df = metrics.query('metric == "mae" & method == "%s"' % method)
    axs[1].plot(met_df.horizon, met_df.value, color='blue', label='Updated VAR')

  ivar_mae = np.array([0.39, 0.49, 0.57, 0.66, 0.74, 0.83, 0.91, 0.98, 1.05,
                       1.12, 1.18, 1.24, 1.29, 1.34, 1.39, 1.43, 1.47, 1.5 ,
                       1.53, 1.56, 1.59, 1.62, 1.64, 1.66, 1.68, 1.7 , 1.72,
                       1.73, 1.75, 1.76, 1.77, 1.78, 1.8 , 1.81, 1.82, 1.83,
                       1.83, 1.84, 1.85, 1.85, 1.86, 1.86, 1.87, 1.87, 1.88,
                       1.88, 1.89, 1.89])
  axs[1].plot(steps, ivar_mae, color='black', label='Initial VAR')

  axs[1].set_xlabel("horizon - half hour steps")
  axs[1].set_ylabel("mae")

  plt.legend(bbox_to_anchor=(1.04, 0.5), loc="center left", borderaxespad=0)
  plt.show()


def update_metrics(metrics, test_data, method, get_metrics,
                   model = None,
                   met_cols = ['type', 'method', 'metric', 'horizon', 'value']):
  metrics_h = []

  if method in ['SES', 'HWES']:
    horizons = [i for i in range(4, 49, 4)]
    horizons.insert(0, 1)
  else:
    horizons = range(1, 49)

  if method in ['VAR']:
    variates = 'multivariate'
  else:
    variates = 'univariate'

  print("h\trmse\tmae")
  for h in horizons:
    if method in ['VAR']:
      rmse_h, mae_h = get_metrics(test_data, h, method, model)
    else:
      rmse_h, mae_h = get_metrics(test_data, h, method)

    metrics_h.append(dict(zip(met_cols, [variates, method, 'rmse', h, rmse_h])))
    metrics_h.append(dict(zip(met_cols, [variates, method,  'mae', h,  mae_h])))

  print("\n")

  metrics_method = pd.DataFrame(metrics_h, columns = met_cols)
  metrics = metrics.append(metrics_method)

  return metrics


# rolling_cv with pre-trained model
def var_rolling_cv(data, horizon, method, model):
    lags = model.k_ar  # lag order
    i = lags
    h = horizon
    rmse_roll, mae_roll = [], []
    endo_vars = ['y', 'dew.point', 'humidity', 'pressure']
    exog_vars = ['day.cos.1', 'day.sin.1', 'year.cos.1', 'year.sin.1',
                 'irradiance', 'azimuth_cos', 'za_rad'
                ]

    while (i + h) < len(data):
        obs_df  = data[endo_vars].iloc[i:(i + h)]
        endo_df = data[endo_vars].iloc[(i - lags):i].values
        exog_df = data[exog_vars].iloc[i:(i + h)]

        y_hat = model.forecast(endo_df, exog_future = exog_df, steps = h)
        preds = pd.DataFrame(y_hat, columns = endo_vars)

        rmse_i = rmse(obs_df.y,      preds.y)
        mae_i  = medianabs(obs_df.y, preds.y)
```

```
        rmse_roll.append(rmse_i)
        mae_roll.append(mae_i)

        i = i + 1

    print(h, '\t', np.nanmean(rmse_roll).round(3), '\t', np.nanmean(mae_roll).round(3))

    return [np.nanmean(rmse_roll).round(2), np.nanmean(mae_roll).round(2)]


def get_var_backtest(model, data, endo_vars, exog_vars, y_col=Y_COL, horizon=HORIZON):
  lags = model.k_ar  # lag order
  i = lags
  h = horizon
  preds = []

  while (i + h) < len(data):
    if i % 1000 == 0:
      print(i)

    obs_df  = data[endo_vars].iloc[i:(i + h)]
    endo_vals = data[endo_vars].iloc[(i - lags):i].values

    if exog_vars is not None:
      exog_df = data[exog_vars].iloc[i:(i + h)]
      y_hat_lol = model.forecast(endo_vals, exog_future = exog_df, steps = h)
    else:
      y_hat_lol = model.forecast(endo_vals, steps = h)

    y_col_pos = 0  # hardcoding is bad mkay - make function param?
    y_hat_series = pd.Series(data  = [y_hat_l[y_col_pos] for y_hat_l in y_hat_lol],
                             index = obs_df.index,
                             name  = y_col)
    y_hat_ts = TimeSeries.from_series(y_hat_series)
    # y_hat_ts = TimeSeries.from_values(np.array([y_hat_l[y_col_pos] for y_hat_l in y_hat_lol]))
    # y_hat = [y_hat_l[y_col_pos] for y_hat_l in y_hat_lol]

    preds.append(y_hat_ts)
    i = i + 1

  return preds
```

Data Setup

Import Pre-calculated Features

See feature_engineering.ipynb notebook for further details.

Load default features:

```
train_df, valid_df, test_df = load_train_valid_test_features('default', location='github')

print('train_df:')
print_df_summary(train_df)
plot_cols = ['y', 'humidity', 'dew.point', 'wind.speed.mean.x',
             'wind.speed.mean.y']  # 'pressure',
plot_observation_examples(train_df, plot_cols)
```

```
WARN: high overlap between train_df and valid_df rows!
Number of shared rows: 5231
Approximate overlap: 29.86 %

WARN: high overlap between train_df and test_df rows!
Number of shared rows: 5707
Approximate overlap: 32.57 %

train_df:
Shape:
(87168, 109)

Total NAs: 0
Rows with NAs: 0
Cols with NAs: 0

Info:
<class 'pandas.core.frame.DataFrame'>
DatetimeIndex: 87168 entries, 2016-01-12 00:00:00 to 2020-12-31 23:30:00
Freq: 30T
Columns: 109 entries, ds to y_des_shadow
dtypes: datetime64[ns](1), float64(89), int64(19)
memory usage: 73.2 MB
None

Summary stats:
```

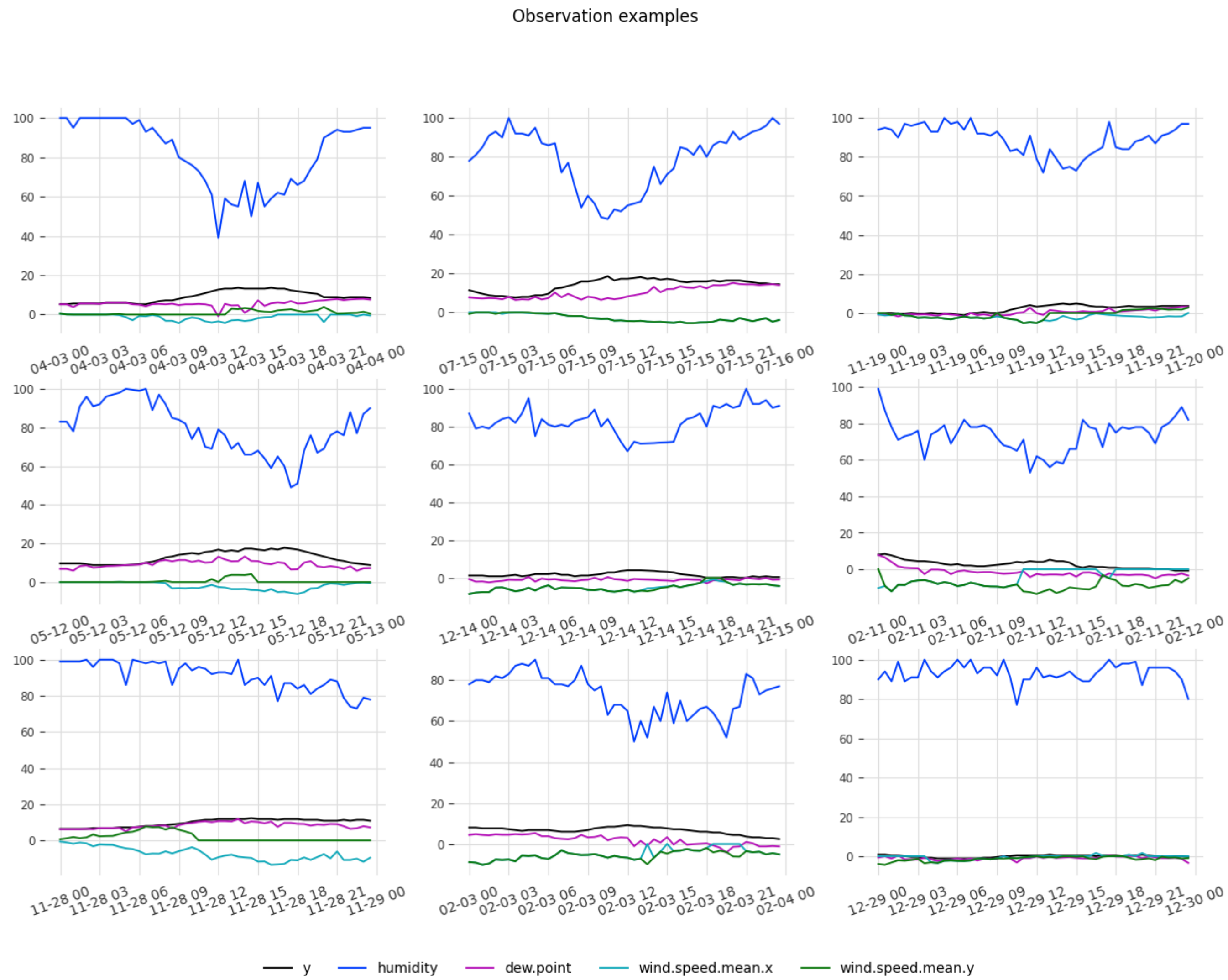
	y	humidity	dew.point	pressure	pressure.log	y_window_48_min_max_diff	wind.speed.mean.sqrt	win
count	87168.000000	87168.000000	87168.000000	87168.000000	87168.000000	87168.000000	87168.000000	
mean	9.652911	77.762962	5.355061	1014.848902	6.922424	8.432414	1.709399	
std	6.503256	18.134399	5.108536	12.102744	0.011968	3.659642	1.071297	
min	-6.800000	7.000000	-10.000000	966.000000	6.873164	0.800000	0.000000	
25%	4.800000	67.000000	1.500000	1008.000000	6.915723	5.700000	0.836660	
50%	8.800000	82.000000	5.300000	1016.000000	6.923629	8.000000	1.732051	
75%	14.100000	92.000000	9.100000	1023.000000	6.930495	10.800000	2.509980	
max	36.100000	100.000000	20.900000	1051.000000	6.957497	21.100000	5.403702	

8 rows × 108 columns

```
Raw data:
```

ds	ds	y	humidity	dew.point	pressure	pressure.log	y_window_48_min_max_diff	wind.speed.mean.sqrt	wind.spe
2016-01-12 00:00:00	2016-01-12 00:00:00	1.6	96.0	1.0	986.0	6.893656	4.422576	2.000000	
2016-01-12 00:30:00	2016-01-12 00:30:00	2.0	94.0	1.1	986.0	6.893656	4.422576	2.428992	
2016-01-12 01:00:00	2016-01-12 01:00:00	2.8	87.0	0.9	987.0	6.894670	4.422576	2.549510	
2016-01-12 01:30:00	2016-01-12 01:30:00	2.0	93.0	1.0	987.0	6.894670	4.422576	2.408319	
2016-01-12 02:00:00	2016-01-12 02:00:00	2.4	89.0	0.8	987.0	6.894670	4.422576	2.626785	
...	...	...	...	...	...	...	...	...	
2020-12-31 21:30:00	2020-12-31 21:30:00	-2.8	96.0	-3.3	1006.0	6.913737	3.200000	0.000000	
2020-12-31 22:00:00	2020-12-31 22:00:00	-3.2	100.0	-3.2	1007.0	6.914731	3.200000	0.000000	
2020-12-31 22:30:00	2020-12-31 22:30:00	-3.6	100.0	-3.6	1007.0	6.914731	3.200000	0.000000	
2020-12-31 23:00:00	2020-12-31 23:00:00	-4.4	97.0	-4.8	1007.0	6.914731	3.200000	0.000000	
2020-12-31 23:30:00	2020-12-31 23:30:00	-4.8	99.0	-4.9	1007.0	6.914731	3.200000	0.632456	

87168 rows × 109 columns



Unfortunately, `pressure` is not included in the variable plots. It ranges in value from approx 950 to 1,050 so does not make sense to plot it along with the other features.

VAR Models

Variables modelled:

- `y` (temperature)
- `dew.point`
- `humidity`
- `pressure`

Exogenous variables included:

- `day.cos.1`
- `day.sin.1`
- `year.cos.1`
- `year.sin.1`
- `irradiance`

- azimuth_cos
- za_rad

Lag selection

Lagged variables up to 96 steps (2 days) were considered.

AIC, BIC and HQIC are used for lag selection.

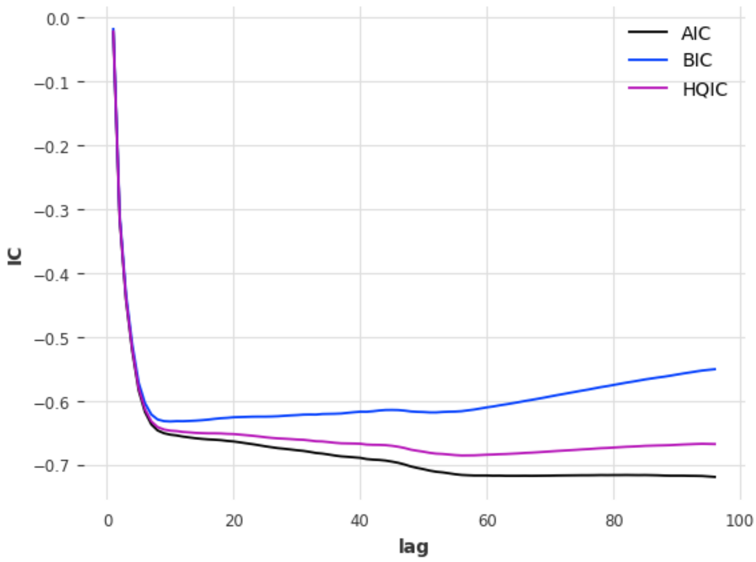
```
# approx. 5 mins

train_df = train_df.asfreq(freq='30min')
valid_df = valid_df.asfreq(freq='30min')
test_df  = test_df.asfreq(freq='30min')

train_df.dropna(inplace=True)

endo_vars = ['y', 'dew.point', 'humidity', 'pressure']
exog_vars = ['day.cos.1', 'day.sin.1', 'year.cos.1', 'year.sin.1',
             'irradiance', 'azimuth_cos', 'za_rad']
endo_df = train_df[endo_vars]
exog_df = train_df[exog_vars]

var_model = VAR(endog = endo_df, exog = exog_df)
MAX_LAGS = 96
lag_order_res = var_model.select_order(MAX_LAGS)
display(lag_order_res.summary())
display(lag_order_res.selected_orders)
print(lag_order_res.selected_orders['bic'])

lag_order_table = lag_order_res.summary().data
headers = lag_order_table.pop(0)
lag_order_df = pd.DataFrame(lag_order_table, columns=headers)
lag_order_df.drop('', axis=1, inplace=True)

with warnings.catch_warnings():
    warnings.simplefilter(action='ignore', category=FutureWarning)
    lag_order_df = pd.concat([lag_order_df[col].str.replace('*', '').astype(float)
                         for col in lag_order_df], axis=1)

lag_order_df.loc[1:, ['AIC','BIC','HQIC']].plot()
plt.xlabel('lag')
plt.ylabel('IC')
plt.show()
```

VAR Order Selection (* highlights the minimums)

	AIC	BIC	FPE	HQIC
0	12.62	12.63	3.040e+05	12.63
1	-0.02283	-0.01766	0.9774	-0.02125
2	-0.3150	-0.3081	0.7298	-0.3129
3	-0.4416	-0.4330	0.6430	-0.4390
4	-0.5225	-0.5122	0.5930	-0.5193
5	-0.5837	-0.5717	0.5578	-0.5801
6	-0.6177	-0.6039	0.5392	-0.6135
7	-0.6372	-0.6217	0.5288	-0.6325
8	-0.6468	-0.6296	0.5237	-0.6415
9	-0.6511	-0.6322	0.5215	-0.6453
10	-0.6534	-0.6327*	0.5203	-0.6471
11	-0.6546	-0.6322	0.5197	-0.6478
12	-0.6564	-0.6323	0.5187	-0.6491
13	-0.6577	-0.6319	0.5180	-0.6498
14	-0.6590	-0.6314	0.5174	-0.6506
15	-0.6600	-0.6307	0.5169	-0.6510
16	-0.6608	-0.6298	0.5165	-0.6513
17	-0.6613	-0.6286	0.5162	-0.6513
18	-0.6620	-0.6276	0.5158	-0.6515
19	-0.6632	-0.6270	0.5152	-0.6521
20	-0.6640	-0.6261	0.5148	-0.6524
21	-0.6654	-0.6258	0.5141	-0.6533
22	-0.6669	-0.6255	0.5133	-0.6542
23	-0.6683	-0.6252	0.5126	-0.6551
24	-0.6699	-0.6251	0.5118	-0.6562
25	-0.6716	-0.6251	0.5109	-0.6574
26	-0.6731	-0.6249	0.5101	-0.6584
27	-0.6745	-0.6245	0.5094	-0.6592
28	-0.6755	-0.6238	0.5089	-0.6597
29	-0.6766	-0.6232	0.5084	-0.6603
30	-0.6778	-0.6227	0.5077	-0.6610
31	-0.6788	-0.6220	0.5072	-0.6614
32	-0.6803	-0.6218	0.5064	-0.6625
33	-0.6821	-0.6219	0.5055	-0.6637
34	-0.6830	-0.6210	0.5051	-0.6640
35	-0.6845	-0.6208	0.5043	-0.6650
36	-0.6861	-0.6206	0.5036	-0.6661
37	-0.6874	-0.6202	0.5029	-0.6669
38	-0.6881	-0.6192	0.5025	-0.6671
39	-0.6889	-0.6182	0.5021	-0.6673
40	-0.6897	-0.6174	0.5017	-0.6676
41	-0.6916	-0.6175	0.5008	-0.6689
42	-0.6925	-0.6167	0.5003	-0.6693
43	-0.6930	-0.6155	0.5001	-0.6694
44	-0.6940	-0.6147	0.4996	-0.6698
45	-0.6956	-0.6146	0.4988	-0.6708
46	-0.6975	-0.6148	0.4978	-0.6723
47	-0.7001	-0.6157	0.4966	-0.6743
48	-0.7032	-0.6170	0.4950	-0.6769
49	-0.7055	-0.6176	0.4939	-0.6786
50	-0.7075	-0.6179	0.4929	-0.6801
51	-0.7098	-0.6185	0.4918	-0.6819
52	-0.7115	-0.6185	0.4909	-0.6831
53	-0.7125	-0.6178	0.4904	-0.6836
54	-0.7139	-0.6174	0.4897	-0.6845
55	-0.7154	-0.6172	0.4890	-0.6854
56	-0.7165	-0.6166	0.4885	-0.6860*
57	-0.7170	-0.6153	0.4882	-0.6859
58	-0.7174	-0.6140	0.4880	-0.6858
59	-0.7174	-0.6123	0.4880	-0.6853
60	-0.7174	-0.6106	0.4880	-0.6848
61	-0.7177	-0.6092	0.4879	-0.6845
62	-0.7176	-0.6074	0.4879	-0.6840
63	-0.7179	-0.6059	0.4878	-0.6837
64	-0.7179	-0.6042	0.4878	-0.6832
65	-0.7179	-0.6025	0.4878	-0.6827
66	-0.7178	-0.6006	0.4878	-0.6820
67	-0.7179	-0.5990	0.4878	-0.6816
68	-0.7178	-0.5972	0.4878	-0.6810
69	-0.7177	-0.5954	0.4879	-0.6803
70	-0.7176	-0.5935	0.4879	-0.6797

71	-0.7174	-0.5916	0.4880	-0.6790
72	-0.7173	-0.5898	0.4881	-0.6784
73	-0.7172	-0.5880	0.4881	-0.6778
74	-0.7171	-0.5862	0.4881	-0.6772
75	-0.7170	-0.5844	0.4882	-0.6765
76	-0.7170	-0.5827	0.4882	-0.6760
77	-0.7169	-0.5808	0.4883	-0.6753
78	-0.7167	-0.5789	0.4884	-0.6746
79	-0.7168	-0.5773	0.4883	-0.6742
80	-0.7167	-0.5755	0.4884	-0.6736
81	-0.7167	-0.5737	0.4884	-0.6730
82	-0.7166	-0.5719	0.4884	-0.6724
83	-0.7167	-0.5703	0.4884	-0.6720
84	-0.7168	-0.5686	0.4883	-0.6715
85	-0.7167	-0.5668	0.4884	-0.6709
86	-0.7169	-0.5653	0.4883	-0.6706
87	-0.7171	-0.5638	0.4882	-0.6703
88	-0.7175	-0.5625	0.4880	-0.6701
89	-0.7177	-0.5609	0.4879	-0.6698
90	-0.7177	-0.5592	0.4879	-0.6693
91	-0.7177	-0.5575	0.4879	-0.6688
92	-0.7179	-0.5560	0.4878	-0.6684
93	-0.7180	-0.5544	0.4877	-0.6680
94	-0.7182	-0.5529	0.4876	-0.6677
95	-0.7189	-0.5518	0.4873	-0.6679
96	-0.7196*	-0.5508	0.4869*	-0.6681

```
{'aic': 96, 'bic': 10, 'hqic': 56, 'fpe': 96}
10
```



The lowest BIC value occurs at 10 lags. I'm going to use `maxlags = 10` because that is where decreasing returns set in.

Rebuild VAR model using `maxlags = 10, ic = 'bic'`.

```
var_fit = var_model.fit(maxlags = 10, ic = 'bic')
print(var_fit.summary())

main_var_col = 'y'
backtest_var = get_var_backtest(var_fit, valid_df, endo_vars, exog_vars, y_col = main_var_col)
hist_comp_var = get_historic_comparison(backtest_var, valid_df, y_col = main_var_col)
summarise_historic_comparison(hist_comp_var, valid_df, y_col = main_var_col)
```

```
# metric_cols = ['type', 'method', 'metric', 'horizon', 'value']
# metrics = pd.DataFrame([], columns = metric_cols)
# metrics = update_metrics(metrics, valid_df, 'VAR', var_rolling_cv, var_fit)
## metrics = update_metrics(metrics, test_df, 'VAR', var_rolling_cv, var_fit)
# plot_baseline_metrics(metrics, 'Multivariate Baseline Comparison - 2021 valid data')
```

```
# 2019 data
# maxlags = 5
# ...
# h    rmse    mae
```

```
# 1    0.39    0.39
# 48   2.45    1.89

# endo_vars = ['y', 'dew.point', 'pressure', 'humidity',]
# maxlags = 52
# h    rmse    mae
# 1    0.37    0.37
# 48   2.253   1.784
# maxlags = 52 substantially better than maxlags = 9

# endo_vars = ['y', 'dew.point', 'humidity',]
# maxlags = 52
# h    rmse    mae
# 1    0.37    0.37
# 48   2.293   1.814
# including pressure is beneficial

# endo_vars = ['y', 'dew.point', 'pressure', 'humidity',]
# exog_vars = ['za_rad', 'irradiance', 'azimuth_cos',]
# maxlags = 51
# h    rmse    mae
# 1    0.369   0.369
# 48   2.163   1.729
# exog_vars is beneficial
# 1 hr 28 mins :-(

# endo_vars = ['y', 'dew.point', 'pressure', 'humidity',]
# exog_vars = ['day.cos.1', 'day.sin.1', 'year.cos.1', 'year.sin.1',]
# maxlags = 52
# h    rmse    mae
# 1    0.37    0.37
# 48   2.133   1.68
# Sinusoidal terms better than irradiance etc!

# endo_vars = ['y', 'dew.point', 'pressure', 'humidity',]
# exog_vars = ['day.cos.1', 'day.sin.1', 'year.cos.1', 'year.sin.1', 'irradiance']
# maxlags = 51
# h    rmse    mae
# 1    0.369   0.369
# 48   2.105   1.667
# irradiance worth adding to sinusoidal terms

# endo_vars = ['y', 'dew.point', 'pressure', 'humidity',]
# exog_vars = ['day.cos.1', 'day.sin.1', 'year.cos.1', 'year.sin.1', 'za_rad']
# maxlags = 51
# h    rmse    mae
# 1    0.37    0.37
# 48   2.134   1.679
# za_rad not as beneficial as irradiance

# endo_vars = ['y', 'dew.point', 'pressure', 'humidity',]
# exog_vars = ['day.cos.1', 'day.sin.1', 'year.cos.1', 'year.sin.1', 'azimuth_cos']
# maxlags = 51
# h    rmse    mae
# 1    0.37    0.37
# 48   2.131   1.675
# azimuth_cos more beneficial than za_rad

# endo_vars = ['y', 'dew.point', 'pressure', 'humidity',]
# exog_vars = ['day.cos.1', 'day.sin.1', 'year.cos.1', 'year.sin.1',
#               'irradiance', 'azimuth_cos']
# maxlags = 51
# h    rmse    mae
# 1    0.368   0.368
# 48   2.098   1.658
# Best model so far

# endo_vars = ['y', 'dew.point', 'pressure', 'humidity',]
# exog_vars = ['day.cos.1', 'day.sin.1', 'year.cos.1', 'year.sin.1',
#               'irradiance', 'azimuth_cos', 'za_rad']
# maxlags = 51
# h    rmse    mae
# 1    0.368   0.368
# 48   2.098   1.657
# Marginally better with za_rad

# valid_df
# endo_vars = ['y', 'dew.point', 'pressure', 'humidity',]
# exog_vars = ['day.cos.1', 'day.sin.1', 'year.cos.1', 'year.sin.1',
#               'irradiance', 'azimuth_cos', 'za_rad']
# maxlags = 51
# h    rmse    mae
# 1    0.347   0.347
```

```
# 48   2.012   1.581
#

# valid_df
# endo_vars = ['y_des', 'dew.point_des', 'pressure', 'humidity',]
# exog_vars = ['day.cos.1', 'day.sin.1', 'year.cos.1', 'year.sin.1',
#              'irradiance', 'azimuth_cos', 'za_rad']
# maxlags = 53
# h    rmse    mae
# 1    0.347   0.347
# 48   2.724   2.132

# valid_df
# endo_vars = ['y_des', 'dew.point_des', 'pressure', 'humidity',]
# exog_vars = ['day.cos.1', 'day.sin.1', 'year.cos.1', 'year.sin.1',
#              'irradiance', 'azimuth_cos', 'za_rad']
# maxlags = 53
# h    rmse    mae
# 1    0.357   0.357
# 48   2.712   2.121

# valid_df
# train_df.loc['2016-01-01':,]
# endo_vars = ['y', 'dew.point', 'pressure', 'humidity',]
# exog_vars = ['day.cos.1', 'day.sin.1', 'year.cos.1', 'year.sin.1',
#              'irradiance', 'azimuth_cos', 'za_rad']
# maxlags = 22
# h    rmse    mae
# 1    0.352   0.352
# 48   2.926   2.305
# Backtest RMSE 48th: 2.92592
# Backtest MAE 48th:  2.304481
# Radical decrease in maxlags!
# Not a great model
```

```
min          -6.000000
25%           4.400000
50%           9.200000
75%          13.600000
max          29.600000
Name: y, dtype: float64
```

The updated VAR model shows substantial improvement. It would benefit from further validation, including residual plots, QQ plots, autocorrelation of residual plots etc

NOTE: Updated VAR validated on 2021 data; initial VAR validated on 2019 data.

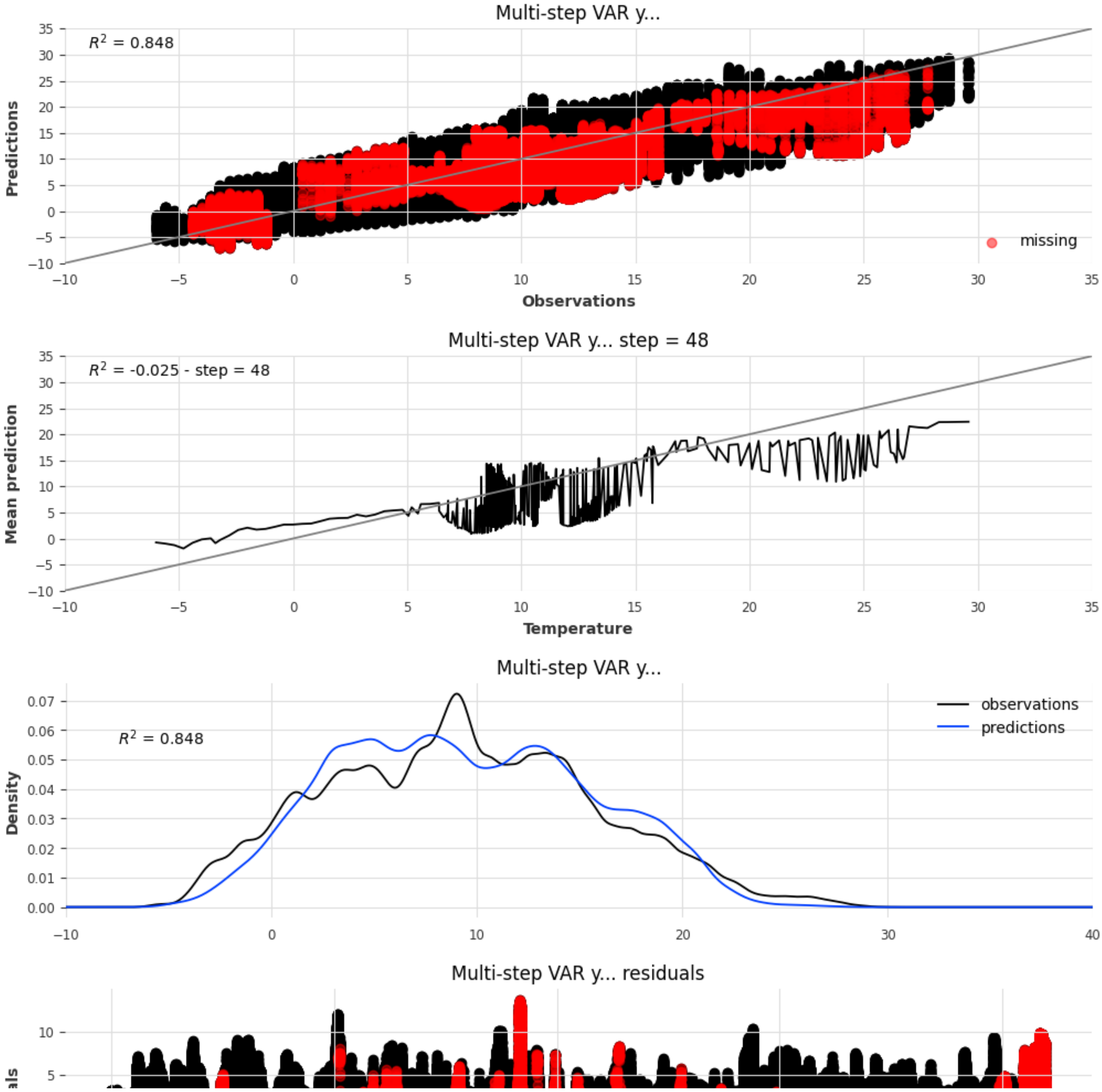
Plot model diagnostics

Next, I plot the old and updated VAR models rmse and mae values for forecast horizons up to 48 (24 hours, each horizon step is equivalent to 30 minutes).

Some points to note regarding diagnostic plots:

- `plot_horizon_metrics`
  - plot rmse and mae values for each individual step-ahead
- `check_residuals`
  - observations against predictions
  - residuals over time
  - residual distribution

```
title_var = 'VAR ' + main_var_col + '...'
plot_multistep_diagnostics(hist_comp_var, title_var, y_col = main_var_col)
```



Multi-step VAR y...
R^2 = 0.848
Predictions
Observations
missing
Multi-step VAR y... step = 48
R^2 = -0.025 - step = 48
Mean prediction
Temperature
Multi-step VAR y...
R^2 = 0.848
Density
observations
predictions
Multi-step VAR y... residuals

Updated VAR baseline metric values:

rmse

```
[0.39, 0.52, 0.64, 0.75, 0.86, 0.96, 1.06, 1.15, 1.23,
 1.31, 1.38, 1.45, 1.51, 1.57, 1.63, 1.68, 1.73, 1.77,
 1.81, 1.85, 1.89, 1.92, 1.96, 1.99, 2.02, 2.05, 2.08,
 2.1 , 2.13, 2.15, 2.18, 2.2 , 2.22, 2.24, 2.26, 2.28,
 2.3 , 2.31, 2.33, 2.35, 2.36, 2.38, 2.39, 2.4 , 2.42,
 2.43, 2.44, 2.45]
```

mae

```
[0.39, 0.49, 0.57, 0.66, 0.74, 0.83, 0.91, 0.98, 1.05,
 1.12, 1.18, 1.24, 1.29, 1.34, 1.39, 1.43, 1.47, 1.5 ,
 1.53, 1.56, 1.59, 1.62, 1.64, 1.66, 1.68, 1.7 , 1.72,
 1.73, 1.75, 1.76, 1.77, 1.78, 1.8 , 1.81, 1.82, 1.83,
 1.83, 1.84, 1.85, 1.85, 1.86, 1.86, 1.87, 1.87, 1.88,
 1.88, 1.89, 1.89]
```

Forecast error variance decomposition

The variance decomposition of forecast errors shows the amount of information each variable contributes to the other variables in the vector autoregression.

The forecast errors can be computed via the statsmodels fevd function up through 48 steps ahead:

```
var_fit.plot()
plt.show()

var_fit.fevd(48).plot()
plt.show()

# var_fit.mse(48)
# var_fit.plot_acorr()
# plt.show()
```