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What exactly is a test statistic?

A test statistic describes how closely the distribution of our data matches the distribution predicted under the null hypothesis of the statistical test you are using.

The distribution of data is how often each observation occurs, and can be described by its central tendency and variation around that central tendency. Different statistical tests predict different types of distributions, so it’s important to choose the right statistical test for your hypothesis.

The test statistic summarizes our observed data into a single number using the central tendency, variation, sample size, and number of predictor variables in our statistical model.

Generally, the test statistic is calculated as the pattern in our data (i.e. the correlation between variables or difference between groups) divided by the variance in the data (i.e. the standard deviation).

Example

we are testing the relationship between temperature and flowering date for a certain type of apple tree. we use a long-term data set that tracks temperature and flowering dates from the past 25 years by randomly sampling 100 trees every year in an experimental field.

Null hypothesis: There is no correlation between temperature and flowering date.

Alternate hypothesis: There is a correlation between temperature and flowering date.

To test this hypothesis we perform a regression test, which generates a t-value as its test statistic. The t-value compares the observed correlation between these variables to the null hypothesis of zero correlation.

Types of test statistics

Below is a summary of the most common test statistics, their hypotheses, and the types of statistical tests that use them.

Different statistical tests will have slightly different ways of calculating these test statistics, but the underlying hypotheses and interpretations of the test statistic stay the same.

Test statistic	Null and alternative hypotheses	Statistical tests that use it
***t*-value**	**Null:** The means of two groups are equal **Alternative:** The means of two groups are not equal	• *T*-test • Regression tests
***z*-value**	**Null:** The means of two groups are equal **Alternative:**The means of two groups are not equal	• *Z*-test
***F*-value**	**Null:** The variation among two or more groups is greater than or equal to the variation between the groups **Alternative:** The variation among two or more groups is smaller than the variation between the groups	• ANOVA • ANCOVA • MANOVA
X^2-value	**Null:** Two samples are independent **Alternative:** Two samples are not independent (i.e. they are correlated)	• Chi-squared test • Non-parametric correlation tests

One sample t-test

Data:

Systolic blood pressures of 14 patients are given below:

183, 152, 178, 157, 194, 163, 144, 114, 178, 152, 118, 158, 172, 138

Test, whether the population mean, is less than 165

Hypothesis

H0: There is no significant mean difference in systolic blood pressure. i.e.,

$\mu = 165$

H1: The population mean is less than 165. i.e., $\mu < 165$

Test statistic

Where,

$\bar{x}$ is sample mean

μ is the population mean

s is sample standard deviation

n is the number of observations;

Code Python Code:

```
sys_bp=[183, 152, 178, 157, 194, 163, 144, 114, 178, 152, 118, 158, 172, 138]
mu=165
from scipy import stats
t_value,p_value=stats.ttest_1samp(sys_bp,mu)

one_tailed_p_value=float("{:.6f}".format(p_value/2)) # Since alternative hypothesis is one tailed, We need to divide the p value by 2
print('Test statistic is %f'%float("{:.6f}".format(t_value)))
print('p-value for one tailed test is %f'%one_tailed_p_value)
alpha = 0.05
if one_tailed_p_value<=alpha:
  print('Conclusion','n','Since p value(=%f)'%p_value,'<','alpha(=%.2f)'%alpha,'''We reject the null hypothesis H0. So we conclude th
else:
  print('Conclusion','n','Sincep-value(=%f)'%one_tailed_p_value, '>', 'alpha(=%.2f)'%alpha,'We do not reject the null hypothesis H0.'
```

```
Test statistic is -1.243183
p-value for one tailed test is 0.117877
Conclusion n Sincep-value(=0.117877) > alpha(=0.05) We do not reject the null hypothesis H0.
```

So we conclude that there is a significant mean difference in systolic blood pressure. i.e., μ < 165 at %.2f level of significance”’%alpha)

Two sample t-test

Data: Compare the effectiveness of ammonium chloride and urea, on the grain yield of paddy, an experiment was conducted. The results are given below:

Ammonium chloride (X_1)	13.4	10.9	11.2	11.8	14	15.3	14.2	12.6	17	16.2	16.5	15.7
Urea (X_2)	12	11.7	10.7	11.2	14.8	14.4	13.9	13.7	16.9	16	15.6	16

Hypothesis

H_0: The effect of ammonium chloride and urea on grain yield of paddy are equal i.e., $\mu_1 = \mu_2$

H_1: The effect of ammonium chloride and urea on grain yield of paddy is not equal i.e., $\mu_1 \neq \mu_2$

Test statistic

Where,

x̄1 and x̄2 are sample means for x1 and x2 respectively.

n1 and n2 are the numbers of observations in x1 and x2 respectively.

s1 and s2 are the sample standard deviation for x1 and x2 respectively.

```
Ammonium_chloride=[13.4,10.9,11.2,11.8,14,15.3,14.2,12.6,17,16.2,16.5,15.7]
```

```
Urea=[12,11.7,10.7,11.2,14.8,14.4,13.9,13.7,16.9,16,15.6,16]

from scipy import stats

t_value,p_value=stats.ttest_ind(Ammonium_chloride,Urea)

print('Test statistic is %f'%float("{:.6f}".format(t_value)))

print('p-value for two tailed test is %f'%p_value)

alpha = 0.05

if p_value<=alpha:

    print('Conclusion','n','Since p-value(=%f)'%p_value,'<','alpha(=%.2f)'%alpha,'''We reject the null hypothesis H0. So we conclude

effect of ammonium chloride and urea on grain yield of paddy are not equal i.e., μ1 = μ2 at %.2f level of significance.'''%alpha)

else:

    print('Conclusion','n','Since p-value(=%f)'%p_value,'>','alpha(=%.2f)'%alpha,'''We do not reject the null hypothesis H0''')
```

```
Test statistic is 0.184650
p-value for two tailed test is 0.855195
Conclusion n Since p-value(=0.855195) > alpha(=0.05) We do not reject the null hypothesis H0
```

So we conclude that the effect of ammonium chloride and urea on grain yield of paddy are equal

paired t-test

Data: Eleven schoolboys were given a test in Statistics. They were given a Month’s tuition and a second test were held at the end of it. Do the marks give evidence that the students have benefited from the exam coaching?

Marks in 1st test: 23 20 19 21 18 20 18 17 23 16 19

Marks in 2nd test: 24 19 22 18 20 22 20 20 23 20 18

Hypothesis

H0: The students have not benefited from the tuition class. i.e., d = 0

H1: The students have benefited from the tuition class. i.e., d < 0

Where, d = x-y; d is the difference between marks in the first test (say x) and marks in the second test (say y).

Test statistic

Where, n is the number of samples ‘s’ is sample standard deviation

COde

```
alpha = 0.05
first_test =[23, 20, 19, 21, 18, 20, 18, 17, 23, 16, 19]
second_test=[24, 19, 22, 18, 20, 22, 20, 20, 23, 20, 18]


from scipy import stats

t_value,p_value=stats.ttest_rel(first_test,second_test)

one_tailed_p_value=float("{:.6f}".format(p_value/2))

print('Test statistic is %f'%float("{:.6f}".format(t_value)))

print('p-value for one_tailed_test is %f'%one_tailed_p_value)

alpha = 0.05

if one_tailed_p_value<=alpha:
```

```
        print('Conclusion','n','Since p-value(=%f)'%one_tailed_p_value,'<','alpha(=%.2f)'%alpha,'''We reject the null hypothesis H0.

    So we conclude that the students have benefited by the tuition class. i.e., d = 0 at %.2f level of significance.'''%alpha)

    else:

        print('Conclusion','n','Since p-value(=%f)'%one_tailed_p_value,'>','alpha(=%.2f)'%alpha,'''We do not reject the null hypothesis H

    So we conclude that the students have not benefited by the tuition class. i.e., d = 0 at %.2f level of significance.'''%alpha)
```

```
Test statistic is -1.707331
p-value for one_tailed_test is 0.059282
Conclusion n Since p-value(=0.059282) > alpha(=0.05) We do not reject the null hypothesis H0.

So we conclude that the students have not benefited by the tuition class. i.e., d = 0 at 0.05 level of significance.
```

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