**Modeling Drying of a Coated Paper**

**Abstract**

Drying of a coated paper is modeled and simulated. The paper sheet is assumed to form three zones, and each zone has its own drying mechanism. Coupling of energy and mass balances must be used in order to solve differential equations. The simulations are carried out in various drying conditions i.e. only hot air drying, only radiant drying, and mixed hot air-radiant drying. Also effect of one side and two side assumption on evaporation is studied. Effect of venting air speed and radiant heat source presence and its distance from the drying surface on the drying of a coated paper has been studied. It’s found that both distance and venting air speed are inversely related to drying in mixed hot air-radiant drying. Both surfaces participate in evaporation however; during last time of drying, no difference between upper and bottom surfaces exists.

**Keyword**: mathematical modeling, coated paper drying, hot air drying, radiant drying, evaporation and penetration.

**1. Introduction**

A detailed modeling of drying process can help the understanding of transport phenomena occurring during the process. The models should be able to produce simulations of industrial process accurately. In fact, in the case of coated paper drying, the whole process should be modeled and optimized in order to obtain product with high quality in an efficient process.

As the modeling of the drying process has to be refined for more in-depth understanding of the process, there're a large number of papers concerning this in the literature. For instance, a review of development of macroscopic models is summarized by Noboa [1]. According to Lampinen [2], Farkas reported a comprehensive model of drying and binder migration [3]. Also Rajala [4] proposed a model for drying and print properties. But these models did not include the penetration of water from coating into the paper. Heikkila [5] was the first one who proposed a macroscopic model contains important transport mechanisms occurring in coated paper drying in absence of radiant heat sources.

Here a complete and detailed modeling of coated paper is presented in presence of convection, conduction, and radiation heat transfer. The effects of heat convection characteristics on drying process have been studied. It found that the air humidity, velocity and temperatures affect the drying process. The effect of radiation is considered in the modeling as well as the effect of distance between drying surface and radiation heat source is studied. The presented model is complete and general enough as the only assumption is the one dimensional heat conduction through the porous media which valid as the other dimensions are large in comparison to the thickness.

**2. Governing Equations**

**2.1 General Remarks**

Here the drying process of a web of paper which enters the radiant drying section to be coated and dried with infrared radiation heat sources, will be studied. It’s assumed that the entering paper is in a uniform initial moisture content of Xp0 (approximately a dry paper’s moisture content) and its moisture content increase as a result of the added coating material on its surface. The coating has water content (moisture) of X1, 0 which is much greater than the paper web moisture beneath. Thus a moisture gradient is created and together with the gravitational force, the water wets the paper web and penetrates into it. Immediately the paper absorb the water, wets and reaches a moisture content of about the coating’s one on its upper parts. The bottom of the paper can be assumed at moisture content of about its initial value or a little higher, as the length of water penetration is limited and also time depended.

The heat received from convection, conduction, and IR heat sources causes’ temperature rise in entire moisture content of coating material and the paper. Due to the temperature rises, water content will evaporate from the coating to the air, and the inner evaporated water moves upward in the paper. In order to calculate evaporation rates, coupling of mass and heat transfer should be considered. However, commonly one side evaporation is assumed, here the effect of evaporation from one and also two surfaces is studied. The derivation of heat and mass transfer governing equations is the scope of following paragraphs.

According to description above, it can be concluded that three zone exist: zone 1 is the coating material, zone 2 is the upper parts of the paper that its thickness should be calculated because of time dependency nature of water penetration, and zone 3 is the bottom of the paper where will never or slightly experience moisture rising. These three zones are illustrated in Fig. 1 together with the heat and mass transfer scheme.

Figure 1. A schematic of modeled coated paper; H.T = Heat Transfer Model, M.T = Mass Transfer Model. Indices Conv. and Cond. and Rad. indicate Convection, Conduction and Radiation Energy Transfer, respectively. Index p refers to Penetration mechanism. λ (t) is the depth of penetration which changes with Time.

In modeling, the paper (porous media/wet web) and coating are assumed homogenous and heat and mass transfer are one dimensional and in direction of thickness as other dimensions (length and width i.e. x and y) are very large in comparison to the thickness and the transfer in those dimensions can be neglected.

**2.2 Overall Models**

**2.2.1 Heat Transfer**

In current modeling, heat (energy) balance includes infrared radiation (IR), convection, and conduction and also phase change enthalpy. Zone 1 (Coating) is in direct contact with radiation and convection sources. Here a part of received heat causes an amount of this layer moisture content evaporates into the air, also another part will be conducted to next zone through conduction. For this zone, as the thickness is small, a uniform temperature profile is assumed. Convection heat transfer coefficient is calculated using Nusselt number correlation proposed by Churchill and Ozoe [6].

 (1)

Operating pressure is constant and as the surrounding fluid is the atmospheric air, one can assume Prandtl number as a constant, Pr=0.708. Thus simplifying, Eq. 1 is rewritten as Eq. 2.

 (2)

Radiation heat transfer occurs among IR heat source, coating surface and surrounding fluid- the air- which are indicated as Node 1, Node 2 and Node 3, respectively in Fig. 2 that illustrates a network scheme for it.

Figure 2. A network scheme of radiation heat transfer among Zone 1 (coating), IR heat source and surroundings.

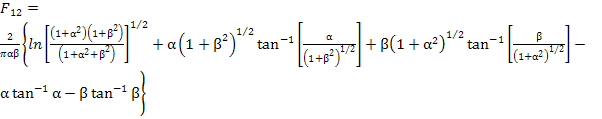
As the shape factors are known, the net heat transfer is calculated using Eq. 5. J1 and J2 are determined by solving a set of two Nodal equations on Node 1 and Node 2. As the coating surface temperature raises gradually, Eb2 also changes in value as well as J1 and J2.

 (3)

 (4)

 (5)

Here, F12 is calculated through Eq. 6 where the effect of IR source distance from the web is of interest.

 (6)

Where α=x/D and β=y/D, are equation dimensionless parameters. x and y indicates width and length of paper respectively. A wide variety of shape factors have been reported by Howell [7]. Eb2 in Eq. 4 is equal to σT4, and σ is 5.669×10-8 W/m2.K4. It’s the well-known Stefan-Boltzmann law.

Fourier's law of heat conduction is applied for calculation of heat conduction between zones i.e. from zone 1 to zone 2, as well as from zone 2 to zone 3, with an effective thermal conductivity, keff and vice versa.

Energy balance on each zone is summarized as follows.

*Zone 1*

 (7)

*Zone 2*

 (8)

*Zone 3*

 (9)

**2.2.2 Mass Transfer**

The detailed consideration on the moisture transfer is delayed till section *drying mechanisms*, however a short totally speaking overview of the mass transfer follows here. There are two mass transfer mechanisms; penetration and evaporation. Evaporation occurs as a result of heat transfer. Here, it's assumed that evaporation is in upward direction for all three zones, but for zone 3 a downward flow into the air also exists as the bottom of the paper is in free contact with the air. Upward evaporation into upper zones affects moisture profiles and makes the calculations complex and cumbersome. Thus, the evaporation is considered once from the upper surface and once from both sides. In order to calculate evaporation rates, coupling of mass and heat transfer should be considered which can be achieve through equations 7 through 9. It should be noted that zone 3 has only one mechanism of drying; i.e. evaporation, other two zones i.e. zone 2 and 1 have penetration and evaporation mechanisms simultaneously.

Each zone has unique moisture content which transfer from upper layer to bottom layer due to penetration. A part of moisture content in zone 1 penetrates into zone 2 as mentioned, which cause an increase in this zone moisture content. However, the penetration depth is limited and time depended. As the paper is a porous media, based on Washburn assumption [8], it is consisted of n-cylindrical capillary pipes, so a modified Lucas-Washburn equation can be used for calculation of the penetration depth [9-13].

Here, Hamraoui and Nylander [9] approach is used as a base to calculate λ, penetration depth, at each time. Assuming water as a non-viscous liquid moving inside porous media (web) and pores radii equal to 0.474 mm (critical value), equation 10 is obtained for calculating λ. Penetration rates are calculated through Fick's law of diffusion with an effective diffusivity Deff with regard to colloidal nature of the paper web and λ as diffusion length.

 (10)

Where γ=-2.8084\*10-8, φ=-3.155\*10-5 and κ=3.16\*1014. Here υ is the kinematic viscosity of water. It can be concluded from Eq. 10 that λ has very small value due to logarithm function constrain.

Finally, Mass Balance on each zone illustrated in Fig 1, is summarized as follows.

*Zone 1*

 (11)

*Zone 2*

 (12)

*Zone 3*

 (13)

**3. Drying Mechanisms**

**3.1 General Remarks**

In an entirely wet substance like paper, a thin layer of liquid film formed on its surface which can be regarded as unbound moisture. This moisture evaporates when it is exposed to the heating sources such as venting air (convection) or radiation. The rate of evaporation can be calculated with the help of heat and mass transfer discussed. During the evaporation, heat is absorbed by moisture as its latent heat and the rate of heat absorption exactly equals the rate of heat flow from the surroundings. Thus for a short period, a constant saturated humidity is expected at the liquid surface interface. The evaporated water (moisture), as rapidly as it evaporates, is compensated by the moisture that is delivered from capillaries and interstices of the paper. So it can be concluded that there exists a constant rate of evaporation, NC, under constant drying conditions which refers as constant rate drying.

Commonly, the surface temperature of paper and the liquid film is colder than the ultimate surface temperature and this leads to an increase in evaporation rate during the surface temperature rise till reaches the temperature of constant rate drying and vice versa. This short period of temperature adjustment is usually so short that commonly can be ignored in the drying analysis.

There exists a limit for constant rate drying where the average moisture content of the paper reaches to its critical moisture content, XC. From this point, dry spots are appeared on the surface of the paper web and as the drying continues larger proportion of the surface is occupied by these spots and the falling rate drying period starts. In the first step, unsaturated surface drying is occurred linearly with constant drying rate till entire surface filled by dry spots.

The next step is the moisture movement (internal-diffusion of moisture) from the lower parts of the paper to its upper parts due to the moisture concentration gradient. The moisture movement rate decreases as the drying cause moisture concentration decreases gradually. The process of drying will be stopped when the moisture content reaches to its equilibrium value, X\* for the air humidity prevails. However, the desired moisture of final paper is more than the equilibrium and the process is stopped sooner [14-21]. Frequently, the entire falling rate drying curve is taken linear due to the lack of detailed data. Depending on the initial moisture content of three described zones and XC, either or both constant and falling rate drying may be involved.

The drying mechanism of each zone is briefly described in following paragraphs.

**3.2 Zone 1**

All three drying period mentioned previously are present in this zone. The heat is supplied to other zones through this zone. Equations 3 through 6 are used when the effect of radiant heat source distance from the drying surface is of interest and studied. For fixed distances between the IR heat source and the coating material surface following equation can be used for simplicity and time saving [22, 23].

 (14)

Where ε is the emissivity of the drying surface and Ts and T1 are IR heat source and drying surface absolute temperatures, respectively. Determining the total heat exchanged in this zone (qrad+qCond+qConv), the NC can be calculated by dividing it on latent heat of evaporation i.e. NC=q/λw. The drying rate is important as with an increase in its value, the critical moisture content will increase. The critical moisture content of many industrial solids is reported by McCormick [15].

**3.3 Zone 2**

Shortly after adding coating material, this zone is formed. The thickness of this zone (λ) is time depended as mentioned previously, but because of rapid wetting, it can be assumed that the final depth of moisture penetration is reached in very short period of time. The moisture content of this zone moves upward to compensate the surface moisture content of zone 1 which is evaporated due to heat adsorption. This process continues even a short time after appearance of falling rate drying in the upper zone. In other words, water internal diffusion occurs in zone 2, and next this diffuses into the coating material to reach the surface.

**3.4 Zone 3**

The only mechanism of drying in this zone is the moisture movement within paper. The moisture moves downward at initial periods before the second step (internal-diffusion of moisture) of falling rate drying period described above reaches as the upper zone pores are full of water (moisture). When the second step of falling rate drying period starts, the moisture moves in upward direction in this zone. The evaporated moisture (vapor) diffuses to the air through the porous media, the paper and also the thin layer of coating.

All these three zones have the final step in common i.e. vapor diffusion through the porous media which is the paper and the coating in zone 2&3, and zone 1 respectively.

**4. Computational Method**

According to obtained mass and energy equation, The unsteady-state calculation of the web temperature and moisture was done for the 3-zone model element of the coated web. The calculation starts at the coater surface as actuator of all mass and energy changes acts on upper boundary of this zone, and ends at the dry base.

For zone 1, it’s assumed that unsteady temperature and moisture profiles exist, in other words its temperature and moisture change with time. The moisture content of zone 1 decreases as a result of penetration and evaporation.

It should be noted that the temperature and moisture in common interface of zone 1 and 2, and also 2 and 3, aren’t the same, in fact there is a small temperature gap between each two layer at contact interface, but in order to avoid complexity of computations, the temperature and moisture at interfaces are assumed to be equal.

At each time step, the diffusion length (λ) is calculated using Eq.10, from this value thickness of zone 2 is calculated. In other words, the depth of zone 2 is time-depended. The zone 3 is assumed to loss its initial moisture content with a constant thickness as a result of heating conducted through zone 2.

Two hierarchies are employed for physical properties evaluation; before change of web composition (before internal diffusion), and after occurring changes in composition. Before the internal diffusion of moisture (vapor), the physical properties of all three zones are assumed variable, after that the available [24] and constant physical properties of individual materials are used i.e. only coating properties for zone 1, only paper properties for zone 2 and 3. The water (moisture content) properties are correlated by data fitting from [24] in the form of empirical equations as follows. [see also Appendix A].

For variable property period, the obtained correlations for Density (ρ), Heat capacity (Cp), Viscosity (μ), and thermal conductivity (k) of water as a function of temperature (θ) in centigrade scale are:

 (15)

 (16)

 (17)

 (18)

To show the applicability of the model system to simulate the drying process, the available setup data has been collected from production setup with an 80 g/m² end product at a web speed of 1450 m/min. The paper web had 47 g/m² and 2% moisture, a thickness of 70 μm and a porosity of about 40 %. The upper surface of paper is coated with 65% solid content and 5 g/m². The venting air had temperatures about 650 and 700 °C. The temperature of IR heat source is about 1000 ºC with an emissivity of unity due to small distance between the emitter and the drying surface.

**5. Result and Discussion**

The effects of heat convection characteristics on drying process have been studied. It found that the air humidity, velocity and temperatures affect the drying process. The effect of radiation is considered in the modeling as well as the effect of distance between drying surface and radiation heat source is studied. The results of this study are discussed in following paragraphs.

**5.1 Hot Air/Gas Drying (Convection)**

Through one side of the dryer, the hot air/gas is introduced to the wet paper surface. By contacting the liquid phase (wet surface) and the gas phase, the water is evaporated and the venting air stream also removes this evaporated water. Here the hot gas is the air with temperatures of about 650 ºC. Figure 3 shows the temperature change of the hot air together with the drying surface’s. The average temperature of the drying surface is constant and at final steps a temperature rise in its temperature is observed where it has already lost almost of its water content and started to dry up. This because that the received heat from the hot air is directly delivered to the paper, thus its temperature rises.

Figure 3. The temperature profile of the paper and the venting hot air. The bold line indicates the hot air temperature profile and so on. The points have equal distance from others and placed on the drying surface from the entrance to the output indicated by 0 and 40 respectively.

The temperature of hot air decreases through the entire the dryer and reaches to temperature of about 120 ºC on the leave and during this it absorbs the evaporated water and removes it. Thus the moisture content of the hot air is increases as shown in Fig 4. Figure 5 shows the moisture change of the paper during hot air drying.

Figure 4. The profile of moisture absorbed by the venting hot air through the dryer.

The moisture content of the paper decreases through the entering and exiting from the dryer which indicates the drying of the paper (Fig. 5).

Figure 5. Paper moisture profile at each point through the dryer.

The end product quality i.e. moisture is strongly depended to the operating condition of the drying process such as the rate by which the wet paper stream supplied. For an example an increase or decrease in the amount of entering wet paper has a great effect on the final moisture content. If the amount of the wet paper increases, in the same drying condition, the final moisture content of dried paper increases. The reason for such behavior is that the initial amount of water increases. Figure 6, for example, shows the response of the system to an increase in the initial wet paper over time.

Figure 6. The moisture change of dried paper due to the change in initial amount of wet paper.

**5.2 Radiant Drying (IR)**

Here the effect of infrared radiant heat transfer for drying of the paper is presented. The surface temperature of the heater is assumed around 1000 ºC with the emissivity of unity. The effect of distance between the emitter surface and the drying surface is studied for three value of D, the distance as shown in Eq. 6, namely 10, 20, 30 and 40 centimeters. It can be seen that as the emitter is further from the drying surface, the lower flux of evaporated water is expected as shown in Fig 7 over the time.

Figure 7. Influence of the distance between infrared emitter and heated surface on the flux of evaporated water.

Figure 8 also shows the drying curves for distances of 10, 20 and 30 centimeters. From Fig. 8 it can be concluded that increase of the distance between the emitter and the drying surface decreases the flux of evaporation. Also the maximum flux depends upon the distance between emitters and the drying surface.

Figure 8. Influence of the distance between emitter and drying surface on the course of rate of drying curves.

The effect of the distance of the emitter from the drying surface is negligible in initial and the final steps but during the drying process the lower the D is, higher evaporation flux.

The figure 9 illustrates the comparison of the effect of radiant drying and hot air drying exposed on the paper.

Figure 9. Influence of infrared (D=20 cm) and hot air (650ºC, 1.5 m/s) drying on the course of drying curves of paper. The bold line indicates the hot air drying curve.

From the Fig. 9, it can be seen that hot air drying requires much more time for the drying of the wet paper. Clearly the evaporated water during the radiant drying is larger than that calculated during hot air drying also in the very beginning of the process.

**5.3 Mixed IR and Convection**

The drying of paper is depended on the speed of venting hot air and also the distance between the emitter and the drying surface. Figure 10 shows the effect of hot air speed on the drying curve for venting speed of 0.5, 1 and 1.5 m/s where the D is 10 cm. higher air speed, longer drying time. Although the coefficient of heat transfer is increased with the venting air speed, it rubs the heat received to the drying surface from the heater surface.

Figure 10. Influence of air speed on the course of drying of paper.

Figure 11 shows the effect of venting speed on rate of water evaporation. Again, higher air speed leads to longer drying time as it rubs the heat received to the drying surface from the heater surface.

Figure 11. Influence of venting air speed on relative evaporation rate water.

The moisture-temperature profile of paper for air speeds of 0.5, 1 and 1.5 is illustrated in Fig. 12. In this figure, it can be seen that the temperature rise at the beginning of the process is fast as in the removal of the final moisture.

**Figure 12.** The moisture-temperature profile of paper.

**5.4 On the evaporation assumption**

The drying rate discussed above is in the case of water evaporation from the upper and bottom surfaces, since upper surface evaporation is not equivalent to the water flux removed from the paper during the infrared drying. Thus, the share of upper and bottom surfaces of the paper must be determined to calculate the water evaporation rate. The upper surface absorbs the infrared energy and the bottom surface exchange heat with its surrounding air by convection.

Figure 13 shows that the drying time in one side evaporation is much longer than the availability of two sides for evaporation. The share of upper and bottom surfaces in the drying is related to the paper water content as illustrated in Fig. 14. The share of upper surface increase as long as the initial surface water is being evaporated (till some 10% of water is evaporated), and then the differences between fluxes decreases gradually. When about 80% of water is evaporated, both surfaces participate is the drying equally.

Thus it can be concluded that both surfaces participate in water evaporation during infrared drying. So water evaporation calculation based on only the upper surface would be inappropriate.

Figure 13. Drying curves of paper heated by infrared energy. The bold line indicates only upper surface evaporation condition.

Figure 14. Relationship between water content and evaporation flux of water. The bold line indicates the evaporation from both surfaces.

**6. Conclusion**

The drying of a coated paper in presence of radiant heat source and convection is studied. It’s found that in hot air drying the temperature of the air at exit decreases up to 120 ºC, and its moisture content rises. In the case of mixed hot air-radiation drying, the speed of venting air has a negative effect on the drying as it removes the heat delivered to drying surface. The increase in distance of heater causes a decrease in evaporation rate as the rate of delivered heat is reduced. Also it’s clear that assuming only upper surface evaporation is inappropriate and during the drying process both surfaces participate in the water evaporation. In fact, a short period after the drying starts, both surfaces participate in water evaporation however; the surface exposed to radiant heat source evaporates much more water than that not heated by infrared. Finally, during last time of drying, no difference between upper and bottom surfaces exists.

**7. Future research opportunities**

The authors wish to mention following suggestions for future research on this topic, as better understanding of drying mechanism and the applicability, accuracy and ability of the models can help to design more efficient controller devices, dryers, etc. for papermaking industry.

* **Porosity study**: the effect of porous structure of paper web on the quality of end-product should be studied. In other words, the feed paper itself is a final product which is a dry and porous media. This porous dry paper is wetted due to adding coating on its surface. So the porosity of initial paper plays an important role on zones alignment. Also the accuracy of models for porosity and moisture diffusion in porous media should be studied to find out which model give better representation of phenomena.
* **Drying models uncertainty factor**: the uncertainty of previous models and current model may be studied with meta-heuristics methods such as fuzzy logic, artificial neural network, evolutionary computing, tabu search, particle warm intelligence, differential evolution, genetic algorithms and hybrid optimization techniques.
* **Moisture penetration visualization**: computational fluid dynamic (CFD) may be used to visualize how the moisture diffuses through the porous structure of the paper. New penetration model can be developed from these finding and etc.
* **Physical properties**: studies on physical property changes during the drying operation could be conducted. These properties have very important effect on drying curves and mechanism.
* …..

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**Appendix A**

1. **Hot Air/Gas Drying (Convection)**

In this case, no radiation is present. Thus corresponding terms in left side of energy balances is set to zero.

While the average moisture content of the paper is greater than its critical moisture content, XC, follow procedure bellow;

1. Set all physical properties equal to their available constant values in literature.
2. From venting air/hot gas speed calculate Reynolds number.
3. Using Re and Pr numbers, calculate Nu number, and from that find the value of h.
4. Find the depth of penetration using modified washburn equation.
5. From value of lambda, each zone thickness is obtained.
6. Solve conservation balances obtained using a numerical method considering coupling of these two equations together, we use ODE45 method in MATLAB which itself uses a fourth-fifth adaptive step size Runge-Kutta.
7. Continue until X equals to XC.

When X is lower than XC, internal diffusion occurs, thus variable properties should be accounted for. So Equations 15-18 should be used for evaluation of physical properties.

1. Evaluate physical properties from obtained correlation.
2. From venting air/hot gas speed calculate Reynolds number.
3. Using Re and Pr numbers, calculate Nu number, and from that find the value of h.
4. Find the depth of penetration using modified washburn equation.
5. From value of lambda, each zone thickness is obtained.
6. For diffusion of vapor to upper zones, using an appropriate relation, calculated the diffusivity, and then evaporation rate to upper zone.
7. Solve conservation balances obtained using a numerical method considering coupling of these two equations together, we use ODE45 method in MATLAB which itself uses a fourth-fifth adaptive step size Runge-Kutta.
8. Stop calculation when X reaches to final product’s desired moisture value, here 0.02.
9. **Radiant Drying (IR)**

In this case, no convection is present. Thus corresponding terms in left side of energy balances is set to zero. Here the effect of distance of heater is to be studied, so equation 6 is used for calculation of shape factor. The procedure is;

While the average moisture content of the paper is greater than its critical moisture content, XC, follow procedure bellow;

1. Set all physical properties equal to their available constant values in literature.
2. From Eq. 6 calculate shape factor for current distance of heater from the web surface.
3. Solve Equations 3 and 4 in each time step in order to find value of Equation 5.
4. Find the depth of penetration using modified Washburn equation.
5. For diffusion of vapor to upper zones, using an appropriate relation, calculated the diffusivity, and then evaporation rate to upper zone.
6. From value of lambda, each zone thickness is obtained.
7. Solve conservation balances obtained using a numerical method considering coupling of these two equations together, we use ODE45 method in MATLAB which itself uses a fourth-fifth adaptive step size Runge-Kutta.
8. Continue until X equals to XC.

When X is lower than XC, internal diffusion occurs, thus variable properties should be accounted for. So Equations 15-18 should be used for evaluation of physical properties. Using same procedure as above, continue calculations till the desired final moisture content, 0.02, reaches.

1. **Mixed IR and Convection**

In this case, mas and energy equations should be solved. Two mentioned procedure in previous section A and B, merged and used together for the distance D and the speed V.

While the average moisture content of the paper is greater than its critical moisture content, XC, follow procedure bellow;

1. Set all physical properties equal to their available constant values in literature.
2. From Eq. 6 calculate shape factor for current distance of heater from the web surface.
3. Solve Equations 3 and 4 in each time step in order to find value of Equation 5.
4. From venting air/hot gas speed calculate Reynolds number.
5. Using Re and Pr numbers, calculate Nu number, and from that find the value of h.
6. Find the depth of penetration using modified Washburn equation.
7. For diffusion of vapor to upper zones, using an appropriate relation, calculated the diffusivity, and then evaporation rate to upper zone.
8. From value of lambda, each zone thickness is obtained.
9. Solve conservation balances obtained using a numerical method considering coupling of these two equations together, we use ODE45 method in MATLAB which itself uses a fourth-fifth adaptive step size Runge-Kutta.
10. Continue until X equals to XC.

The same procedure but with variable physical properties is used for X lower than Xc until the final desired moisture content reaches.

**Nomenclature**

A surface area

Cp specific heat capacity

D distance from heater

E energy

Eb energy of bulk body

F shape factor

g gravity 9.86

H.T heat transfer

IR infrared radiation

J radiosity

k thermal conductivity

m mass

M.T mass transfer

N rate of evaporation

Nu nusselt number

Pr prandtl number

Q,q heat energy

Re Reynolds number

T temperature

t time

x length of IR heat source

X moisture content

y width of IR heat source

**Greek symbols**

 constant in Eq. 10

 constant in Eq. 10

 constant in Eq. 10

 Density

 dimensionless parameters

 emissivity

 kinematic viscosity

 Lambda, depth of penetration

 temperature in Eqs. 15-18

 viscosity

**Superscripts**

\* equilibrium value

**Subscripts**

1, 2, 3 zones number

c constant condition

Cond conduction

Conv convection

e evaporation

eff effective

ev evaporation

net net energy

p penetration

p0 initial paper (entering paper)

Rad radiation

s heat source

v vaporization

w water

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