Generalized similarity transformation method applied to partial differential equations (PDEs) in falling film mass transfer

Abstract

The governing equation describing wetted wall column is partial differential equation (PDE) which can be solved by similarity method. The selection of the combined variable to transform the PDE to an ordinary differential equation is very important subject in this method and mainly is selected based on experiences. But, in this work, a general combined variable was introduced for the PDEs transformation leading to a generalized analytical solution. The obtained analytical solutions were compared to literature and examined using some reported experimental data. According to the results, the deviations of calculated and collected experimental values were small which demonstrates accuracy and reliability of the presented method besides its highly computationally consistency and easy-to-use features.

Keywords**:** similarity transformation; modeling; falling film; absorption; solid dissolution

1. Problem definition

Two phase contactors play main role in separation processes where one phase is usually dispersed in the other one to increase mass transfer surface and mass transfer coefficient ([Asano, 2006](#_ENREF_1); [Ernest J. Henley & Seader, 1981](#_ENREF_12); [Glasgow, 2010](#_ENREF_15); ["Green Separation Processes : Fundamentals and Applications," 2005](#_ENREF_16); ["Mass Transfer in Chemical Engineering Processes," 2011](#_ENREF_30); [Prieve, 2000](#_ENREF_38); [Seader et al., 2010](#_ENREF_43); [R. Taylor & Krishna, 1993](#_ENREF_47); ["Transport Phenomena and Kinetic Theory: Applications to Gases, Semiconductors, Photons, and Biological Systems," 2007](#_ENREF_50)). The wetted wall columns have been used frequently in chemical processes such as absorption or solid dissolution processes, especially in lab scale applications ([Seader et al., 2010](#_ENREF_43); [Treybal, 1980](#_ENREF_51)). From mathematical point of view, wetted wall column is an interesting contactor since mass transfer coefficients can be measured experimentally and correlated to a reliable relationship or compared with modeling results ([Assad & Lampinen, 2002](#_ENREF_2); [Bo et al., 2010](#_ENREF_5); [Chermiti et al., 2011](#_ENREF_8); [Fujita & Hihara, 2005](#_ENREF_13); [Killion & Garimella, 2001](#_ENREF_24); [Kim & Infante Ferreira, 2009a](#_ENREF_25), [2009b](#_ENREF_26); [Raisul Islam et al., 2004](#_ENREF_39); [Rehfeldt & Stichlmair, 2007](#_ENREF_40); [Sisoev et al., 2005](#_ENREF_45); [Sobieszuk & Pohorecki, 2010](#_ENREF_46); [Xu et al., 2008](#_ENREF_53); [Yiǧit, 1999](#_ENREF_55)). Generally speaking, having a mathematical model or an analytical method to predict transport properties, one would confidently proceed to a reliable design of such equipment ([Khansary et al., 2014](#_ENREF_23)). Wetted wall columns are commonly used in gas absorption (**Fig. 1**) or solid dissolution processes (**Fig. 2**) ([Asano, 2006](#_ENREF_1); [McCabe & Smith, 1967](#_ENREF_31); [Prieve, 2000](#_ENREF_38); [Treybal, 1980](#_ENREF_51)).

In absorption operation as shown in **Fig. 1** ([Treybal, 1980](#_ENREF_51)), the free surface of the liquid falling film on a vertically aligned flat solid surface is exposed to a gas phase containing an unwanted component (*A)*. Therefore, component “*A*” is transferred and dissolved into the falling liquid film. At the top of the falling liquid film (e.g. *y*=*0*), a uniform concentration profile () could be assumed for transferring component “” ([Bird et al., 2006](#_ENREF_4); [Treybal, 1980](#_ENREF_51)). Although the interface composition of component “” is affected by thermodynamic equilibrium, and liquid and gas phase mass transfer coefficients ([Treybal, 1980](#_ENREF_51)), it can be assumed constant () through the gas-liquid interface (from  to).

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|  |
| **Fig. 1.** Schematic illustration of a wetted wall column in absorption operation |

Similarly, in solid wall dissolution operation, a thin liquid film is formed by flowing down a liquid on a vertically aligned flat solid surface and the solid wall, component “*A*”, is dissolved in the flowing down thin liquid film, as shown in **Fig. 2**, by diffusional mass transfer ([McCabe & Smith, 1967](#_ENREF_31); [Treybal, 1980](#_ENREF_51)). At the top of the falling film (e.g. *y*=*0*), a uniform and constant concentration profile () for transferring component “” can be assumed ([McCabe & Smith, 1967](#_ENREF_31); [Treybal, 1980](#_ENREF_51)). The interface composition of component “” is affected by thermodynamic equilibrium and solid to liquid mass transfer coefficients ratio ([Treybal, 1980](#_ENREF_51)) and for a simple calculation can be assumed constant () throughout the solid-liquid interface (e.g. from  to).

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| **Fig. 2.** Schematic illustration of a wetted wall column in solid wall dissolution operation |

To calculate the amount of dissolved gas in liquid phase over the lengthor the amount of the dissolved solid in liquid film over the length, determination of liquid mass transfer coefficient is necessary ([Treybal, 1980](#_ENREF_51)). For this purpose, both the continuity equation of component and Navier-Stokes equation must be considered to obtain the governing partial differential equations (PDEs) ([Bird et al., 2006](#_ENREF_4); [Kessler & Greenkorn, 1999](#_ENREF_22)), which then must be solved simultaneously using proper boundary conditions. Using short contact time, steady state, no chemical reaction, no diffusion in flow direction, no convective in parallel to the diffusion direction (), constant diffusivity, constant density and two dimensional mass transfer assumptions ([Bird et al., 2006](#_ENREF_4); [Treybal, 1980](#_ENREF_51)), following set of PDEs would be obtained by using equation of continuity for component “” and the equation of motion (Navier-Stokes equation) respectively for absorption operation (Eq. 1) and solid wall dissolution operation (Eq. 2) ([Bird et al., 2006](#_ENREF_4); [Treybal, 1980](#_ENREF_51));

|  |  |
| --- | --- |
|  | 1 |
|  | 2 |

Where,  represents average liquid film velocity from  to.

These PDEs describing the concentration distribution of penetrant in falling liquid film, have been solved conventionally by similarity (or combination of variables) technique ([Haberman, 2004](#_ENREF_19); [Schiesser & Griffiths, 2009](#_ENREF_42)) in which the obtained PDE together with its boundary conditions are simplified to an ordinary differential equation (ODE) using a variable which combines the independent variables of system ([Bird et al., 2006](#_ENREF_4); [Debnath, 2005](#_ENREF_9); [Dubin, 2003](#_ENREF_10); ["Encyclopedia of Mathematics," 2007](#_ENREF_11); [H.J. Lee & Schiesser, 2003](#_ENREF_18); [Haberman, 2004](#_ENREF_19); ["Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables," 1970](#_ENREF_20); ["Numerical Methods for Hyperbolic and Kinetic Problems," 2005](#_ENREF_34); [Polyanin & Manzhirov, 2007](#_ENREF_36); [Shingareva & Lizárraga-Celaya, 2011](#_ENREF_44); [Yarin, 2012](#_ENREF_54)). New independent variable (combination of independent variables) should mainly satisfy the following criteria ([Polyanin & Manzhirov, 2007](#_ENREF_36); [Shingareva & Lizárraga-Celaya, 2011](#_ENREF_44); [Yarin, 2012](#_ENREF_54)):

1. Should be dimensionless
2. Should be able to transform PDE to an ODE
3. Should be able to reduce PDE’s boundary conditions to ODE’s necessary boundary conditions

Some certain type of combined variables have been used in literature to transfer the PDEs of these two considered cases into ODEs which are mainly selected based on experience in literature ([Assad & Lampinen, 2002](#_ENREF_2); [Bird et al., 2006](#_ENREF_4); [Bo et al., 2010](#_ENREF_5); [Budd et al., 2006](#_ENREF_7); [Debnath, 2005](#_ENREF_9); [Dubin, 2003](#_ENREF_10); ["Encyclopedia of Mathematics," 2007](#_ENREF_11); [Guedda & Ouahsine, 2012](#_ENREF_17); [H.J. Lee & Schiesser, 2003](#_ENREF_18); [Haberman, 2004](#_ENREF_19); ["Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables," 1970](#_ENREF_20); [Killion & Garimella, 2001](#_ENREF_24); [Knabner & Angermann, 2003](#_ENREF_27); [Kumar et al., 2014](#_ENREF_28); [LeVeque, 1992](#_ENREF_29); [Murray R. Spiegel & Liu, 1999](#_ENREF_32); [Myint-U & Debnath, 2007](#_ENREF_33); ["Numerical Methods for Hyperbolic and Kinetic Problems," 2005](#_ENREF_34); [Olsen-Kettle](#_ENREF_35); [Polyanin & Manzhirov, 2007](#_ENREF_36); [Polyanin & Zaitsev, 2002](#_ENREF_37); [Schiesser & Griffiths, 2009](#_ENREF_42); [Shingareva & Lizárraga-Celaya, 2011](#_ENREF_44); [Xu et al., 2008](#_ENREF_53); [Yarin, 2012](#_ENREF_54); [Zhang & Chaolu, 2013](#_ENREF_56)). For absorption process, new independent variable,  can be obtained using a dimensional analysis as presented by Eq. 3 in which  substituted as and D as  ([Bird et al., 2006](#_ENREF_4); [Treybal, 1980](#_ENREF_51));

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|  | 3 |

Eq. 3 shows that  is a dimensionless number. Conventionally, as a rule in literature, the transforming independent variable,, is chosen as (i.e. square root of dimensionless variable represented by Eq. 3) ([Asano, 2006](#_ENREF_1); [Baehr & Stephan, 2006](#_ENREF_3); [Bird et al., 2006](#_ENREF_4); [Boyadjiev & Babak, 2000](#_ENREF_6); [Glasgow, 2010](#_ENREF_15); ["Heat and Mass Transfer – Modeling and Simulation," 2011](#_ENREF_21); ["Mass Transfer in Chemical Engineering Processes," 2011](#_ENREF_30); [McCabe & Smith, 1967](#_ENREF_31); [Prieve, 2000](#_ENREF_38); [Seader et al., 2010](#_ENREF_43); [R. Taylor & Krishna, 1993](#_ENREF_47); [Tosun, 2007](#_ENREF_49); ["Transport Phenomena and Kinetic Theory: Applications to Gases, Semiconductors, Photons, and Biological Systems," 2007](#_ENREF_50); [Treybal, 1980](#_ENREF_51); [Yarin, 2012](#_ENREF_54)).

Similarly, for solid wall dissolution operation, new independent variable,  can be obtained using dimensional analysis as presented by Eq. 4 in which substituted as and  as ;

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|  | 4 |

Eq. 4 shows that  is a dimensionless term. Conventionally, as a rule in literature, the transforming independent variable,, is chosen as  ([Asano, 2006](#_ENREF_1); [Baehr & Stephan, 2006](#_ENREF_3); [Bird et al., 2006](#_ENREF_4); [Boyadjiev & Babak, 2000](#_ENREF_6); [Glasgow, 2010](#_ENREF_15); ["Heat and Mass Transfer – Modeling and Simulation," 2011](#_ENREF_21); ["Mass Transfer in Chemical Engineering Processes," 2011](#_ENREF_30); [McCabe & Smith, 1967](#_ENREF_31); [Prieve, 2000](#_ENREF_38); [Seader et al., 2010](#_ENREF_43); [R. Taylor & Krishna, 1993](#_ENREF_47); [Tosun, 2007](#_ENREF_49); ["Transport Phenomena and Kinetic Theory: Applications to Gases, Semiconductors, Photons, and Biological Systems," 2007](#_ENREF_50); [Treybal, 1980](#_ENREF_51); [Yarin, 2012](#_ENREF_54)).

However, here we’ve shown that a general combined variable can be introduced and there’s no constraint or recommendation on which combined variable to be used. Our proposed method leads to a general solution for governing PDEs. The method was then applied for two special cases as discussed.

1. Generalized similarity transformation technique

In our proposed generalized similarity method, the combined (or transforming) variable  is introduced and defined as a (*arbitrary*) power of the dimensionless combined variable , as presented by Eq. 5;

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|  | 5 |

Using this general combined variable, the obtained governing equations (PDEs) can be transformed to the following two second order ordinary differential equations given by Eqs.6 and 7 respectively;

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|  | 6 |
|  | 7 |

These two second orders ordinary differential equations given by Eqs. 6 and 7 can be rewritten in a generalized form as given by Eq. 8;

|  |  |
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|  | 8 |

In which  and  represent Eqs. 6 and 7 respectively.

The general solution for this ODE, representing concentration profile of the component “*A*”, can be obtained by using Wolfram Mathematica© Software (v9.0) (["Wolfram Mathematica," 2014](#_ENREF_52)), as shown by Eq. 9;

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|  | 9 |

Where, is the upper incomplete gamma function and given by  ([Dubin, 2003](#_ENREF_10); ["Wolfram Mathematica," 2014](#_ENREF_52)). It is necessary to note that there is also a lower incomplete gamma function and given by  ([Dubin, 2003](#_ENREF_10); ["Wolfram Mathematica," 2014](#_ENREF_52)). By definition, the lower and upper incomplete gamma functions satisfy Eq. 10 ([Dubin, 2003](#_ENREF_10); ["Wolfram Mathematica," 2014](#_ENREF_52)), in which  is the complete gamma function. The incomplete gamma functions ( and ) can be related to the error function,  by relationship given by Eq. 11 ([Dubin, 2003](#_ENREF_10); ["Wolfram Mathematica," 2014](#_ENREF_52)).

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|  | 10 |
|  | 11 |

It can be shown that the recommended and commonly used combined variables of literature eliminate the term  from the obtained ODEs. By substituting  in Eq. 16 and  in Eq. 17, following ordinary differential equations would be obtained as shown by Eqs. 12 and 13 respectively;

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|  | 12 |
|  | 13 |

Such selection of combined variable should have been recommended as results in simplest form of governing ODE as no successive computer and computational facilities were available in old days. The proposed method of this work unifies and generalizes the solution of PDEs and especially those considered in current work.

1. Application of proposed method for considered cases

Substituting for **gas absorption** in Eq. 9, the general solution is as presented by Eq. 14;

|  |  |
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|  | 14 |

Applying the boundary conditions and noting that from properties of upper incomplete gamma function, we have  and ([Dubin, 2003](#_ENREF_10); ["Wolfram Mathematica," 2014](#_ENREF_52)), one would obtain the parameter  and  as given by Eqs. 15-16,

|  |  |
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|  | 15 |
|  | 16 |

Substituting  and  from Eqs. 15-16 and simplifying resulting equation, one can derive concentration profile of component “*A*” in liquid falling film as given by Eq. 17:

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|  | 17 |

It is worthy to note that using Eq. 11, one would write the obtained concentration profile in terms of error function as presented in Eq. 18;

|  |  |
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|  | 18 |

Therefore, the concentration profile of component “*A*” in liquid falling film in gas absorption process can be obtained as given by Eq. 20;

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|  | 19 |
|  | 20 |

Therefore, the solution of PDE using the proposed method is in accord with reported solutions in literature ([Bird et al., 2006](#_ENREF_4); [Glasgow, 2010](#_ENREF_15); [Tosun, 2007](#_ENREF_49); [Treybal, 1980](#_ENREF_51); [Yarin, 2012](#_ENREF_54)). As presented by Eq. 5, although there is a wide variety of choice on the power *n* (i.e. combined variable), there’s no requirement on initial selection of such power as the solution, provided here, is independent of the choice on the power “*n*”. For the case of absorption operation, the recommendation in text books is a choice of  ([Asano, 2006](#_ENREF_1); [Baehr & Stephan, 2006](#_ENREF_3); [Bird et al., 2006](#_ENREF_4); [Boyadjiev & Babak, 2000](#_ENREF_6); [Glasgow, 2010](#_ENREF_15); ["Heat and Mass Transfer – Modeling and Simulation," 2011](#_ENREF_21); ["Mass Transfer in Chemical Engineering Processes," 2011](#_ENREF_30); [McCabe & Smith, 1967](#_ENREF_31); [Prieve, 2000](#_ENREF_38); [Seader et al., 2010](#_ENREF_43); [R. Taylor & Krishna, 1993](#_ENREF_47); [Tosun, 2007](#_ENREF_49); ["Transport Phenomena and Kinetic Theory: Applications to Gases, Semiconductors, Photons, and Biological Systems," 2007](#_ENREF_50); [Treybal, 1980](#_ENREF_51); [Yarin, 2012](#_ENREF_54)), which conceptually has been advised as it scales the combined variables to the coordinate  by removing its power ().

The molar mass transfer flux of component ‘’ (respect to stagnant coordinate) can be given by Eq. 21;

|  |  |
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|  | 21 |

The mass transfers flux of “” at the gas-liquid interface () in liquid film (absorption flux) would be calculated as given by Eq. 22;

|  |  |
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|  | 22 |

The mass transfer coefficient () can be derived () as presented by Eq. 23;

|  |  |
| --- | --- |
|  | 23 |

The average mass transfer coefficient along the falling film wall (e.g. from *y*=*0* to *y*=*L*) then can be calculated;

|  |  |
| --- | --- |
|  | 24 |

The calculated and  are exactly same as reported in the textbooks for mass transfer coefficient from gas phase to liquid falling film (gas absorption process) in vertical wetted wall column ([Asano, 2006](#_ENREF_1); [McCabe & Smith, 1967](#_ENREF_31); [Tosun, 2007](#_ENREF_49); [Yarin, 2012](#_ENREF_54)). It is necessary to note that textbooks ([Asano, 2006](#_ENREF_1); [Baehr & Stephan, 2006](#_ENREF_3); [Bird et al., 2006](#_ENREF_4); [Boyadjiev & Babak, 2000](#_ENREF_6); [Glasgow, 2010](#_ENREF_15); ["Heat and Mass Transfer – Modeling and Simulation," 2011](#_ENREF_21); ["Mass Transfer in Chemical Engineering Processes," 2011](#_ENREF_30); [McCabe & Smith, 1967](#_ENREF_31); [Prieve, 2000](#_ENREF_38); [Seader et al., 2010](#_ENREF_43); [R. Taylor & Krishna, 1993](#_ENREF_47); [Tosun, 2007](#_ENREF_49); ["Transport Phenomena and Kinetic Theory: Applications to Gases, Semiconductors, Photons, and Biological Systems," 2007](#_ENREF_50); [Treybal, 1980](#_ENREF_51); [Yarin, 2012](#_ENREF_54)) have used the conventional transforming independent variable to calculate concentration profile and mass transfer coefficient but we showed that all choice of power “” can be used and the result is exactly the same.

The average concentration profile of component “*A*” in liquid film at each -cross-section (), can be calculated by using component “*A*” mole balance as given in Eq. 25;

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|  | 25 |

The average concentration of component “*A*” in liquid film at the bottom of wall () can be obtained by substituting  in Eq. 25 as given:

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|  | 26 |

For **solid wall dissolution** operation at short contact time condition, the general solution is given as presented by Eq. 27;

|  |  |
| --- | --- |
|  | 27 |

Applying the boundary conditions and noting that from properties of upper incomplete gamma function ([Dubin, 2003](#_ENREF_10); ["Wolfram Mathematica," 2014](#_ENREF_52)) we have and, one would obtain the parameter  and  as given by Eqs. 28-29,

|  |  |
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|  | 28 |
|  | 29 |

Finally, one can derive concentration profile of component “*A*” in falling liquid film, as given by Eq. 30:

|  |  |
| --- | --- |
|  | 30 |

The molar mass transfer flux of component ‘’ (respect to stagnant coordinate) can be obtained as given by Eq. 31;

|  |  |
| --- | --- |
|  | 31 |

The molar flux of component “” at the solid-liquid interface () in liquid film (solid wall dissolution flux) would be calculated as given by Eq. 32;

|  |  |
| --- | --- |
|  | 32 |

The mass transfer coefficient () can be derived as presented by Eq. 33 ();

|  |  |
| --- | --- |
|  | 33 |

Using Eq. 33, the average mass transfer along the falling film wall (from  to) is calculated as follows:

|  |  |
| --- | --- |
|  | 34 |

The calculated and  () are exactly same as reported in the textbooks for mass transfer coefficient in vertical wall falling film ([Bird et al., 2006](#_ENREF_4); [McCabe & Smith, 1967](#_ENREF_31); [Tosun, 2007](#_ENREF_49)). It is necessary to note that all textbooks ([Asano, 2006](#_ENREF_1); [Baehr & Stephan, 2006](#_ENREF_3); [Bird et al., 2006](#_ENREF_4); [Boyadjiev & Babak, 2000](#_ENREF_6); [Glasgow, 2010](#_ENREF_15); ["Heat and Mass Transfer – Modeling and Simulation," 2011](#_ENREF_21); ["Mass Transfer in Chemical Engineering Processes," 2011](#_ENREF_30); [McCabe & Smith, 1967](#_ENREF_31); [Prieve, 2000](#_ENREF_38); [Seader et al., 2010](#_ENREF_43); [R. Taylor & Krishna, 1993](#_ENREF_47); [Tosun, 2007](#_ENREF_49); ["Transport Phenomena and Kinetic Theory: Applications to Gases, Semiconductors, Photons, and Biological Systems," 2007](#_ENREF_50); [Treybal, 1980](#_ENREF_51); [Yarin, 2012](#_ENREF_54)) have used the conventional transforming independent variable  to calculate concentration profile and mass transfer coefficient but we showed that all choice of power“” can be used and the results are exactly the same.

Similar to gas absorption process, the average concentration profile of component “*A*” in liquid film at each -cross-section (), can be calculated by using component “*A*” mole balance as given by Eq. 35;

|  |  |
| --- | --- |
|  | 35 |

Or in simplified form:

|  |  |
| --- | --- |
|  | 36 |

The average concentration of component “*A*” in liquid film at the bottom of wall () can be obtained by substituting  in Eq. 36 as follows:

|  |  |
| --- | --- |
|  | 37 |

1. Validation of method using numerical example

Here, as an example, mass transfer of from gas phase to the falling liquid film (water considered as component “*B*”) on a vertical wetted wall column has been considered, for which the required information are summarized in **Table 1** ([Treybal, 1980](#_ENREF_51)) where  is the  (component “*A*”) concentration at,  is  concentration at gas-liquid interface, is the maximum velocity of flowing fluid, is the liquid film thickness and  is the binary diffusivity (diffusion coefficient of  (*A*) in water (*B*)).

**Table 1.** The required information for mass transfer on a vertical wetted wall column ([Treybal, 1980](#_ENREF_51))

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Parameter** |  |  |  |  |  |  |  |
| **Value** |  |  |  |  |  |  |  |
| **Unit** |  | | |  |  |  | |

Using the presented general solution and the data in **Table 1**, the concentration profile of component “*A*” can be written as Eq. 38;

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|  | 38 |

In **Fig. 3**, the concentration profile of component “*A*” is plotted for  and;

|  |  |
| --- | --- |
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|  | |
| **Fig. 3**. concentration profile in liquid phase plotted for  and : ***upper left***, the 3D concentration meshes on domain; ***upper right***, the 3D concentration contours on domain; ***bottom***, contour of concentration | |

The average concentration profile in liquid film at each cross-section can be given by Eq. 39 which is plotted for  in **Fig. 4**;

|  |  |
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|  | 39 |

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| **Fig. 4.** Average concentration profile in liquid film at each y plotted for |

It is obvious that the average  concentration in liquid film increases as y increases. The average  concentration in liquid film at the bottom of wall () can be obtained by substituting  in Eq. 40 as follows:

|  |  |
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|  | 40 |

The reported average  concentration in liquid film at the bottom of wall is **0.02404** *kmol/m3* ([Treybal, 1980](#_ENREF_51)), so deviation of calculation is **0.0009**.

Average mass transfer coefficient from  to () can be calculated as presented by Eq. 41;

|  |  |
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For the case of **wall dissolution**, as an example, consider a slab of trinitrotoluene, or TNT, with 100 cm in length and 100 cm in width where a film of water is falling down the slab with 0.5 cm wide ([Germanese, 2002](#_ENREF_14); [Ro et al., 1996](#_ENREF_41); [S. Taylor et al., 2009](#_ENREF_48)). The solubility of TNT in water is about 0.035g per 100g of water. In addition, at 20 °C, the solubility of the TNT-water solution can be approximated by the viscosity of water. The required information are summarized in **Table 2** ([Germanese, 2002](#_ENREF_14); [Ro et al., 1996](#_ENREF_41); [S. Taylor et al., 2009](#_ENREF_48)) where  is the TNT (component “*A*”) concentration at,  is TNT concentration at solid-liquid interface, is the maximum velocity of flowing fluid, is the liquid film thickness and  is the binary diffusivity (diffusion coefficient of TNT (*A*) in water (*B*)).

**Table 2.** The required information for TNT mass transfer on a vertical wetted wall column ([Germanese, 2002](#_ENREF_14); [Ro et al., 1996](#_ENREF_41); [S. Taylor et al., 2009](#_ENREF_48))

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Parameter** |  |  |  |  |  |  |  |
| **Value** |  |  |  |  |  |  |  |
| **Unit** |  | | |  |  |  | |

Using the presented general solution and considering data in **Table 2**, the concentration profile of component “*A*” can be written as Eq. 43;

|  |  |
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In **Fig. 5**, the concentration profile of component “*A*” is plotted for  and;

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| **Fig. 5**. TNT (“*A*”) concentration profile in liquid phase plotted for  and : ***upper left***, the 3D concentration meshes on domain; ***upper right***, the 3D concentration contours on domain; ***bottom***, contour of concentration | |

The average concentration of component “*A*” can be given by Eq. 44 which is plotted for  in **Fig. 6**;

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|  | 44 |

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| **Fig. 6.** Average TNT concentration profile in liquid film at each y plotted for |

The average concentration at the bottom of wall () can be obtained by substituting  in Eq. 44 as follows:

|  |  |
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|  | 45 |

Average mass transfer coefficient from  to () can be calculated as presented by Eq. 46;

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|  | 46 |

1. Conclusion

A general similarity solution for partial differential equations (PDEs) describing falling film operations (gas absorption and solid dissolution) were presented using conventional similarity transformation technique. For this purpose, selection of combined variable reviewed critically and it was shown that a wide choice of combined variable can be used in similarity method in contrast to the recommendations in open literatures. Introducing a general combined variable, generalized similarity solution was developed and applied for mass transfer coefficient prediction of gas absorption and solid wall dissolution processes. According to obtained results, the deviations of calculated and collected experimental values were small which demonstrates accuracy and reliability of the presented method.

List of Symbols

See supplementary file

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