Normal Performance Measurement (POW)

1. Introduction

There are at least two different algorithms that can compute X^N for some positive integer *N*.

Algorithm 1 is to use *N*–1 multiplications.

Algorithm 2 works in the following way: if *N* is even, $X^N = X^{N/2} \times X^{N/2}$; and if *N* is odd, $X^N = X^{(N-1)/2} \times X^{(N-1)/2} \times X$. Figure 2.11 in your textbook gives the recursive version of this algorithm.

Your tasks are:

- (1) Implement Algorithm 1 and an **iterative** version of Algorithm 2;
- (2) Analyze the complexities of the two algorithms;
- (3) Measure and compare the performances of Algorithm 1 and the iterative and recursive implementations of Algorithm 2 for *X*=1.0001 and *N* = 1000, 5000, 10000, 20000, 40000, 60000, 80000, 100000.

2. Algorithm Specification

Algorithm1: Using *N*–1 multiplications for computing X^N

Input: $X \in R$, and $N \in N^+$

Output: The result and time cost

Main Idea: As mentioned above

Pseudo Code:

```
int f1(int n,double x)
{
  p=1
  for (i = 0; i < n; i++) {
     p*=x;
  }
  return p
}
```

Algorithm2: Recursive version of computing X^N

Input: $X \in R$, and $N \in N^+$

Output: The result and time cost

Main Idea: if *N* is even, $X^N = X^{N/2} \times X^{N/2}$; and if *N* is odd, $X^N = X^{(N-1)/2} \times X^{(N-1)/2} \times X$. Figure 2.11 in your textbook gives the recursive version of this algorithm.

Pseudo Code:

```
int f2(int n, double x)
{
  if(n==0)
    return 1;
  else if(n is even)
    return f2(n/2,x)*f2(n/2,x);
  else
    return f2((n-1)/2,x)*f2((n-1)/2,x)*x;
}
//noticed that it could be optimized as follows:
int f2(int n, double x)
{
  if(n==0)
    return 1;
  else if(n is even)
    return f2(n/2,x*x);
  else
    return f2((n-1)/2,x*x)*x;
}
```

Algorithm3: Iterative version of computing X^N

Input: $X \in R$, and $N \in N^+$

Output: The result and time cost

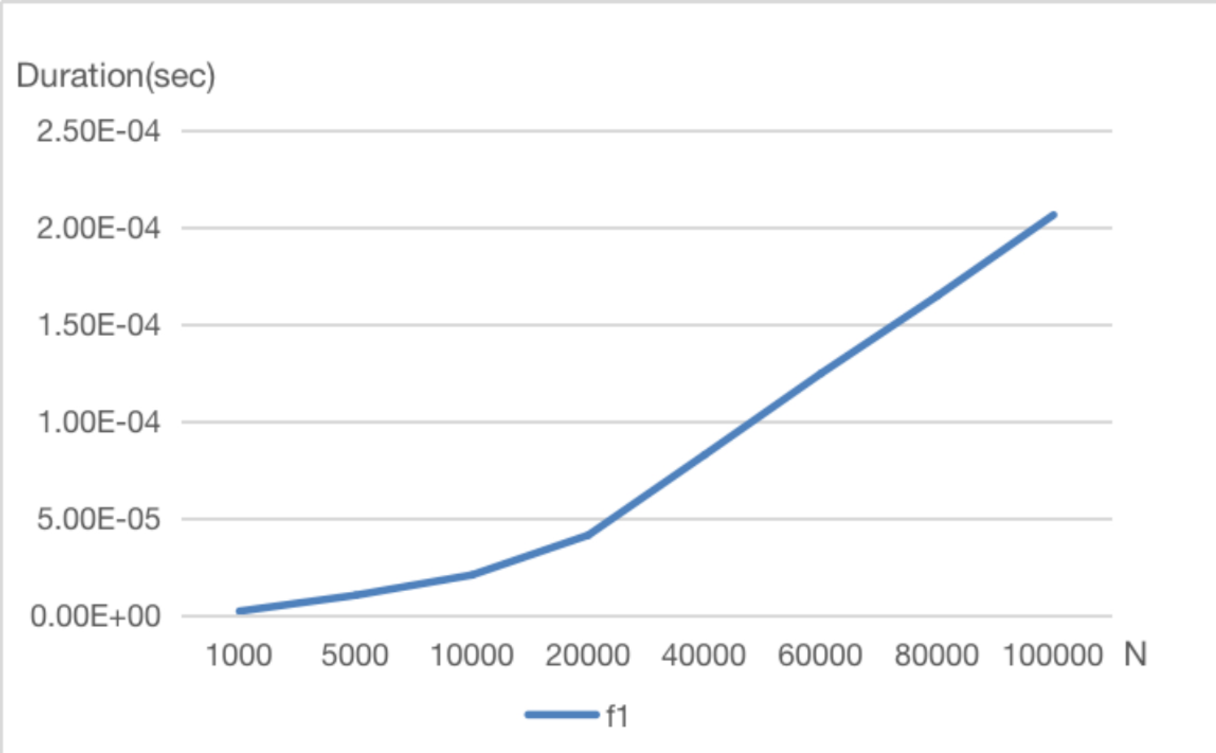
Main Idea: if *N* is even, $X^N = X^{N/2} \times X^{N/2}$; and if *N* is odd, $X^N = X^{(N-1)/2} \times X^{(N-1)/2} \times X$.

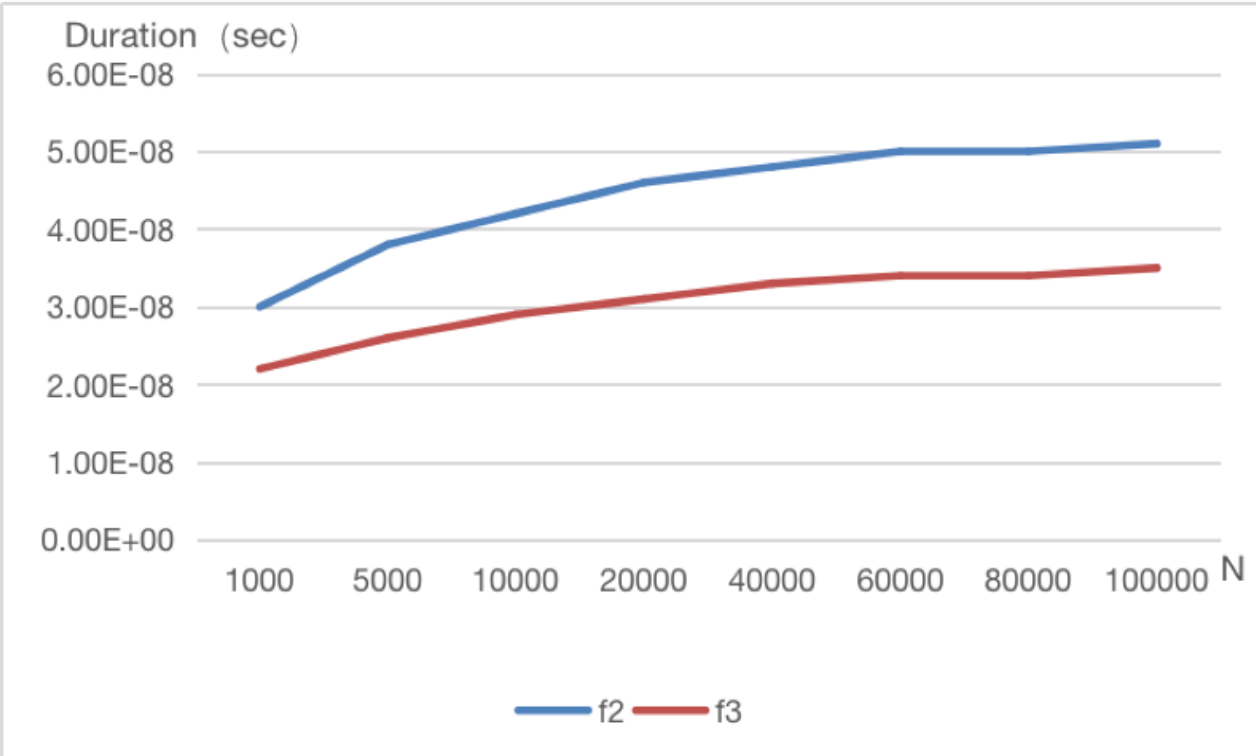
Pseudo Code:

```
int f3(int n, double x)
{
  p=1;
  while(n>0){
    if(n is odd) p*=x;
      x*=x;
      n/=2;
  }
}
```

3. Testing Results

N	1000	5000	10000	20000	40000	60000	80000	100000
Iterations(K) of f1	1e6	1e6	1e6	1e6	1e6	1e6	1e6	1e6
Ticks of f1	2047	10278	20725	41247	82650	124542	164522	206263
Total time(sec) of f1	2.047	10.278	20.725	41.247	82.650	124,542	164.522	206.263
Duration(sec) of f1	2.047e-6	10.278e-6	20.725e-6	41.247e-6	82.650e-6	124.542e-6	164.522e-6	206.263e-6
Iterations(K) of f2	1e6	1e6	1e6	1e6	1e6	1e6	1e6	1e6
Ticks of f2	30.00	38.00	41.00	46.00	48.00	50.00	50.00	51.00
Total time(sec) of f2	0.030	0.038	0.042	0.046	0.048	0.050	0.050	0.051
Duration(sec) of f2	0.030e-6	0.038e-6	0.042e-6	0.046e-6	0.048e-6	0.050e-6	0.050e-6	0.051e-6
Iterations(K) of f3	1e6	1e6	1e6	1e6	1e6	1e6	1e6	1e6
Ticks of f3	22.00	26.00	29.00	31.00	33.00	34.00	34.00	35.00
Total time(sec) of f3	0.022	0.026	0.029	0.031	0.033	0.034	0.034	0.035
Duration(sec) of f3	0.022e-6	0.026e-6	0.029e-6	0.031e-6	0.033e-6	0.034e-6	0.034e-6	0.035e-6





4. Analysis and Comments

Time complexity

- f1: $O(N)$
- f2: $O(logN)$
- f3: $O(logN)$

Space complexity

- f1: $O(1)$
- f2: $O(logN)$ because every time to be recursived it would need an additional space of memory
- f3: $O(1)$

Appendix: Source Code (in C++)

Tips: The code should be compiled as a .cpp file or it will go wrong!

> to compile it, just input `cc code.cpp` in your terminal and run the a.out

```
//code.cpp
#include <stdio.h>
#include<time.h>
#include<math.h>
#define CLK_TCK 1000
clock_t start, stop;
double duration1, duration2, duration3;
double f1(int n, double x);
double f2(int n, double x);
double f3(int n, double x);
void time(int N, int k);
int main() {
  int N, k;//N is the pow of x, k is the number of repeating
  printf("Plese input N and k:");
  scanf("%d %d", &N, &k);
  time(N, k);
  return 0;
}
double f1(int n, double x) {
  double p = 1;
  for (int i = 0; i < n; i++)
    p *= x;
  return p;
}
double f2(int n, double x) {
```

```
  if (n == 0)//if n==0 return 1, the initialization
    return 1;
  if (n % 2 == 0 && n > 1)//if n is even return f(n/2,x^2)
    return f2(n / 2, x * x);
  else//if n is odd return f((n-1)/2,x^2)*x
    return f2((n - 1) / 2, x * x) * x;
}

double f3(int n, double x) { //The recursive algorithm, where you get n down to 0 in half,
you can't just multiply x by x in half, you can square x in half
  double p = 1;
  for (; n > 0;) {
    if (n % 2 == 1)
      p *= x;
    x *= x;
    n /= 2;
  }
  return p;
}
void time(int N, int k) {//For convenience, the statistics time is treated as a function, and
only the power N and the number of runs k need to be entered at each call
    printf("%lf\n", f1(N, 1.0001));//To check if the result of f1 is right
  printf("%lf\n", f2(N, 1.0001));//To check if the result of f2 is right
  printf("%lf\n", f3(N, 1.0001));//To check if the result of f3 is right. Check the three
against each other to see if the procedure is wrong
  start = clock();
  for (int i = 0; i < k; i++)//cycle f1 k times
    f1(N, 1.0001);
  stop = clock();
  duration1 = ((double)(stop - start)) / CLK_TCK;//computing the time of cycling
  start = clock();
  for (int i = 0; i < k; i++)//cycle f2 k times
    f2(N, 1.0001);
  stop = clock();
  duration2 = ((double)(stop - start)) / CLK_TCK;//computing the time of cycling
  start = clock();
  for (int i = 0; i < k; i++)//cycle f3 k times
    f3(N, 1.0001);
  stop = clock();
  duration3 = ((double)(stop - start)) / CLK_TCK;//computing f3 k times
  printf("duration1=%.30lf\nduration2=%.30lf\nduration3=%.30lf\n", duration1, duration2,
duration3);//Show the time costs of the three functions
}
```

Declaration

I hereby declare that all the work done in this project is of my independent effort.