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Article *in* Journal of Economic Structures · May 2016

DOI: 10.1186/s40008-017-0065-z

Stelios Kotsios
National and Kapodistrian University of Athens

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Ilias Kostarakos - Stelios Kotsios

Department of Economics,
Division of Mathematics and Informatics,
National and Kapodistrian University of Athens
Aristidou 11 & Sofokleous 1, Athens 10559, Greece
hkostarak@econ.uoa.gr

Working Paper

Keywords: Fiscal Policy; Public Debt; Linear Feedback Control; Algorithmic Control
JEL classification: C60; E61; E62; H63

Abstract

We propose an algorithmic feedback control approach for the design of fiscal policy rules. In particular, we calculate linear feedback policy rules such that predetermined target levels for GDP and public debt are simultaneously, exactly tracked. We run a number of simulations in order to examine the effects of different policy response rates and the overall effectiveness of the proposed methodology.

1 Introduction

One of the most important objectives of economic policy is to ensure, via the appropriate manipulation of the available policy instruments (control variables), that the economic system tracks, as closely as possible, a desired path for the policy targets (outputs). One of the approaches that has been used in the relevant literature for the design of economic policy is the feedback approach, stemming from the mathematical control theory. Various aspects of the feedback methodology have been utilized for the purposes of policy design for more than 50 years, starting with the use of PID controllers in the seminal paper by Phillips [14]. These aspects range from (stochastic) optimal feedback control (see, among others, Amman and Kendrick [1], Christodoulakis and Levine[5], Christodoulakis and Van Der Ploeg [6]) to nonlinear (Athanasiou et.al. [3], Athanasiou and Kotsios [4], Kotsios and Leventidis [13]) and stochastic control applications (Dassios et.al. [7]).

The importance of feedback policy rules for policy design is evident from the fact that for more than 20 years, monetary policy decisions have been, to a large extent, based on the Taylor rule (see Taylor [16]); this is a linear feedback policy rule stipulating (in its simplest form) that the interest rate is set based on deviations of inflation and GDP from target inflation levels and potential GDP, respectively; so, for example, if GDP exceeds the full-employment level, then nominal interest rates need to be increased.

One of the advantages of adopting the feedback framework is that it allows to explicitly take into account the time-lags associated with the conduct of economic policy, since they can be incorporated into the dynamics of the model and the feedback policy rule (Kendrick [8]). Most importantly, the feedback methodology allows for more frequent (and, possibly, smaller) interventions by the policymaker, which are likely to result in a smoother transition path for the economy (see Kendrick and Amman [9], Kendrick and Shoukry [10]).

Our aim in this paper is to utilize the algorithmic feedback control framework for the design of fiscal policy. That is, we want to design linear feedback policy rules for the fiscal policy instruments available so that predetermined (fixed) desired sequences for the policy targets (GDP and public debt levels) are simultaneously tracked. In particular, we assume that the policymaker has at his disposal two instruments: expenditures related to compensation of public sector employees, social benefits etc (i.e. expenditures that cover individual and collective consumption) and expenditures related to investment projects (e.g. infrastructure) that will be funded by the government. However, these investment expenditures are subject to several time lags (including, among others, legislative, design and implementation lags) and, as a result, their effects will affect the economy with a possibly substantial delay; however, the feedback mechanism used allows us to explicitly incorporate these lags into the calculation of the fiscal policy rules. These rules will provide the exact sequence of the policy instruments necessary to ensure that the target levels of GDP and public debt will be simultaneously met, without any deviation (thus, the tracking error will be equal to zero).

In order to design the policy rules we use an algorithmic linear feedback control technique known as (exact) *model matching*. It is a completely parameterized technique allowing us to develop appropriate symbolic algorithms in order to design the requested policy rules. One of the main advantages of this approach is that we obtain as a solution a class of feedback policy rules; this grants the policymaker the ability to choose the most appropriate policy rule, depending on the particular case at hand. Moreover, the policy rules are responsive, taking into account the state of the economy, and thus they represent a more discretionary approach to the design of fiscal policy.

Our analysis is conducted in the context of a linear, deterministic variant of the standard multiplier-accelerator model proposed by Samuelson [15]. The main reason for choosing this simple linear model is its tractability, as it will allow us to examine the effects of the proposed methodology on the workings of the system.

The rest of this paper is organized as follows: section 2 presents the model, in section 3 we develop the proposed the methodology and section 4 contains some simulation results. Section 5 concludes.

2 Formulation of the Model

As already stated in the introduction, the model we chose is a linear, deterministic variant of the multiplier-accelerator model coupled with the government budget constraint.

The multiplier-accelerator part of the model consists of an income identity and four behavioral equations. Assuming a closed economy, the income identity is given by:

$$Y(t) = C(t) + I(t) + \lambda_0 G^I(t) + \lambda_1 G^I(t-1) + \lambda_2 G^I(t-2) + G^w(t) \tag{1}$$

where $t \in \mathbb{N}$ is the time index and the real sequences $C(t)$, $I(t)$, $G^I(t)$ and $G^w(t)$ denote consumption, private investment, general government expenditures (compensation of employees, social benefits etc) and government investment, respectively. For the λ_i parameters we assume that $\lambda_i \in (0,1)$, $\lambda_0 + \lambda_1 + \lambda_2 = 1$. Government investment-related expenditures are subject to various time-lags, such as legislative (the time until the projects to be funded by the government are approved by the relevant parliamentary committee) and implementation lags (the time until the funds are actually disbursed) and, as a result, the policy will hit the economy and its effects will become apparent in subsequent periods. These lags are captured by the λ_i parameters, which indicate the percentage of the government's decision to invest in period t that is realized in period $t+i$; that is, the λ_i parameters represent the percentage of the funds that the government aims to invest in period t that are actually disbursed in period $t+i$.

Moving on to the behavioral equations, the consumption function is given by:

$$C(t) = (1-s)Y^d(t-1) + sY^d(t-2) \tag{2}$$

where $s \in (0,1)$ is the marginal propensity to save and

$$Y^d(t) = Y(t) - T(t) \tag{3}$$

is the disposable income. For the tax function, we assume that it takes the following tax-on-income form:

$$T(t) = \tau Y(t) \tag{4}$$

where $\tau \in (0,1)$ is the constant tax rate. Investment depends on the accelerator principle:

$$I(t) = \nu(Y(t-1) - Y(t-2)) \tag{5}$$

where $\nu > 0$ denotes the accelerator.
Finally, the government budget constraint has the standard form:

$$B(t) - (1+r)B(t-1) = G^I(t) + G^w(t) - T(t) \tag{6}$$

where $B(t)$ denotes debt outstanding and r is the (constant) interest on public debt.
After all the necessary substitutions among equations (1)-(6) and some necessary algebra, we end up with the following pair of equations:

$$\begin{aligned} &Y(t) - a_1 Y(t-1) - a_2 Y(t-2) = \lambda_0 G^I(t) + \lambda_1 G^I(t-1) \\ &+ \lambda_2 G^I(t-2) + G^w(t) \\ &B(t) - (1+r)B(t-1) + (1-\tau)Y(t) = G^I(t) + G^w(t) \end{aligned} \tag{7}$$

where, $a_1 = (1-s)(1-\tau) + \nu$, $a_2 = s(1-\tau) - \nu$. This is the input-output form of the model, with $Y(t)$, $B(t)$ being the outputs (policy targets) and $G^I(t)$, $G^w(t)$ being the inputs (policy instruments).

3 Solution Technique

Our aim is to design linear feedback policy rules for short-term fiscal policy interventions (that is, for the next 4 to 6 quarters). In particular, we want to design policy rules for general government expenditure (G_t^w) and government investment (G_t^I) which, once implemented, they will modify the dynamics of the system in such a way that predetermined, desired sequences for the levels of GDP and public debt will be simultaneously, exactly tracked. The feedback rules relate the current value of the policy instruments to lagged values of both the instruments and the targets; thus, the requested linear feedback policy rules will be functions of the form:

$$\begin{aligned} G^I(t) &= \sum a_i G^I(t-i) + \sum c_j G^w(t-j) + \sum d_h Y(t-h) + \sum e_f B(t-f) \\ G^w(t) &= \sum k_p G^I(t-p) + \sum l_q G^w(t-q) + \sum m_r Y(t-r) + \sum n_s B(t-s) \end{aligned} \tag{8}$$

where $a_i, c_j, \ldots, n_s$ are unknown, real parameters to be determined. The dependence of the current values of the instruments on lagged values of both the instruments and the targets is a fundamental property of feedback rules known as *causality* (see Astrom and Wittenmark [2]). This property ensures that the policy rules are responsive (see Taylor [16]), that is, the instruments change in response to changes in the target values. This is in contrast to rules that specify fixed settings for the instruments (e.g. the *k%* rule proposed by Friedman) and thus, they represent a more discretionary approach to the exercise of fiscal policy.
In order to design the feedback policy rules we use a technique from the control theory literature known as (exact) *model matching*; it is a completely parameterized technique, allowing for proper symbolic solution algorithms to be developed. In what follows we provide a brief description of the workings of the model matching approach (see Appendix 1 for the mathematical formulation and the papers by Kostarakos and Kotsios [11, 12] for the relevant theorems and proofs): we want to design policy rules of the form (8) which will modify the dynamics of the original system (7) - the *open loop* system - in such a way that predetermined, fixed targets are reached. We work as follows: first, the policymaker decides on the desired sequence for the policy targets, say $\vec{x}^*(t) = (Y^*(t), B*(t))^T$. Then, using an appropriate symbolic algorithm we construct a linear (artificial) system, which has the property that its output is identical to the desired sequence $\vec{x}^*(t)$; this is known as the *desired* system. Now, the problem at hand reduces to that of calculating the unknown parameters of the policy rules (8) such that the original system becomes identical, i.e. *matched* to the desired one. Again, the parameters are calculated using an appropriate symbolic algorithm (see Kostarakos and Kotsios [11] and Kotsios and Leventidis [13] for a detailed analysis of both algorithms). The most important advantage of this approach is that the algorithms provide as a solution a class of feedback policy rules (the

coefficients of the algebraic expressions of the resulting policy rules are not fixed), which essentially constitutes a set of potential policies. Thus, the policymaker is able to choose from this set those rules he deems more appropriate, depending on the problem at hand. This class of policy rules can be augmented by calculating more complex rules (e.g. rules that contain more lags for the instruments and the targets). Another important advantage is that we can calculate the exact sequence of the policy instruments necessary for tracking the desired target levels; that is, the sequence necessary for reaching the targets without any deviations. Moreover, the rules are such that the system immediately settles on the desired path: if the rule is applied in period t, then the system starts following the desired trajectory from $t+1$. Therefore, these policy rules are 'optimal', in the sense that they ensure zero settling time to the desired path. Finally, we can simulate the model under different policy rules in order to obtain a better insight as to how the policy rules affect the working of the system, under different specifications and policy scenarios.

4 An Application

From an economic policy point of view, the timing of the policy action is a central issue. In particular, given the lags associated with policy conduct, should the government immediately react to signs of a downturn in economic activity via, for example, a frontloaded disbursement of investment funds, or would it be preferable to adopt a more gradual response? Moreover, how do the lags associated with policy conduct affect the actual implementation of the policy? In the analysis presented in section 2, we saw that the policy lags are incorporated into our model via the λ_i parameters, using the following mechanism:

$$\lambda_0 G^I(t) + \lambda_1 G^I(t-1) + \lambda_2 G^I(t-2)$$

Then, for the cases where $\lambda_2 > \lambda_0$, the government manages to immediately react to signs of a crisis (since a bigger percentage of the expenditures will be disbursed in period t), while for $\lambda_0 > \lambda_2$ the bulk of the expenditures will actually be disbursed in period $t+2$; this implies that the changes in the size of government investment will have an effect in the economy with a two-period delay.
In order to examine the effects of different policy response times, we conducted a series of simulations. In particular, we examine the following cases for the λ_i parameters:

1. The *immediate* response case: $\lambda_0 = 0.2,\ \lambda_1 = 0.2,\ \lambda_2 = 0.6$
2. The *gradual* response case: $\lambda_0 = 0.4,\ \lambda_1 = 0.3,\ \lambda_2 = 0.3$
3. The *delayed* response case: $\lambda_0 = 0.5,\ \lambda_1 = 0.25,\ \lambda_2 = 0.25$

We assume the following plausible values for the rest of the parameters of the open loop system (7):

$$\nu = 1,\ s = 0.2,\ \tau = 0.4,\ r = 0.04$$

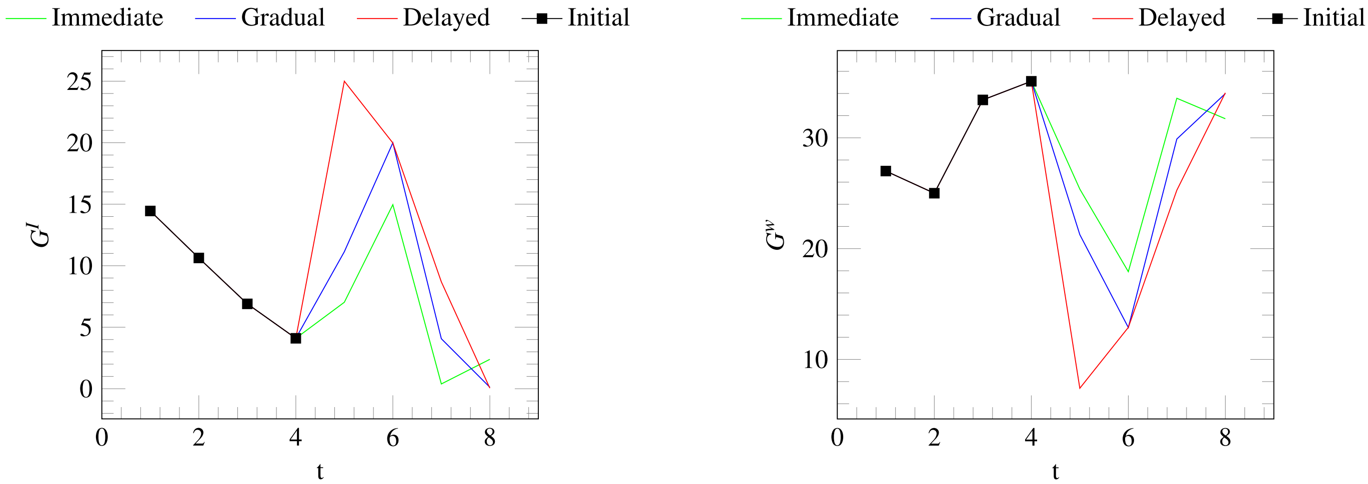
Then, the open-loop system is:

$$\begin{aligned} &Y(t) - 1.54Y(t-1) + 0.94Y(t-2) = \lambda_0 G^I(t) + \lambda_1 G^I(t-1) + \lambda_2 G^I(t-2) + G^w(t) \\ &B(t) - 1.04B(t-1) + 0.4Y(t) = G^I(t) + G^w(t) \end{aligned} \tag{9}$$

Finally, the initial conditions are given in table 1 and they depict an economy facing a severe downturn, with large decreases in GDP levels, accompanied with large increases in debt levels. Moreover, government investment-related expenditures (G^I) exhibited a large decline.
We assume that the government aims for a 1% per period increase in GDP levels, and a corresponding decrease in the levels of debt. Figure 1 presents the time-paths of the control variables under all policy scenarios (a table with the results can be found in Appendix 2).
As we can see in the figure 1, the response time is critical regarding the composition of total government expenditures, i.e. the allocation between investment related expenditures ($G^I(t)$) and general government expenditures ($G^w(t)$) as well as regarding the necessary changes in the size of the instruments. In particular, when the government

Table 1: Initial Conditions

Time	Y	B	G^I	G^w
1	120	135	14.45	27
2	112	142	10.63	25
3	105	145	6.9	33.42
4	100	150	4.1	35.1



(a) G^I path

(b) G^w path

Figure 1: G^I, G^w paths under all policy scenarios

is able to immediately respond to a downturn ($\lambda_2 > \lambda_0$ case), then $G^I(t)$ needs to slightly increase in the first two periods of policy implementation, in order to provide a boost to the economy via the multiplier principle. At the same time, $G^w(t)$ needs to be cumulatively decreased by 10%, to ensure that a surplus is generated so that a reduction in debt levels can be achieved. On the contrary, when the response time entails considerable lags ($\lambda_0 > \lambda_2$ case), the size of the changes of the policy instruments is significantly larger. Government investment funds need to be immediately increased due to the fact that this change will only hit the economy after 2 periods, and they exhibit a cumulative increase of 48,8%. As a result of these increases, $G^w(t)$ exhibits a sharp decline over the entire period (almost a 34% reduction), in order to achieve the debt reduction target. In the gradual response case, the necessary changes are smaller and both instruments exhibit smooth transition paths.

5 Conclusion

In this paper we presented an application of algorithmic linear feedback control for the design of short-term fiscal policy. In particular, in the context of a linear variant of the multiplier-accelerator model, using an algebraic control theory technique known as model matching, we calculated a class of linear feedback laws such that the system will immediately, track a predetermined, desired trajectory for both policy targets, without any deviation. Moreover, in order to examine the effects of lags we run some simulations under different policy response rates. The results of the policy experiments indicate that immediate response allows the government to achieve the policy targets with relatively small policy interventions, compared to cases where there are larger delays in the disbursement of funds.

6 Appendix 1 - Mathematical Formulation of the Model Matching Problem

In this appendix we provide the basic mathematical formulation of the model matching technique, elaborated in section 3.
First, we have to rewrite the input-output model (7) in a more compact form. This is done via using the notion of the q-operator, a lag operator defined as $q^i f(t) = f(t-i)$, for any sequence $f(t)$ (see Astrom and Wittenmark [2]). Then, the system (7) can be written in the so called algebraic form:

$$A(q)\vec{x}(t) = K(q)\vec{u}(t) \tag{10}$$

where $\vec{x}(t) = (Y(t), B(t))^T$, $\vec{u}(t) = (G^I(t), G^w(t))^T$ and

$$A = \begin{bmatrix} 1 - a_1 q - a_2 q^2 & 0 \\ \tau & 1-(1+r)q \end{bmatrix}, \; K = \begin{bmatrix} \lambda_0 + \lambda_1 q + \lambda_2 q^2 & 1 \\ 1 & 1 \end{bmatrix}$$

This is also known as the *open-loop* system i.e. the system before the policy intervention. The desired system, that is the system having the property that its output is exactly equal to the desired sequence $\vec{x}^*(t)$ is of the form:

$$A^d(q)\vec{x}^*(t) = K^d(q)\vec{u}_c(t) \tag{11}$$

where A^d, K^d are $2x2$ polynomial matrices constructed using an appropriate symbolic algorithm. Then, we want to calculate feedback policy rules which, once applied to the open-loop system (10) they will modify its dynamics in such a way that it will be identical, i.e. *matched*, to the desired system (11). The feedback rules (8) can be written, using the q-operator, as:

$$R(q)\vec{u}(t) = T(q)\vec{u}_c(t) - S(q)\vec{x}(t) \tag{12}$$

where $R(q)$, $S(q)$ and $T(q)$ are unknown polynomial matrices in q to be designed. It turns out that, if the following set of equations holds:

$$\begin{aligned} &R(q)A(q) + K(q)S(q) = A^d(q) \\ &K(q)T(q) = K^d(q) \\ &R(q)K(q) = K(q)R(q) \end{aligned}$$

then the policy rules (12) can ensure that the open-loop system (10) will be matched to the desired one. These equations are solved using appropriate symbolic algorithms developed in Mathematica (see Kostarakos and Kotsios [11, 12]).

7 Appendix 2 - Table of Results

References

[1] Amman, H.M. & Kendrick, D.A. (2003). 'Mitigation of the Lucas Critique with Stochastic Control Methods', *Journal of Economic Dynamics and Control*, 27:11-12, 2035-2057

[2] Astrom, K.J. & Wittenmark, B (1996). *Computer-Controlled Systems: Theory and Design*, (3rd ed.). Prentice Hall

[3] Athanasiou, G., Karafyllis, I. & Kotsios, S. (2008) 'Price Stabilization using Buffer Stocks', *Journal of Economic Dynamics and Control*, 32:4, 1212-1235

[4] Athanasiou, G. & Kotsios, S. (2008). 'An Algorithmic Approach to Exchange Rate Stabilisation', *Economic Modelling*, 25:6, 1246-1260

Table 2: G^I, G^w values for $r = 4\%$

Time	Immediate		Gradual		Delayed	
	G^I	G^w	G^I	G^2	G^I	G^2
5	7.023	25.37	11.138	21.26	24.998	7.4
6	14.954	17.91	19.991	12.88	19.981	12.89
7	0.388	33.57	4.065	29.89	8.689	25.27
8	2.39	31.72	0.121	33.99	0.047	34.06
Sum	24.765	108.58	35.315	98.02	53.715	79.62

[5] Christodoulakis, N. & Levine, P. (1987). 'The Trade-Off between Simplicity and Optimality in Macroeconomic Policy Design', *Journal of Economic Dynamics and Control*, 11:2, 173-178

[6] Christodoulakis, N. & Van Der Ploeg, F. (1987). 'Macrodynamic Formulation with Conflicting Views of the Economy: A Synthesis of Optimal Control and Feedback Design', *International Journal of Systems Science*, 18:3, 449-479

[7] Dassios, I.K, Zimbidis, A.A & Kontzalis, C.P. (2014). 'The Delay Effect in a Stochastic MultiplierAccelerator Model', *Journal of Economic Structures*, 3:7

[8] Kendrick, D.A. (1988). *Feedback: A New Framework for Macroeconomic Policy*, Springer Netherlands

[9] Kendrick, D. & Amman, H.M. (2014). 'Quarterly Fiscal Policy'. *The Economists' Voice*, 11:1, 7-12

[10] Kendrick, D. & Shoukry, G. (2014). 'Quarterly Fiscal Policy Experiments with a Multiplier-Accelerator Model', *Computational Economics*, 44:3, 269-293

[11] Kostarakos, I. & Kotsios, S. (2015) 'Fiscal Policy Design in Greece in the Aftermath of the Crisis: An Algorithmic Approach', *submitted*, Presented at the 7th Biennial HO PhD Symposium on Contemporary Greece and Cyprus, LSE, 4-5 June 2015

[12] Kostarakos, I. & Kotsios, S. (2015) 'Controlling GDP and Public Debt Using Feedback Policy Rules for Government Expenditures and Taxation', *submitted*

[13] Kotsios, S. & Leventidis, J. (2004). 'A Feedback Policy for a Modified Samuelson-Hicks Model', *International Journal of Systems Science*, 35:6, 331-341

[14] Phillips, A.W. (1954). 'Stabilisation Policy in a Closed Economy', *Economic Journal*, 64:254, 290-323

[15] Samuelson, P. (1939). 'Interactions between the Multiplier Analysis and the Principle of Acceleration', *Review of Economics and Statistics*, 21:2, 75-78

[16] Taylor, J.B. (1993). 'Discretion versus Policy Rules in Practice', *Carnegie-Rochester Conference Series on Public Policy*, 39, 195-214