Homework 1
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1 Solve $y' = (x \times y^2 + x)/(y - x^2 \times y)$ using Euler and Runge–Kutta 3 methods

Euler's method for numerically approximating the solution of a first-order initial value problem $y' = f(x, y)$, $y(x_0) = y_0$ as a table of values. To start, we must decide the interval $[x_0, x_f]$ that we want to find a solution on, as well as the number of points n that we wish to approximate in that interval. Then taking $\Delta x = (x_f - x_0)/(n - 1)$ we have n evenly spaced points $x_0, x_1, ..., x_n$, with $x_j = x_0 + j \times \Delta x$. Then our objective is then to fill in the values of $y(x_i)$ in the table. Let's use Euler's method to approximate the value of the function in the interval [0.0, 0.9] with 10 points. Then $x_0 = 0, y_0 = 1, x_f = 0.9, n = 10$, and $\Delta x = (x_f - x_0)/(n - 1) = 0.1$. I have the following Python script to evaluate these values:

```
import numpy as np
from matplotlib import pyplot as plt
x0 = 0
y0 = 1
xf = 0.9
n = 10
deltax = (xf-x0)/(n-1)
x = np.linspace(x0, xf, n)
y = np.zeros([n])
y[0] = y0
for i in range(1,n):
    y[i]=deltax*((x[i]*y[i-1]^2+x[i])/(y[i-1]-x[i]^2*y[i-1]))+y[i-1]
for i in range(n):
    print(x[i], y[i])

plt.plot(x,y,'o')
plt.xlabel("Value of x")
plt.ylabel("Value of y")
plt.title("Approximate solution of with Euler's Method")
plt.show()
```

2 Solve $y' = (1 - 2 * x)/y^2$ using Modified Euler method 1 and Runge-Kutta 3 method

3 Solve $y' = e^x - 1$ using Modified Euler method 2 and Runge-Kutta 4 method

4 Solve $y' = (y^2 - y)/x^2$ using Euler method and Runge-Kutta 4 method

5 Solve $y' = (1 + y)/tanx$ using Modified Euler method 1 and Runge-Kutta 4 method

References

[1] Michel Goossens, Frank Mittelbach, and Alexander Samarin. *The LaTeX Companion*. Addison-Wesley, Reading, Massachusetts, 1993.

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