Homework 4
Marlen Akimaliev
BIL622-Numerical Analysis II

Friday 21st April, 2017

1 Problem

Solve the following Boundary Value Problem:
$x^2 \times y'' - x \times y = 3 \times x^3$
$y(1) = 2$
$y(2) = 9$
Use shooting method for solution.

2 Solution

In order to solve second order BVP problem it has to be converted into a first order system of two equations:
Higher order differential equations can be written as a system with a very simple change of variable. We'll start by defining the following two new functions.
$x_1(t) = y(t)$
$x_2(t) = y'(t)$
If we differentiate both sides of these we get;
$x_1' = y' = x_2$
$x_2' = y'' = (3 \times x^3 + x \times y')/x^2 = (3 \times x^3 + x \times x_1)/x^2$
Putting all of this together gives the following system of differential equations.
$x_1' = x_2$, $x_1(1) = 2$
$x_2' = (3 \times x^3 + x \times x_1)/x^2$, $x_1(2) = 9$
Following is the table after getting results at the range[2,9].

t	x
2.0	2.0000000000000000
2.8	−13.4076702532885363
3.6	−26.5513372930734732
4.3	−37.6932727986715363
5.1	−46.1898472357814711
5.9	−50.8412316172525038
6.7	−49.9920471001467845
7.4	−41.5416864109323285
8.2	−22.9134425699207362
9.0	9.0000000000004121

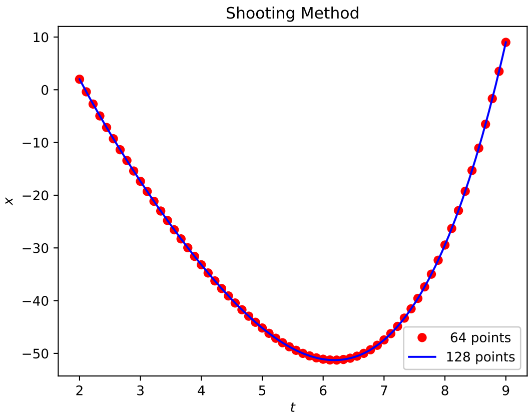
Following Python code [1] is used to evaluate values and plot the graph.

```
import numpy
def shoot( f, a, b, z1, z2, t, tol ):
    from diffeq import rk4
    max_iter = 25   # Maximum number of shooting iterations
    n = len( t )    # Determine the size of the arrays we will generate
    y = rk4( f, [a,z1], t )
    w1 = y[n-1,0]
    for i in xrange( max_iter ):
        y = rk4( f, [a,z2], t )
        w2 = y[n-1,0]
        if abs( b - w2 ) < tol:
            break
        z1, z2 = ( z2, z2 + ( z2 - z1 ) / ( w2 - w1 ) * ( b - w2 ) )
        w1 = w2
    if abs( b - w2 ) >= tol:
        print "\a**** ERROR ****"
        print "Maximum number of iterations (%d) exceeded" % max_iter
        print "Returned values may not have desired accuracy"
        print "Error estimate of returned solution is %e" % ( b - w2 )
    return y[:,0]

if __name__ == "__main__":

    import math
    from pylab import *
    a = 2
    b = 9
    n1 = 10
    t1 = linspace( a, b, n1 )
    # Compute shooting method solutions
    def f(x,t):
        return array( [x[1], (3*t**3+t*x[0])/t**2] )
    xs1 = shoot( f, 2, 9, 1.0, 10.0, t1, 1e-5 )
    # Prepare for display; set interactive mode to true so each plot
    # is shown as it is generated
    interactive( True )
    for p1, p2 in list(zip(t1, xs1))[::1]:
        print("%4.1f %10.16f" % (p1, p2))
    # Plot solutions
    plot( t1, xs1, 'ro')
    title( 'Shooting Method' )
    xlabel( "t" )
    ylabel( "x" )
    #legend( ( '%3d points' % n1, '%3d points' % n2 ), loc='lower right' )
    savefig( '1_2.eps', fmt='EPS', dpi=100 )
    draw()
    z = raw_input( "Press ENTER to quit..." )
```

If we try to plot these values as a graph result is as follows:



References

[1] MAT342/CPS342 Python Demos, gordon.edu,
`http://www.math-cs.gordon.edu/courses/ma342/python/bvp.py`