Mid Term Exam
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BIL622-Numerical Analysis II

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1 Solve the following Cauchy problem:

1.1 Description

$y'' + 1/5 \times y' + 3.125 \times y = 0$
$y(0) = 0$
$y'(0) = 1$

1.2 Solution

Solution for 100 points as a table in the interval of t at $[0; 10]$ with step size 0.1 is as follows:

t	$y1$	$y2$
0.00000	0.00000	1.00000
0.10000	0.10000	1.00000
0.20000	0.20000	0.96875
0.40000	0.38750	0.81348
0.50000	0.46885	0.69238
0.60000	0.53809	0.54587
0.70000	0.59267	0.37772
0.80000	0.63044	0.19251
0.90000	0.64969	−0.00451
1.00000	0.64924	−0.20754
1.10000	0.62849	−0.41043
1.20000	0.58745	−0.60683
1.30000	0.52676	−0.79041
1.40000	0.44772	−0.95502
1.50000	0.35222	−1.09493
1.60000	0.24273	−1.20500
1.70000	0.12223	−1.28086
1.80000	−0.00586	−1.31905
1.90000	−0.13776	−1.31722
2.00000	−0.26949	−1.27417
2.10000	−0.39690	−1.18996

2.20000	−0.51590	−1.06593
2.30000	−0.62249	−0.90471
2.40000	−0.71296	−0.71018
2.50000	−0.78398	−0.48738
2.60000	−0.83272	−0.24238
2.70000	−0.85696	0.01784
2.80000	−0.85517	0.28564
2.90000	−0.82661	0.55288
3.00000	−0.77132	0.81119
3.10000	−0.69020	1.05223
3.20000	−0.58498	1.26792
3.30000	−0.45819	1.45072
3.40000	−0.31311	1.59391
3.50000	−0.15372	1.69176
3.60000	0.01545	1.73979
3.70000	0.18943	1.73496
3.80000	0.36293	1.67577
3.90000	0.53051	1.56235
4.00000	0.68674	1.39657
4.10000	0.82640	1.18196
4.20000	0.94459	0.92371
4.30000	1.03697	0.62853
4.40000	1.09982	0.30448
4.50000	1.13027	−0.03922
4.60000	1.12634	−0.39243
4.70000	1.08710	−0.74441
4.80000	1.01266	−1.08413
4.90000	0.90425	−1.40058
5.00000	0.76419	−1.68316
5.10000	0.59587	−1.92197
5.20000	0.40368	−2.10818
5.30000	0.19286	−2.23433
5.40000	−0.03057	−2.29460
5.50000	−0.26003	−2.28504
5.60000	−0.48854	−2.20378
5.70000	−0.70892	−2.05111
5.80000	−0.91403	−1.82958
5.90000	−1.09699	−1.54394
6.00000	−1.25138	−1.20114
6.10000	−1.37149	−0.81008
6.20000	−1.45250	−0.38149
6.30000	−1.49065	0.07242
6.40000	−1.48341	0.53825
6.50000	−1.42958	1.00181
6.60000	−1.32940	1.44856
6.70000	−1.18455	1.86400
6.80000	−0.99815	2.23417
6.90000	−0.77473	2.54609
7.00000	−0.52012	2.78819
7.10000	−0.24130	2.95073

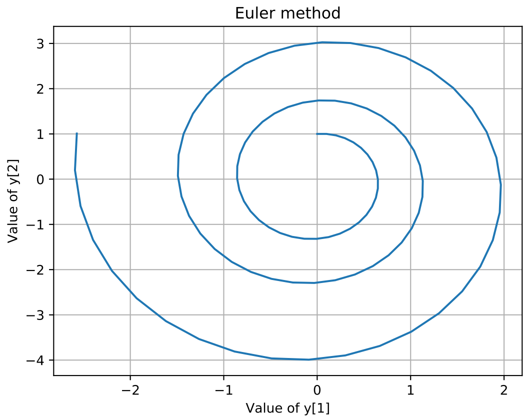
7.20000	0.05377	3.02614
7.30000	0.35638	3.00933
7.40000	0.65732	2.89796
7.50000	0.94711	2.69255
7.60000	1.21637	2.39658
7.70000	1.45603	2.01646
7.80000	1.65767	1.56145
7.90000	1.81382	1.04343
8.00000	1.91816	0.47661
8.10000	1.96582	−0.12281
8.20000	1.95354	−0.73713
8.30000	1.87983	−1.34761
8.40000	1.74507	−1.93506
8.50000	1.55156	−2.48039
8.60000	1.30352	−2.96526
8.70000	1.00700	−3.37261
8.80000	0.66974	−3.68729
8.90000	0.30101	−3.89659
9.00000	−0.08865	−3.99065
9.10000	−0.48772	−3.96295
9.20000	−0.88401	−3.81053
9.30000	−1.26507	−3.53428
9.40000	−1.61849	−3.13895
9.50000	−1.93239	−2.63317
9.60000	−2.19571	−2.02930
9.70000	−2.39863	−1.34314
9.80000	−2.53295	−0.59357
9.90000	−2.59231	0.19798
10.00000	−2.57251	1.00808

I have used the following Python code [1] to evaluate values and plot the graph.

```
#  Solves a Cauchy problem for a 2nd order ODE by Euler's method
#                 y" + 1/5y'+3.125y = 0, y(0) = 0, y'(0) = 1
#  Equivalent problem: y[1] = y, y[2] = y'
#     y[1]' =  y[2],     y[1](0) = 0
#     y[2]' = -3.125y[1]-1/5y[2],     y[2](0) = 1
#----------------------------------------------------------------------------
from math import *
from ode import *
from matplotlib import pyplot as plt

def Func(t, y, f):                                    # Right-hand sides of ODEs
        f[1] =  y[2]
        f[2] = -3.125*y[1]-1/5*y[2]

# main

y0 = 0; dy0 = 1                          # initial values => y(t) = sin(t)
```

```
tmax = 10e0
# time span
ht = 0.1e0
# step size

n = 2                                         # number of 1st order ODEs
nt = int(tmax/ht + 0.5) + 1                   # number of time steps
y = [0]*(n+1)                                 # solution components

out = open("ode.txt","w")
# open output file
out.write("      t          y1         y2\n")

t = 0e0
y_1 = []
y_2 = []

y[1] = y0; y[2] = dy0
# initial values
out.write(("{0:10.5f}{1:10.5f}{2:10.5f}\n"). \
          format(t,y[1],y[2]))

y_1.append(y[1])
y_2.append(y[2])

while (t+ht <= tmax):
# propagation loop
    Euler(t,ht,y,n,Func)
    t += ht

    y_1.append(y[1])
    y_2.append(y[2])

    out.write(("{0:10.5f}{1:10.5f}{2:10.5f}\n"). \
              format(t,y[1],y[2]))
out.close()

plt.plot(y_1, y_2)
plt.title( "Euler method")
plt.xlabel('Value of y[1]')
plt.ylabel('Value of y[2]')
plt.legend(loc=4)
plt.grid()
plt.savefig( '1_1.eps', fmt='EPS', dpi=100 )
plt.show()
```

Plot for these values will be as follows:



2 Solve the following Boundary Value Problem:

2.1 Description

$y'' + (x-1) \times y' + 3.125 \times y = 4 \times x$
$y(0) = 1$
$y(1) = 1.368$

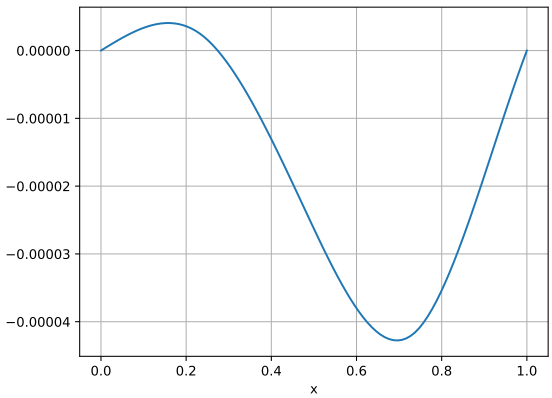
2.2 Solution

I have used the following Python code [2] to evaluate values and plot the graph.

```
#       $y''+ (x-1)y'+ 3.125y = 4x$\\
#       y(0) = 1
#       y(1) = 1.368
#       y1' = y2
#       y2' = 4x-(x-1)y2 - 3.125y1

import numpy as np
from scipy.integrate import solve_bvp
import matplotlib.pyplot as plt

def fun(x, y, p):
        x = p[0]
        return np.vstack((y[1], 4*x-(x-1)*y[1] - 3.125*y[0]))

def bc(ya, yb, p):
        k = p[0]
        return np.array([ya[0], yb[0], ya[1] - k])

x = np.linspace(0, 1, 5)
y = np.zeros((2, x.size))
y[0, 1] = 1
y[0, 3] = 1.3
```

```
sol = solve_bvp(fun, bc, x, y, p=[5])
print sol.p[0]

x_plot = np.linspace(0, 1, 100)
y_plot = sol.sol(x_plot)[0]
plt.plot(x_plot, y_plot)
plt.grid()
plt.xlabel("x")
plt.ylabel("y")
plt.show()
```

Plot for these values will be as follows:



References

[1] Introduction to Numerical Programming,
`http://phys.ubbcluj.ro/ tbeu/INP/programs.html`

[2] scipy.integrate.solve-bvp,
`https://docs.scipy.org/doc/scipy-0.18.1/reference/generated/scipy.integrate.solve-bvp.h`