25 - VI - 2022
SITGES

TASKS

(1) COMPLEX NUMBERS $\to$ study book

complex number algebra

$$a, b \in \mathbb{C} \quad \to \quad a + b \in \mathbb{C}$$
$$a \cdot b \in \mathbb{C}$$

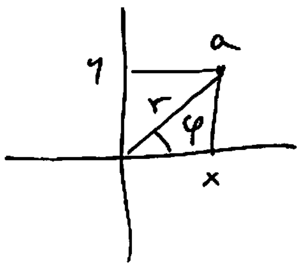
$$a \in \mathbb{C} \quad \to \quad a = x + iy, \quad x, y \in \mathbb{R}$$

$$i^2 = -1 \qquad b = x' + iy'$$

$$a + b = (x + x') + i(y + y')$$

$$a \cdot b = (xx' - yy') + i(xy' + x'y)$$

$a \cancel{b} \in \mathbb{C}$



$$a = x + iy = r e^{i\varphi}$$

$$r = \sqrt{x^2 + y^2} \qquad r = |a|$$

complex modulus

$$\varphi = tg^{-1}(y/x)$$

$$e^{i\varphi} = \cos\varphi + i \sin\varphi \quad \leftarrow \text{Euler formula}$$

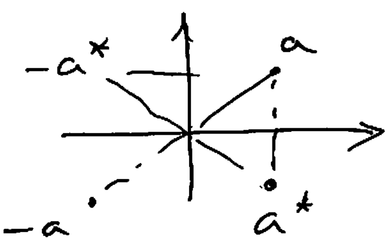
$$x = r\cos\varphi$$
$$y = r\sin\varphi$$

$$a = x + iy \qquad (e^{i\varphi})^* = e^{-i\varphi}$$

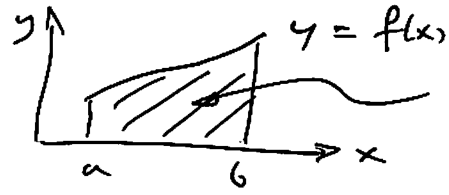
$$a^* = x - iy$$

(complex conjugate)

$$a a^* = |a|^2$$



② Definite integrals



area $= \int_a^b dx\, f(x)$

Indefinite integrals

$$\int dx\, f(x) = F(x) + C$$

$$F'(x) = f(x) \qquad (C' = 0)$$

$$\int_a^b f(x)\, dx = F(b) - F(a) = F(x)\Big|_a^b$$

→ properties

→ simple integrals $\qquad \int dx\, x^n = \dots$

$\int dx\, \cos x = —$

$\boxed{\text{book}}$

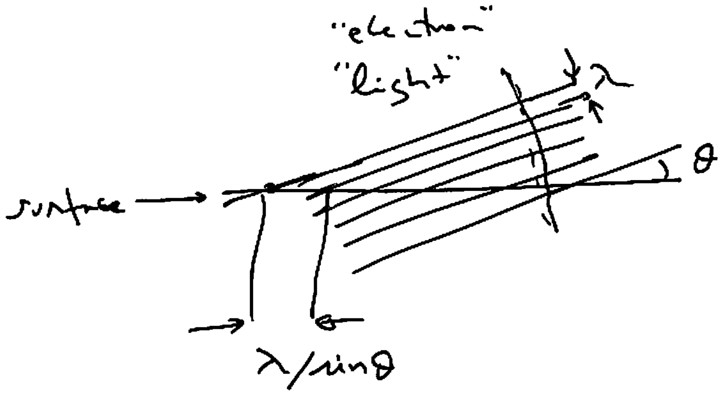
③ → $\boxed{\text{Abbe's diffraction limit}}$

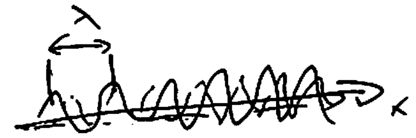
→ wave diffraction / waves

wave interference

→ ("de Broglie")

④ study the following example: formation of a focal spot

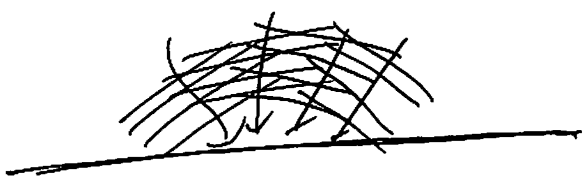




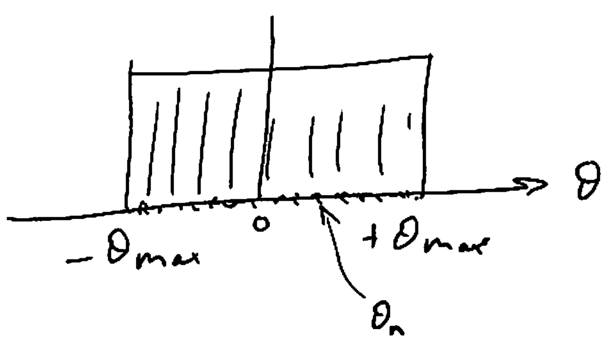
$\cos \frac{2\pi x}{\lambda}$

$\sin \frac{2\pi x}{\lambda}$

$\left(\cos\left(\theta + \frac{\pi}{2}\right) = -\sin \theta\right)$



$$f(x) = \frac{1}{N} \sum_{n=0}^{N-1} \cos\left[\frac{2\pi x}{(\lambda/\sin\theta_n)}\right]$$

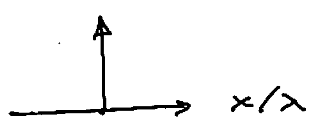


$$\theta_n = -\theta_{max} + \Delta\theta\,(n + 1/2)$$

$$\Delta\theta = \frac{2\theta_{max}}{N}$$

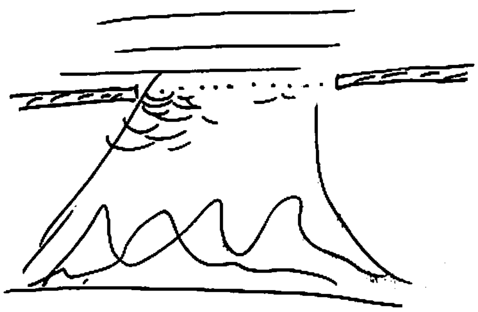
plot $f(x)$ for $N = \underbrace{10, 100, 1000}$

$\theta_{max} = \frac{\pi}{20}, \frac{\pi}{10}, \frac{\pi}{5}, \frac{\pi}{3}$



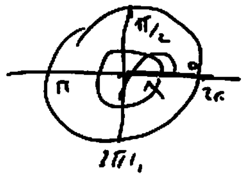
(41) Connect with double ~~s~~lit, etc.

"Huyens"
↳ wiki



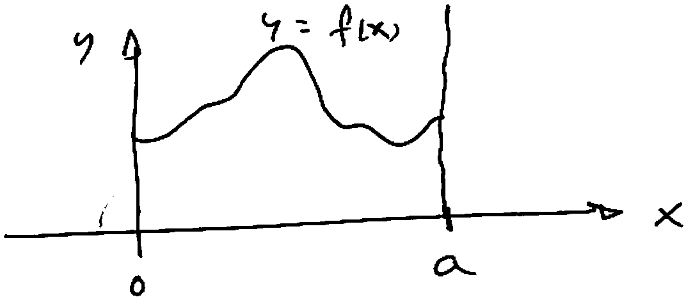
Fourier analysis

$$e^{i\varphi} = \cos\varphi + i\sin\varphi$$



$$e^{i\,2\pi n} = \cos(2\pi n) + i\sin(2\pi n) = 1$$

$n \in \mathbb{Z}$



$$\boxed{f(x) = \sum_n f_n \, e^{i\,2\pi n\,x/a}}$$

$$\int_0^a dx \; e^{2\pi i\, m\,x/a} \underset{\downarrow}{\overset{m\neq 0}{=}} \left.\frac{e^{2\pi i m\,x/a}}{(2\pi i m/a)}\right|_0^a$$

$$= \frac{1}{(2\pi i m/a)}\left(\underbrace{e^{2\pi i m}}_{1} - 1\right)$$

$$= 0$$

$\cancel{\int_0 dx\, e^{\alpha x} = \frac{e^{\alpha x}}{\alpha} + C}$
$(\alpha = 2\pi i m/a)$

$$(e^x)' = e^x$$
$$(e^{\alpha x})' = \alpha\, e^{\alpha x}$$

(δ of Kronecker)

$$\frac{1}{a}\int_0^a dx \; e^{2\pi i m\,x/a} = \delta_{m,0}$$

exercise

$$\boxed{f_n = \frac{1}{a}\int_0^a dx \; f(x)\, e^{-i\,2\pi n\,x/a}}$$