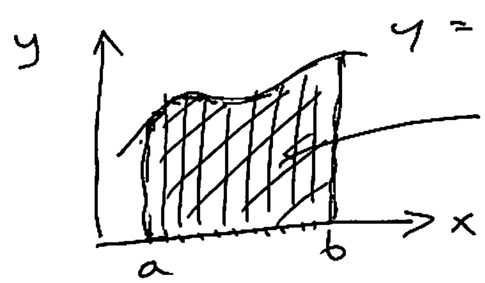
23 - VI - 2022
SITGES

- → Integrals
- → Complex numbers
- → Fourier analysis

Integrals



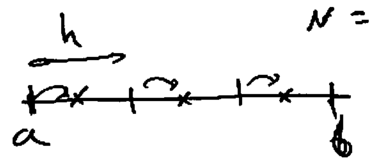
$$\text{Area} = \underbrace{\int_a^b dx\, f(x)}_{} \circledcirc$$

$$\circledcirc \underbrace{\sum_{i=0}^{N-1} h\, f(x_i)}_{N\to\infty}$$

trapezoid rule

$$x_i = \frac{(i+1/2)}{N}(b-a) + a$$

$i = 0, \ldots, N-1$

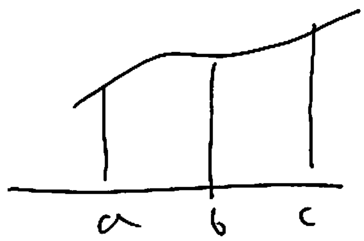


$$h = \frac{b-a}{N}$$

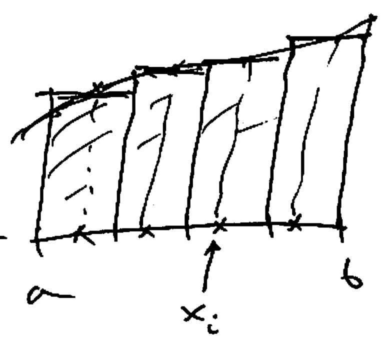
$$x_i \circledcirc a + i\,h + 0.5h$$

$i = 0, 1, 2$

$$= a + (i+0.5)\underbrace{\frac{b-a}{N}}_{h}$$



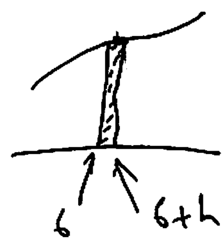
$$\int_a^c dx\, f(x) = \int_a^b dx\, f(x) + \int_b^c dx\, f(x)$$



$$\boxed{\frac{d}{db}\left[\int_a^b dx\, f(x)\right]} \circledcirc$$

$$\circledcirc \lim_{h\to 0} \frac{1}{h}\left[\int_a^{b+h} dx\, f(x) - \int_a^b dx\, f(x)\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\int_b^{b+h} dx\, f(x) = \cancel{\frac{h}{h}}\boxed{f(b)}$$

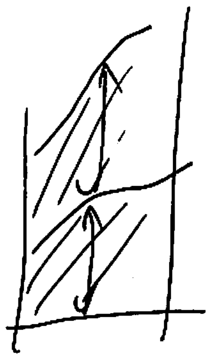


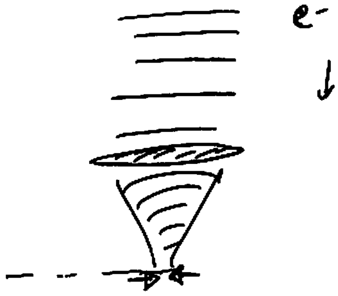
integration by parts

$$(f(x)\ g(x))' = f'(x)\ g(x) + f(x)\ g'(x)$$

$$\int_a^b f'(x)\, g(x)\, dx = \int_a^b (f(x)\ g(x))'\, dx - \int_a^b f(x)\, g'(x)\, dx$$

$$= \underbrace{f(x)\ g(x)\Big|_a^b}_{f(b)\, g(b) \,-\, f(a)\, g(a)} - \int_a^b f(x)\, g'(x)\, dx$$

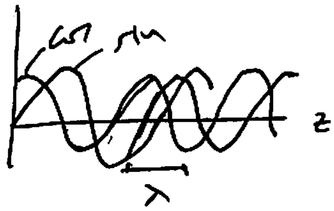


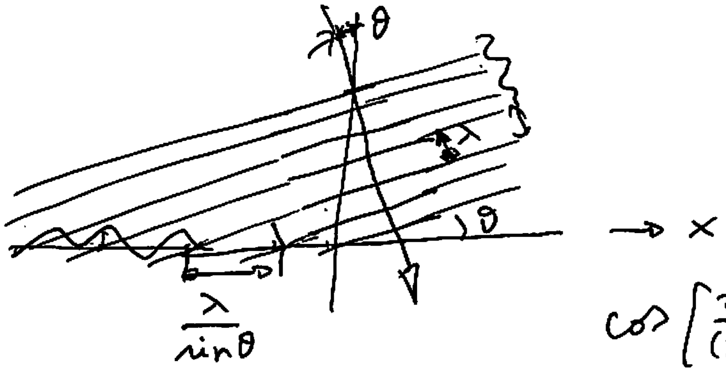


$\cos\left(\frac{2\pi z}{\lambda}\right)$

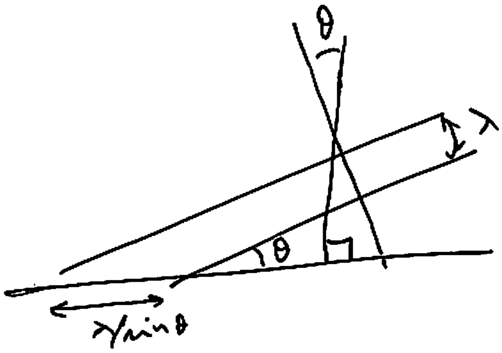
$\sin\left(\frac{2\pi z}{\lambda}\right)$

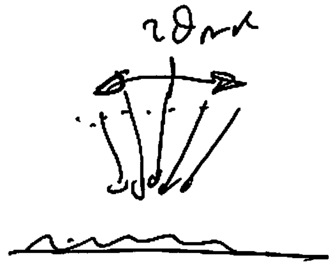
Q.M. $\rightarrow$ particle $\leftrightarrow$ wave
λ

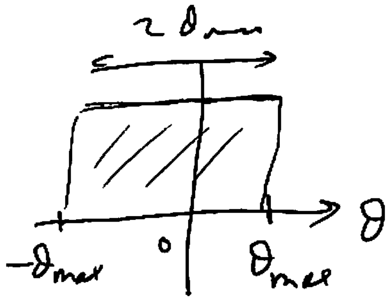




$\cos\left[\frac{2\pi x}{(\lambda/\sin\theta)}\right]$





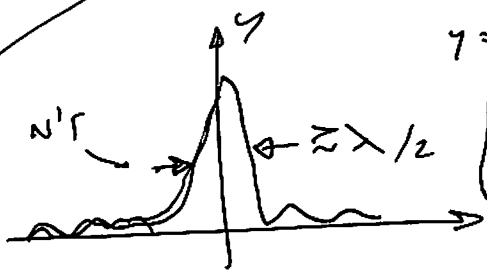


$$f(x) = \frac{1}{N}\sum_{n=0}^{N-1} \cos\left[\frac{2\pi x}{(\lambda/\sin\theta_n)}\right]$$

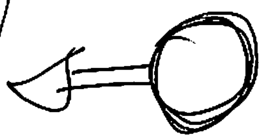
$$\theta_n = -\theta_{max} + \left(n+\tfrac{1}{2}\right)\Delta\theta$$

$$\Delta\theta = \frac{2\theta_{max}}{N}$$

$y = f(x)$



Abbe's diffraction limit



$N = 100, 1000$ / $\theta_{max} = \frac{\pi}{5}$, $\frac{\pi}{3}$ ⋮

$$\cos\left[\left(2\pi \sin\theta_n\right)\left(\frac{x}{\lambda}\right)\right]$$