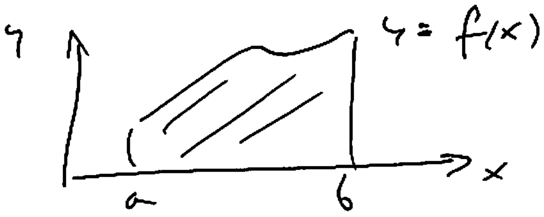
PLAN FOR RADI'S COURSE

11-VI-2022
SITGES

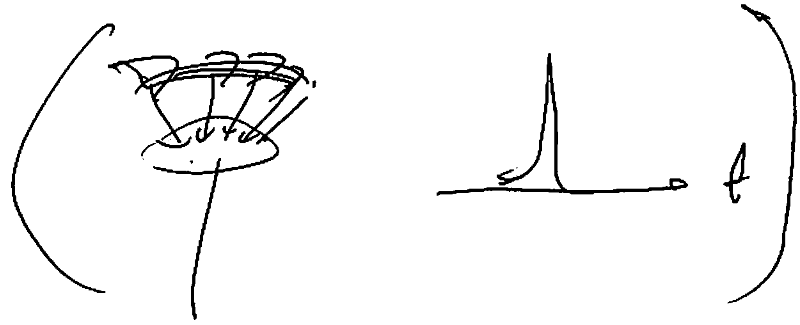
<u>def.</u>: $\int f(x)\,dx = F(x) + C$ $\quad F'(x) = f(x)$

$\longrightarrow$ $\int_a^b f(x)\,dx = F(b) - F(a) = F(x)\Big|_a^b$



$\longrightarrow$ $\mathbb{C}$ . . .

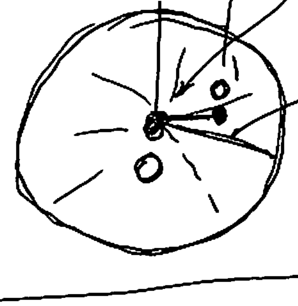
$\longrightarrow$ $\left(\int dz\, f(z)\right)$ $\underline{z \in \mathbb{C}}$ $\longrightarrow$



$\longrightarrow$ special relativity theory

O:

$\rightarrow vt \leftarrow$ light burst at $t=0$



$\underline{r = c\,t}$

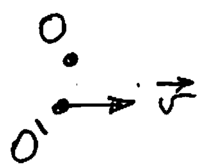
t

O':

t'

$c \rightarrow$ light speed in vacuum

$t=0$



Complex numbers intro

11-VI-2022
ПТ GES

$$x^2 + 1 = 0 \rightarrow x = \sqrt{-1} = \pm i$$

$$\boxed{i^2 = -1} \rightarrow i \rightarrow \text{imaginary unit} \qquad (\sqrt{4} = \pm 2)$$

$$z = \underbrace{a}_{\text{real part}} + i \underbrace{b}_{\text{imaginary part}} \qquad a, b \in \mathbb{R}, \quad z \in \mathbb{C}$$

$$z' = a' + i b'$$

$$\rightarrow z z' = (a + ib)(a' + ib')$$
$$\overset{!}{=} aa' + \underbrace{i^2}_{-1} bb' + i(ab' + a'b)$$
$$\overset{!}{=} \underbrace{(aa' - bb')}_{\in \mathbb{R}} + i \underbrace{(ab' + a'b)}_{\in \mathbb{R}} \in \mathbb{C}$$

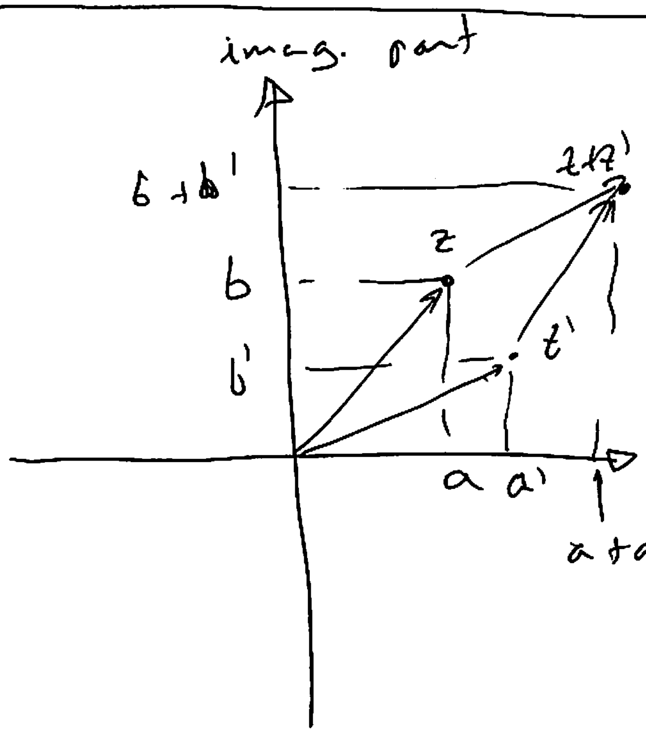
$$\rightarrow z \pm z' = (a \pm a') + i(b \pm b') \in \mathbb{C}$$

$$x^2 + 1 = (\underset{}{\overset{x = i}{x - i}})(\overset{x = -i}{x + i})$$
$$\overset{!}{=} x^2 - i^2 = x^2 + 1 \quad \underline{\underline{\text{OK}}}$$

$$a x^2 + bx + c = 0 \rightarrow x = \frac{-b}{2} \pm \frac{1}{2}\sqrt{\underbrace{b^2 - 4ac}_{< 0}} \rightarrow$$
$$\overset{!}{=} -\frac{b}{2} \pm i \, \frac{1}{2}\sqrt{4ac - b^2}$$

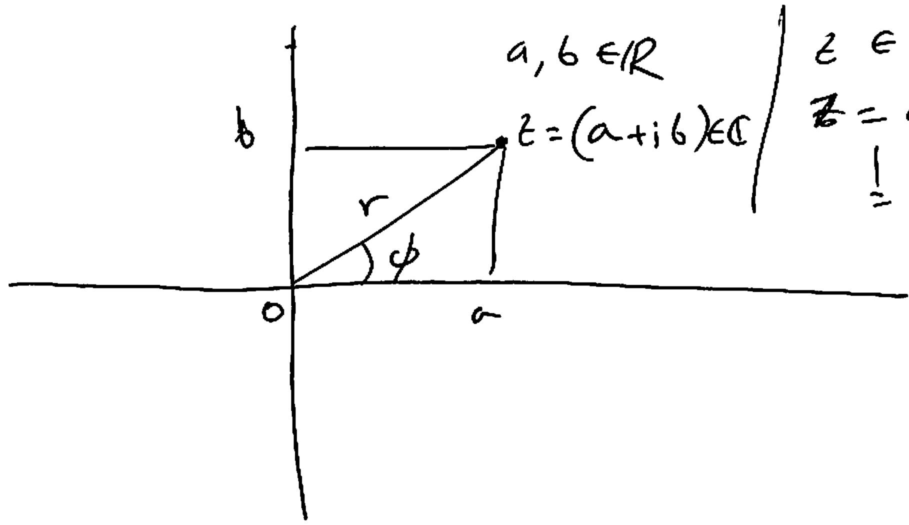
$$(\sqrt{-4} = \pm i \sqrt{4}) \longrightarrow$$

Complex number representation



$z \in \mathbb{C}$

$\boxed{z = a + ib}$



$z \in \mathbb{C}$

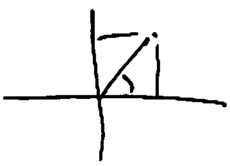
$z = a + ib \boxed{= r e^{i\phi}}$

$\overset{!}{=} r\cos\phi + i r \sin\phi$

$r = \sqrt{a^2 + b^2}$

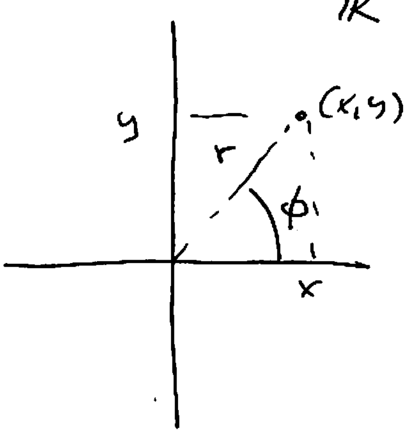
$a = r\cos\phi$

$b = r\sin\phi$



$$\boxed{e^{\pm i\phi} = \cos\phi \pm i \sin\phi}$$

$\mathbb{R}^2$

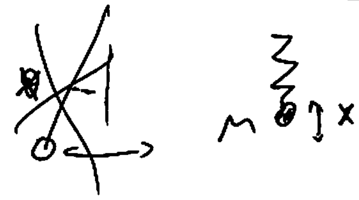


$r = \sqrt{x^2 + y^2}$ $\qquad x = r\cos\phi$

$\phi = tg^{-1}\left(\frac{y}{x}\right)$ $\qquad y = r\sin\phi$

C++ "ϕ = tan2(y, x);"

$\boxed{\text{Oscillations with complex numbers}}$



$$M \frac{d^2 x}{dt^2} = -kx - \gamma' \frac{dx}{dt}$$

$$\frac{d^2 x}{dt^2} = -\overbrace{\boxed{\frac{k}{M}}}^{\cancel{=\omega_0^2}} x - \overbrace{\boxed{\frac{\gamma'}{M}}}^{=\gamma} \frac{dx}{dt}$$

$$\boxed{x = e^{-i\omega t}}$$

$$\frac{dx}{dt} = -i\omega e^{-i\omega t}$$

$$\frac{d^2 x}{dt^2} = (-i\omega)^2 e^{-i\omega t} = -\omega^2 e^{-i\omega t}$$

$$-\omega^2 \cancel{e^{i\omega t}} = -\frac{k}{M} \cancel{e^{i\omega t}} + \gamma i\omega \cancel{e^{i\omega t}}$$

$$\omega^2 + i\omega\gamma - \boxed{\frac{k}{M}} = 0$$

$\gamma = 0$ $\quad \omega^2 = \frac{k}{M} = \omega_0^2, \quad \omega_0 = \sqrt{\frac{k}{M}}$

$$\underline{\underline{\omega}}^2 + i\underline{\underline{\omega}}\gamma - \omega_0^2 = 0 \qquad (i\gamma)^2 = -\gamma^2$$

$$\omega = \frac{-i\gamma \pm \sqrt{4\omega_0^2 - \gamma^2}}{2}$$

$\gamma, \omega_0 > 0$

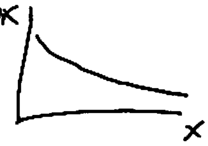
$$\Rightarrow \quad \gamma^2 < 4\omega_0^2 \to \boxed{\gamma < 2\omega_0} \Rightarrow \sqrt{\ } \in \mathbb{R}$$

$$x = e^{-iat} = \exp\left\{-\frac{\gamma}{2}t \mp i\sqrt{\omega_0^2-\left(\frac{\gamma}{2}\right)^2}\,t\right\}$$

$$= \underbrace{e^{-\gamma t/2}}_{\xrightarrow{t\to\infty} 0}\; e^{\mp i \underbrace{\sqrt{\omega_0^2-(\gamma/2)^2}}\, t}$$

$(e^{A+B} = e^A e^B)$

$\to \cos\sqrt{\ }\,t$
$\sin\sqrt{\ }\,t$



$$\Rightarrow \quad \gamma > 2\omega_0 \to \sqrt{\ } \notin \mathbb{R}$$

$$\to \sqrt{\omega_0^2-(\gamma/2)^2} = i\sqrt{(\gamma/2)^2-\omega_0^2}$$