Answer

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1st Chapter Problem

1.1.

$$\begin{aligned}\gamma_{-k} :=& \mathrm{Cov}(y_t, y_{t-(-k)}) \\ =& \mathrm{Cov}(y_t, y_{t+k}) \\ =& \mathrm{Cov}(y_{t+k}, y_t) \\ =& \mathrm{Cov}(y_{(t+k)}, y_{(t+k)-k}) = \gamma_k . \square\end{aligned}$$

1.2.

$$\begin{aligned} E(y_t) = \mu + E(\epsilon) = \mu. \\ \mathrm{Cov}(y_t, y_{t-k}) = E(\epsilon_t \epsilon_{t-k}) = \gamma_k. \\ \gamma_k = 0, \forall k \in \mathbb{N}. \qquad \square\end{aligned}$$

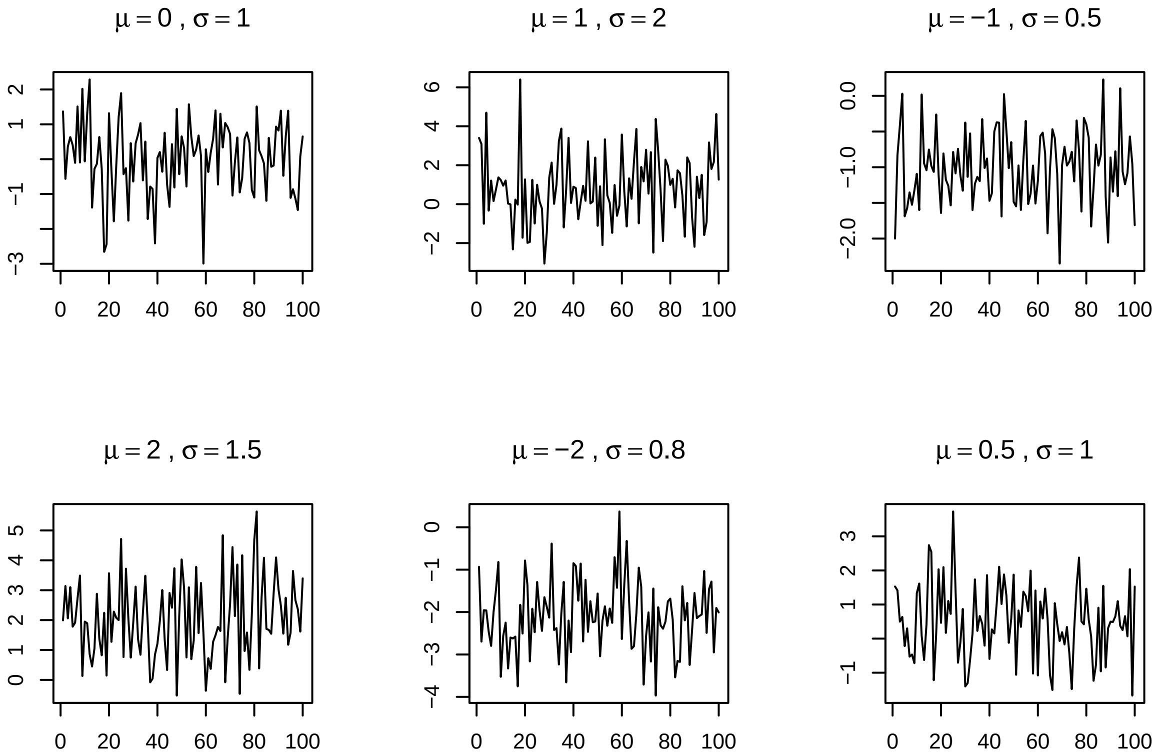
1.3.

```r
set.seed(42)

par(mfrow=c(2,3))

n <- 100
mu <- c(0, 1, -1, 2, -2, 0.5)
sigma <- c(1, 2, 0.5, 1.5, 0.8, 1)

for (i in 1:6) {
  eps <- rnorm(n, mean=mu[i], sd=sigma[i])
  time_axis <- 1:n
  plot(
    time_axis, eps,
    main=bquote(mu == .(mu[i]) ~ "," ~ sigma == .(sigma[i])),
    type="l", xlab = "", ylab = "")
}
```



1.4.

At first, I think the answer is Auto-regressive Process. However, this process have the same expectation which doesn't depend on time.