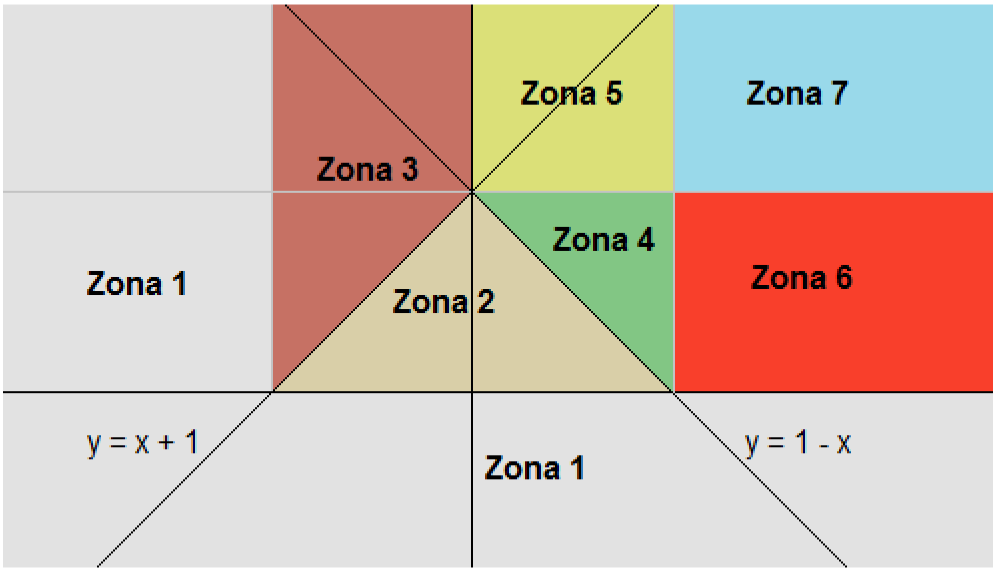
CÁLCULO DE PROBABILIDADES II, GRADO EN ESTADÍSTICA

RELACIÓN 1, EJERCICIO 4: FUNCIÓN DE DISTRIBUCIÓN

$$F(x_0, y_0) = \begin{cases} 0 & x_0 < 1 \text{ ó } y_0 < 0 \ (\mathbf{1}) \\ \int_0^{y_0} \int_{y-1}^{x_0} 1 dx dy = x_0 y_0 - \frac{y_0^2}{2} + y_0 & y_0 - 1 \leq x_0 < 1 - y_0, 0 \leq y_0 < 1 \ (\mathbf{2}) \\ \int_{-1}^{x_0} \int_0^{x+1} 1 dy dx = \frac{x_0^2+1}{2} + x_0 & -1 \leq x_0 < 0, y_0 \geq x_0 + 1 \ (\mathbf{3}) \\ \int_0^{y_0} \int_{y-1}^{1-y_0} 1 dx dy + \int_{1-y_0}^{x_0} \int_0^{1-x} 1 dy dx = & \\ = 2y_0 - y_0^2 + x_0(1 - \frac{x_0}{2}) - \frac{1}{2} & 0 \leq x_0 < 1, 1 - x_0 \leq y_0 < 1 \ (\mathbf{4}) \\ \int_{-1}^{0} \int_0^{x+1} 1 dy dx + \int_0^{x_0} \int_0^{1-x} 1 dy dx = \frac{1}{2} + x_0 - \frac{x_0^2}{2} & 0 \leq x_0 < 1, y_0 \geq 1 \ (\mathbf{5}) \\ \int_0^{y_0} \int_{y-1}^{1-y} 1 dx dy = 2y_0 - y_0^2 & x_0 \geq 1, 0 \leq y_0 < 1 \ (\mathbf{6}) \\ 1 & x_0 \geq 1, y_0 \geq 1 \ (\mathbf{7}) \end{cases}$$



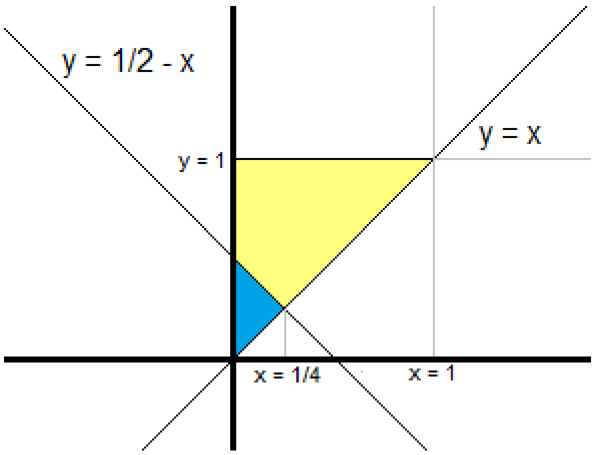
RELACIÓN 1, EJERCICIO 5: RESOLUCIÓN COMPLETA

- Constante k:

$$\int_{-\infty}^{+\infty} f(x,y)dx\,dy = 1 \Rightarrow \int_0^1 \int_x^1 \frac{k}{\sqrt{xy}}dy\,dx = \int_0^1 \frac{k}{\sqrt{x}}\left[2\sqrt{y}\right]_x^1 dx = \int_0^1 \frac{k}{\sqrt{x}}(2-2\sqrt{x})dx = \int_0^1 \frac{2k}{\sqrt{x}} - 2k\,dx =$$

$$= 2k\int_0^1 \frac{1}{\sqrt{x}}dx - 2k\int_0^1 1dx = 2k\left[2\sqrt{x}\right]_0^1 - 2k = 4k - 2k = 2k = 1 \Rightarrow k = \frac{1}{2}$$

- $P[X+Y \leq 1/2]$:



$$P(X+Y\leq 1/2) = P(Y \leq 1/2 - X) = \int_{-\infty}^{+\infty}\int_{-\infty}^{1/2-x} \frac{1}{2\sqrt{xy}}dy\,dx = \int_0^{1/4}\int_x^{1/2-x} \frac{1}{2\sqrt{xy}}dy\,dx =$$

$$= \int_0^{1/4} \frac{1}{2\sqrt{x}} \int_x^{1/2-x} \frac{1}{\sqrt{y}}dy\,dx = \int_0^{1/4} \frac{1}{2\sqrt{x}}\left[2\sqrt{y}\right]_x^{1/2-x} dx = \int_0^{1/4} \sqrt{\frac{1/2-x}{x}} - 1dx =$$

$$= \int_0^{1/4} \sqrt{\frac{1/2}{x} - 1} - 1\,dx \overset{(*)}{\approx} 0.3927$$

(*) Para resolver esta integral sería necesario hacer el cambio de variable $u = \frac{1/2}{x}$.

- Función de distribución del vector (X, Y):

$$F(x_0,y_0)=\begin{cases} 0 & x_0<0 \text{ ó } y_0<0\ (\mathbf{1}) \\ \int_0^{x_0}\int_x^{y_0}\frac{1}{2\sqrt{xy}}dy\,dx=\int_0^{x_0}\frac{1}{2\sqrt{x}}\left[2\sqrt{y}\right]_x^{y_0}dx= & \\ =\int_0^{x_0}\frac{\sqrt{y_0}}{\sqrt{x}}-1dx=2\sqrt{x_0y_0}-x_0 & 0\le x_0<1, x_0\le y_0<1\ (\mathbf{2}) \\ \int_0^{y_0}\int_0^{y}\frac{1}{2\sqrt{xy}}dxdy=\int_0^{y_0}\frac{1}{2\sqrt{y}}\left[2\sqrt{x}\right]_0^{y}dy=y_0 & x_0\ge y_0, 0\le y_0<1\ (\mathbf{3}) \\ \int_0^{x_0}\int_x^{1}\frac{1}{2\sqrt{xy}}dydx=\int_0^{x_0}\frac{1}{2\sqrt{x}}\left[2\sqrt{y}\right]_x^{1}dx=2\sqrt{x_0}-x_0 & 0\le x_0<1, y_0\ge 1\ (\mathbf{4}) \\ 1 & x_0\ge 1, y_0\ge 1\ (\mathbf{5}) \end{cases}$$