Wireless Mid-range Power Transfer: Design Challenges

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***Abstract*—This report elaborates Wireless Power Transfer (WPT) with which has regained attention in last 10-15 years; due to proliferation of portable devices operated on battery. A brief history of attempts made so far in the field of WPT is presented. More elaborate treatment is given for WPT systems designed for Battery powered vehicles (BPVs). These pose challenges because of increased power levels (several kW), higher target efficiencies (>90%) and stronger constraints on coil design. In particular, design of coils for WPT systems is studied in detail. The *bifurcation-free* design of coils is verified by FE simulations and the results are presented. Open ended issues in the research of these systems are enumerated at the end.**

***Index Terms*—Wireless Power Transfer, Battery Charging, IPT**

I. Introduction

Wireless power transfer (WPT) has been a topic of investigation of electrical engineers and scientists since almost 120 years. Experimental investigation into WPT began with Tesla [1]. It has gained an increased attention since past 10-15 years. As the name indicates the systems consist of a transmitter of power and a receiver of power which are not connected by wires. WPT systems are mainly classified by two criteria - a) Type of coupling - inductive or electrostatic and b) Phenomenon of coupling - radiative or non-radiative. It is also classified as short-range, mid-range and long-range. The term ”radiative” is used for frequencies of the order of GHz and distance is in kilometers. One of its main applications is powering of satellites [2]. This report mainly focuses on *near-field inductive* type of WPT systems.

A. Brief History of WPT systems

The discovery of electromagnetic induction is attributed to Faraday and Henry. They both discovered it almost simultaneously in 1832. Maxwell predicted the existence of electromagnetic fields in 1862. Tesla extensively experimented with wireless transfer of power using resonant inductive coupling. He also built the famous Wardenclyffe Tower to transmit power across continents, which could not be completed. One of his famous inventions - the “Tesla Coils” are resonant transfomers but operating at high voltage and low currents besides high frequencies. The induction motor also involves in a way transfer of power without wires across the air-gap between stator and rotor. However, the topic never gained momentum from commercial perspective later, except for induction heating application. Since 1960s increased attention was given to inductive power transfer systems (IPTs) as they are used in medical implants especially in pacemakers and insulin pumps [3]. Noteworthy efforts have also gone into other types of systems like electrostatic coupling, dynamic capacitive [4]. A recent paper by Joannopoulo, Soljacic and Karalis from MIT, have proposed a generic theory to model transfer of power between systems operating at resonance and which are coupled without wires [5]. Besides magnetically coupled systems, the paper has also discussed dielectric disks and capacitively loaded conducting wire loops.

The principle of induction is used for making cookers on commercial scale since 1970s. The same principle is used to transfer power without wires. With advent of fast switching power electronic devices, the WPT systems were investigated more closely for industrial applications. They were typically applied in industries for loads such as lighting, instrumentation for electronic systems, clean factories, etc [6]. The motivation behind this development was primarily to develop a charging system which will operate in adverse weather and environmetal conditions (including dust, ice etc.) [6]. It should be noted both static and dynamic charging systems were being investigated. Monorails and automatic guided vehicles are examples of dynamic systems.

Use of laptops, mobile phones and other battery powered *portable* devices has multiplied several times since 1990’s. By the time these gadgets were proliferated worldwide, the investigations of WPT for medical systems had also reached certain matured stage. This triggered R and D activities for application of WPTs to charging devices for such portable gadgets [3], [7]. Wireless Power Consortium was formed in 2008. They have formulated famous Qi standards which are the leading set of standards available today. Several commercial products are available in market today, an example of which is found in [8]. Thus, it can be said that WPT systems for these portable gadgets has reached first phase of maturity. The typical air-gap between the transmitter and receiver coil for such applications is very small i.e. of the order of few mm *(2 to 10 mm)* and the efficiency of the systems is lesser i.e. from 40 to 70 % [3].

B. Motivation for the topic today

In past few years, vehicle OEMs (Original Equipments Manufacturers) have shifted focus from conventional IC (Internal Combustion) Engine powered vehicles (ICV) to Battery Powered Vehicles (BPV) due to depletion of fossil fuels and problem of pollution. Wireless charging can be used as an alternative to plug-in type of BPVs. There are three advantages of these systems -

1) Lower sized battery management systems within vehicle
2) Immunity to adverse weather and environment conditions
3) Effective utilization of time e.g. during temporary halts of public vehicles

Parameter	BPV requirements	Commercially available products
Air gap	150 to 300 mm	4 - 10 mm
Power	few kW to 50 kW	< 500 W
Efficiency	>90%	approx 70%

TABLE I: BPV requirements vs commercial feasibility as of today

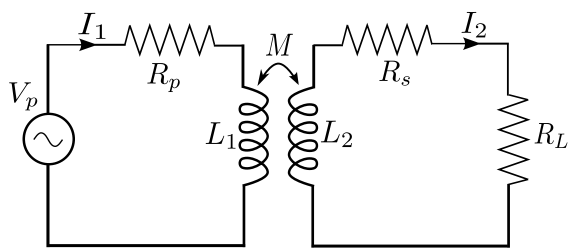


Fig. 1: Central circuit of WPT system

The engineering specifications of WPT systems required for BPVs differ from those that are commerciallly feasible today. This is clearly pointed out in table I. Further, the issue of misalignment is also cruicial in case of vehicles because of large air-gap and insufficient visual feedback indicating the alignment of transmitter and receiver coils. It is for the particular separation of coils ($\sim$ 150 mm) that the term *mid-range* is used for WPT systems.

C. Focus and overview of the report

To this end, this report focuses on WPT systems targeted for BPVs. The following section will elaborate on basic topologies used for wireless charging systems. In particular, the author has studied design methods proposed by S Chopra [9] and [10]. The methods will be discussed in section III. The next section throws some light on fundamentals of the WPT systems. With the methods explained in [9], [10] a sample system is considered for design of transmitter and receiver coils. The coil design is carried out by means of closed form solution for inductance as well as FEA (Finite Element Analysis) software. Last section before conclusion discusses open issues in this area and scope for research work.

II. WPT Fundamentals

An central part of an IPT system has two or more coils which are separated by air. In other words, IPT system can be looked at as an air-cored transformer. As air has relative permeability μ_0 of 1, the inductance of air-cored transformer is very less compared to the usual transformer with a core made up of magnetic material. If M is mutual inductance between two coils and L_p and L_s are self inductances of primary and secondary coils respectively, then mutual coupling coefficient, k, expressed by (1) is of the order of 0.05-0.3 in case of air cored transformer.

$$M = k\sqrt{L_p L_s} \tag{1}$$

Consider two inductive coils separated by air and the secondary is connected to a load resistance R_L. The governing equations in frequency domain are -

$$V_p = jX_{L_p} I_1 - jX_M I_s \tag{2}$$

$$jX_M I_1 = jX_{L_p} I_2 + I_p R_L \tag{3}$$

However, physical conductors used to make the coils have a finite resistance as shown in Fig. 1 and hence the equations get modified as -

$$V_p = R_p I_1 + jX_{L_p} I_1 - jX_M I_2 \tag{4}$$

$$jX_M I_1 = R_s I_2 + jX_{L_s} I_2 + I_2 R_L \tag{5}$$

where R_p and R_s represent the resistances of primary and secondary coils respectively. Efficiency η of this system is given by [9]

$$\eta = \frac{R_L}{(R_L + R_s)\left(1 + \frac{R_p(R_s+R_L)}{\omega^2 M^2}\right) + R_p\left(\frac{L_s}{M}\right)^2} \tag{6}$$

Maximum efficiency is possible only for a frequency given by

$$\omega_e >> \frac{\sqrt{R_p\left(R_s + R_L\right)}}{M} \tag{7}$$

And maximum efficiency is given by

$$\eta_{max} = \frac{R_L}{R_L + R_s + R_p\left(\dfrac{L_s}{M}\right)^2} \tag{8}$$

A. System architecture and topologies

Equation (7) suggests that the frequency of operation needs to be set above ω_e for higher efficiency. As it will be seen in section IV , the mutual inductance of the two coils is of the order of few μH. Consequently, the required frequencies for high efficiency operation are much higher than available utility power frequency. Higher frequency of operation is also desirable from control point of view. Hence complete WPT system architecture, from utility supply to the load, is as shown in Fig. 2. The Figure depicts a WPT system used for battery charging of BPVs, for example. It should be noted that this involves additional converters C1, C2 on the input side and C3 on the secondary side. C1 can be a diode rectifier. C2 and C3 are high frequency inverters and require high frequency switches. As frequency increases the inductive reactance increases which results in poor power factor. This consequently results in increased VA rating of the power electronic inverter required at the input. The power factor can be improved by connecting capacitances in primary and secondary circuits. Thus, by operating at almost unity power factor, the VA ratings of power electronic converters C1 and C2 can be reduced. Consequently, four circuit topologies are suggested [3], [9] as follows. The topologies are also depicted in Fig. 3.

1) Series - Series (SS)
2) Series - Parallel (SP)
3) Parallel - Series (PS)
4) Parallel - Parallel (PS)

Here, capacitances are so chosen that they form resonant circuit with the primary and secondary coils of the system. A particular application can demand a particular type of topology. For example, for battery charging application there

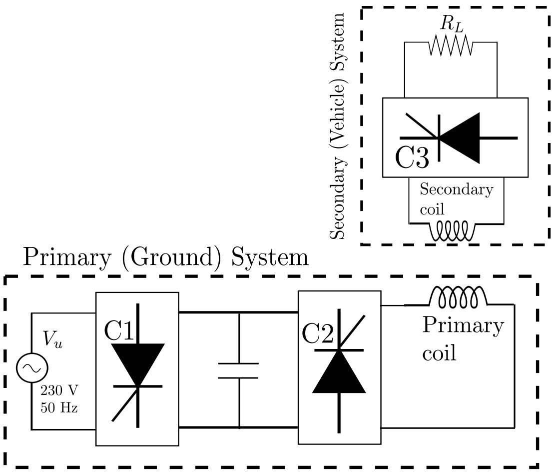
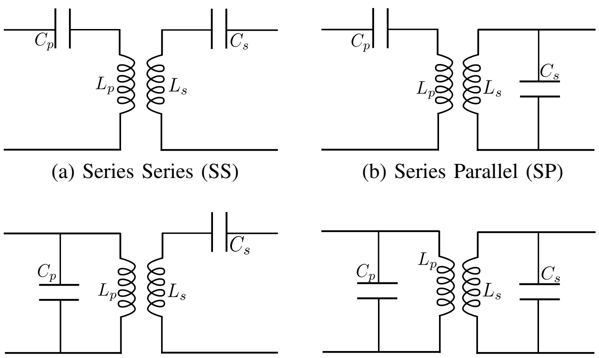


Fig. 2: Complete WPT System: C1 is rectifier, C2 and C3 are high frequency inverter and converter respectively



(a) Series Series (SS) (b) Series Parallel (SP)

(c) Parallel Series (PS) (d) Parallel Parallel (PP)

Fig. 3: Topologies commonly employed for WPT

may be a requirement of constant voltage charging or constant current charging. The series compensated topology resembles a voltage source while a parallel compensated topology resembles a current source [10]. Appropriate topology can then be implemented depending on requirement.

III. Design Issues and Methodology

Design of WPT systems is far from trivial design of systems. There are several choices to be made in the design process in addition to design of sophisticated control system which will improve the overall reliability of the system. In this report only certain issues as illustrated by subsections below are discussed.

A. Choice of capacitance

WPT systems consist of two resonant circuits which are magnetically coupled. The primary and secondary capacitances are selected such that the power is delivered at unity power factor. Hence they have to operate at the same resonant frequency. The magnetic coupling makes the choice of capacitances non-trivial for the given application. There can be multiple ways to select the compensation capacitance. One of the ways is given by Chopra *et. al.* [9] and Wang *et. al.* [10]. They suggest to

Topology	Primary Capacitance
SS	$\dfrac{C_s L_s}{L_p}$
SP	$\dfrac{C_s L_s^2}{L_p L_s - M^2}$
PS	$\dfrac{(L_p L_s - M^2) C_s L_s^2}{\dfrac{M^4 C_s R_L}{L_s} + (L_p L_s - M^2)^2}$
PP	$\dfrac{C_s L_s}{\dfrac{M^4}{L_p C_s L_s R_L} + L_p}$

TABLE II: Formulae for primary capacitance for unity power factor [9] [10]

first select the secondary capacitance by compensating for the secondary inductance as -

$$\omega_0 = \frac{1}{\sqrt{L_s C_s}} \tag{9}$$

Then, the primary capacitance is selected such that the imaginary part of total impedance as seen by the primary source V_p is zero. The formulae to calculate primary capacitance are given in table II [10]. For SP, PS and PP topology the capacitance depends on the inductance of secondary coil. For parallel compensated primary, the load resistance also plays a role in value of C_p. This is particularly not desirable from control point of view. The load resistance might change in case of battery charging and hence the pre-decided value of C_p may not be optimum from point of view of high efficiency. Hence, for simplicity, in this report, a sample SS topology is selected which will be discussed in the upcoming sections.

B. Effect of variation of frequency on operating point of the system: bifurcation

Primary and secondary inductances and capacitances are designed to operate at a particular frequency. However, there might be a change in operating frequency due to various factors. It is important to see the effect of such variation on operating point of the system. Chopra and Bauer [9] has shown this effect by considering a phenomenon called as bifurcation.

The equivalent impedance as seen by primary source is given by,

$$Z_{1eq} = R_p + \left(j\omega L_p - \frac{1}{j\omega C_p}\right) + \frac{\omega^2 M^2}{(R_2 + R_L) + \left(j\omega L_s - \frac{1}{j\omega C_s}\right)} \tag{10}$$

where ω is the operating frequency which may be different from the resonant frequency ω_0. The phase plot of Z_{1eq} has more than one zero crossing. This is quite unusual behaviour, although it is expected because the system has four energy storage elements, out of which two are coupled to each other. Further, as it will be shown in next section, a deviation from zero phase angle frequency results in large deviation in power factor. This large deviation in power factor can be avoided if the system is designed such that equation (11) is satisfied [9].

$$\begin{gathered} Q_2 < Q_d \\ \text{where, } Q_d = \sqrt{\frac{1}{2(1 - \sqrt{1 - k^2})}} \\ \text{and } Q_2 = \frac{\omega_0 L_s}{R_L} \end{gathered} \tag{11}$$

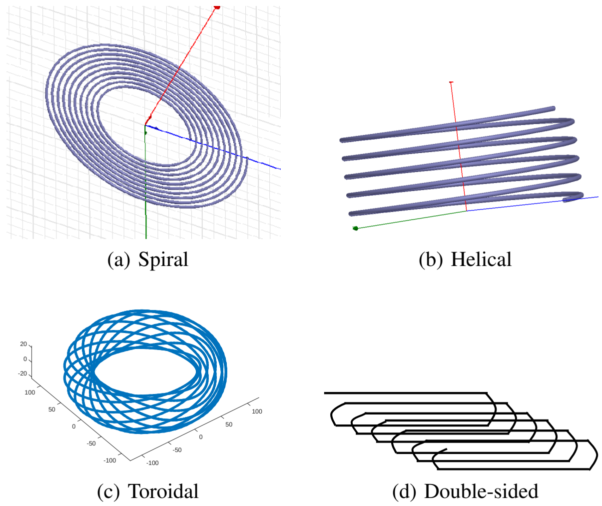


Fig. 4: Coil geometries employed for WPT

Here Q_2 is the quality factor of the secondary system. Thus, this *bifurcation* phenomenon puts a constraint on the design of secondary coils with respect to the load resistance. Its effect will be illustrated in the next section by some sample simulation results.

C. Choice of Geometry of Coils

Geometry of coils for WPT is a cruicial point in the design of WPT systems. As indicated in the last section, the inductance of secondary is limited by the factor Q_d for bifurcation-free operation. Further, the coil geometry itself is constrained by available space and weight, particularly in case of BPV systems. The most common geometries employed for the systems are [3], [6], [11]

1) Spiral
2) Helical
3) Toroidal
4) Double-sided

Each of these coils is illustrated in Fig. 4. To overcome the limit of leakage and permeability of air, there have been several attempts to design structures made of ferrites that will confine the flux to the two coils. Thus, there are further variants possible of the above geometries when ferrites are employed in the design. Budhia *et. al.* [12] have considered number and dimension of ferrite spokes during the optimization of the coils that they have used. Eventually, the design of coils is constrained by following factors -

1) Dimensions
2) Leakage flux
3) Immunity of misalignment
4) Weight

Considering all of these factors, there are several papers available which have worked on either one or all of these factors towards development of better coils. For example, Sakamoto *et. al.* [13] have suggested a large air-gap coupler for inductive charger. Recently, there are different types of coil structures being proposed for the WPT systems [11], [14]. Sampath *et. al.* [15] have deviced a different toroidal geometry which is better immune to misalignment.

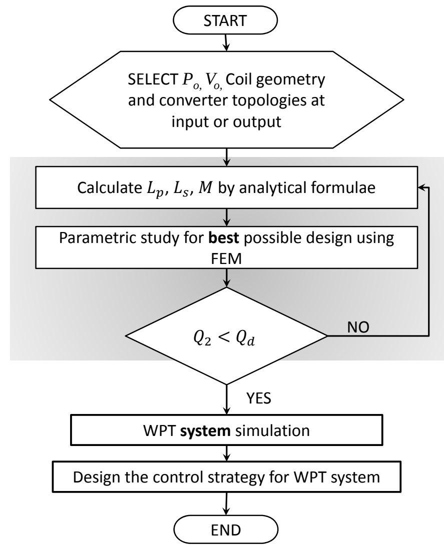


Fig. 5: Flowchart for design methodology of WPT system

Parameter	Symbol	Value
High frequency AC input voltage rms (V)	V_p	30
Frequency of operation (kHz)	f	15
Output voltage (V)	V_{dc}	48
Output power (kW)	P_o	1
Load Resistance (Ω)	R_L	1
Maximum outer diameter of coils (cm)	d_{out}	30
Distance between coils (cm)	v_{dist}	10
Coil Geometry		Spiral

TABLE III: System parameters used for design

Some preliminary simulations are done to see effect of coil geometry on inductances. That will be elaborated in the next section.

D. Design methodology

Based on the topics dealt with in earlier sections, Fig. 5 shows flowchart for design of a WPT system. This is just one of the methods. It can be seen that unlike design of other electrical machines which rely on simple formulae derived out of first principles of electromagnetics, this method is more empirical and involves many empirical decision steps.

In particular, the design of coils (the shaded portion in flowchart) was investigated for this duration. It is decided by the factor Q_d as denoted in equation (11). The results of simulations carried out for design will be discussed in the next section.

IV. Design of coils: Simulation results

Following the system design methodology enlisted in the last section, a system is designed with particular focus on parametric study of coil design and variation. System parameters are taken to start with the design are given in Table III. Various parameteric variations of coil design are carried out as explained in the following sections.

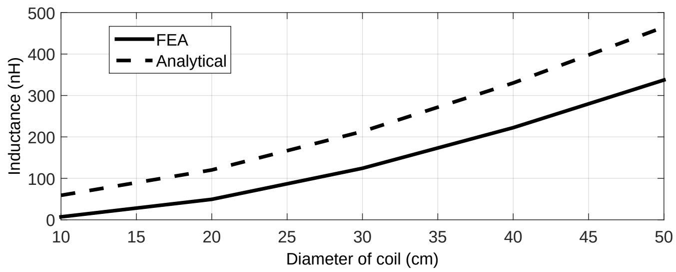


Fig. 6: Comparison of Mutual inductance of filametal conductors: These can be summed up to get M for spiral conductor [17]

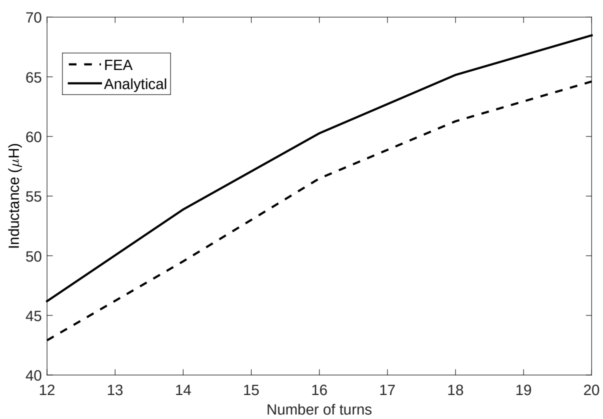


Fig. 7: Comparison of inductance values calculated by analytical formula and FE simulation: Number of turns is proportional to inductance as expected. An offset in both methods is seen because, analytical method in [19] is developed for planar coils

A. Comparison of analytical and FE simulation results

The analytical expression for inductance relies on path taken by flux. In case of iron-core transformers, the path is *directed* through the core and hence it is easy to evaluate the inductance. However in air-core case the closed form expression for inductance is not so *neat*. Grover [16] has given an elaborate treatment for calculating inductances for various geometries. There is a lot of literature available for calculation of inductances of various geometries [11], [12], [17], [18].

To calculate inductance, the planar spiral geometry is modelled as composed of several concentric circular coils. Inductance of several Inductances of these coils are calculated by analytical formulae which involves elliptic integrals as given by [17]. As the concentric turns are in series in actual coil, the matrix of inductances so formed is summed blockwise to get full inductance matrix. A comparison of mutual inductances of two filamental conductors of 2.0168 mm diameters is shown in Fig. 6. The self inductance can be formulated analytically in many ways. An approach given in [19] is used here. A comparison is presented in Fig. 7.

It can be noted that from Fig. 6 and 7 that analytical methods give higher values. Also the analytical formulae approximate the original geometry by some either filamental or planar structure and hence they give results that are higher than the FE simulated results. This is expected as stated in [17].

Parameter	Symbol
Primary or Ground coil	$C_{[g]}$
Secondary or Vehicle coil	$C_{[v]}$
Inner radius of vehicle coil $C_{[v]}$	r_{iv}
Outer radius of ground coil $C_{[g]}$	r_{og}
Outer radius of vehicle coil $C_{[v]}$	r_{ov}
Inner radius of ground coil $C_{[g]}$	r_{ig}
Number of turns in vehicle coil $C_{[v]}$	n_v
Number of turns in vehicle coil $C_{[g]}$	n_g

TABLE IV: List of symbols

Parameter kept constant	Parameter varied	Variation
n_v, n_g, r_{iv}, r_{ov}, r_{ig}	r_{og}	20 cm to 64 cm
n_v, r_{og}, r_{iv}, r_{ov}, r_{ig}	n_g	12 to 14

TABLE V: Parameters of coils used for FE simulation: The simulation shows effects of various factors on inductance values

Analytical closed form solutions are important because they provide a handle on the design of inductors. Closed form solutions can give quick results as agains FE results which consume significant amount of time. However, for the results reported in next section only FE results are used.

B. Effect of variation of coil parameters on inductance values

To select a best coil design, a parametric study of coil parameters is helpful. Following factors are studied for this work -

1) Radius of primary coil
2) Number of turns of coil

A list of symbols used is presented in table IV for convenience. The parametric study is summarized in Table V. The variation of parameters is applied only to the ground coil only, because usually the the vehicle coil is limited usually limited by more strictly by weight and size. The details of the simulation results are given in following subsections.

1) Radius of coil: In case of ferrite core transformer, the coil radius is inversly proportional to the inductance of coils. However, in case of air-cored coils, it is found that as the coil radius increases the inductance of the coils goes on increasing. This is particularly useful because that will reduce the value of choice of capacitors in the system. However, the radius itself is limited by the space available for coils. Fig. 10 shows the variation of inductance with radius of the coil. It can be seen that the although L increases monotonically with increase in coil radius, the M drops after reaching a threshold. Thus the M can be increased only to a limited extent by such variation.

2) Number of turns in the coil: The results of this parameteric variation are already stated in Fig. 7. It should be noted that the inductance resistance of the coil also increases with the increase in number of turns. This also increases the weight of the coils.

C. Bifurcation and coil design

It is shown in equation (11) that the inductance of the coil must be within limits for zero phase angle operation. It is noted that for increasing number of turns in the coil, the quality factor of the coil increases. As a result, it becomes greater than the factor Q_d. This is depicted in Fig. 8. The effect

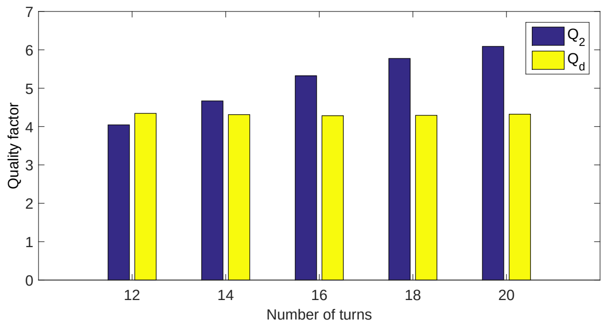


Fig. 8: Comparison of Q_2 and Q_d as in equation (11)

of this phenomenon can be clearly seen if we plot the phase angle of input impedance as seen by the high frequency AC source. A MATLAB code was written to plot the same. The system parameters are taken from table III. The inductance values are taken as calculated in 7. Appropriate capacitance values are calculated for operating frequency of 15 kHz.

The results of simulation are shown in Fig. 9. Following observations can be noted from the figure -

1) $Q_2 < Q_d$ - There is exactly one resonant frequency. However, any deviation from that frequency does not hamper the efficiency to a great extent.
2) $Q_2 = Q_d + \epsilon$ - When Q_2 is slightly greater than Q_d, there are more than one frequencies for zero power factor operation. However, the efficiency does not vary significantly in close vicinity of resonant frequency
3) $Q_2 > Q_d$ In this case the efficiency is badly affected if the operating frequency deviates even in close vicinity of the resonant frequency.

Thus, control of secondary quality factor is quite important from view of zero power factor operation.

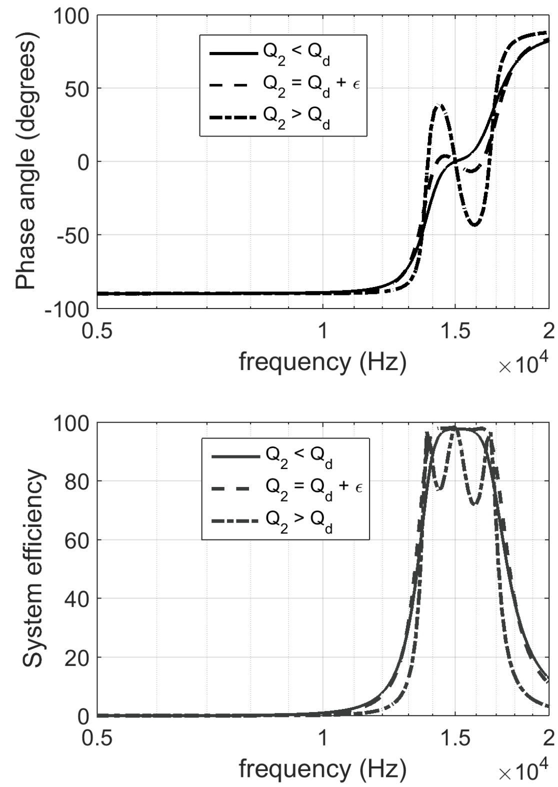


Fig. 9: Effect of bifurcation on zero power factor operation and operating efficiency

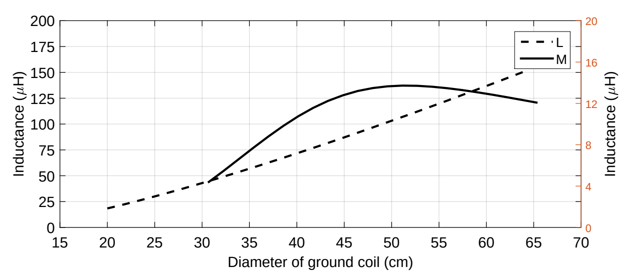


Fig. 10: Effect of radius on inductance of the coil: results of FE simulation in Maxwell

V. Unaddressed issues and open problems

There are several other practical and safety issues in WPT systems. This section will briefly touch upon these topics.

A. *Design of coils*

Some light was thrown on design of coils in the last section. As it can be seen to some extent, design of coil forms a part of design of the whole system. Hence, it is always an iterative process. Following are problems which can be further analyzed

1) A design should be checked for its **reliability against misalignment** which can occur in case of BPVs.
2) A coil capacitance itself can form resonance with its inductance. A design can be targetted with a focus to use **the coil capacitance for resonance**.
3) Various **ferrite geometries** for *directing* the flux through the coil can be tested.

B. *Input and output converters*

This report mainly focussed on the central WPT system. As indicated in Fig. 2, a complete system consists of converters C1, C2 and C3. Out of these much work is being done in the topologies of the converters C1, C2 and C3. Hence, there is a scope to further improve on the topologies.

C. *Control strategies*

It can also be recognized that the complete system is fairly complex. There are several parameters to control viz., the operating frequency, output voltage, output power and input power factor. Thus, this system also poses a multivariable control problem. The control problem is particularly difficult because as the primary and second system need to be physically isolated. Hence, a control strategy should be able to control the output voltage without really sensing it. Otherwise, there is a necessity if wireless communication between the two subsystems. Hence there is a scope for control strategies here.

D. *Choice of components*

The coils in WPT need to carry significant amount of current at high frequencies. Hence, a soild conductor cannot be used for coils, because these conductors would exhibit an increased resistance and inductance at higher frequencies. Hence, Litz wires are used for these coils. The Litz wire consist of a large number of strands which are twisted and weaved such that the skin and proximity effects are nullified. Such wires are costlier than copper wires and are not availble in India. As the circuit is a resonant circuit, voltage drop across capacitors is very large and is an AC circuit. The AC capacitors with such high value of withstanding voltage are not common in the market. Polypropylene capacitors need to be used for such application. In short, newer component development is possible especially for the WPT systems.

E. Safety issues

It is natural to investigate safety of humans as they can be exposed to electromagnetic fields generated by WPT systems [3]. ICNIRP (International Commosion on Non-ionizing Radiation Protection) has given several guidelines for safety related to WPT systems. The guidelines are classified in two frequency classes - a) from 1 Hz to 100 kHz and b) from 100 kHz to 300 GHz. IEEE has also guidelines for Safety Levels with Respect to Human Exposure to Radio Frequency EMF 3 kHz to 300 GHz [3].

Wireless power consortium has also developed some guidelines for the same. However, in most of the near field WPT systems, as the coils are in air, the electric and magnetic fields are within safety limits [3]. Nevertheless, the safety of such systems need to be guarenteed by quatitative tests like SAR (Specific Absorption Rate) test devloped for the mobile devices.

Apart from safety of humans, the environment where WPT systems are implemented, also needs to be protected from EMI and EMC of WPT systems. Hence there is a scope to develop special magnetic shields that can help to reduce the leakage fields and also help to increase coupling coefficients [3], [11], from both the above perspectives. It should be noted that the exposure limits become more stringent as frequency increases. Hence, this also puts additional constraint on choice of frequency for the system.

F. Effect on Power Systems

If the wireless charging facilities grow in number and become widespread, it will change the load profile both from the temporal and spatial scales. This can pose the stability problems for power systems. Hence, there is a scope for investigating the behaviour of power systems with respect to these changes in the load pattern.

VI. Conclusion

As indicated in the introduction, WPT systems for power transfer in range of few kW is in infant stage. This report has given a brief historical review in general of the WPT systems. Major important topologies are discussed. In particular, the works of S Chopra *et al* [9] and Covic and Boys [6] is studied in detail.

The focus of study is design of coils and the associated electromagnetics. However, complete system is also introduced. Open ended issues in the research for WPT systems are stated at the end. It can be concluded that there are more interdisciplinary open ended problems in wireless power transfer which can be taken up for the research.

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