~~N1~~ $f_\xi(x) = a e^{-\lambda|x|}$, $a>0$, $\lambda>0$

Домашнє завдання

N1 $f_\xi(x) = a e^{-\lambda|x|}$ $a>0$, $\lambda>0$

$$\int_{-\infty}^{+\infty} f_\xi(x)\,dx = 1$$

$$\int_{-\infty}^{\infty} a e^{-\lambda|x|} dx = a\left(\int_{-\infty}^{0} e^{\lambda x} dx + \int_{0}^{\infty} e^{-\lambda x} dx\right) = 2a\int_0^{\infty} e^{-\lambda x} dx =$$

$$= 2a \cdot \left.\frac{e^{-\lambda x}}{-\lambda}\right|_0^{\infty} = \frac{2a}{\lambda}$$

$$\frac{2a}{\lambda} = 1 \Rightarrow a = \frac{\lambda}{2}$$ – знайшли a.

Шукаємо функцію розподілу

$$F_\xi(x) = \int_{-\infty}^{x} f_\xi(u)\,du$$

$\cdot\ x \geq 0 \quad \int_{-\infty}^{x} \frac{\lambda}{2} e^{-\lambda|u|} du = \frac{\lambda}{2}\left(\int_{-\infty}^{0} e^{\lambda u} du + \int_{0}^{x} e^{-\lambda u} du\right) =$

$= \frac{\lambda}{2}\left(\frac{e^{\lambda u}}{\lambda}\Big|_{-\infty}^{0} + \frac{e^{-\lambda u}}{-\lambda}\Big|_{0}^{x}\right) = \frac{\lambda}{2}\left(\frac{2}{\lambda} - \frac{e^{-\lambda x}}{\lambda}\right) = 1 - \frac{e^{-\lambda x}}{2}$

$\cdot\ x < 0 \quad \frac{\lambda}{2}\int_{-\infty}^{x} e^{\lambda u} du = \frac{\lambda}{2} \cdot \frac{e^{\lambda u}}{\lambda}\Big|_{-\infty}^{x} = \frac{e^{\lambda x}}{2}$

$$F_\xi(x) = \begin{cases} 1 - \frac{e^{-\lambda x}}{2} & x \geq 0 \\ \frac{e^{\lambda x}}{2} & x < 0 \end{cases}$$

Шукаємо мат. сподівання

$\underset{\sim}{M\xi} = \frac{\lambda}{2}\int_{-\infty}^{\infty} x \cdot e^{-\lambda|x|} dx = \frac{\lambda}{2}\left(\int_{-\infty}^{0} x \cdot e^{\lambda x} dx + \int_{0}^{\infty} x \cdot e^{-\lambda x} dx\right) =$

$= \frac{\lambda}{2}\left(-\frac{1}{\lambda^2} + \frac{1}{\lambda^2}\right) = 0$

$\rightarrow \int_{-\infty}^{0} x \cdot e^{\lambda x} dx = \frac{x}{\lambda} \cdot e^{\lambda x}\Big|_{-\infty}^{0} - \int \frac{e^{\lambda x}}{\lambda} dx = -\frac{e^{\lambda x}}{\lambda^2}\Big|_{-\infty}^{0} = -\frac{1}{\lambda^2}$

$\rightarrow \int_{0}^{\infty} x \cdot e^{-\lambda x} = -\frac{x}{\lambda} e^{-\lambda x}\Big|_{0}^{\infty} - \int_{0}^{\infty} \frac{e^{-\lambda x}}{-\lambda} dx = -\frac{e^{-\lambda x}}{\lambda^2} = \frac{1}{\lambda^2}$

Шукаємо дисперсію

$M\xi^2 = \frac{\lambda}{2}\int_{-\infty}^{\infty} x^2 \cdot e^{-\lambda|x|} dx = \frac{\lambda}{2}\left(\int_{-\infty}^{0} x^2 e^{+\lambda x} dx + \int_{0}^{\infty} x^2 e^{-\lambda x} dx\right) =$

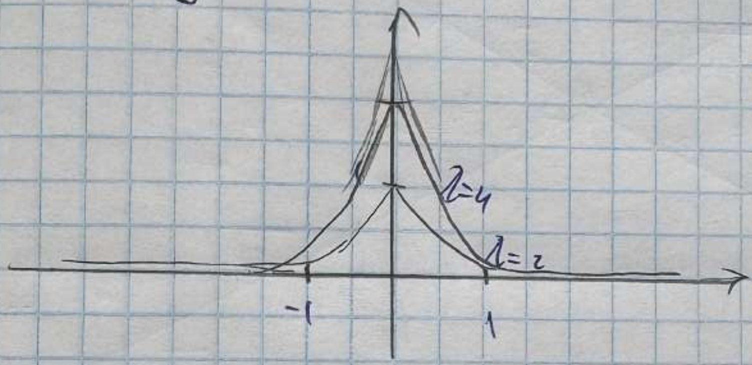
$= \frac{\lambda}{2}\left(\frac{2}{\lambda^3} + \frac{2}{\lambda^3}\right) = \frac{2}{\lambda^2}$

$\cdot \int_{-\infty}^{0} x^2 e^{\lambda x} dx = \frac{x^2}{\lambda} \cdot e^{\lambda x}\Big|_{-\infty}^{0} - 2\int x e^{\lambda x}\frac{1}{\lambda} dx = \frac{2}{\lambda^3}.$

$$\int_0^\infty x^2 e^{-\lambda x} dx = \frac{x^2}{-\lambda} e^{-\lambda x}\Big|_0^\infty + \frac{2}{\lambda}\cdot\int_0^\infty x e^{-\lambda x} dx = \frac{2}{\lambda^3}.$$

$$D\xi = M\xi^2 - (M\xi)^2 = \frac{2}{\lambda^2}.$$

Графік для $f_\xi(x)$



Графік для $F_\xi(x)$