Домашняя робота

1) $$S^2 = \frac{\sum_{i=1}^{n}(\xi_i - \bar{\xi})^2}{n} \rightarrow MS^2 = \frac{n-1}{n}\sigma^2$$

$$\sum_{i=1}^{n}(\xi_i - \bar{\xi})^2 = \sum_{i=1}^{n}(\xi_i - a)^2 - n(\bar{\xi} - a)^2.$$

$$S_2 = \frac{1}{n}\left(\sum_{i=1}^{n}(\xi_i - a)^2 - n(\bar{\xi} - a)^2\right) = \frac{\sum_{i=1}^{n}(\xi_i - a)^2}{n} - (\bar{\xi} - a)^2$$

$$MS_2 = \frac{\sum_{i=1}^{n}M(\xi_i - a)^2}{n} - M(\bar{\xi} - a)^2 = \frac{n\sigma^2}{n} - \frac{\sigma^2}{n} = \sigma^2\left(1 - \frac{1}{n}\right) =$$

$$= \frac{n-1}{n}\sigma^2$$

2. $P(\xi=k)=\frac{\theta^k}{(1+\theta)^{k+1}}$, $\theta>0$, $k=0,1\ldots$

$$D\hat{\theta}=D\frac{1}{n}\sum_{i=1}^{n}\xi_i=\frac{1}{n^2}\cdot\sum_{i=1}^{n}D\xi_i=\frac{1}{n^2}\cdot n(\theta^2+\theta)=\frac{1}{n}(\theta^2+\theta)=$$

$$=\frac{n\sigma^2}{n^2}=\frac{\sigma^2}{n}$$

$$M\hat{\theta}=M\frac{\sum_{i=1}^{n}\xi_i}{n}=\theta$$

$$L(x,\theta)=\prod_{k=1}^{n}P(\xi=x_k)=\prod_{k=1}^{n}\frac{\theta^{x_k}}{(1+\theta)^{x_k+1}}=\frac{\theta^{\sum_{k=1}^{n}x_k}}{(1+\theta)^{\sum_{k=1}^{n}(x_k+1)}}$$

$$\ln L(x,\theta)=\sum_{k=1}^{n}x_k\cdot\ln\theta+\ln\left(1+\theta^{-\sum_{k=1}^{n}(x_k+1)}\right)=$$

$$=\sum_{k=1}^{n}x_k\cdot\ln\theta-\sum_{k=1}^{n}(x_k+1)\cdot\ln(1+\theta)=\sum_{k=1}^{n}x_k\cdot\ln\theta-$$

$$-n\cdot\ln(1+\theta)-\sum_{k=1}^{n}x_k(\ln(1+\theta))$$

$$\frac{\partial}{\partial\theta}\ln L(x,\theta)=\frac{\sum_{k=1}^{n}x_k}{\theta}-\frac{n+\sum_{k=1}^{n}x_k}{1+\theta}$$

$$J(\theta)=-M\frac{\partial^2}{\partial^2\theta}\ln L(\xi,\theta)=-M\left(-\frac{\sum_{k=1}^{n}\xi_k}{\theta^2}+\frac{n+\sum_{k=1}^{n}\xi_k}{(1+\theta)^2}\right)=$$

$$=\frac{1}{\theta^2}\cdot\sum_{k=1}^{n}M\xi_k-\frac{1}{(1+\theta)^2}\left(M\sum_{k=1}^{n}\xi_k+n\right)=\frac{1}{\theta^2}\cdot n\theta-$$

$$-\frac{1}{(1+\theta)}(n\theta+n)=\frac{n}{\theta}-\frac{n}{1+\theta}=\frac{n}{\theta+\theta^2}$$

$D\hat{\theta}=\frac{\theta^2+\theta}{n}=(J(\theta))^{-1}$ – нижня межа

Отже, оцінка є ефективною