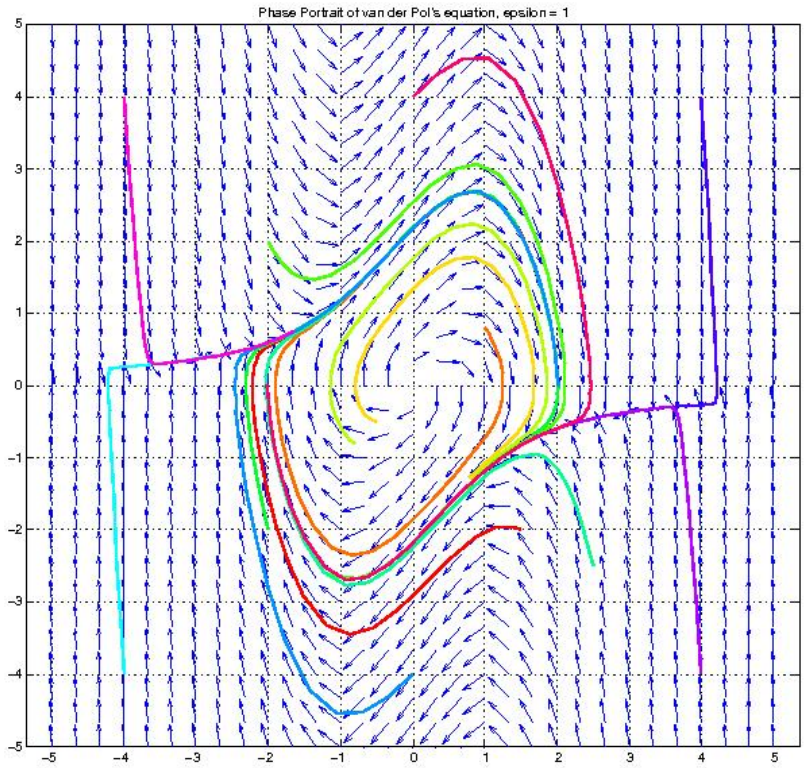
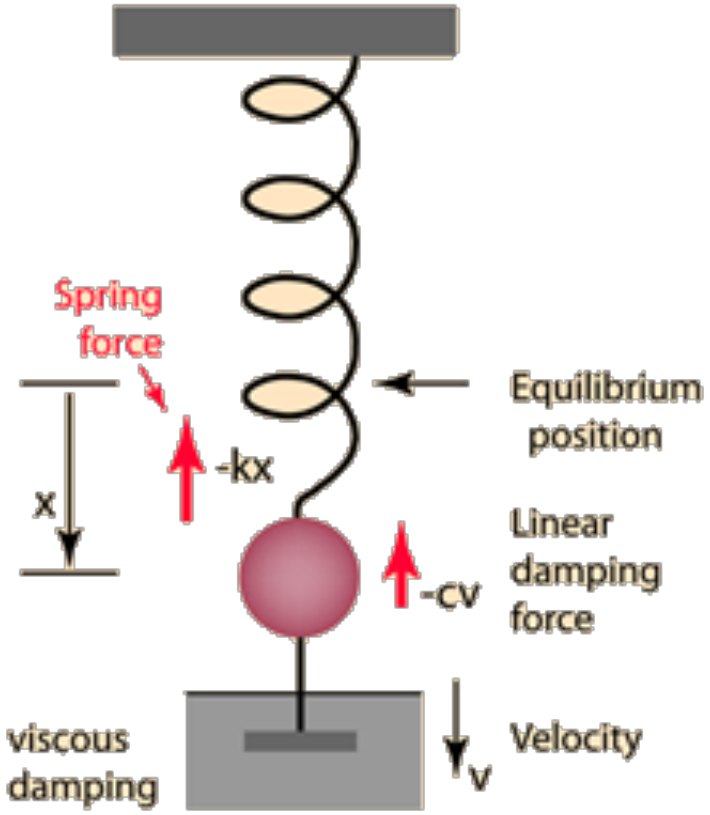
Van der Pol Equation

Ngoc Luu



Linear damped mass – spring system



$$x'' + \mu x' + \omega_0^2 x = A\omega_0^2 sin(\omega t)$$

Van der Pol Oscillators

$$x'' + \mu(x^2 - 1)x' + \omega_0^2 x = A\omega_0^2 sin(\omega t)$$

non-linear damping force

Standard form

- x: position coordinate at time t
- : damping coefficient
- : negative damped force
- : positive damped force

Solutions

- unique periodic solution
- as $t \to \infty$, all nearby solutions tend toward the unique solutions
- is key

$= 0$

General Solution:

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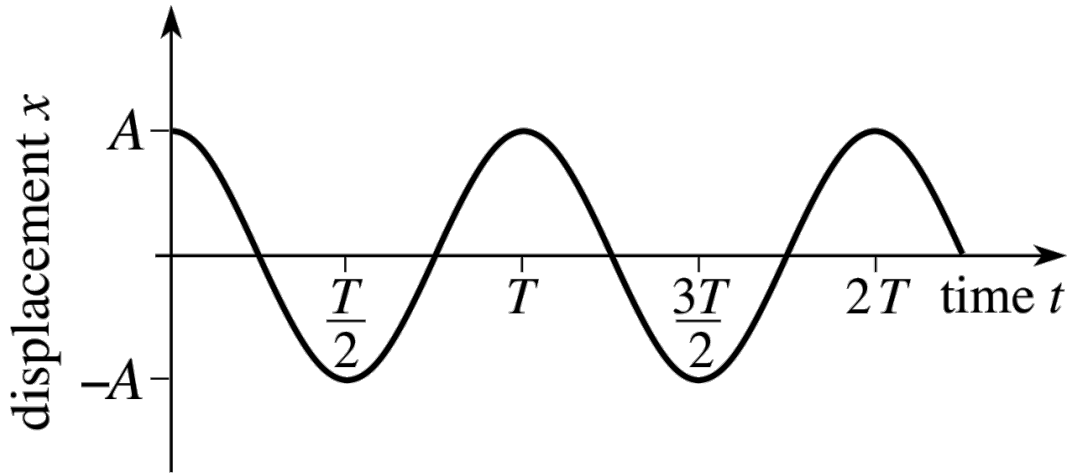
$$x'' + x = 0$$

SIMPLE HARMONIC MOTION



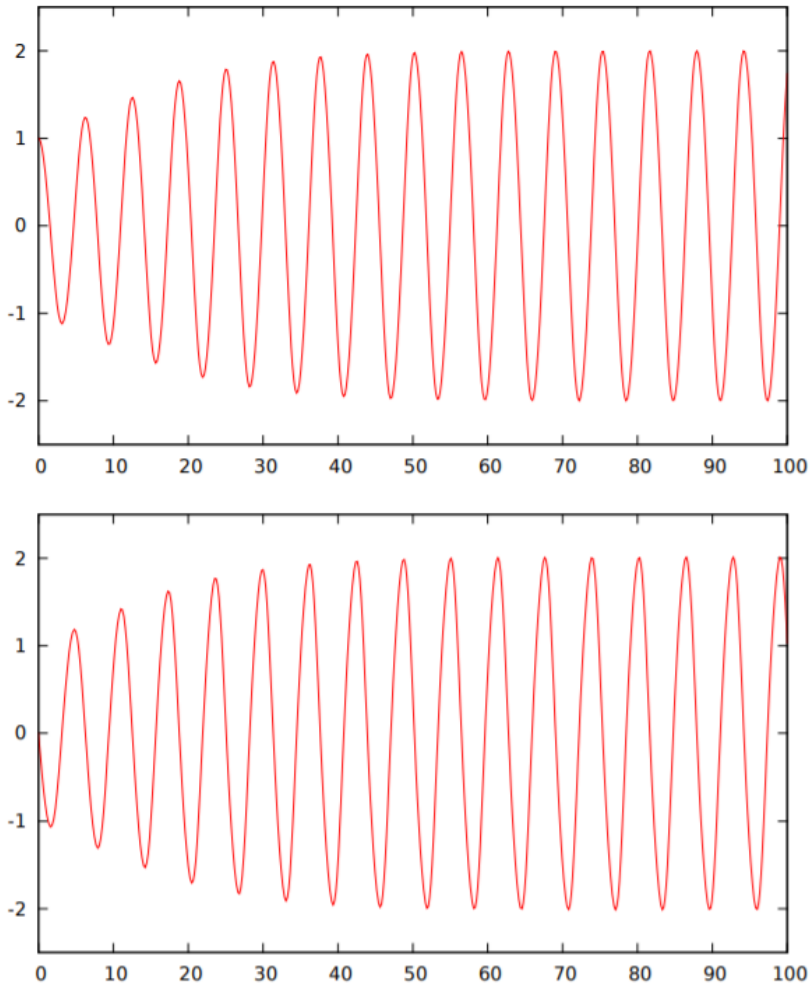


Figure 1: Typical solution of van der Pol equation for small values of μ; top graph – $x(t)$, bottom graph – $\dot{x}(t)$; $\mu = 0.1$

Large

- Relaxation oscillation

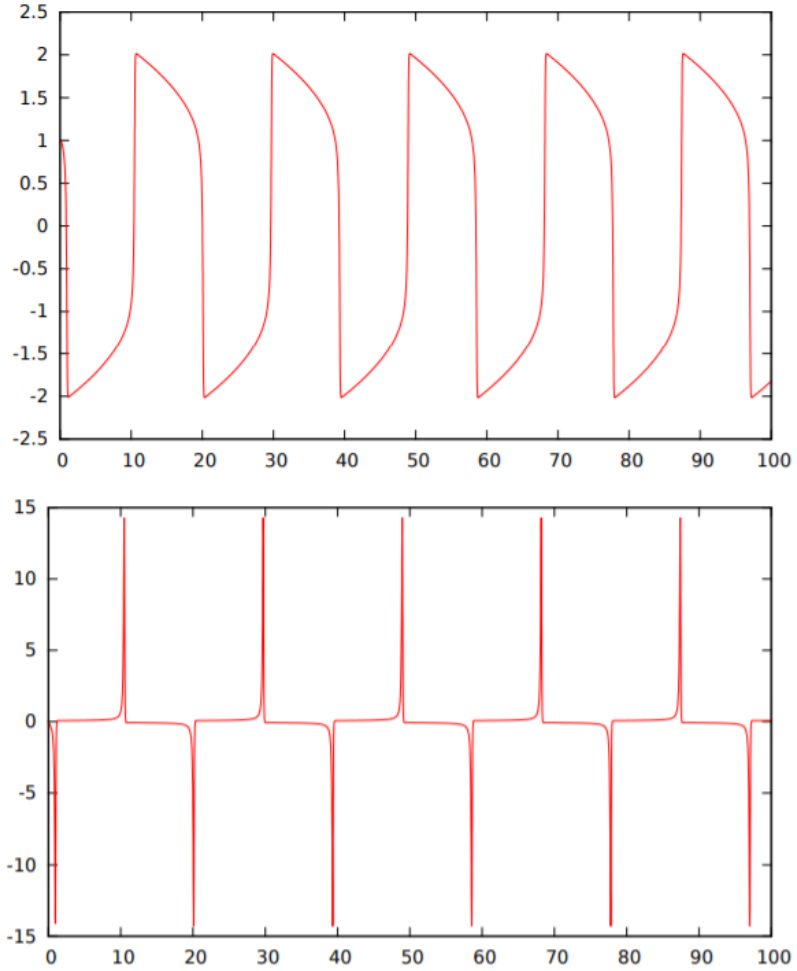
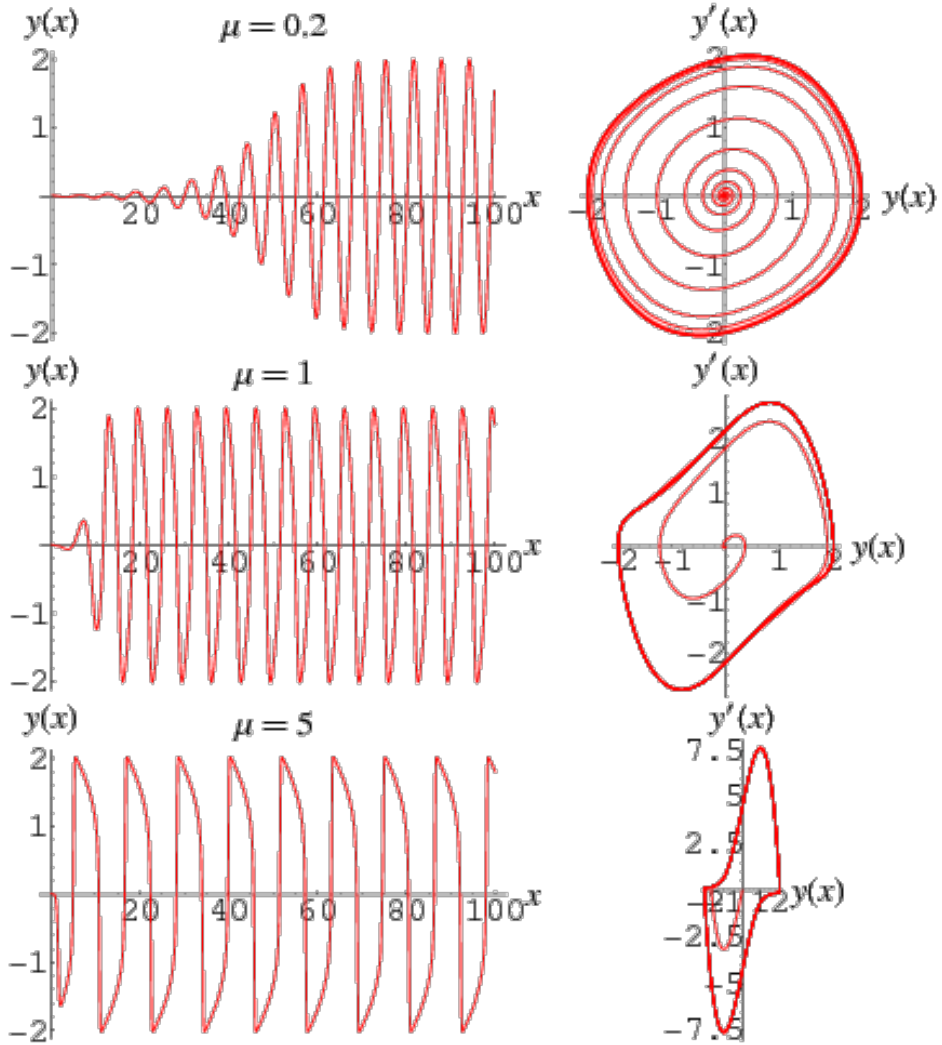


Figure 3: Typical solution of van der Pol equation for large values of μ; top graph – $x(t)$, bottom graph – $\dot{x}(t)$; $\mu = 10$.

Limit cycles

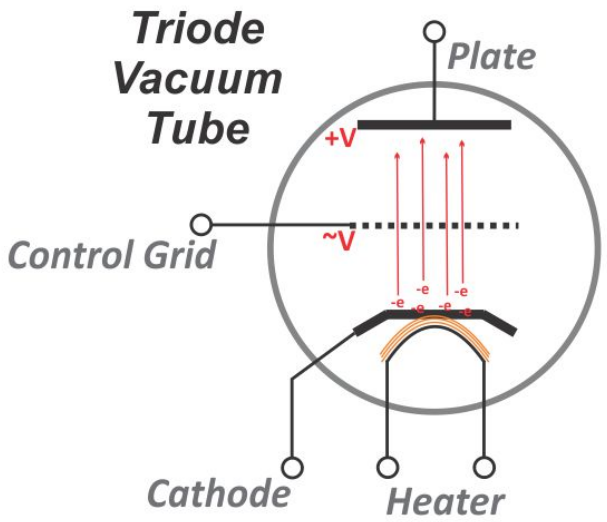


History

- Balthazar van der Pol (1889-1959)
- Described mathematical model for triode oscillations in electrical circuits (1927)
- Van der Pol equation

Application

- vacuum tubes triode circuit
- physics, electronics, biology, neurology, sociology and economics



Reference

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