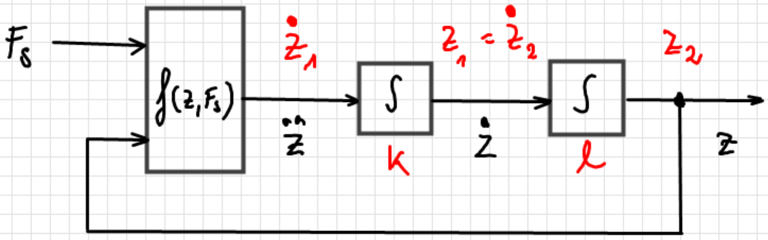
Aufgabe 1 :

a)



$$\underline{\underline{f(z,F_s) = 0.001\, F_s - 2 - \frac{5}{(z+1)^2}}}$$

b) abh. Var. : z Eingangsgröße : F_s

unabh. Var. : t

c)

$$\dot{z}_1 = 0.001\, F_s - 2 - \frac{5}{(z_2+1)^2}$$

$$\dot{z}_2 = z_1$$

d)

$$\boxed{\begin{aligned} z_{1,n+1} &= z_{1,n} + h \cdot \left(0.001 \cdot F_s - 2 - \frac{5}{(z_{2,n}+1)^2}\right) \\ z_{2,n+1} &= z_{2,n} + h \cdot z_{1,n} \\ t_{n+1} &= t_n + h \end{aligned}}$$

e)

$$k_1 = h \cdot \left[0.001\, F_s - 2 - \frac{5}{(z_{2,n}+1)^2} \right]$$

$$l_1 = h \cdot z_{1,n}$$

$$k_2 = h \cdot \left[0.001\, F_s - 2 - \frac{5}{\left(z_{2,n} + \frac{l_1}{2} + 1\right)^2} \right]$$

$$l_2 = h \cdot \left(z_{1,n} + \frac{k_1}{2}\right)$$

$$z_{1,n+1} = z_{1,n} + k_2$$

$$z_{2,n+1} = z_{2,n} + l_2$$

$$t_{n+1} = t_n + h$$

Aufgabe 2 :

a)

$$\sum F = m \cdot \ddot{x} = m \dot{v}$$

$$F_S - F_R = (m_0 - Dt) \cdot \dot{v}$$

$$K \cdot D - \frac{1}{2} c_W \cdot A \cdot \rho \cdot v^2 = (m_0 - Dt) \cdot \dot{v}$$

$$\underline{\underline{\dot{v} = \frac{1}{(m_0 - Dt)} \cdot \left[K \cdot D - \frac{1}{2} c_W \cdot A \, \rho \cdot v^2 \right]}}$$

b)

$$K \cdot D = 1000 \, \frac{N\cancel{s}}{\cancel{kg}} \cdot 1 \, \frac{\cancel{kg}}{\cancel{s}} = \underline{\underline{1000 \, N}}$$

$$\frac{1}{2} c_W \cdot A \cdot \rho = \frac{1}{2} \cdot 0.8 \cdot 1 \cancel{m^2} \cdot 1.29 \, \frac{kg}{m^{\cancel{3}}} = 0.52 \, \frac{kg}{m}$$

$$\underline{\underline{\dot{v} = \frac{1}{\left(200 \, kg - 1 \frac{kg}{s} \cdot t\right)} \cdot \left[1000 \, N - 0.52 \, \frac{kg}{m} \cdot v^2 \right]}}$$

c) auf SI-Einheiten normiert

$$\underline{\underline{\dot{v} = \frac{1}{(200 - t)} \cdot \left[1000 - 0.52 \cdot v^2 \right]}}$$

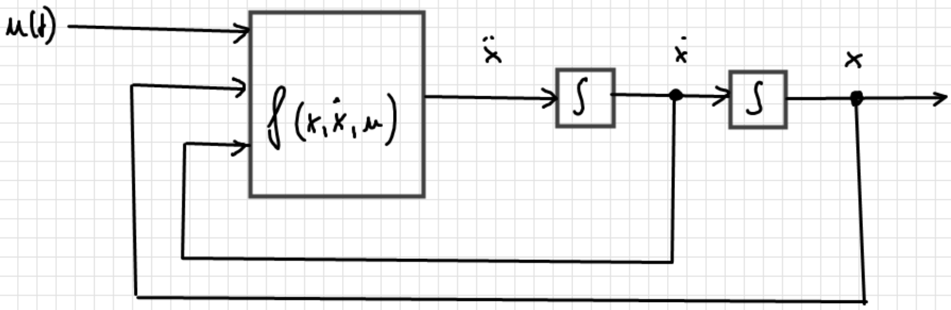
d) im Gleichgewichtszustand gilt: $\dot{v} = 0$

$$1000 - 0.52 \cdot v_\infty^2 = 0 \quad \Leftrightarrow \quad v_\infty = \sqrt{\frac{1000}{0.52}} \, \frac{m}{s}$$

$$v_\infty = 43{,}85 \, \frac{m}{s} = 43{,}85 \, \frac{km}{1000} \cdot \frac{3600}{h} = \underline{\underline{157{,}8 \, \frac{km}{h}}}$$

Aufgabe 3 :

a) $$\ddot{x} = \underbrace{-4x^2\dot{x} - 7x\cdot u - 26 - u}_{f(x,\dot{x},u)}$$



b) stat. Punkt $x_A = -2 \Rightarrow \ddot{x} = \dot{x} = 0$

$$7x_A u + u = -26$$

$$-14u + u = -26 \Rightarrow \underline{\underline{u_A = +2}}$$

c)

$$\left.\frac{\partial f}{\partial u}\right|_{AP} = -7x - 1 = \underline{\underline{13}}$$

$$\left.\frac{\partial f}{\partial \dot{x}}\right|_{AP} = -4x^2 = \underline{\underline{-16}}$$

$$\left.\frac{\partial f}{\partial x}\right|_{AP} = -\overset{0}{\cancel{8}}x\dot{x} - 7u_A = \underline{\underline{-14}}$$

$$\Delta \ddot{x} = -16\,\Delta\dot{x} - 14\,\Delta x + 13\,\Delta u$$

d) $\ddot{x} + 14\,\dot{x} + 14\,x = 12\,u$

$$X(s)\left[s^2 + 14s + 14\right] = 12\,U(s)$$

$$G(s) = \frac{12}{s^2 + 14s + 14} = \frac{X(s)}{U(s)}$$

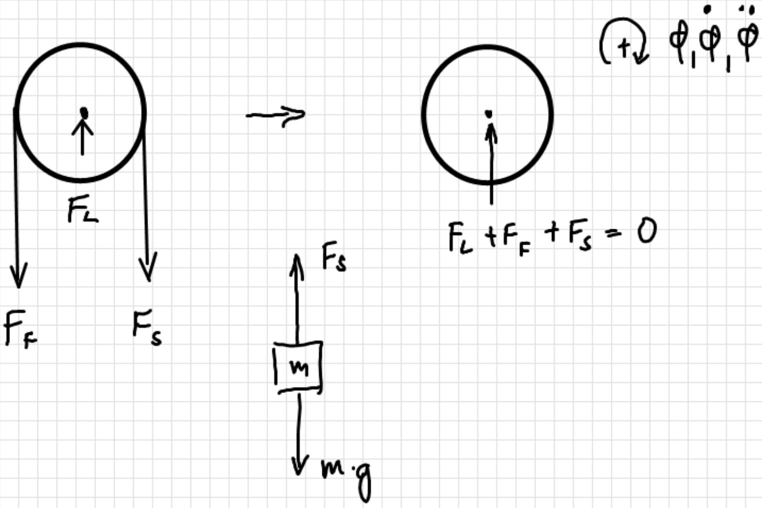
e) $s_{1,2} = -7 \pm \sqrt{7^2 - 14} = -7 \pm 5.92$

$s_1 = -1.08$ $\quad$ $s_2 = -12.9$

Beide Pole mit neg. Realteil $\Rightarrow$ stabil

Aufgabe 4 :

a)



b) Für die Scheibe gilt :

$$\sum M = J \cdot \ddot{\varphi} \quad (1) \qquad J = \frac{1}{2} m_R r^2 \quad (2)$$

$$\sum M = (F_s - F_F) \cdot r \quad (3)$$

Für die Masse gilt: $m \cdot g - F_s = m \ddot{x}$

bzw. $F_s = m(g - \ddot{x}) \quad (4)$

Für die Feder gilt : $F_F = c \cdot x \quad (5)$

Kompatibilitätsbedingung : $\ddot{\varphi} = \frac{\ddot{x}}{r} \quad (6)$

(2),(3) → (1)

$$(F_S - F_F)\cdot \cancel{r} = \frac{1}{2} m_R r^{\cancel{2}} \cdot \ddot{\varphi} \qquad (7)$$

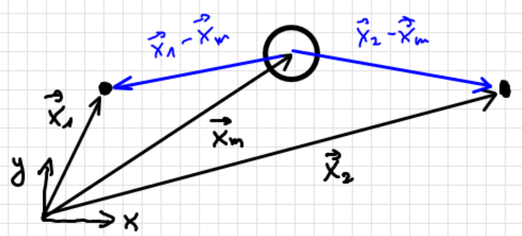
(4),(5),(6) → (7)

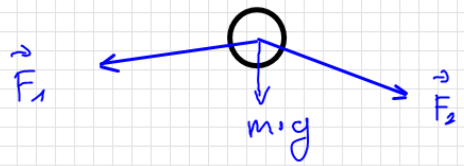
$$mg - m\ddot{x} - cx = \frac{1}{2} m_R \cdot \cancel{r} \cdot \frac{\ddot{x}}{\cancel{r}}$$

$$mg - cx = \ddot{x}\left(\frac{1}{2} m_R + m\right)$$

$$\ddot{x} = -\frac{c}{\frac{1}{2} m_R + m}\cdot x + \frac{m\cdot g}{\frac{1}{2} m_R + m}$$

Aufgabe 5 :





$$\vec{F}_1 = \underbrace{\left(|\vec{x}_1 - \vec{x}_m| - l_0 \right)}_{\text{Dehnung}} \cdot c \cdot \underbrace{\frac{\vec{x}_1 - \vec{x}_m}{|\vec{x}_1 - \vec{x}_m|}}_{\text{Richtung der Kraft}} \qquad (1)$$

$$\vec{F}_2 = \left(|\vec{x}_2 - \vec{x}_m| - l_0 \right) \cdot c \cdot \frac{\vec{x}_2 - \vec{x}_m}{|\vec{x}_2 - \vec{x}_m|} \qquad (2)$$

$$\sum \vec{F} = m \ddot{\vec{x}}_m \qquad (3)$$

$$\Rightarrow \quad \underline{\underline{\vec{F}_1 + \vec{F}_2 + (0, \; m \cdot g)^T = m \ddot{\vec{x}}_m}}$$