Aufgabe 1: a) $\dddot{y} + 8\ddot{y} + 7\dot{y} + 10 = 14\,u(t)$

$$\Big\Downarrow \text{ in DGL für } \Delta \text{ umwandeln}$$

$$\Delta\dddot{y} + 8\,\Delta\ddot{y} + 7\,\Delta\dot{y} = 14\,\Delta u(t)$$

$$\circ\!\!-\!\!\bullet \quad \text{Laplace-Transf.}$$

$$\left(s^3 + 8s^2 + 7s\right) Y(s) = 14\,U(s)$$

$$\underline{\underline{G(s) = \frac{Y(s)}{U(s)} = \frac{14}{s\left(s^2 + 8s + 7\right)}}}$$

b) Nullstellen für $s^2 + 8s + 7 = 0$ bestimmen

$$s_{1,2} = -4 \pm \sqrt{4^2 - 7} = -4 \pm \sqrt{9} = -4 \pm 3$$

$\underline{\underline{s_1 = -7}}$ $\quad$ $\underline{\underline{s_2 = -1}}$

$$\Longrightarrow \quad s^2 + 8s + 7 = (s+7)(s+1)$$

auf Standardform bringen

$$\Longrightarrow \quad G(s) = \frac{14}{s\,(s+1)(s+7)} = \frac{2}{s\,(s+1)\left(\frac{1}{7}s + 1\right)}$$

$\Rightarrow$ Reglerentwurf mit sym. Optimum

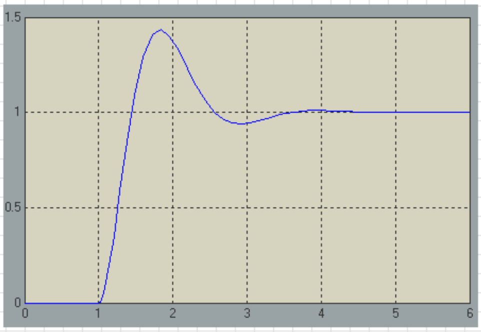
$T_1 = 1$, $T_E = \frac{1}{7}$, $K_S \cdot K_0 = 2$, $\beta = 2$ (schnell)

kleine Zeitkonstante

große Zeitkonstante

$\Rightarrow$ PID-Regler $\quad \underline{\underline{T_V = 1}}$, $\underline{\underline{T_N = \beta^2 \cdot T_E = \frac{4}{7}}}$

$$\underline{\underline{K_P}} = \frac{1}{\beta K_S K_0 T_E} = \frac{1}{2 \cdot 2 \cdot \frac{1}{7}} = \underline{\underline{\frac{7}{4}}}$$



Sprungantwort des Regelkreises

Aufgabe 2:

a) $$G_s(s) = \frac{1}{(12s+4)(s+2)} = \frac{\frac{1}{4}\cdot\frac{1}{2}}{(3s+1)(\frac{1}{2}s+1)}$$

$$= \frac{1}{8}\cdot\frac{1}{(3s+1)}\cdot\frac{1}{(\frac{1}{2}s+1)} \qquad T_1 = 3 \qquad K_s = \frac{1}{8}$$

$$T_2 = \frac{1}{2}$$

b) $$G_s(s) = \frac{1}{12s^2+28s+8} \qquad G_R(s) = \frac{K}{s}$$

$$G_o(s) = \frac{K}{s(12s^2+28s+8)}$$ gesamt, offen

$$G(s) = \frac{G_o}{1+G_o} = \frac{\frac{K}{s(12s^2+28s+8)}}{1+\frac{K}{s(12s^2+28s+8)}}$$

$$= \frac{K}{\underset{a_0}{12s^3}+\underset{a_1}{28s^2}+\underset{a_2}{8s}+\underset{a_3}{K}}$$

Hurwitz: $$H_2 = \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix} = \begin{vmatrix} 28 & K \\ 12 & 8 \end{vmatrix}$$

$$H_2 = 224 - 12K > 0 \Rightarrow K < 18\tfrac{2}{3}$$

$$\Rightarrow \quad \underline{\underline{K > 0 \wedge K < 18\tfrac{2}{3}}}$$

Aufgabe 3 :

Digitaler (quasikontinuierlicher) Regler ist ein additiv realisierter PID-Regler

Formeln zur Umrechnung multipl. Form ⟹ add. Form des PID-Reglers

$$K_{Pa} = K_P \cdot \frac{T_N + T_V}{T_N}$$

$$T_{Na} = T_N + T_V \qquad (1)$$

$$T_{Va} = K_P \cdot \frac{T_N \cdot T_V}{T_N + T_V}$$

$$\boxed{u(k) = u(k-1) + q_0 \cdot e(k) + q_1 \cdot e(k-1) + q_2 \cdot e(k-2)}$$

T = Abtastzeit

$$\boxed{q_0 = K_P \cdot \left(1 + \frac{T_V}{T}\right)} \quad \boxed{q_1 = -K_P \cdot \left(1 - \frac{T}{T_N} + 2\frac{T_V}{T}\right)} \quad \boxed{q_2 = K_P \cdot \frac{T_V}{T}} \qquad (2)$$

mit $K_P = 5$, $T_N = 4$, $T_V = 2$ (multiplikativ)

und (1) $\Rightarrow$ $\underline{\underline{K_{Pa}}} = 5 \cdot \frac{(4+2)}{4} = \underline{\underline{7{,}5}}$

$\underline{\underline{T_{Na}}} = 4 + 2 = \underline{\underline{6}}$

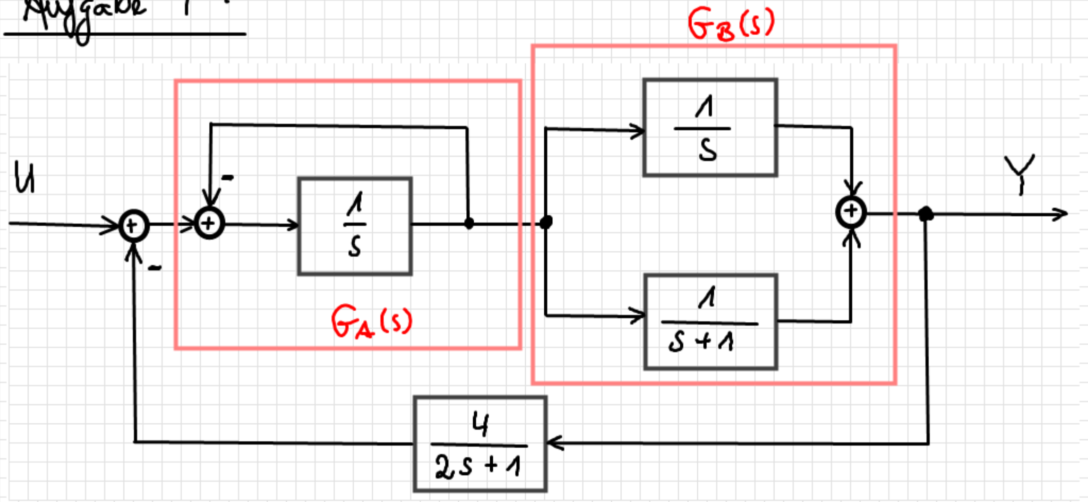
$\underline{\underline{T_{Va}}} = 5 \cdot \frac{4 \cdot 2}{4 + 2} = \frac{40}{6} = \underline{\underline{6\tfrac{2}{3}}}$

mit (2) $\Rightarrow$ $\underline{\underline{q_0}} = 7.5 \cdot \left(1 + \frac{6\frac{2}{3}}{1}\right) = \underline{\underline{57{,}5}}$

$\underline{\underline{q_1}} = -7.5 \cdot \left(1 - \frac{1}{6} + 2\,\frac{6\frac{2}{3}}{1}\right) = \underline{\underline{-106\tfrac{1}{4}}}$

$\underline{\underline{q_2}} = 7{,}5\,\frac{6\frac{2}{3}}{1} = \underline{\underline{50}}$

Aufgabe 4 :



$$G_A(s) = \frac{\frac{1}{s}}{1 + \frac{1}{s}} = \frac{1}{(s+1)}$$

$$G_B(s) = \frac{1}{s} + \frac{1}{s+1} = \frac{(s+1)+s}{s(s+1)} = \frac{2s+1}{s(s+1)}$$

$$\underline{\underline{G_{AB}(s)}} = G_A(s) \cdot G_B(s) = \frac{1}{(s+1)} \cdot \frac{(2s+1)}{s(s+1)} = \underline{\underline{\frac{2s+1}{s \cdot (s+1)^2}}}$$

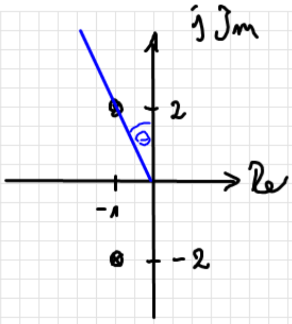
$$\underline{\underline{G(s)}} = \frac{G_{AB}(s)}{1 + \frac{4}{(2s+1)} \cdot G_{AB}} = \frac{\frac{2s+1}{s \cdot (s+1)^2}}{1 + \frac{4}{\cancel{(2s+1)}} \cdot \frac{\cancel{(2s+1)}}{s \cdot (s+1)^2}}$$

$$= \frac{(2s+1)}{s \cdot (s+1)^2 + 4} = \underline{\underline{\frac{2s+1}{s^3 + 2s^2 + s + 4}}}$$

b) $s^2 + 2s + 5 = 0$ $\qquad \underline{\underline{s_{1,2}}} = -1 \pm \sqrt{1^2 - 5} = \underline{\underline{-1 \pm 2j}}$

Pole bei : $\underline{\underline{s_1 = -1 + 2j}}$ $\qquad \underline{\underline{s_2 = -1 - 2j}}$

c)



$$D = \sin\Theta = \frac{1}{\sqrt{2^2+1}} = \frac{1}{\sqrt{5}}$$

$$\underline{\underline{D = 0.447}}$$

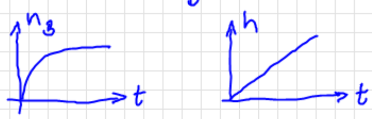
d)

$$G(s) = \frac{2s+1}{s^2+2s+5}$$

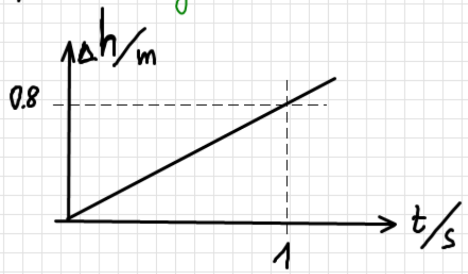
$$\underline{\underline{y_\infty}} = \lim_{s\to 0} G(s) = \lim_{s \to 0} \frac{2s+1}{s^2+2s+5} = \underline{\underline{0.2}}$$

Aufgabe 5 :

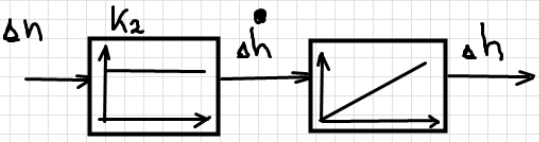
Wirkungskette: $X \xrightarrow{1} n_B \xrightarrow{2} h$



a) Änderung der Höhe in Abhängigkeit von der Drehzahl:



Strecke ohne Ausgleich (I-Verhalten)



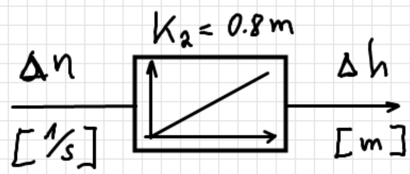
bei $\Delta n_b = 1\,\frac{U}{s}$

$$\boxed{\Delta\dot{h} = K_2 \cdot \Delta n}$$

DGL der Teilregelstrecke

$$\Rightarrow \quad \underline{\underline{K_2}} = \frac{\Delta\dot{h}}{\Delta n} = \frac{0.8\,\frac{m}{s}}{1\,\frac{1}{s}} = \underline{\underline{0.8\,m}}$$

$\Rightarrow$

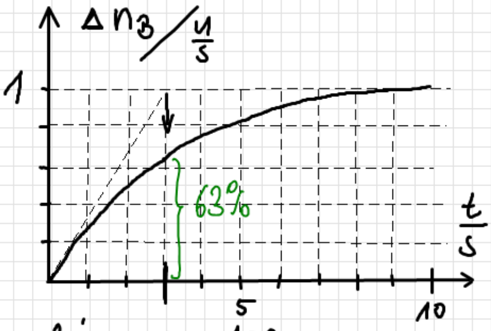


$$\Delta\dot{h} = K_2 \cdot \Delta n \quad ○\!\!-\!\!\bullet \quad s \cdot H(s) = 0.8 \cdot N(s)$$

$$\Rightarrow \quad \underline{\underline{G_2(s) = \frac{H(s)}{N(s)} = 0.8 \cdot \frac{1}{s}}}$$

(SI-Normierung)

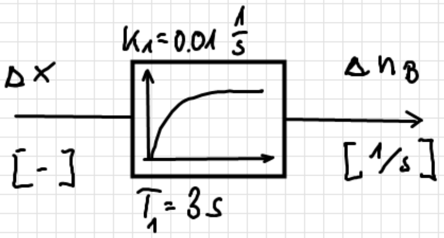
Änderung der Drehzahl in Abhängigkeit vom Registerwert:



bei $\Delta x = 100$

$$\boxed{T_1 \Delta \dot{n}_B + \Delta n_B = K_1 \cdot \Delta x}$$

$$\underline{\underline{K_1}} = \frac{\Delta n_B(t\to\infty)}{\Delta x} = \frac{1\,\frac{1}{s}}{100} \approx \underline{\underline{0.01\,\frac{1}{s}}}$$



PT_1

$$\Rightarrow \quad G_1(s) = \frac{0.01}{(3s+1)} \qquad (\text{SI-Normierung})$$

Gesamtübertragungsfunktion

$$\underline{\underline{G(s)}} = \frac{0.8}{s} \cdot \frac{0.01}{(3s+1)} = \underline{\underline{8 \cdot 10^{-3} \cdot \frac{1}{s\,(3s+1)}}}$$

b) PI-Regler mit symm. Optimum

$$G_S(s) = \frac{K_0 K_S}{s(sT_E+1)}$$

$$T_E = \sum_{i=2}^{n} T_i$$

PI

$$G_R(s) = \frac{K_P(sT_N+1)}{sT_N}$$

$$T_N = \beta^2 T_E, \quad K_P = \frac{1}{\beta K_S T_E K_0}$$

$$\left.\begin{aligned} K_0 \cdot K_S &= 8 \cdot 10^{-3} \\ T_E &= 3 \\ \beta &= 2 \end{aligned}\right\}$$

$$\boxed{\begin{aligned} \underline{\underline{T_N}} &= 4 \cdot 3 = \underline{\underline{12}} \\ \underline{\underline{K_P}} &= \frac{1}{2 \cdot 8 \cdot 10^{-3} \cdot 3} \approx \underline{\underline{20.8}} \end{aligned}}$$