Aufgabe 1:

a)

$$\dot{x}_1 = -\frac{c}{m}\sqrt{x_1^2 + y_1^2} \cdot x_1 \qquad (1)$$

$$\dot{x}_2 = x_1 \qquad (2)$$

$$\dot{y}_1 = -g - \frac{c}{m}\sqrt{x_1^2 + y_1^2} \cdot y_1 \qquad (3)$$

$$\dot{y}_2 = y_1 \qquad (4)$$

b)

$$x_{1_{n+1}} = x_{1_n} + h \cdot \left[-\frac{c}{m}\sqrt{x_{1_n}^2 + y_{1_n}^2} \cdot x_{1_n} \right. \qquad (5)$$

$$x_{2_{n+1}} = x_{2_n} + h \cdot x_{1_n} \qquad (6)$$

c)

$$k_1 = h \cdot \left[-\frac{c}{m}\sqrt{x_{1_n}^2 + y_{1_n}^2} \cdot x_{1_n} \right] \qquad (7)$$

$$k_2 = h \cdot \left[-\frac{c}{m}\sqrt{\left(x_{1_n} + \frac{k_1}{2}\right)^2 + \left(y_{1_n} + \frac{p_1}{2}\right)^2} \cdot \left(x_{1_n} + \frac{k_1}{2}\right) \right. \qquad (8)$$

$$l_1 = h \cdot x_{1_n} \qquad (9)$$

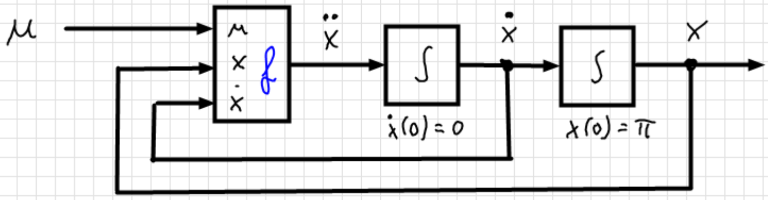
$$l_2 = h \cdot \left(x_{1_n} + \frac{k_1}{2} \right) \qquad (10)$$

$$x_{1_{n+1}} = x_{1_n} + l_1 \qquad (11)$$

$$x_{2_{n+1}} = x_{2_n} + l_2 \qquad (12)$$

Aufgabe 2

a) $\ddot{x} = \underbrace{\frac{1}{2}\mu^2 - 2x^2\cdot\dot{x} + 4\cdot x\cdot\sin(x) - 8}_{f(\mu,x,\dot{x})}$



b) im AP (stat. Zustand) gilt: $\dot{x} = \ddot{x} = 0$

$$\mu_0^2 = -8x_0\cdot\sin(x_0) + 16 = 16 \quad \Rightarrow \quad \underline{\underline{\mu_0 = 4}}$$

c) $\left.\frac{\partial f}{\partial \mu}\right|_{AP} = \mu_0 = \underline{\underline{4}}$

$$\left.\frac{\partial f}{\partial x}\right|_{AP} = -4\overset{0}{\cancel{x_0\dot{x}_0}} + 4\sin(x_0) + 4x_0\cos(x_0)$$

$$= 4\cdot 0 + 4\cdot\pi\cdot(-1) = \underline{\underline{-4\pi}}$$

$$\left.\frac{\partial f}{\partial \dot{x}}\right|_{AP} = -2x_0^2 = \underline{\underline{-2\pi^2}}$$

$$\Delta\ddot{x} = \frac{\partial f}{\partial \mu}\Delta\mu + \frac{\partial f}{\partial x}\cdot\Delta x + \frac{\partial f}{\partial \dot{x}}\Delta\dot{x}$$

$$\underline{\underline{\Delta\ddot{x} = 4\Delta\mu - 4\pi\Delta x - 2\pi^2\Delta\dot{x}}}$$

Vereinfachte lin. DGL (Δ weggelassen)

$$\ddot{x} = 4u - 4\pi x - 2\pi^2 \dot{x}$$

d) $\ddot{x} + 20\dot{x} + 12x = 4u$

$$X(s)\left[s^2 + 20s + 12\right] = 4\,U(s)$$

$$G(s) = \frac{X(s)}{U(s)} = \frac{4}{s^2 + 20s + 12}$$

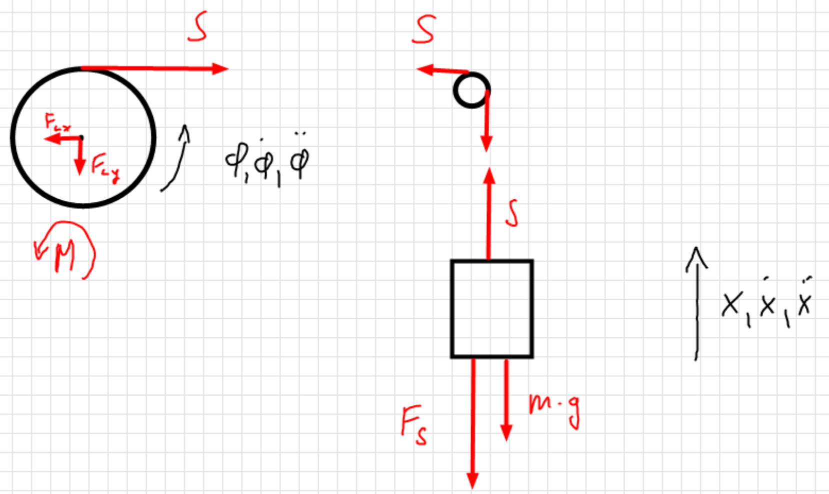
e) $N_{1,2} = -10 \pm \sqrt{10^2 - 12} = -10 \pm 9.38$

$$\left.\begin{aligned} N_1 &= -0.62 \\ N_2 &= -19.38 \end{aligned}\right\} \text{beide Pole negativ}$$

$\Rightarrow$ System ist stabil

Aufgabe 3

a)



b) Trommel:

$$M - S \cdot r = J_T \cdot \ddot\varphi \qquad (1)$$

$$\ddot\varphi = \frac{\ddot x}{r} \qquad (2)$$

$$\Rightarrow M - S \cdot r = J \frac{\ddot x}{r} \qquad (3)$$

Masse:

$$S - m \cdot g - F_S = m \ddot x \qquad (4)$$

(3) und (4) nach S umformen und gleichsetzen:

$$\frac{M}{r} - J \frac{\ddot x}{r^2} = m \cdot g + F_S + m \ddot x \qquad (5)$$

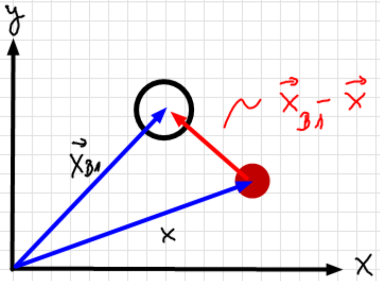
$$\ddot x \left[m + \frac{J}{r^2} \right] = \frac{M}{r} - m \cdot g - F_S \qquad (5')$$

$$F_R = \frac{1}{2} c_w \cdot \rho \cdot A \cdot \dot{x}^2 \qquad (6)$$

$$\Rightarrow \quad \ddot{x} = \frac{1}{\left(m + \frac{J}{r^2}\right)} \cdot \left[\frac{M}{r} - m \cdot g - \frac{1}{2} c_w \cdot \rho \cdot A \cdot \dot{x}^2 \right]$$

Aufgabe 4

a) $\vec{F} = \vec{F}_k + \vec{F}_{B1} + \vec{F}_{B2} + \vec{F}_{B3}$



$$\vec{F}_{B1} = \underbrace{\frac{A}{|\vec{x}_{B1} - \vec{x}|}}_{\text{Betrag}} \cdot \underbrace{\frac{(\vec{x}_{B1} - \vec{x})}{|\vec{x}_{B1} - \vec{x}|}}_{\text{Richtung}}$$

$$\Rightarrow \vec{F} = 7m \cdot (\sin\varphi_x, \sin\varphi_y) + A \cdot \left[\frac{\vec{x}_{B1} - \vec{x}}{|\vec{x}_{B1} - \vec{x}|^2} + \frac{\vec{x}_{B2} - \vec{x}}{|\vec{x}_{B2} - \vec{x}|^2} + \frac{\vec{x}_{B3} - \vec{x}}{|\vec{x}_{B3} - \vec{x}|^2} \right]$$

b) $\vec{F} = m\ddot{\vec{x}} \Rightarrow \ddot{\vec{x}} = \frac{1}{m} \cdot F(x, \varphi_x, \varphi_y)$

c) Der Strömungswiderstand wirkt gegen die Bewegungsrichtung (gegen $\dot{\vec{x}}$)

$$\ddot{\vec{x}} = \frac{1}{m}\left[F(x, \varphi_x, \varphi_y) - \frac{1}{2} c_w \cdot A \cdot \rho \cdot \dot{x}^2 \cdot \frac{\dot{\vec{x}}}{|\dot{\vec{x}}|} \right]$$

Aufgabe 5

$$G_{34} = G_3 + G_4 = 2 + \frac{2}{s} = \frac{2s+2}{s} = 2\,\frac{s+1}{s}$$

$$G_{234_0} = G_2 \cdot G_{34} = 2\,\frac{\cancel{(s+1)}}{s} \cdot \frac{1}{\cancel{(s+1)}} = \frac{2}{s}$$

$$G_{234_{gr}} = \frac{G_{234_0}}{1 + G_{234_0}} = \frac{\frac{2}{s}}{\left(1 + \frac{2}{s}\right)} \begin{matrix} \cdot s \\ \cdot s \end{matrix} = \frac{2}{(s+2)}$$

$$= \frac{1}{\left(\frac{1}{2}s + 1\right)}$$

$$G_0 = G_5 \cdot G_{234_{gr}} \cdot G_1$$

$$= 2 \cdot \frac{1}{\left(\frac{1}{2}s + 1\right)} \cdot \frac{2}{s} = \frac{4}{s\left(\frac{1}{2}s + 1\right)}$$

$$G = \frac{G_0}{1 + G_0} = \frac{\frac{4}{s \cdot \left(\frac{1}{2}s + 1\right)}}{1 + \frac{4}{s \cdot \left(\frac{1}{2}s + 1\right)}}$$

$$= \frac{4}{s\left(\frac{1}{2}s + 1\right) + 4} = \frac{4}{\frac{1}{2}s^2 + s + 4}$$

$$= \frac{8}{s^2 + 2s + 8}$$

Aufgabe 6

Die ÜF des geregelten Systems lautet:

$$G(s) = \frac{K_R \frac{(s+2)}{(s+10)} \cdot \frac{1}{(10s^2+1)}}{1 + K_R \frac{(s+2)}{(s+10)(10s^2+1)}}$$

$$= \frac{K_R (s+2)}{(s+10)(10s^2+1) + K_R \cdot (s+2)}$$

$$= \frac{K_R (s+2)}{\underset{a_0}{10 s^3} + \underset{a_1}{100 s^2} + \underset{a_2}{(K_R+1)\cdot s} + \underset{a_3}{(2K_R+10)}}$$

$$H_2 = \begin{vmatrix} 100 & 2K_R+10 \\ 10 & K_R+1 \end{vmatrix} = 100 K_R + \cancel{100} - 20 K_R - \cancel{100}$$

$$= 80 K_R > 0$$

$\Rightarrow$ stabil, wenn $\underline{\underline{K_R > 0}}$

Aufgabe 7

a) schweben heißt : $\ddot{x} = 0$

$$\Rightarrow F(n) = G$$

$$a n^2 = G \Rightarrow \underline{\underline{n_0}} = \sqrt{\frac{G}{a}} = \sqrt{\frac{m \cdot g}{a}} = \underline{\underline{150 \frac{1}{s}}}$$

b) $a n^2 - G = m \ddot{x}$

$$\ddot{x} = \underbrace{\frac{a}{m} n^2 - g}_{f(n)}$$

$$\underline{\underline{\Delta \ddot{x}}} = \frac{\partial f}{\partial n}\Big|_{AP} \cdot \Delta n = 2 \frac{a}{m} n_0 \cdot \Delta n = \underline{\underline{0.1308 \cdot \Delta n}}$$

$$\underline{\underline{G_2(s)}} = \frac{X(s)}{N(s)} = \underline{\underline{0.1308 \frac{1}{s^2}}}$$

c) $T \dot{n} + n = K_s \cdot u$ $\qquad K_s = \frac{x_a(t \to \infty)}{x_e} = 0.2 \frac{1}{s}$

$T = 0.5 s$

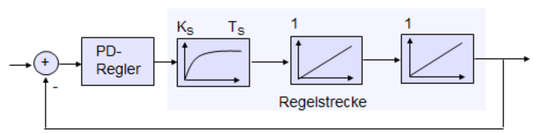
$\Rightarrow$ SI-norm. $\underline{\underline{0.5 \dot{n} + n = 0.2 \cdot u}}$

d) $N(s)\,[0.5 s + 1] = 0.2\, U(s)$

$$G_1(s) = \frac{N(s)}{U(s)} = \underline{\underline{\frac{0.2}{0.5 s + 1}}}$$

e)
$$\underline{\underline{G_2(s)}} = \frac{X(s)}{U(s)} = \underline{\underline{\frac{0.02616}{s^2\,(0.5 \cdot s + 1)}}}$$

f) $\Rightarrow$ PD - Regler



$$G_{PD}(s) = K_R \cdot \frac{sT_V + 1}{s\frac{T_V}{\alpha} + 1}$$

$$T_V = 10 \cdot T_S \sqrt{\alpha} \qquad K_R = \frac{\sqrt{\alpha}}{K_S \cdot T_V^2}$$

α	Überschw.
5	45%
10	30%
20	20%

$K_s = 0.02616$

$T_s = 0.5$

$\Rightarrow \underline{\underline{T_v = 11.18}}$

$\Rightarrow \underline{\underline{K_R = 0.684}}$