

Computergrafik

Lisa Kassebaum - 199856

Andreas Pfohl - 184670

Martin Knoll - 193321

1

1.1

$$|v| = \left| \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} \right| = \sqrt{1^2 + 4^2 + 3^2} = \sqrt{26} = 5,1$$

$$v \text{ normalisiert : } \frac{1}{\sqrt{26}} = 0,2 \quad \frac{4}{\sqrt{26}} = 0,78 \quad \frac{3}{\sqrt{26}} = 0,58$$

$$|v \text{ normalisiert}| : \sqrt{0,2^2 + 0,78^2 + 0,58^2} = 1$$

$$|v| = \left| \begin{pmatrix} 0 \\ 0 \\ 12 \end{pmatrix} \right| = \sqrt{0^2 + 0^2 + 12^2} = \sqrt{144} = 12$$

$$v \text{ normalisiert : } \frac{0}{\sqrt{12}} = 0 \quad \frac{0}{\sqrt{12}} = 0 \quad \frac{12}{\sqrt{12}} = 1$$

$$|v \text{ normalisiert}| : \sqrt{0^2 + 0^2 + 1^2} = 1$$

$$|v| = \left| \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} \right| = \sqrt{(-2)^2 + 0^2 + 1^2} = \sqrt{5} = 2,24$$

$$v \text{ normalisiert : } \frac{-2}{\sqrt{5}} = -0,89 \quad \frac{0}{\sqrt{5}} = 0 \quad \frac{1}{\sqrt{5}} = 0,45$$

$$|v \text{ normalisiert}| : \sqrt{(-0,89)^2 + 0^2 + 0,45^2} = 1$$

1.2

$$\begin{pmatrix} 1 \\ 4 \\ 7 \end{pmatrix} \times \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix} = 19$$

$$\begin{pmatrix} -5 \\ 1 \\ 3 \end{pmatrix} \times \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} = -19$$

$$\begin{pmatrix} -5 \\ 1 \\ 3 \end{pmatrix} \times \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} = 0$$

1.3

$$\begin{pmatrix} 5 \\ 3 \\ 0 \end{pmatrix} \times \begin{pmatrix} -2 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 26 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

1.4

$$c = \vec{a} \times \vec{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$$

$$c \cdot \vec{a} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

$$= (a_2 b_3 - a_3 b_2)(a_1) + (a_3 b_1 - a_1 b_3)(a_2) + (a_1 b_2 - a_2 b_1)(a_3)$$

$$= a_1 a_2 b_3 - a_1 a_3 b_2 + a_2 a_3 b_1 - a_1 a_2 b_3 + a_1 a_3 b_2 - a_2 a_3 b_1 = 0$$

$$c \cdot \vec{b} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$= (a_2 b_3 - a_3 b_2)(b_1) + (a_3 b_1 - a_1 b_3)(b_2) + (a_1 b_2 - a_2 b_1)(b_3)$$

$$= a_2 b_1 b_3 - a_3 b_1 b_2 + a_3 b_1 b_2 - a_1 b_2 b_3 + a_1 b_3 b_2 - a_2 b_1 b_3 = 0$$

1.5

$$\vec{a} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} -4 \\ -3 \\ 0 \end{pmatrix}$$

$$\text{a) } a^T b = \|a\| \|b\| \cos(a) \rightarrow \alpha = \cos^{-1}\left(\frac{a^T b}{\|a\| \|b\|}\right)$$

$$\|a\| = 5$$

$$\|b\| = 5$$

$$\cos(a) \rightarrow \cos^{-1}\left(\frac{-12}{25}\right) = 118,69^\circ$$

$$\text{b) } \|a \times b\| = \|a\| \|b\| \sin(a) \rightarrow \alpha = \sin^{-1}\left(\frac{\|a \times b\|}{\|a\| \|b\|}\right)$$

$$\|a\| = 5$$

$$\|b\| = 5$$

$$\begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} \times \begin{pmatrix} -4 \\ -3 \\ 0 \end{pmatrix} = \left| \begin{pmatrix} 12 \\ 16 \\ -9 \end{pmatrix} \right| = \sqrt{481}$$

$$\sin(a) \rightarrow \sin^{-1}\left(\frac{\sqrt{481}}{25}\right) = 61,31^\circ$$

1.6

$$\vec{a} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} \frac{3}{5} \\ \frac{4}{5} \end{pmatrix}$$

$$a = a_{\parallel} + a_{\perp}$$

$$a_{\parallel} = (a \circ e_b) \cdot e_b$$

$$= \left[\begin{pmatrix} 3 \\ 0 \end{pmatrix} \circ \begin{pmatrix} \frac{3}{5} \\ \frac{4}{5} \end{pmatrix} \right] \cdot \begin{pmatrix} \frac{3}{5} \\ \frac{4}{5} \end{pmatrix} = \begin{pmatrix} \frac{27}{25} \\ \frac{36}{25} \end{pmatrix}$$

$$a_{\perp} = a - a_{\parallel}$$

$$= \begin{pmatrix} 3 \\ 0 \end{pmatrix} - \begin{pmatrix} \frac{27}{25} \\ \frac{36}{25} \end{pmatrix} = \begin{pmatrix} \frac{48}{25} \\ -\frac{36}{25} \end{pmatrix}$$