Computergrafik

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1.1

$$|v| = \left| \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} \right| = \sqrt{1^2 + 4^2 + 3^2} = \sqrt{26} = \qquad 5,1$$

$$v \text{ normalisiert}: \quad \frac{1}{\sqrt{26}} = 0,2 \quad \frac{4}{\sqrt{26}} = 0,78 \quad \frac{3}{\sqrt{26}} = 0,58$$

$$|v \text{ normalisiert}|: \quad \sqrt{0,2^2 + 0,78^2 + 0,58^2} = \qquad 1$$

$$|v| = \left| \begin{pmatrix} 0 \\ 0 \\ 12 \end{pmatrix} \right| = \sqrt{0^2 + 0^2 + 12^2} = \sqrt{144} = \qquad 12$$

$$v \text{ normalisiert}: \quad \frac{0}{\sqrt{12}} = 0 \quad \frac{0}{\sqrt{12}} = 0 \quad \frac{12}{\sqrt{12}} = 1$$

$$|v \text{ normalisiert}|: \quad \sqrt{0^2 + 0^2 + 1^2} = \qquad 1$$

$$|v| = \left| \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} \right| = \sqrt{(-2)^2 + 0^2 + 1^2} = \sqrt{5} = \qquad 2,24$$

$$v \text{ normalisiert}: \quad \frac{-2}{\sqrt{5}} = -0,89 \quad \frac{0}{\sqrt{5}} = 0 \quad \frac{1}{\sqrt{5}} = 0,45$$

$$|v \text{ normalisiert}|: \quad \sqrt{(-0,89)^2 + 0^2 + 0,45^2} = \qquad 1$$

1.2

$$\begin{pmatrix}1\\4\\7\end{pmatrix} \times \begin{pmatrix}-2\\0\\3\end{pmatrix} = 19$$

$$\begin{pmatrix}-5\\1\\3\end{pmatrix} \times \begin{pmatrix}4\\-2\\1\end{pmatrix} = -19$$

$$\begin{pmatrix}-5\\1\\3\end{pmatrix} \times \begin{pmatrix}4\\-2\\1\end{pmatrix} = 0$$

1.3

$$\begin{pmatrix}5\\3\\0\end{pmatrix} \times \begin{pmatrix}-2\\4\\0\end{pmatrix} = \begin{pmatrix}0\\0\\26\end{pmatrix}$$

$$\begin{pmatrix}1\\0\\0\end{pmatrix} \times \begin{pmatrix}0\\1\\1\end{pmatrix} = \begin{pmatrix}0\\-1\\1\end{pmatrix}$$

1.4

$$c = \vec{a} \times \vec{b} = \begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix} \times \begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix} = \begin{pmatrix}a_2b_3 - a_3b_2\\a_3b_1 - a_1b_3\\a_1b_2 - a_2b_1\end{pmatrix}$$

$$c \cdot \vec{a} = \begin{pmatrix}a_2b_3 - a_3b_2\\a_3b_1 - a_1b_3\\a_1b_2 - a_2b_1\end{pmatrix} \begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}$$

$$= (a_2b_3 - a_3b_1)(a_1) + (a_3b_1 - a_1b_3)(a_2) + (a_1b_2 - a_2b_1)(a_3)$$

$$= a_1a_2b_3 - a_1a_3b_2 + a_2a_3b_1 - a_1a_2b_3 + a_1a_3b_2 - a_2a_3b_1 = 0$$

$$c \cdot \vec{b} = \begin{pmatrix}a_2b_3 - a_3b_2\\a_3b_1 - a_1b_3\\a_1b_2 - a_2b_1\end{pmatrix} \begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$$

$$= (a_2b_3 - a_3b_1)(b_1) + (a_3b_1 - a_1b_3)(b_2) + (a_1b_2 - a_2b_1)(b_3)$$

$$= a_2b_1b_3 - a_3b_1b_2 + a_3b_1b_2 - a_1b_2b_3 + a_1b_3b_2 - a_2b_1b_3 = 0$$

1.5

$$\vec{a} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} -4 \\ -3 \\ 0 \end{pmatrix}$$

$$\text{a)} \quad a^T b = \|a\| \|b\| \cos{(a)} \to \alpha = \cos^{-1}(\frac{a^T b}{\|a\| \|b\|})$$

$$\|a\| = 5$$

$$\|b\| = 5$$

$$\cos{(a)} \to \cos^{-1}(\frac{-12}{25}) = 118,69°$$

$$\text{b)} \quad \|a \times b\| = \|a\| \|b\| \sin{(a)} \to \alpha = \sin^{-1}(\frac{\|a \times b\|}{\|a\| \|b\|})$$

$$\|a\| = 5$$

$$\|b\| = 5$$

$$\begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} \times \begin{pmatrix} -4 \\ -3 \\ 0 \end{pmatrix} = \left| \begin{pmatrix} 12 \\ 16 \\ -9 \end{pmatrix} \right| = \sqrt{481}$$

$$\sin{(a)} \to \sin^{-1}(\frac{\sqrt{481}}{25}) = 61,31°$$