

Assignment 1

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Question taken from

cbse/math/10/2006/set1-Q17

1 QUESTION 1

Find the value of p for which the points

$\mathbf{A} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 \\ p \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$ are collinear

2 SOLUTION

Given:- Given:- $\mathbf{A} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 \\ p \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

Given that the points are collinear, so we create a matrix

$$\mathbf{M} = (\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A})^T \quad (2.0.1)$$

where $\text{rank}(\mathbf{M}) = 1$. We have the matrix \mathbf{M} as,

$$\mathbf{M} = \begin{pmatrix} 2 + 1 & p - 3 \\ 5 + 1 & -1 - 3 \end{pmatrix} \quad (2.0.2)$$

$$\Rightarrow \mathbf{M} = \begin{pmatrix} 3 & p - 3 \\ 6 & -4 \end{pmatrix} \quad (2.0.3)$$

Now we row reduce the matrix \mathbf{M} ,

$$\begin{pmatrix} 3 & p - 3 \\ 6 & -4 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 6 & -4 \\ 3 & p - 3 \end{pmatrix} \quad (2.0.4)$$

$$\xrightarrow{R_1 \rightarrow \frac{R_1}{2}} \begin{pmatrix} 3 & -2 \\ 3 & p - 3 \end{pmatrix} \quad (2.0.5)$$

$$\xrightarrow{R_2 \rightarrow R_2 - R_1} \begin{pmatrix} 3 & -2 \\ 0 & p - 1 \end{pmatrix} \quad (2.0.6)$$

$$\xrightarrow{R_1 \rightarrow \frac{R_1}{3}} \begin{pmatrix} 1 & -\frac{2}{3} \\ 0 & p - 1 \end{pmatrix} \quad (2.0.7)$$

Since $\text{rank}(\mathbf{M}) = 1$, we have

$$p - 1 = 0 \quad (2.0.8)$$

$$\Rightarrow p = 1 \quad (2.0.9)$$

Fig.2.1 verifies that the points are indeed collinear for $p = -1$

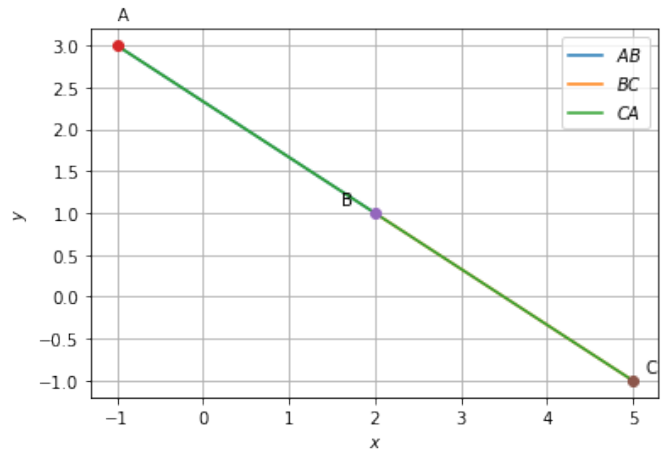


Fig. 2.1: Collinear