Чуриков Нисар РИС-22-1

1. 2, 3, 6, 3, 1
1, 2, 3, 3, 6
$M_0 = 3$ $Me = 3$
Ответ: 5) 3

$M(X) = \frac{15}{5} = 3$

$3 - 3 + 3 = 3$

2. 1) $X_1 + X_2 - X_3$
$M(X_1) + M(X_2) - M(X_3) = 4 + 4 - 4 = 4$
Ответ: 1) $X_1 + X_2 - X_3$

3. $M(X) = \frac{2 \cdot 12 + 5 \cdot 16 + 7 \cdot 14 + 10 \cdot 8}{12 + 16 + 14 + 8} = \frac{24 + 80 + 98 + 80}{50} =$

$= \frac{282}{50} = 5,64$ Ответ: 2) 5,64

4. $p = \frac{1}{7}$ $q = \frac{6}{7}$
$n = 40$

$np - q \leq m_0 \leq np + p$

$\frac{40}{7} - \frac{6}{7} \leq m_0 \leq \frac{40}{7} + \frac{1}{7}$

$\frac{34}{7} \leq m_0 \leq \frac{41}{7}$

$4,86 \leq m_0 \leq 5,86 \Rightarrow m_0 = 5$

Ответ: 5) 5

5. $p = 0,002$ $q = 0,998$
$n = 4000$ $npq = 7,984 \leq 10 \Rightarrow$ Пуассон $\lambda = np = 8$

для 0 человек $P(0) = \frac{\lambda^m}{m!} \cdot e^{-\lambda} = \frac{8^0}{0!} \cdot e^{-8} = e^{-8}$

для 1 человека $P(1) = \frac{\lambda^m}{m!} \cdot e^{-\lambda} = \frac{8^1}{1!} \cdot e^{-8} = 8e^{-8}$

для 2 человек $P(2) = \frac{\lambda^m}{m!} e^{-\lambda} = \frac{8^2}{2!} e^{-8} = 32e^{-8}$

$P = P(0) + P(1) + P(2) = e^{-8} + 8e^{-8} + 32e^{-8} = 41e^{-8}$

Ответ: $41e^{-8}$

6. $n = 10000$ $npq = 900 > 1 \Rightarrow$ можно интегральную

$p = 0{,}1 \quad q = 0{,}9$

$P(950 < X < 1050) = \Phi(x'') - \Phi(x')$

$x'' = \frac{1050 - 1000}{\sqrt{900}} = \frac{50}{30} = \frac{5}{3}$

$x' = \frac{950 - np}{\sqrt{npq}} = \frac{950 - 1000}{\sqrt{900}}$

$= -\frac{5}{3}$ Ответ: 1) $2\Phi(\frac{5}{3}) - 1$

$P(950 < X < 1050) = \Phi(\frac{5}{3}) - \Phi(-\frac{5}{3}) =$

$= \Phi(\frac{5}{3}) - (1 - \Phi(\frac{5}{3})) = 2\Phi(\frac{5}{3}) - 1$

7.

X	2	4	3	4	7
Y	2	5	5	7	9

$M(X) = \bar{x} = 4$

$M(Y) = \bar{y} = \frac{28}{5}$

$K_{X,Y} = M(X \cdot Y) - M(X) \cdot M(Y)$

$M(X \cdot Y) = \frac{4 + 20 + 15 + 28 + 63}{5} = \frac{130}{5}$

$K_{X,Y} = \frac{130}{5} - 4 \cdot \frac{28}{5} = \frac{130}{5} - \frac{112}{5} = \frac{18}{5} = 3{,}6$

Ответ: 1) 3,6

8. $P(X \geq 2 \mid X \leq 5) = \frac{P(X \geq 2 \cap X \leq 5)}{P(X \leq 5)} =$

$= \frac{0{,}2 + 0{,}4}{0{,}2 + 0{,}2 + 0{,}4} = \frac{0{,}6}{0{,}8} = \frac{6}{8} = \frac{3}{4} = 0{,}75$

Ответ: 4) 0,75

9. $1 \cdot (0{,}1 - 0) + 3(0{,}9 - 0{,}1) + 5(1 - 0{,}9) =$

$= 0{,}1 + 3 \cdot 0{,}8 + 5 \cdot 0{,}1 = 0{,}1 + 2{,}4 + 0{,}5 = 3$

Ответ: 4) 3

10. $P(X < 3{,}5) = \frac{max - 3{,}5}{max - min} = \frac{5 - 3{,}5}{5 - 1} = \frac{1{,}5}{4} =$

$= 0{,}375$ Ответ: 2) 0,375

11. $P(5<X<7) = \int_5^7 f(x)\,dx = \int_6^7 \left(\frac{x}{2}-3\right)dx =$

$= \left(\frac{x^2}{4} - 3x\right)\Big|_6^7 = \frac{49}{4} - 21 - \frac{36}{4} + 18 = \frac{13}{4} - 3\frac{1}{4} =$

$= \frac{13-12}{4} = \frac{1}{4} = 0{,}25$ Ответ: 5) 0,25

12.

13. $M(X) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \int_{-1}^{0} (2x^2+2x)\,dx =$

$= \left(\frac{2x^3}{3} + x^2\right)\Big|_{-1}^{0} = \frac{2}{3} - 1 = -\frac{1}{3}$

$P\left(X + \frac{1}{3} < 0\right) = P\left(-\infty < X < -\frac{1}{3}\right) = \int_{-\infty}^{-\frac{1}{3}} f(x)\,dx =$

$= \int_{-1}^{-\frac{1}{3}} (2x+2)\,dx = (x^2+2x)\Big|_{-1}^{-\frac{1}{3}} = \frac{1}{9} - \frac{2}{3} - 1 + 2 =$

$= \frac{1-6}{9} + 1 = -\frac{5}{9} + 1 = \frac{4}{9}$ Ответ: 5) $\frac{4}{9}$

14. $x^2 = 0{,}49 \Rightarrow x = 0{,}7$ Ответ: 3) 0,7

16. $X \sim N(-2;4) \Rightarrow M(X) = a = -2$

$D(X) = \sigma^2 = 4 \Rightarrow \sigma = 2$

$P(X \geq 1) = P(1 < X < \infty) = \Phi(\infty) - \Phi\left(\frac{1+2}{2}\right) =$

$= 0{,}5 - \Phi(1{,}5) = 0{,}5 - 0{,}4332 = 0{,}0668$

Ответ: 5) 0,0668

17. $P(-\infty < X < 15) = 0,35$

$P(30 < X < \infty) = 0,32$

$$\begin{cases} \Phi\left(\frac{15-a}{\sigma}\right) - \Phi(-\infty) = 0,35 \\ \Phi(\infty) - \Phi\left(\frac{30-a}{\sigma}\right) = 0,32 \end{cases}$$

$$\begin{cases} \Phi\left(\frac{15-a}{\sigma}\right) + 0,5 = 0,35 \\ 0,5 - \Phi\left(\frac{30-a}{\sigma}\right) = 0,32 \end{cases} \Leftrightarrow \begin{cases} \Phi\left(\frac{15-a}{\sigma}\right) = -0,15 \\ \Phi\left(\frac{30-a}{\sigma}\right) = 0,18 \end{cases}$$

$$\begin{cases} \frac{15-a}{\sigma} = -0,39 \\ \frac{30-a}{\sigma} = 0,47 \end{cases} \Leftrightarrow \begin{cases} 15-a = -0,39\sigma \\ 30-a = 0,47\sigma \end{cases}$$

$$\begin{cases} a = 15 + 0,39\sigma \\ 30 - 15 - 0,39\sigma = 0,47\sigma \end{cases} \qquad 0,86\sigma = 15$$

$$\sigma \approx 17,44$$

Ombem: 3) 17,58

18. $M(X) = 2$, $D(X) = 3 \Rightarrow M(X^2) = 3 + 4 = 7$

$K_{2X,2-5X} = M(2X(2-5X)) - M(2X) \cdot M(2-5X)$

$M(2X(2-5X)) = M(4X - 10X^2) = 4M(X) - 10M(X^2) =$

$= 4 \cdot 2 - 10 \cdot 7 = 8 - 70 = -62$

$M(2X) = 2M(X) = 4$

$M(2-5X) = 2 - 5M(X) = 2 - 10 = -8$

$K_{2X,2-5X} = -62 + 8 \cdot 4 = -62 + 32 = -30$

$D(2-5X) = 25D(X) = 25 \cdot 3 = 75$; $D(2X) = 4D(X) = 12$

Ombem: 2) $\begin{pmatrix} 12 & -30 \\ -30 & 75 \end{pmatrix}$

19. $a = 2 = M(X) \mid \sigma^2 = 4 = D(X) \Rightarrow M(X^2) = 4 + 4 = 8$

$M(8 - 2X - X^2) = 8 - 2M(X) - M(X^2) = 8 - 4 - 8 =$

$= -4$

Ombem: 5) -4

20. $r = \frac{K_{x,y}}{\sigma_x \sigma_y}$ $\quad K_{x,y} = M(XY) - M(X)M(Y)$

$M(Y) = \frac{\sum Y_i}{30} = \frac{30}{30} = 1$

$M(X) = \frac{\sum X_i}{30} = \frac{90}{30} = 3$

$M(XY) = \frac{\sum X_i Y_i}{30} = \frac{135}{30} = 4{,}5$

$M(Y^2) = \frac{\sum Y_i^2}{30} = \frac{30}{30} = 1$

$M(X^2) = \frac{\sum X_i^2}{30} = \frac{360}{30} = 12$

$K_{x,y} = 4{,}5 - 3 \cdot 1 = 1{,}5$

$D(X) = M(X^2) - M^2(X) = 12 - 9 = 3$

$D(Y) = M(Y^2) - M^2(Y) = 11 - 1 = 10$

$r = \frac{1{,}5}{\sqrt{3}\sqrt{10}} \approx \frac{1{,}5}{5{,}48} \approx 0{,}274$

Ответ: 4) 0,274

12. $M(X) = \frac{1}{5}(2+3+4+5+6+7) = \frac{27}{5}$

Ответ: 5) $\frac{25}{12}$

$M(X^2) = \frac{1}{5}(4+9+16+25+36+49) = \frac{139}{5}$

$D(X) = \frac{139}{5} - \frac{729}{25} = \frac{69}{25}$

$M(X) = \int_2^7 \frac{1}{5} x \, dx = \frac{x^2}{10}\Big|_2^7 = \frac{49}{10} - \frac{4}{10} = \frac{45}{10} = \frac{9}{2} = 4{,}5$

$M(X^2) = \int_2^7 \frac{x^2}{5} dx = \frac{x^3}{15}\Big|_2^7 = \frac{343}{15} - \frac{8}{15} = \frac{335}{15} = \frac{67}{3}$