Endogenous Production Networks and the Business Cycle

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Abstract

This paper develops a structural model of trade between heterogeneous firms in which the network of firm-level input-output linkages is endogenously determined. Firms vary in the size of their customer and supplier bases and adjust their sets of trade partners over time. Using data on buyer-seller relationships between US firms, I structurally estimate the model's key parameters, and apply the theory to quantify how important the endogenous adjustment of the production network is for US business cycles. Simulations of the model show that extensive margin adjustments of the production network over horizons of one year account for around 4% of observed real gross output and welfare, while adjustments over longer horizons account for larger components of these outcomes. Furthermore, adjustment of the production network is quantitatively important primarily as a response to idiosyncratic shocks at the relationship level rather than the firm level.

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1 Introduction

1.1 Motivation and overview

A growing body of economic research studies how input-output networks matter for the aggregate effects of shocks at the level of the production unit (sectors, industries, or firms). However, many theories of the role that production networks play in this aggregation take the fundamental structure of the network as exogenous. Typically, the underlying network of the economy is conceived of as the technological input-output matrix between production units, and hence the network is treated as a fixed technology that does not change in response to shocks. Buyer-seller relationships that constitute the production network at the firm-to-firm level, however, are the result of endogenous firm choices that depend on and respond to the macroeconomic environment. In this paper, I study the importance of endogenous production network adjustment at the firm-to-firm level for business cycle fluctuations in the United States.

To motivate the analysis, it is useful to first consider a version of the theorem of Hulten (1978), which states that under very general assumptions, the elasticity of real value-added with respect to labor productivity of a set of firms S can be expressed as:

$$\epsilon_S = \begin{array}{c}\text{labor share of}\\ \text{firms in } S\end{array} \tag{1.1}$$

As discussed in Baqaee and Farhi (2018), this is true even when firms in the economy are connected via heterogeneous input-output linkages, *as long as the network of linkages is exogenous.* Equation (1.1) therefore implies that to a first-order approximation, the aggregation of firm-level shocks depends only on observable firm-level characteristics, and that beyond this, the structure of the underlying production network is irrelevant. When the network of buyer-seller linkages itself responds to shocks, however, I show that the corresponding elasticity also depends on an additional term capturing the role of endogenous production networks:

$$\epsilon_S = \begin{array}{c}\text{labor share of}\\ \text{firms in } S\end{array} + \begin{array}{c}\text{network}\\ \text{adjustment}\end{array} \tag{1.2}$$

The goal of this paper is hence to quantify the importance of network adjustments in generating US business cycle fluctuations - to a first-order approximation, the second term in equation (1.2), but more generally the global effects of such adjustments as well.

To do so, I first develop a structural model of trade between heterogeneous firms in which the network of firm-level input-output linkages is endogenously determined. In the model, firms trade off the benefits of additional relationships (lower costs and higher profits) against a time-varying cost of maintaining each relationship. These assumptions deliver a model of a production network where firms vary in the size of their customer and supplier bases, occupy heterogeneous positions in supply chains, and adjust their sets of active relationships dynamically. I develop a tractable computational algorithm to solve for the model's equilibrium, and use panel data on firm-level trading relationships in the US to structurally estimate the model's key parameters. I then show that the model matches both cross-sectional and dynamic moments characterizing the production network amongst firms in the data. Finally, simulations of the model are used to quantify the importance of endogenous production network adjustment for business cycle fluctuations. I find that endogenous adjustment of the production network over horizons of one year account for around 4% of observed real gross output and welfare, while adjustments over horizons of two and three years account for around 9% and 16% respectively of these outcomes. Furthermore, network adjustment is important primarily as a response to idiosyncratic shocks at the relationship-level rather than at the firm-level.

1.2 Relation to literature

This paper builds on a growing literature that has documented various forms of evidence that input-output networks are quantitatively important for the aggregation of microeconomic shocks into macroeconomic fluctuations. Acemoglu et al. (2012) and Grassi (2018), for example, argue that the network structure of linkages between sectors matters for how idiosyncratic sector-level shocks translate into aggregate changes in output. Magerman et al. (2016) make an analogous argument by studying the production network between Belgian firms, while Barrot and Sauvagnat (2016) find that buyer-seller relationships are important for transmitting firm-level shocks resulting from natural disasters in the US. Similarly, Baqaee and Farhi (2018) extend the first-order characterization of the analysis in Hulten (1978) to show how the existence of production network linkages can matter for the aggregation of idiosyncratic shocks. However, these papers do not seek to explain what determines the network structure of the economy in the first place, nor how the network structure evolves in response to changes in the economic environment. The theory developed in this paper endogeneizes the formation of the production network, and therefore allows us to extend the analysis to address these questions.

In modeling the endogenous formation of firm-level networks, this paper hence adds to a relatively new body of theoretical work that has sought to develop models of firm-to-firm input-output linkages that are computationally tractable and thus suitable for empirical work. In this vein of the literature, three main classes of models have emerged, which one might refer to for brevity as: (i) *costly relationship network* models, (ii) *extreme-value network* models, and (iii) *stochastic network* models.

The theory developed in this paper is most closely related to the first class of network formation models based on costly relationships. The key idea is that firms face a resource cost in forming or maintaining relationships with other firms. Firms' decisions about how to trade off the costs and benefits of these relationships then generate the extensive margin of firm-to-firm trade. Papers that adopt this approach to modeling firm-to-firm network formation include Tintelnot et al. (2018) and Kikkawa et al. (2018), who study the production network amongst Belgian firms, as well as Taschereau-Dumouchel (2018), who studies the production network amongst US firms. In these models, solving for equilibria with endogenous networks is computationally challenging because firms are assumed to be large enough to affect aggregate outcomes. Tractability is then obtained by exogenously restricting the set of relationships that can form. In contrast, the model developed in this paper allows for arbitrary relationship formation, and instead achieves tractability by assuming that the set of firms is large enough that each individual firm has no effect on aggregate variables. This approach affords the model the tractability that is needed to be applied to the study of business cycles.[1]

In its empirical application, this paper is also most closely related to the work of Tintelnot et al. (2018) and Kikkawa et al. (2018). These papers focus on how endogenous production network linkages matter for the aggregate effects of international

[1] The assumption of costly relationships with a large set of infinitesimally small firms is also adopted by Bernard et al. (2018a,b) to study buyer-seller relationship formation by Japanese and Norwegian firms, although these papers study linkage formation between a single set of buyers and a single set of sellers rather than the entire production network.

trade shocks, and argue that since very few firms directly import or export, accounting for changes in domestic production linkages can dramatically alter the predicted gains from openness to international trade. The quantitative results obtained in this paper hence provide complementary evidence in a different context - macroeconomic business cycles - that the rewiring of firm-to-firm production networks matters for the aggregate response of the economy to shocks.

The modeling of firm-to-firm linkages through costly relationships then contrasts with the extreme-value class of network models. A leading example of this type of theory is the model of production networks developed by Oberfield (2018). The central idea is that every firm in the economy requires a fixed number of suppliers in order to produce, and can choose from amongst an exogenous set of potential suppliers, with each of which it receives a relationship-specific productivity draw. Tractability in the model is then obtained by assuming a specific functional form for these draws from the extreme-value class of distributions, as in the spirit of the Eaton and Kortum (2002) model of international trade.[2] As the number of suppliers per firm is exogenously fixed, however, this class of models restricts the extensive margin of firm-to-firm relationship formation, which is a key object of study in this paper.

Finally, papers that adopt the *stochastic network* approach posit an exogenous stochastic algorithm for the formation of linkages between firms, and then proceed to study the resulting properties of the network. These theories draw from classic network formation models in graph theory, and apply the same ideas to economic models of firm-to-firm relationships. For example, Atalay et al. (2011) combine elements from the Erdös and Rényi (1959) random attachment algorithm and the Barabási and Albert (1999) preferential attachment algorithm to model the buyer-supplier network in the US economy, while Chaney (2014, 2018) draws on similar ideas to explain the distribution of a firm's customers across geographic locations. Although these models are able to replicate important features of empirical firm-to-firm networks, they do not take into account firms' optimizing behavior with regard to either network formation or production decisions, and hence cannot be used to study how production networks respond to changes in economic incentives.

Collectively, the models of firm-to-firm linkages described above also build on an older literature studying how network formation results from strategic interacti-

[2] In recent work, Eaton et al. (2018) have reintroduced the key insights of Oberfield (2018) into an international trade model to study firm-to-firm linkages in an open economy.

ons between agents in models with generally few structural assumptions. For example, Aumann and Myerson (1988) and Myerson (1991) model network formation as extensive-form and simultaneous move games respectively, Jackson and Wolinsky (1996) adopt a cooperative game theoretic approach, while Kranton and Minehart (2001) study buyer-seller networks in which ascending-bid auctions are used to determine the formation of links.[3] These game-theoretic approaches therefore explicitly take into account optimizing behavior by the agents constituting the network, but primarily due to the lack of structure, the complexity of solving these models beyond simple illustrative examples precludes quantitative analysis.

1.3 Outline

The outline of this paper is as follows. I begin in section 2 by developing a static version of the theoretical model, in which the set of buyer-supplier relationships is taken as given. I characterize how firm size, firm-to-firm trade volumes, and aggregate outcomes such as household welfare depend on the existing production network, and show how to solve for the market equilibrium of the model given any network of relationships. In section 3, I then endogeneize the formation of linkages between firms in the economy and introduce sources of firm-level and relationship-level dynamics. In section 4, I discuss the data used for structural estimation of the model's parameters, the estimation strategy, and the fit of the model to data. Section 5 then discusses the counterfactual exercises used to quantify the importance of endogenous production network adjustment for the US business cycle, and section 6 concludes.

2 Exogenous Network Model

To study the endogenous formation of firm-to-firm linkages, it is useful to first understand how firms behave conditional on these relationships. I therefore begin by developing a static version of the model in which the network of trading relationships between firms is taken as given.

[3] See Jackson (2005, 2011) for a more in-depth survey of this branch of the literature.

2.1 Model environment

The economy consists of a representative household and an exogenously-given unit continuum of firms that each produce a unique good. Firms are owned by the household, and are heterogeneous over states $\chi = (\phi, \delta)$, where ϕ and δ are the *fundamental productivity* of a firm's production process and the *fundamental demand* for a firm's product respectively. One can interpret these characteristics as exogenous supply- and demand-side firm-level heterogeneity. The exogenous cumulative distribution function over firm states is denoted by G_χ, with density g_χ and support S_χ a bounded subset of $\mathbb{R}^2_+$.[4] For brevity, firms with state χ are also referred to as χ-firms.

2.1.1 Households

The representative household supplies L units of labor inelastically and has constant-elasticity-of-substitution (CES) preferences over all goods in the economy, given by:

$$U = \left[\int_{S_\chi} \left[\delta x_H(\chi) \right]^{\frac{\sigma-1}{\sigma}} dG_\chi(\chi) \right]^{\frac{\sigma}{\sigma-1}} \tag{2.1}$$

Here, σ denotes the elasticity of substitution across varieties, and $x_H(\chi)$ is the household's consumption of χ-firm varieties. Given the price $p_H(\chi)$ charged by χ-firms to the household, household demand is given by:

$$x_H(\chi) = \Delta_H \delta^{\sigma-1} \left[p_H(\chi) \right]^{-\sigma} \tag{2.2}$$

Note that conditional on prices, households demand a greater amount of goods for which fundamental demand δ is higher. The household's demand shifter can then be written as:

$$\Delta_H \equiv U P_H^{\sigma} \tag{2.3}$$

and the consumer price index is equal to:

$$P_H = \left[\int_{S_\chi} \left[\frac{p_H(\chi)}{\delta} \right]^{1-\sigma} dG_\chi(\chi) \right]^{\frac{1}{1-\sigma}} \tag{2.4}$$

[4] Note that given the unit mass of firms, integrals of all firm-level variables over the distribution G_χ are equal to both the average as well as the total value of that variable across firms.

2.1.2 Firm production technology

Each firm produces its output using labor and intermediate inputs sourced from other firms. However, firm-to-firm trade is characterized by relationship frictions, such that every $\chi-$firm is only able to purchase inputs from a given $\chi^{'}$-firm with probability $m\left(\chi,\chi^{'}\right)$. Given that there exists a continuum of firms of every state, $m\left(\chi,\chi^{'}\right)$ is also equal to the fraction of $\chi^{'}$-firms that supply a given χ-firm, as well as the fraction of χ-firms that purchase from a given $\chi^{'}$-firm. I refer to m as the *matching function* of the economy, which completely specifies the extensive margin of firm-to-firm trading relationships in the economy.

Given the matching function, the output of a χ-firm is then given by the following constant returns to scale CES production function:

$$X\left(\chi\right)=\left[\left[\phi l\left(\chi\right)\right]^{\frac{\sigma-1}{\sigma}}+\int_{S_{\chi}}m\left(\chi,\chi^{'}\right)\left[\alpha x\left(\chi,\chi^{'}\right)\right]^{\frac{\sigma-1}{\sigma}}dG_{\chi}\left(\chi^{'}\right)\right]^{\frac{\sigma}{\sigma-1}} \tag{2.5}$$

where $l\left(\chi\right)$ is the quantity of labor demanded and $x\left(\chi,\chi^{'}\right)$ is the quantity of each $\chi^{'}$-good used as inputs. Several features of the production function are worth noting. First, observe that the fundamental productivity ϕ of the firm can be interpreted as a measure of its labor productivity, while the parameter α captures how efficiently the output of one firm can be transformed into the output of another firm.[5] Second, to rule out explosive production, it is necessary and sufficient to impose that $\alpha<1$.[6] Finally, the elasticity of substitution across inputs for intermediate demand is assumed to be the same as that for final demand. As discussed below, this simplifying assumption is motivated mainly by data limitations that preclude the identification of different elasticities.

Taking the wage as numeraire and given prices $\left\{p\left(\chi,\chi^{'}\right)\right\}_{\chi^{'}\in S_{\chi}}$ charged by other

[5] It is also possible to extend the model to allow for multiple sources of exogenous supply-side heterogeneity, for example by allowing firms to differ in terms of both labor productivity and total factor productivity. The analytic characterization of the model below will remain largely unchanged, and the only cost of such an extension is added computational complexity due to an increase in the dimension of the firm state space.

[6] When $\alpha\geq1$, it becomes feasible for a pair of firms that are connected to each other both as buyer and seller to use only each other's output as inputs for production, thereby generating infinite output and profits.

firms, the marginal cost of each χ-firm is then given by:

$$\eta(\chi) = \left[\phi^{\sigma-1} + \alpha^{\sigma-1} \int_{S_\chi} m\left(\chi, \chi^{'}\right) \left[p\left(\chi, \chi^{'}\right)\right]^{1-\sigma} dG_\chi(\chi)\right]^{\frac{1}{1-\sigma}} \tag{2.6}$$

while the quantities of labor and intermediate inputs demanded are given respectively by:

$$l(\chi) = X(\chi)\,\eta(\chi)^{\sigma}\,\phi^{\sigma-1} \tag{2.7}$$

$$x\left(\chi, \chi^{'}\right) = X(\chi)\,\eta(\chi)^{\sigma}\,\alpha^{\sigma-1} p\left(\chi, \chi^{'}\right)^{-\sigma} \tag{2.8}$$

Note that conditional on prices, firms with greater fundamental productivity ϕ have lower marginal costs.

2.1.3 Market structure and firm pricing

The market structure for all firm sales is assumed to be monopolistic competition. This implies that regardless of the complexity of the matching function, the markups that firms charge over marginal costs are identical in equilibrium. This follows from the fact that every buyer (including the household) faces a continuum of sellers and that the demand functions (2.2) and (2.8) exhibit a constant price elasticity. Consequently, the profit-maximizing prices charged by each firm are given by:

$$p_H(\chi) = \mu\eta(\chi) \tag{2.9}$$

$$p\left(\chi, \chi^{'}\right) = \mu\eta\left(\chi^{'}\right) \tag{2.10}$$

where $\mu \equiv \frac{\sigma}{\sigma-1}$ is the standard CES markup over marginal cost.

Note that constant equilibrium markups are important for the computational tractability of the model. Although recent research in the production networks literature has begun to explore alternative market structures that deliver variable markups at the firm-level, these models remain computationally difficult to work with. Furthermore, empirical results based on production network models with variable markups suggest that the variable markup margin in fact contributes quantitatively little to the effect of shock aggregation in production networks.[7]

[7] See for example Kikkawa et al. (2018), who develop a model of endogenous production networks

2.1.4 Market clearing

Market clearing for labor requires:

$$\int_{S_\chi} l(\chi)\, dG_\chi(\chi) = L - L_f \tag{2.11}$$

where $L_f < L$ is the aggregate quantity of labor hired to maintain firm-to-firm relationships in the economy. In this section, we take L_f as given, whereas in section 3 when the dynamic formation of the production network is considered, L_f becomes an endogenous variable. Finally, market clearing for the output of a χ-firm requires:

$$X(\chi) = x_H(\chi) + \int_{S_\chi} m\left(\chi', \chi\right) x\left(\chi', \chi\right) dG_\chi(\chi') \tag{2.12}$$

2.2 Market equilibrium

2.2.1 Firm network characteristics

As described above, the parameters ϕ and δ capture exogenous productivity and demand characteristics that are fundamental to the firm, in the sense that they are independent of the firm's connection to other firms. Firm-level outcomes in equilibrium, however, such as the overall size and profit of a firm, depend not only on a firm's fundamental characteristics but also on the characteristics of other firms that it is connected to in the production network. For an arbitrary matching function, a given firm-level outcome may therefore in principle be a function of very complicated moments of the production network, which would render the model intractable.

To circumvent this problem, I rely on the structure of the CES production function specified in (2.5) to derive sufficient statistics at the firm level, from which all variables of interest can be easily computed. In contrast with firm fundamental characteristics ϕ and δ, it is therefore useful to characterize the static market equilibrium of the model in terms of what I call a χ-firm's *network productivity and demand*, defined

with oligopolistic competition. Given the complexity of dealing with both endogenous networks and variable markups, the model can only be solved with around 30 firms.

respectively by:

$$\Phi(\chi) \equiv \eta(\chi)^{1-\sigma} \tag{2.13}$$

$$\Delta(\chi) \equiv \frac{1}{\Delta_H} X(\chi)\, \eta(\chi)^{\sigma} \tag{2.14}$$

Note that $\Phi(\chi)$ is an inverse measure of a χ-firm's marginal cost, while $\Delta(\chi)$ is the demand shifter in a χ-firm's intermediate demand function (2.8) relative to the household's demand shifter Δ_H.

Combining the demand equations (2.2) and (2.8), the firm marginal cost equation (2.6), the goods market clearing condition (2.12), and the pricing conditions (2.9) and (2.10), we obtain the following system of equations that determines firms' network characteristics:

$$\Phi(\chi) = \phi^{\sigma-1} + \mu^{1-\sigma}\alpha^{\sigma-1} \int_{S_\chi} m\left(\chi, \chi'\right) \Phi\left(\chi'\right) dG_\chi\left(\chi'\right) \tag{2.15}$$

$$\Delta(\chi) = \mu^{-\sigma}\delta^{\sigma-1} + \mu^{-\sigma}\alpha^{\sigma-1} \int_{S_\chi} m\left(\chi', \chi\right) \Delta\left(\chi'\right) dG_\chi\left(\chi'\right) \tag{2.16}$$

Note that (2.15) and (2.16) constitute a pair of decoupled linear functional equations in Φ and Δ respectively, and show how a firm's network characteristics depend on both its fundamental characteristics as well as on the network characteristics of its suppliers and customers. Conditional on ϕ and δ, firms that are connected to firms with larger network productivities and demands also have higher network productivities and demands themselves.

Furthermore, since $\alpha < 1$, $\mu > 1$, and $m\left(\chi, \chi'\right) \leq 1$ for all $\left(\chi, \chi'\right) \in S_\chi^2$, it is easily verified via Blackwell's sufficient conditions that (2.15) and (2.16) constitute decoupled contraction mappings in Φ and Δ. The contraction mapping theorem therefore immediately implies the existence and uniqueness of a solution to the firm network characteristic functions, and also guarantees that iteration on Φ and Δ converges to this solution. This offers a tractable method of solving for the model's static equilibrium regardless of the complexity of the matching function.

Proposition 1. *There exist unique network productivity and demand functions* $\Phi : S_\chi \to \mathbb{R}_+$ *and* $\Delta : S_\chi \to \mathbb{R}_+$ *for any matching function* $m : S_\chi \times S_\chi \to [0,1]$.

Note that we can also rewrite equations (2.15) and (2.16) to express the network productivity and demand of a χ-firm respectively as:

$$\Phi\left(\chi\right)=\int_{S_\chi}\left[\sum_{d=0}^{\infty}\left(\frac{\alpha}{\mu}\right)^{d(\sigma-1)}m^{(d)}\left(\chi,\chi^{'}\right)\right]\left(\phi^{'}\right)^{\sigma-1}dG_\chi\left(\chi^{'}\right) \tag{2.17}$$

$$\Delta\left(\chi\right)=\mu^{-\sigma}\int_{S_\chi}\left[\sum_{d=0}^{\infty}\frac{1}{\mu^d}\left(\frac{\alpha}{\mu}\right)^{d(\sigma-1)}m^{(d)}\left(\chi^{'},\chi\right)\right]\left(\delta^{'}\right)^{\sigma-1}dG_\chi\left(\chi^{'}\right) \tag{2.18}$$

where $m^{(d)}$ is the d^{th}-degree matching function, defined recursively by:

$$m^{(0)}\left(\chi,\chi^{'}\right)=\begin{cases}\frac{1}{g_\chi(\chi)}, & \text{if } \chi=\chi^{'}\\ 0, & \text{if } \chi\neq\chi^{'}\end{cases} \tag{2.19}$$

$$m^{(1)}\left(\chi,\chi^{'}\right)=m\left(\chi,\chi^{'}\right) \tag{2.20}$$

$$m^{(d)}\left(\chi,\chi^{'}\right)=\int_{S_\chi}m^{(d-1)}\left(\chi,\chi^{''}\right)m\left(\chi^{''},\chi^{'}\right)dG_\chi\left(\chi^{''}\right) \tag{2.21}$$

Intuitively, one can think of $m^{(d)}\left(\chi,\chi^{'}\right)$ for $d\geq 1$ as the probability that a χ-firm buys indirectly from a $\chi^{'}$-firm through a supply chain that is of length d. With this interpretation, equations (2.17) and (2.18) show how the network characteristics of a firm depend on its connections to all other firms via supply chains of all lengths. Note that the rate at which the value of an indirect relationship decays with the length of the supply chain is decreasing in input suitability α and increasing in the markup μ.

2.2.2 Firm size and inter-firm trade

Once firm network characteristics are known, the total revenue, variable profit, and variable employment of a χ-firm are completely determined up to the scale factor Δ_H. These are given respectively by:

$$R\left(\chi\right)=\mu\Delta_H\Delta\left(\chi\right)\Phi\left(\chi\right) \tag{2.22}$$

$$\pi\left(\chi\right)=\left(\mu-1\right)\Delta_H\Delta\left(\chi\right)\Phi\left(\chi\right) \tag{2.23}$$

$$l\left(\chi\right)=\Delta_H\Delta\left(\chi\right)\phi^{\sigma-1} \tag{2.24}$$

Intuitively, if a firm is twice as productive and produces a product for which there is twice as much demand from the perspective of the entire networked economy, its revenue and profit (gross of fixed operating costs) is quadrupled. Total output of a

χ-firm is also completely determined by firm fundamental and network characteristics up to a scale factor:

$$X(\chi) = \Delta_H \Delta(\chi) \Phi(\chi)^{\frac{\sigma}{\sigma-1}} \tag{2.25}$$

as are the value and quantity of output traded from $\chi^{'}$- to χ-firms:

$$r\left(\chi, \chi^{'}\right) = \left(\frac{\alpha}{\mu}\right)^{\sigma-1} \Delta_H \Delta(\chi) \Phi\left(\chi^{'}\right) \tag{2.26}$$

$$x\left(\chi, \chi^{'}\right) = \frac{\alpha^{\sigma-1}}{\mu^{\sigma}} \Delta_H \Delta(\chi) \Phi\left(\chi^{'}\right)^{\frac{\sigma}{\sigma-1}} \tag{2.27}$$

2.2.3 Household welfare and demand

To complete characterization of the static market equilibrium, it remains to determine the scale factor Δ_H. From the labor market clearing condition (2.11) and the firm variable employment equation (2.24), this is given by:

$$\Delta_H = \frac{L - L_f}{\int_{S_\chi} \Delta(\chi) \phi^{\sigma-1} dG_\chi(\chi)} \tag{2.28}$$

Equations (2.3) and (2.4) then give the consumer price index and household welfare respectively as:

$$P_H = \mu \left[\int_{S_\chi} \Phi(\chi) \delta^{\sigma-1} dG_\chi(\chi)\right]^{\frac{1}{1-\sigma}} \tag{2.29}$$

$$U = \mu^{-\sigma} (L - L_f) \frac{\left[\int_{S_\chi} \Phi(\chi) \delta^{\sigma-1} dG_\chi(\chi)\right]^{\frac{\sigma}{\sigma-1}}}{\int_{S_\chi} \Delta(\chi) \phi^{\sigma-1} dG_\chi(\chi)} \tag{2.30}$$

while household demand is given by:

$$x_H(\chi) = \mu^{-\sigma} \Delta_H \delta^{\sigma-1} \Phi(\chi)^{\frac{\sigma}{\sigma-1}} \tag{2.31}$$

Note that since households are assumed to own all the firms in the economy, aggregate profits in general equilibrium are also a source of household income. One can then easily verify from the equilibrium conditions that the expression for welfare (2.30) is the same as what would be obtained by solving instead for real household expenditure.

Using equations (2.17) and (2.18) to substitute for $\Phi(\chi)$ and $\Delta(\chi)$ respectively in equation (2.30), we can also express household welfare as:

$$U = (L - L_f) \frac{\left[\int_{S_\chi}\int_{S_\chi}\left[\sum_{d=0}^{\infty}\left(\frac{\alpha}{\mu}\right)^{d(\sigma-1)} m^{(d)}\left(\chi,\chi^{'}\right)\right]\left(\delta\phi^{'}\right)^{\sigma-1} dG_\chi\left(\chi\right) dG_\chi\left(\chi^{'}\right)\right]^{\frac{\sigma}{\sigma-1}}}{\int_{S_\chi}\int_{S_\chi}\left[\sum_{d=0}^{\infty}\frac{1}{\mu^d}\left(\frac{\alpha}{\mu}\right)^{d(\sigma-1)} m^{(d)}\left(\chi,\chi^{'}\right)\right]\left(\delta\phi^{'}\right)^{\sigma-1} dG_\chi\left(\chi\right) dG_\chi\left(\chi^{'}\right)} \tag{2.32}$$

Note that the integrands in the numerator and denominator of (2.32) are identical except for the term μ^{-d}. An intuitive approximation to the value of household welfare is therefore:

$$U \approx (L - L_f)\,\mathcal{C} \tag{2.33}$$

where $\mathcal{C}$ is a measure of the total connectivity between firms in the economy:

$$\mathcal{C} \equiv \left[\int_{S_\chi}\int_{S_\chi}\left[\sum_{d=0}^{\infty}\left(\frac{\alpha}{\mu}\right)^{d(\sigma-1)} m^{(d)}\left(\chi,\chi^{'}\right)\right]\left(\delta\phi^{'}\right)^{\sigma-1} dG_\chi\left(\chi\right) dG_\chi\left(\chi^{'}\right)\right]^{\frac{1}{\sigma-1}} \tag{2.34}$$

Equation (2.33) shows how household welfare is greater when buyers of greater fundamental quality δ are better connected with sellers of greater fundamental productivity $\phi^{'}$, with the welfare cost of additional relationships captured by the term $L - L_f$. The approximation (2.33) is exact only in the limit as $\mu \to 1$ (perfect competition), but when $\mu > 1$, the same general intuition applies.

2.2.4 Market equilibrium definition

We can now define an exogenous network market equilibrium of the economy as follows.

Definition 1. Given a matching function m and a quantity of labor L_f used for relationship costs, a static market equilibrium of the economy is a pair of firm network characteristic functions $\Phi : S_\chi \to \mathbb{R}_+$ and $\Delta : S_\chi \to \mathbb{R}_+$ satisfying equations (2.15) and (2.16), a scalar household demand shifter Δ_H satisfying (2.28), and allocation functions $\{l(\cdot), X(\cdot), x(\cdot,\cdot), x_H(\cdot)\}$ given respectively as side equations by (2.24), (2.25), (2.27), and (2.31).

The computational algorithm used to solve for the exogenous network market equilibrium is described in detail in section A.1 of the online appendix. Since Proposition 1 guarantees that the network characteristic functions Φ and Δ are uniquely determined, uniqueness of the market equilibrium follows immediately.

Proposition 2. *The exogenous network market equilibrium exists and is unique.*

2.2.5 Market equilibrium efficiency

To characterize the efficiency of the static market equilibrium, one can compare the resulting allocation with the allocation that would be chosen by a social planner seeking to maximize household welfare subject to the same exogenous matching function, production technology, and resource constraints. The following proposition (proved in section B.1 of the online appendix) summarizes the solution to the planner's problem.

Proposition 3. *Given a matching function* $m : S_\chi \times S_\chi \to [0,1]$*, the network characteristic functions under the social planner's allocation satisfy:*

$$\Phi^{SP}(\chi) = \phi^{\sigma-1} + \alpha^{\sigma-1} \int_{S_\chi} m\left(\chi, \chi^{'}\right) \Phi^{SP}\left(\chi^{'}\right) dG_\chi\left(\chi^{'}\right) \tag{2.35}$$

$$\Delta^{SP}(\chi) = \delta^{\sigma-1} + \alpha^{\sigma-1} \int_{S_\chi} m\left(\chi^{'}, \chi\right) \Delta^{SP}\left(\chi^{'}\right) dG_\chi\left(\chi^{'}\right) \tag{2.36}$$

and the allocations of output and labor are given by equations (2.24), (2.25), (2.27), and (2.31) with μ *set equal to* 1.

This result implies that that any static market equilibrium allocation coincides with the corresponding planner's allocation if and only if all firms in the decentralized equilibrium charge zero markups. With monopolistically-competitive firms, the static market equilibrium allocation is therefore inefficient because of the distortion arising from multiple marginalization along each supply chain. Note that the introduction of relationship frictions into the model through the exogenous matching function m imposes no additional inefficiency beyond this standard distortion.

3 Endogenous Network Formation and Model Dynamics

I now introduce a dynamic process of firm matching to study how the production network is determined and how it evolves over time.

3.1 Model environment

3.1.1 Households

Time is discrete and the representative household has preferences at date t defined by:

$$V_t = \sum_{s=t}^{\infty} \beta^{s-t} U_s \tag{3.1}$$

where U_t is given by the date t equivalent of (2.1). Since the household's value function is linear in per-period utility, household decisions in every period are characterized exactly as in the static model, and the discount factor β only affects how firms discount the future.

3.1.2 Costly relationships

The CES production technology (2.5) implies two things. First, access to additional suppliers always lowers the marginal cost of a firm, which follows from the love of variety feature of the production function.[8] Second, access to additional customers always increases a firm's variable profit, which follows from the production technology exhibiting constant returns to scale. These forces generate incentives for firms to form as many upstream and downstream trading relationships as possible. To counterbalance these incentives and thereby model the endogenous selection of firm-to-firm relationships, I therefore assume that relationships are costly.[9]

In particular, it is assumed that in order for the relationship between buyer i and seller j to be active at date t, a quantity of labor $\xi_{ij,t}$ must be hired by the selling firm j. The assumption that the selling firm always pays the full share of the relationship cost implies that the constant-markup pricing described in section 2 remains optimal in the dynamic setting.[10] Furthermore, this assumption implies that firms are always willing to form upstream relationships, and hence analysis of the network formation

[8]Note that love of variety in the production technology can be reinterpreted as a firm facing convex costs of producing intermediate inputs using goods from any one supplier, which leads to the same demand functions and marginal costs.

[9]As a practical example of the form that this cost might take, market analysts estimate that US firms spent more than $10bn in 2014 on relationship management software systems alone (Gartner (2014a,b)).

[10]If a buying firm had to pay a positive fixed cost and found a relationship undesirable given CES markup pricing by the seller, the seller might then find it optimal to reduce its markup so as to incentivize the buyer to form the relationship.

process can be considered solely from the perspective of potential sellers. In what follows, we refer to $\xi_{ij,t}$ as the *relationship cost shock*.

Note that modeling the extensive margin of firm-to-firm relationships through fixed costs paid by the selling firm is similar to the approach adopted in Bernard et al. (2018a,b). One might also think of these relationship costs as akin to the fixed costs of exporting in Melitz (2003), but applied at the firm-to-firm rather than the country-to-country level. Furthermore, it is also important to note that although the input suitability parameter α is assumed to be constant across relationships, this is simply a normalization, as heterogeneity at the relationship-level can be modeled either through input suitability or the cost of relationships. In other words, relationships may be asymmetric in one of two equivalent ways: (i) inputs may differ in suitability for production while relationship costs are homogeneous; or (ii) inputs may be homogeneous in terms of suitability for production conditional on the existence of the relationship, while the costs of forming the relationship in the first place may be heterogeneous.[11]

3.1.3 Dynamics of the network

We now introduce two sources of dynamics in the model. First, to model firm-level dynamics, we allow the vector of fundamental firm characteristics to be time-varying. Specifically, the fundamental characteristic vector $\chi_{i,t}$ for any given firm i is assumed to follow a first-order Markov process:

$$\chi_{i,t+1} = T_{\chi,t+1}\left(\chi_{i,t}, \epsilon^{\chi}_{i,t+1}\right) \tag{3.2}$$

where $T_{\chi,t+1}$ is a time-varying transition function and $\epsilon^{\chi}_{i,t+1}$ is an idiosyncratic shock with distribution G^{χ}_{ϵ} that is *iid* across firms and across time. Note that the time-varying nature of T_{χ} allows for aggregate shocks, and will be used to capture business cycle fluctuations in the empirical application.

Second, to model the dynamics of firm-to-firm relationships, it is also assumed

[11] To account for variation in relationship formation at the industry-pair-level, for example, one might differentiate firms by industry and then allow either α or the distribution of ξ to vary by industry–pair. Alternatively, it is also possible to extend the model in this direction by imposing an upper-tier nested CES or Cobb-Douglas structure on the production function. The only cost of such extensions is computational complexity, as then the matching function differs by industry-pair.

that relationship costs follow a first-order Markov process:

$$\xi_{ij,t+1} = T_\xi \left(\xi_{ij,t}, \epsilon^{\xi}_{ij,t+1} \right) \tag{3.3}$$

where T_ξ is a transition function and $\epsilon^{\xi}_{ij,t+1}$ is an idiosyncratic shock with distribution G^{ξ}_{ϵ} that is *iid* across firm relationships and across time. Since aggregate shocks are accommodated in the model through (3.2), we assume here for parsimony that the relationship cost transition function T_ξ is constant over time.

In the initial period of the model, firm fundamental characteristics and relationship fixed costs are assumed to be drawn from distributions $G_{\chi,0}$ and $G_{\xi,0}$ respectively, with all realizations independent across firms and relationships.

$$\chi_{i,0} \sim G_{\chi,0} \tag{3.4}$$

$$\xi_{ij,0} \sim G_{\xi,0} \tag{3.5}$$

3.2 Endogenous network market equilibrium

3.2.1 Relationship selection

As discussed above, the assumption that relationship costs are borne solely by selling firms implies that the desirability of a relationship depends only the profit that can be generated for the seller. For a $\chi^{'}$-firm selling to a χ-firm at date t, this profit value is the same as in the static market equilibrium, given by equations (2.16) and (2.23) as:

$$\pi_t \left(\chi, \chi^{'} \right) = \mu^{-\sigma} \left(\mu - 1 \right) \alpha^{\sigma-1} \Delta_{H,t} \Delta_t \left(\chi \right) \Phi_t \left(\chi^{'} \right) \tag{3.6}$$

Note that Φ_t, Δ_t, and $\Delta_{H,t}$ are defined by the date t equivalents of equations (2.15), (2.16), and (2.28), given the distribution of fundamental firm characteristics $G_{\chi,t}$ that is implied by the stochastic process described by (3.2) and (3.4). A relationship is therefore active as long as the static profits accruing to the selling firm covers the relationship cost in each period. The probability that a $\chi^{'}$-firm sells to a χ-firm at date t is then given by:

$$m_t \left(\chi, \chi^{'} \right) = G_{\xi,t} \left[\pi_t \left(\chi, \chi^{'} \right) \right] \tag{3.7}$$

where $G_{\xi,t}$ is the unconditional distribution of ξ at date t that is implied by the stochastic process described by (3.3) and (3.5).

Note that equation (3.7) fully characterizes the endogenous matching function at date t. In particular, solution of the model at each date only requires knowledge of the unconditional distributions $G_{\chi,t}$ and $G_{\xi,t}$; beyond this, the stochastic processes that determine these distributions are irrelevant. This follows from the fact that firms make production and relationship formation decisions period by period based on the contemporaneous realizations of χ and ξ, as there are no option values associated with production or to being in or out of a relationship.[12]

However, the stochastic processes that govern the dynamic behavior of χ and ξ are crucial for determining the extent of churning in firm-to-firm relationships over time. For example, more volatile shocks or less persistence in these processes will generally imply less persistence in buyer-seller relationships. As a result, the specifics of these stochastic processes matter for the role of production network adjustment in generating aggregate fluctuations, an issue that we revisit in sections 5.3.1 and 5.3.2 once the empirical application of the theory is discussed.

3.2.2 Aggregate relationship costs

To close the model, it remains to determine the aggregate quantity of labor $L_{f,t}$ used to pay for relationship costs at date t, which enters into the labor market clearing condition (2.11). Note that even though the stochastic process for the relationship cost shock $\xi_{ij,t}$ is assumed to be independent across relationships, firms in the dynamic market equilibrium select relationships based on the realized values of the relationship cost shocks. Therefore, the total mass of labor used to pay for relationship fixed costs is given by:

$$L_{f,t} = \int_{S_\chi} \int_{S_\chi} \bar{\xi}_t \left(\chi, \chi^{'}\right) dG_{\chi,t} \left(\chi\right) dG_{\chi,t} \left(\chi^{'}\right) \tag{3.8}$$

[12]This would not be true if, for example, firms were only able to adjust the status of each trading relationship in each period with some probability between zero and one, which might be interpreted as a reduced-form way of modeling information frictions in the formation and termination of buyer-seller relationships.

where the term $\bar{\xi}_t\left(\chi,\chi^{'}\right)$ denotes the average value of the idiosyncratic component of the cost shock amongst $\chi-\chi^{'}$ firm pairs:

$$\bar{\xi}_t\left(\chi,\chi^{'}\right)=\int_0^{\xi_{max,t}\left(\chi,\chi^{'}\right)}\xi dG_{\xi,t}\left(\xi\right) \tag{3.9}$$

and $\xi_{max,t}\left(\chi,\chi^{'}\right)$ is the maximum value of the cost shock for which $\chi-\chi^{'}$ relationships are selected, which is simply given by:

$$\xi_{max,t}\left(\chi,\chi^{'}\right)=\pi_t\left(\chi,\chi^{'}\right) \tag{3.10}$$

3.2.3 Market equilibrium definition

We can now define a endogenous network market equilibrium as follows.

Definition 2. Given stochastic processes for firm fundamental characteristics and relationship costs, an endogenous network market equilibrium of the model for periods $t\in\{0,\cdots,T\}$ is a list of sequences of matching functions $\{m_t\}_{t=0}^{T}$, profit functions $\{\pi_t\}_{t=0}^{T}$, and network characteristic functions $\{\Phi_t,\Delta_t\}_{t=0}^{T}$, as well as a list of scalars $\{\Delta_{Ht}\}_{t=0}^{T}$, all of which satisfy equations (2.15), (2.16), (2.28), (3.6), and (3.7). Given the matching function m_t, the allocation at date t in a dynamic equilibrium is as defined in the static model.

The computational algorithm used to solve for the market equilibrium with endogenous production networks is described in detail in section A.2 of the online appendix. Note that once the matching function is endogeneized, Blackwell's conditions can no longer be applied to establish the contraction mapping property of the network characteristic equations (2.15) and (2.16). Therefore, establishing uniqueness of the solution to these equations and hence of the end market equilibrium is not trivial. Nonetheless, numerical solution of the steady-state of the dynamic market equilibrium is only marginally more computationally demanding than solving for the static market equilibrium, and numerical simulations reveal no counterexample to the supposition of uniqueness.

3.2.4 Market equilibrium efficiency

Recall from Proposition 3 that the exogenous network market equilibrium is inefficient relative to the social planner's allocation because of the monopoly markups charged by firms. The same inefficiency naturally carries over to the market equilibrium allocation in each period of the dynamic model. With endogenous networks, however, there is an additional source of inefficiency, as firms' choices about which relationships to form are also not socially optimal.

To examine this, we compare the cutoff value for the relationship cost shock chosen by firms (given by equation (3.10)) to the cutoff value that would be chosen by a social planner that seeks to maximize household welfare. To distinguish the extensive margin distortion from the multiple marginalization distortion present in the exogenous network model, we consider the optimal selection of relationships conditional on the allocation of labor and output then being determined as in the exogenous network market equilibrium with firms charging some constant markup μ.[13] In section B.2 of the appendix, I show that the planner's solution to this problem is characterized by the following proposition.

Proposition 4. *Conditional on a value for firm markups μ, the cutoff values for relationship costs chosen by the social planner are given by:*

$$\xi^{SP}_{max,t}\left(\chi^{*},\chi^{*'}\right)=f^{SP}\left(\mu\right)\alpha^{\sigma-1}\Delta_{H,t}\Delta_{t}\left(\chi^{*}\right)\Phi_{t}\left(\chi^{*'}\right) \tag{3.11}$$

where:

$$f^{SP}\left(\mu\right)\equiv\frac{\sigma}{\sigma-1}\mu^{1-\sigma}\left(\frac{\sum_{d=0}^{\infty}A_{d}}{\sum_{d=0}^{\infty}\mu^{-d}A_{d}}\right)\left(\frac{\sum_{d=0}^{\infty}\mu^{-d}B_{d}}{\sum_{d=0}^{\infty}B_{d}}\right)-\mu^{-\sigma}\frac{\sum_{d=0}^{\infty}\mu^{-d}C_{d}}{\sum_{d=0}^{\infty}C_{d}} \tag{3.12}$$

and:

$$A_{d}\equiv\int_{S_{\chi}}\left(\frac{\alpha}{\mu}\right)^{d(\sigma-1)}m^{(d)}\left(\chi,\chi^{*}\right)\delta^{\sigma-1}dG_{\chi}\left(\chi\right) \tag{3.13}$$

$$B_{d}\equiv\int_{S_{\chi}}\int_{S_{\chi}}\left(\frac{\alpha}{\mu}\right)^{d(\sigma-1)}m^{(d)}\left(\chi,\chi^{'}\right)\left(\delta\phi^{'}\right)^{\sigma-1}dG_{\chi}\left(\chi\right)dG_{\chi}\left(\chi^{'}\right) \tag{3.14}$$

$$C_{d}\equiv\int_{S_{\chi}}\left(\frac{\alpha}{\mu}\right)^{d(\sigma-1)}m^{(d)}\left(\chi^{*'},\chi\right)\phi^{\sigma-1}dG_{\chi}\left(\chi\right) \tag{3.15}$$

Note that the expressions (3.10) and (3.11) differ because of the term $f^{SP}\left(\mu\right)$, which in the market equilibrium is given instead by $f\left(\mu\right)\equiv\mu^{1-\sigma}-\mu^{-\sigma}$. Hence,

[13] Given the results of Proposition 3, the case in which $\mu=1$ corresponds to the complete social planner's problem, in which the planner is allowed to choose both the allocations of labor and output as well as the set of active relationships.

relationship formation in the market equilibrium is inefficient, even conditional on the intensive margin distortion arising from multiple marginalization at any markup $\mu > 1$.[14] In particular, this is true at the equilibrium markup $\mu = \frac{\sigma}{\sigma-1}$. This is the result of what one might refer to as a *network externality*, as firms in the market equilibrium value relationships based only on the potential profits generated by the linkage, and do not internalize the effect of forming that relationship on the profits and marginal costs of other firms downstream and upstream of the link respectively.

4 Data and Structural Estimation

4.1 Data

The primary dataset used for structural estimation of the model's parameters is obtained from Standard and Poor's Compustat platform. The Compustat database contains fundamental firm-level information compiled from financial disclosure forms for publicly-listed firms in the US, with 103,379 firm-year observations from 1979 to 2007. This includes information on variables such as firm revenue and employment, and importantly also contains records of firms' own reports of who their major customers are.[15]

Several features of this dataset are worth noting. First, the dataset provides panel data on the extensive margin of firm-to-firm trading relationships, which is essential for studying the role of production networks in business cycle fluctuations. Therefore, while other datasets may provide cross-sectional observations of firm-level production networks, the temporal nature of the Compustat relationship data is invaluable for the empirical application.

Second, the Compustat data is subject to important censoring along two dimensions: only publicly-traded firms are included in the dataset, and only relationships involving a firm's major customers are observed. With regard to selection on firms, the firms in the Compustat database account for just under 50% of US gross output

[14] This is immediately obvious for the case in which $\mu = 1$. In the market equilibrium, firms earn zero profits, and hence no relationships are formed, whereas $f^{SP}(1) = \frac{1}{\sigma-1} > 0$, and the social planner still optimally chooses a finite measure of active relationships.

[15] The Compustat relationship data was first processed and studied by Atalay et al. (2011), who posit a stochastic matching algorithm that can rationalize some of the observed features of the US production network.

in 2007.[16] Hence, while the dataset covers only a small fraction of the total number of firms in the US economy, it nonetheless captures a sizable fraction of economic activity. With regard to selection on relationships, US Financial Accounting Standards No. 131 defines a "major customer" as a firm that accounts for at least 10% of the reporting seller's revenue. This implies that a firm cannot have more than 10 customers reported in a given year, although there is still substantial variation in the number of suppliers that a firm is reported to have.

To address both of these censoring issues, I also employ data obtained from Standard and Poor's Capital IQ platform, which collects fundamental data for both public and private firms, and contains records of buyer-seller relationships based on a variety of sources, such as publicly available financial forms, company reports, and press announcements. From this database, I select all firms in the continental US for which relationship data is available and average revenue from 2003-2007 is positive, which results in a dataset comprising 8,592 firms with $16.3 trillion in total revenue, accounting for 65% of US gross output in 2007. The Capital IQ platform offers greater coverage of firms and relationships than the Compustat data, and hence is utilized as a check on the key moments predicted by the estimated model. However, the main drawback of the Capital IQ dataset is that it is not possible to tell whether a particular relationship reported in a given year is still active at a later date. In contrast with Compustat, the Capital IQ data hence cannot be used to track the creation and destruction of trading relationships across time.

Finally, it is important to note that the Compustat data does not contain records of transaction values between firms. In other words, it allows observation of the extensive margin of the production network but not the intensive margin. The unavailability of transaction data motivates two important choices with respect to modeling and estimation. First, without transaction data (and in particular prices at the firm-to-firm level), identifying markups at the relationship level becomes impossible. Hence, the assumed market structure of monopolistic competition and the resulting constant CES markups not only afford the model computational tractability; it is also a simplifying assumption made due to the absence of data that would speak to the contrary. Second, the lack of transaction data limits the extent to which some of the structural parameters of the model can be estimated empirically, as described

[16] Based on gross output data of all US industries from the Federal Reserve Bank of St. Louis FRED Database.

below.

4.2 Estimation procedure

The endogenous network model developed above is parameterized by the following: the stochastic process for fundamental firm characteristics described by (3.2) and (3.4); the stochastic process for relationship cost shocks described by (3.3) and (3.5); the input suitability parameter, α; the elasticity of substitution, σ; the household discount factor, β; and the labor supply, L.

I first describe the set of parameters for which values are not estimated or calibrated from data. First, since the Compustat data is of annual frequency, I set $\beta = .95$. Second, since the model is scale invariant, I set the labor supply L to match the units of sales in the model and data. Third, since the available datasets do not contain trade transaction values from which substitution elasticities are typically estimated, I set the value of σ to 3 in the baseline empirical specification,[17] and explore robustness of the empirical results under different values of σ in section 5.3.3. Finally, to calibrate firm-level shocks from sales data alone, only one dimension of firm-level heterogeneity can be accommodated. Hence, while the model can in principle allow for heterogeneity in both fundamental demand δ and productivity ϕ, the empirical analysis will simply impose $\delta = 1$ for all firms.

The structural model can now be thought of as a mapping from the firm productivity process, the relationship cost shock process, and the input suitability parameter α to variables of interest. The structural estimation and calibration of model parameters can then be broken up into three steps. First, given observed data on firm sales and relationships, the relationship cost shock process can be estimated via a maximum likelihood approach, independent of firm fundamental productivities. This is described below in section 4.2.1. Second, the firm-level productivity process can be estimated from observed data on firm sales and relationships, as described in section 4.2.2. Finally, the input suitability parameter can be calibrated to match an aggregate target for the labor share, as described in section 4.2.3.

[17]See, for example, Broda and Weinstein (2006) for a range of values for σ that are typically estimated in the literature.

4.2.1 Estimation of relationship cost process

To estimate the relationship cost shock process described by equations (3.3) and (3.5), we first posit a parametric form. In particular, we assume that the log cost shocks follow a first-order autoregressive process:

$$\log \xi_{ij,t+1} - m_\xi = \rho_\xi \left(\log \xi_{ij,t} - m_\xi\right) + \log \epsilon^{\xi}_{ij,t+1} \tag{4.1}$$

$$\log \epsilon_{ij,t} \sim \mathscr{N}\left(0, s_\xi^2\right) \tag{4.2}$$

Furthermore, we assume that values of ξ in the initial period of the model are drawn from the stationary distribution corresponding to (4.1). The stochastic process for ξ is therefore described by three parameters: the unconditional mean, m_ξ; the serial correlation, ρ_ξ; and the standard deviation of the Gaussian shock, s_ξ. To estimate these parameters, we then first construct the likelihood of observing the set of firm-to-firm relationships.

Note that under the assumptions discussed above, the probability of observing a relationship between buyer i and seller j at date t is the probability that potential profits generated by the relationship at least equal the cost of the relationship:

$$\Pr\left[m_{ij,t} = 1\right] = \Pr\left[\xi_{ij,t} \leq \pi_{ij,t}\right] \tag{4.3}$$

While potential profits are not observed directly, equations (2.22) and (3.6) can be used to express profits as function of observed firm sales and network connections:

$$\pi_{ij,t} = \frac{1}{\sigma}\mu^{-\sigma}\alpha^{\sigma-1}\left(\Delta_{i,t}/\Delta_{j,t}\right) R_{j,t} \tag{4.4}$$

where $R_{i,t}$ and $\Delta_{i,t}$ are sales and network demand respectively for firm i at date t. The former is directly observed in the data, while the latter can be easily computed from equation (2.16) as:

$$\overrightarrow{\Delta}_t = \mu^{-\sigma}\left[I - \mu^{-\sigma}\alpha^{\sigma-1}M_t^T\right]^{-1}\mathbf{1} \tag{4.5}$$

where $\overrightarrow{\Delta}_t$ is a vector with i^{th}-element equal to $\Delta_{i,t}$, M_t is the observed adjacency matrix with ij^{th} element $m_{ij,t} = 1$ if firm i buys from firm j at date t, and $\mathbf{1}$ is the unit vector.

Since the relationship cost shocks are *iid* across relationships, the likelihood of observing the entire panel of relationship statuses $\{M_t\}_{t=1}^{T}$ can be computed for each relationship separately. For a given ij-pair, let m_t^{t+s} denote the vector of observations $\{m_\tau\}_{\tau=t}^{t+s}$ and let $G_{\xi,t}\left(\cdot|m_1^t\right)$ denote the distribution of ξ_t conditional on observing m_1^t. Then the likelihood of observing the sequence of relationship statuses m_1^T can be expressed as follows:

$$\Pr\left[m_1^T\right] = \Pr\left[m_1\right]\Pr\left[m_2|m_1\right]\cdots\Pr\left[m_T|m_1^{T-1}\right] \tag{4.6}$$

where the conditional likelihoods are:

$$\Pr\left[m_1\right] = G_{\xi,1}\left(\pi_1\right)^{m_1}\left[1-G_{\xi,1}\left(\pi_1\right)\right]^{1-m_1} \tag{4.7}$$

$$\Pr\left[m_t|m_1^{t-1}\right] = \int G_\epsilon^\xi\left[T_\xi^{-1}\left(\pi_t|\xi_{t-1}\right)\right]^{m_t}\left[1-G_\epsilon^\xi\left[T_\xi^{-1}\left(\pi_t|\xi_{t-1}\right)\right]\right]^{1-m_t} dG_{\xi,t-1}\left(\xi_{t-1}|m_1^{t-1}\right) \tag{4.8}$$

Here, $G_{\xi,1}$ denotes the distribution of relationship costs in the first period, and T_ξ^{-1} denotes the inverse of T_ξ as a function of ϵ, which is defined by (3.3) and (4.1). The conditional distributions can then be computed from the following recursion:

$$G_{\xi,t}\left(x|m_1^t\right) = \begin{cases} \frac{G_{\xi,t}\left(x|m_1^{t-1}\right)}{G_{\xi,t}\left(\pi_t|m_1^{t-1}\right)} & \text{, if } m_t = 1,\ x \leq \pi_t \\ \frac{G_{\xi,t}\left(x|m_1^{t-1}\right)-G_{\xi,t}\left(\pi_t|m_1^{t-1}\right)}{1-G_{\xi,t}\left(\pi_t|m_1^{t-1}\right)} & \text{, if } m_t = 0,\ x > \pi_t \\ 0 & \text{, otherwise} \end{cases} \tag{4.9}$$

$$G_{\xi,t}\left(x|m_1^{t-1}\right) = \int G_\epsilon^\xi\left[T_\xi^{-1}\left(x|\xi_{t-1}\right)\right] dG_{\xi,t-1}\left(\xi_{t-1}|m_1^{t-1}\right) \tag{4.10}$$

with initial condition $G_{\xi,1}\left(x|m_1^0\right) = G_{\xi,1}\left(x\right)$.

Given observed data on firm sales and firm-to-firm relationships as well as values for the structural parameters $\{\alpha, m_\xi, \rho_\xi, s_\xi\}$, the above procedure therefore allows computation of the likelihood of observing the network panel $\{M_t\}_{t=1}^{T}$. Fixing a value for the input suitability parameter α, the relationship cost shock process parameters are then estimated by maximizing this likelihood.

4.2.2 Estimation of firm productivity process

Turning next to the estimation of the firm-level productivity process described by equations (3.2) and (3.4), we again first posit a parametric form. In particular, we assume that firm-level productivities evolve according to a first-order autoregressive process

$$\log \phi_{i,t+1} - m_{\phi,t+1} = \rho_{\phi,t+1} \left(\log \phi_{i,t} - m_{\phi,t} \right) + \log \epsilon^{\phi}_{i,t+1} \tag{4.11}$$

$$\log \epsilon^{\phi}_{i,t} \sim \mathcal{N} \left(0, s^2_{\phi,t} \right) \tag{4.12}$$

with initial values of $\log \phi$ drawn from a Gaussian distribution with mean $m_{\phi,0}$ and variance $s^2_{\phi,0}$. Note that the unconditional means $\{m_{\phi,t}\}_{t=0}^{T}$, autocorrelation parameters $\{\rho_{\phi,t}\}_{t=1}^{T}$, and standard deviations $\{s_{\phi,t}\}_{t=0}^{T}$ are allowed to vary over time.

To estimate these parameters, we first observe that data on firm sales and firm-to-firm relationships can be used to infer productivity values for each firm. From equations (2.15) and (2.22), one obtains:

$$\overrightarrow{\phi}_t^{\sigma-1} = \frac{1}{\mu \Delta_{H,t}} \left[I - (\alpha/\mu)^{\sigma-1} M \right]^{-1} \left(\overrightarrow{R/\Delta}_t \right) \tag{4.13}$$

where $\overrightarrow{\phi}_t$ and $\overrightarrow{R/\Delta}_t$ are vectors with i^{th}-elements equal to $\phi_{i,t}$ and $R_{i,t}/\Delta_{i,t}$ respectively. Again, $R_{i,t}$ is observed firm sales, and $\Delta_{i,t}$ is recovered from (4.5). Equation (4.13) therefore determines firm-level productivities up to a normalizing constant, from which the autocorrelation parameters $\{\rho_{\phi,t}\}_{t=1}^{T}$ and standard deviations $\{s_{\phi,t}\}_{t=0}^{T}$ can be easily estimated via linear regression. To determine the unconditional means $\{m_{\phi,t}\}_{t=0}^{T}$, we first normalize $m_{\phi,0} = 1$, and then calibrate the mean value of $\log \phi$ in subsequent periods to match the growth rate of real gross output in the data.[18] Conditional on a value for the input suitability parameter α, this procedure therefore enables the model to match aggregate business cycle fluctuations in gross output exactly, while inferring the remaining parameters of the firm-level productivity process from data.

4.2.3 Calibration of input suitability

Finally, it remains to calibrate the input suitability parameter α. Note that this parameter governs, amongst other variables, the shares of labor and intermediates used in production. Since capital is not explicitly modeled in the theory, we attribute payments to capital as profits earned by firms. Hence, labor's share of value-added

[18]Real gross output is calculated by deflating gross output in the Compustat data by the US gross domestic product deflator, where the latter is obtained from the Federal Reserve Bank of St. Louis FRED Database.

in the model can be computed as:

$$\lambda_t = \frac{L}{L + \Pi_t} \tag{4.14}$$

where Π_t denotes aggregate firm profits (net of relationship fixed costs):

$$\Pi_t \equiv \int \pi(\phi)\, dG_{\phi,t}(\phi) - L_{f,t} \tag{4.15}$$

We thus calibrate α by targeting a value for the labor share (averaged across time) of 65%, which is approximately the mean value of the US labor share over the sample period.

4.3 Estimation results

4.3.1 Parameter values

The estimated parameter values are shown in Table 1 and Figure 1, together with standard errors that are computed using a bootstrapping technique.[19] Several aspects of the estimation results are worth emphasizing. First, the time series of mean log firm productivity $m_{\phi,t}$ closely tracks the business cycle (see Figure 7), as should be expected. Second, both firm productivity and relationship costs are estimated to be highly persistent, with estimates of $\rho_{\phi,t}$ very close to unity in all years and an esitimate of ρ_ξ of 0.78. This is unsurprising, given that both firm size and relationship status are highly persistent as well. Third, although the estimated relationship cost parameters imply a large unconditional mean of the relationship cost, only relationships with small costs are endogenously selected. Hence, in equilibrium, the estimated parameters imply that around 0.8% of the total labor endowment is used for relationship costs, which is a small but non-trivial fraction.

4.3.2 Model fit

Before using the model to quantify the importance of production network adjustment, I first assess the fit of the estimated model to data along several key dimensions.

[19] I resample (with replacement) the set of firms in the Compustat dataset, and re-estimate the model parameters according to the procedure described in section 4.2 for each resample. The standard errors in Table 1 and Figure 1 are the standard deviations of the parameter estimates from resampling 20 times.

Parameter		Value	Standard Error
productivity process parameters			
mean	$m_{\phi,t}$	see Figure 1	
autocorrleation	$\rho_{\phi,t}$		
shock standard deviation	$s_{\phi,t}$		
relationship cost process parameters			
mean	m_{ξ}	1.90	0.185
autocorrleation	ρ_{ξ}	0.78	0.018
shock standard deviation	s_{ξ}	2.02	0.041
macro parameters			
input suitability	α	0.62	0.005
substitution elasticity	σ	3	-
discount factor	β	0.95	-
labor supply	L	1500	-

Table 1: Baseline parameter values

First, Figure 2 shows the fit of the model with respect to the firm size distribution, where the distributions from both the simulated and Compustat data are shown for the last year of the sample period. Note that given the assumed log-normal distribution for firm productivity ϕ, firm size in the model also inherits a distributional shape that resembles a log normal form, which matches up well with both the Compustat and Capital IQ data. However, the distribution of firm size in the model deviates from log–normality for two reasons. First, the contribution of firm-to-firm relationships to network productivity Φ and network demand Δ enters linearly in equations (2.15) and (2.16). Since firm size is proportional to the product $\Phi\Delta$, the size of a firm in the model is given by the sum of a log-normal random variable and a constant term. The second reason for the deviation from log-normality is that this constant term differs across firms, as firms are also heterogeneous in their network connectivity.

Second, Figure 3 shows the fit of the model with respect to the distributions of suppliers (in-degree) and customers (out-degree). Around their respective medians, the theoretical and empirical distributions match up well. However, the model fails to replicate the fact that there is a very small number of firms in the data with very large numbers of connections. As discussed in Atalay et al. (2011), replicating this particular feature of the production network in the Compustat data likely requires allowing for a form of preferential attachment, whereby firms with many connections

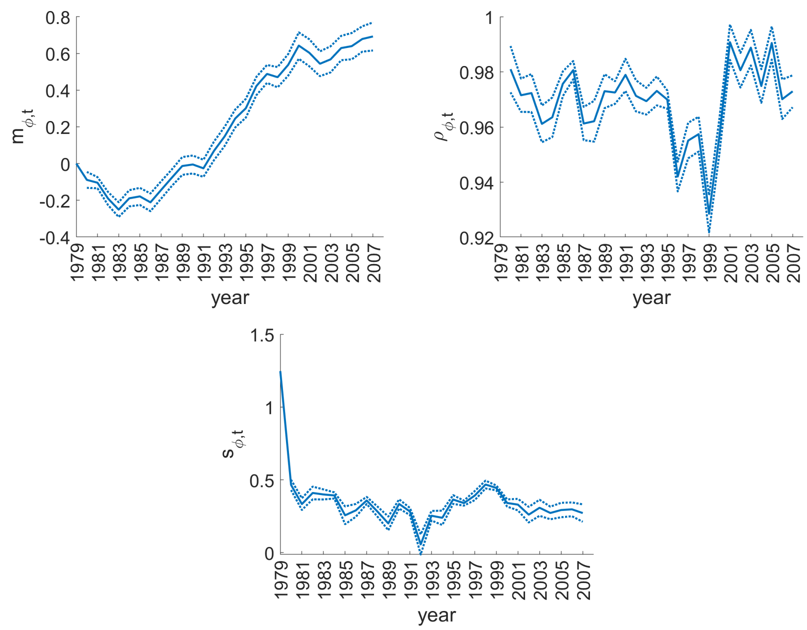

Figure 1: Firm productivity process parameter estimates
Note: The dotted lines show the ±1 standard error bounds of the estimated parameter values.

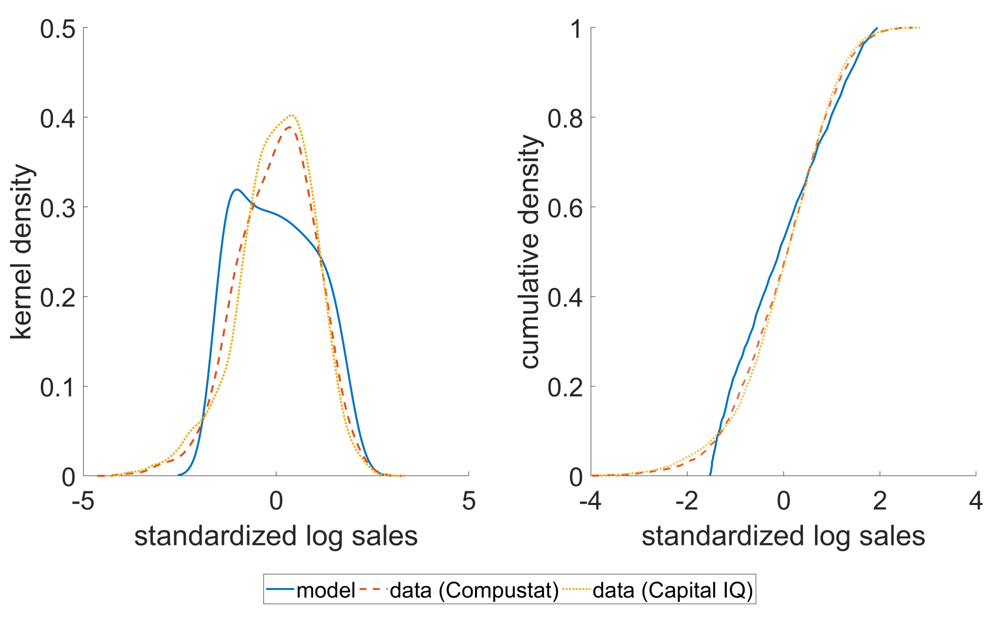

Figure 2: Firm sales distribution
Note: Log sales are standardized by subtracting the mean and dividing by the standard deviation of the respective distribution. Distributions from simulated and Compustat data are for 2007.

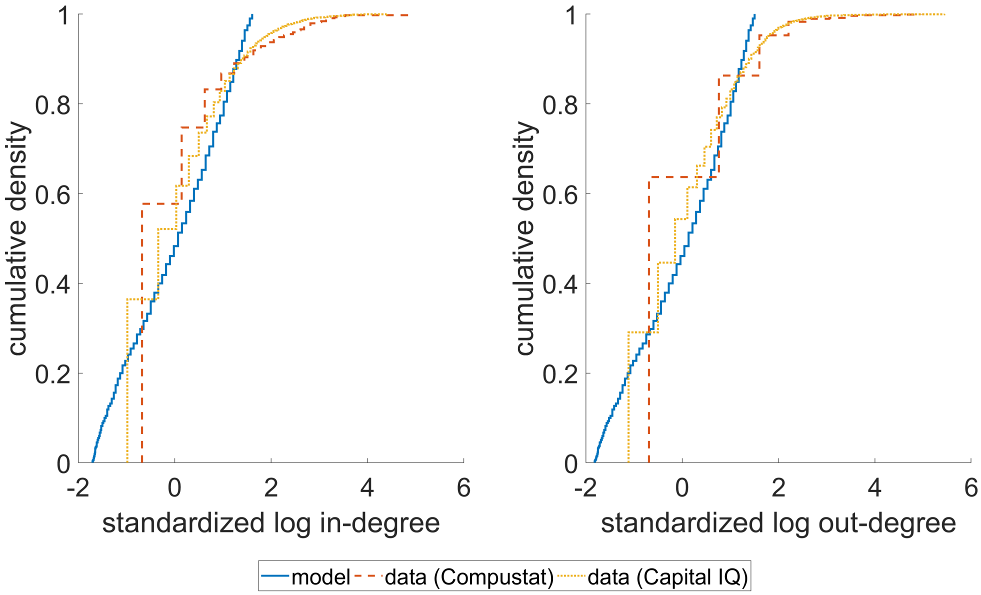

Figure 3: Firm degree distributions

Note: Log degree is standardized by subtracting the mean and dividing by the standard deviation of the respective distribution. Distributions from simulated and Compustat data are for 2007.

also find it easier to form new connections. Since relationship costs in the model are independent of other firm characteristics, such a form of preferential attachment is absent. Nonetheless, in terms of aggregate connectivity, the model predicts a gross output to value added ratio of 1.72, which is almost exactly equal to the value of 1.71 estimated for the US in 2016.[20]

Third, Figure 4 shows the fit of the model with respect to the joint distributions of firm size and degree. Note that in the data, larger firms generally tend to have greater numbers of suppliers and customers. The one exception is the relation between sales and number of customers in the Compustat data. As described in section 4.1, Compustat firms report only customers that account for at least 10% of their sales, and hence even for larger firms, there is a limit on the number of customer relationships that observed. This truncation is partially inherited by the Capital IQ data, which contains Compustat firms as a subset of the sample. Nonetheless, the

[20] One might also consider testing the fit of the model to the firm-level distribution of labor shares. However, this is infeasible since firm-to-firm transaction values are not observed in the Compustat data, and most firms do not report total input purchases. Nonetheless, the model has definite predictions about the labor share distribution that could be tested using other datasets in which firm-level labor shares are observed.

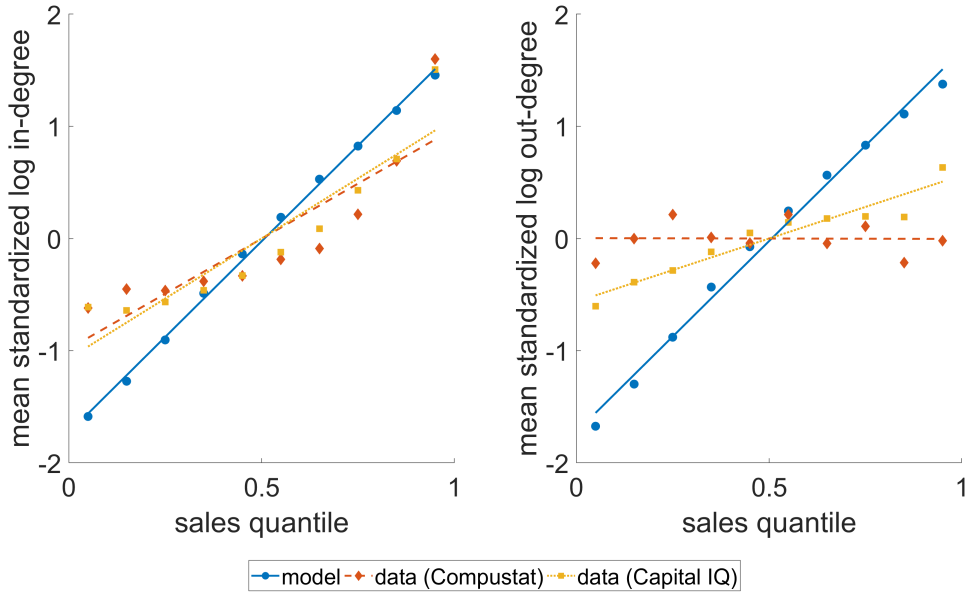

Figure 4: Joint distribution of firm size and degree

Note: Log degree is standardized by subtracting the mean and dividing by the standard deviation of the respective distribution. Distributions from simulated and Compustat data are for 2007.

model replicates the positive relationship between firm size and degree, and the slope of the relationship between firm size and in-degree is almost identical to that observed in the data.

Fourth, Figures 5 and 6 show the fit of the model with respect to matching assortativity, in terms of degree and sales respectively. Note from Figure 5 that both the Compustat and Capital IQ data exhibit *negative* assortative matching in terms of degree - firms that are connected to more suppliers (customers) also tend to be connected to suppliers (customers) that have *fewer* customers (suppliers) on average.[21] This pattern of negative matching is also predicted by the model, which is an intuitive implication of the assumption of costly relationships in the theory. From the relationship formation equations (3.6) and (3.7), the probability of a relationship forming between a buyer i and seller j is increasing in the product $\Delta_i \Phi_j$. Hence, only sellers with the highest values of network productivity will match (in a probabilistic sense) with buyers with the smallest values of network demand, and vice-versa. Since

[21] This appears to be a robust feature of several production network datasets. For example, negative degree matching is documented by Bernard et al. (2018a) for Norwegian export transactions, by Bernard et al. (2018b) for Japanese buyer-seller relationships, and by Tintelnot et al. (2018) for Belgian buyer-seller relationships.

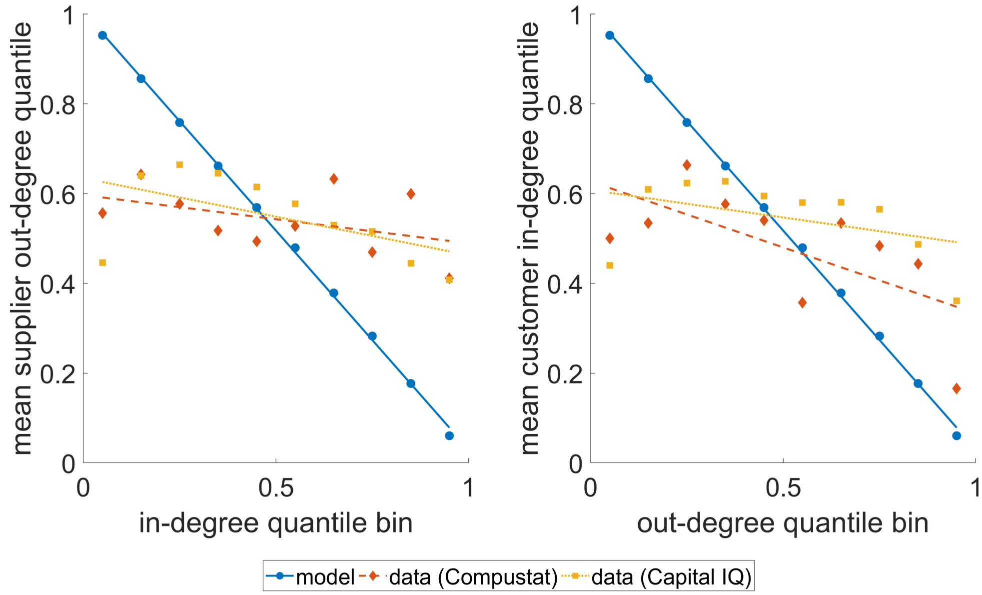

Figure 5: Matching assortativity in degree

firm network characteristics, size, and degree are all increasing in firm fundamental productivity ϕ, the model naturally predicts negative matching on all these variables.

Note from Figure 6, however, that matching assortativity in terms of sales is *positive* in the data - larger firms tend to match with larger firms on average. This is a feature of the data that most likely cannot be rationalized by a model of many-to-many matching based on costly relationships with costs independent of other firm characteristics.[22] As it is as yet unclear whether positive assortative matching on sales is a unique feature of the Compustat and Capital IQ data (which covers mostly larger firms) or is a robust feature of production networks in general, I do not pursue this issue further here, and simply note that it is an aspect of model fit that bears further consideration in future work.[23]

Finally, to assess the fit of the model with respect to dynamic moments in the data, we observe that the average predicted growth rate of firm sales is 13.5% in

[22] As pointed out by Bernard et al. (2018a), however, positive matching would be a natural feature of a version of the model with one-to-one matching, since profits are supermodular in the buyer's network demand and the sellers network productivity, both of which are positively correlated with firm size.

[23] For example, Sugita et al. (2014), Dragusanu (2014), and Benguria (2015) find positive matching on sales, Bernard et al. (2018a) find almost neutral (or weakly positive) matching, while Tintelnot et al. (2018) document negative matching.

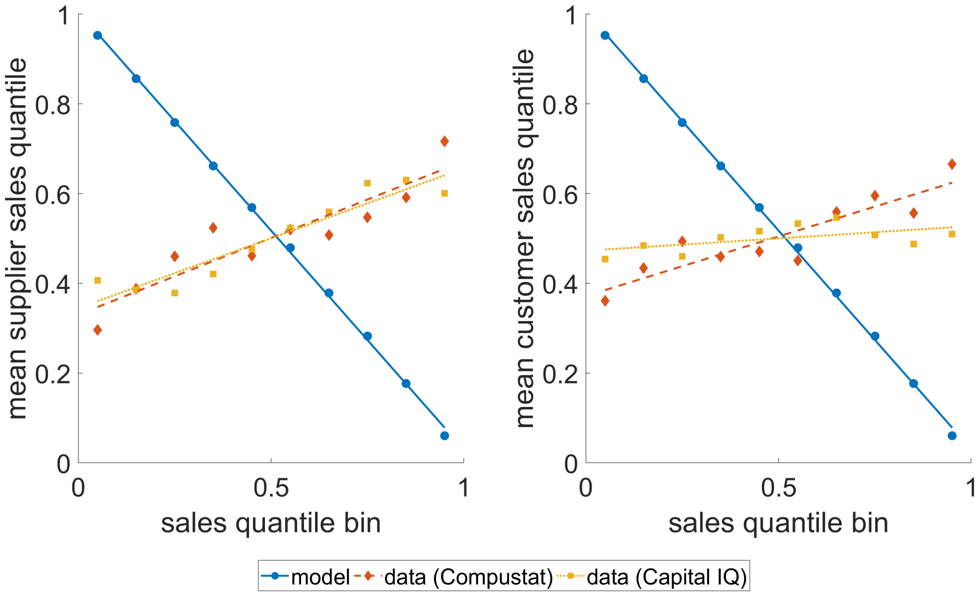

Figure 6: Matching assortativity in firm size

the model. This is close to the average growth rate of 14.8% in the data. Similarly, with regard to relationship dynamics, the estimated model predicts that relationships survive year-to-year at an average rate of around 85%. This is close to but slightly higher than the actual average survival rate observed in the data of about 76%.[24] In general, therefore, the simulated model matches up reasonably well with aggregate moments concerning firm and relationship dynamics in the data.

5 Endogenous Network Adjustment and Business Cycles

Having assessed the fit of the model to data, I now apply the theory to quantify precisely how important the endogenous adjustment of the production network is for US business cycle fluctuations.

[24]The relationship retention rates in the Compustat data are slightly higher than those documented by Monarch (2018), who finds survival rates of around 50% for relationships between US importers and Chinese suppliers. One conjecture for this difference is that domestic relationships are more stable than cross-border relationships. A second potential explanation is that firms in Compustat are large firms, and relationships involving large firms are more stable (as predicted by the model).

To derive clearer intuition for how the extensive margin adjustment of the production network might matter, note from equation (2.33) that household welfare can be approximately expressed (with a slight change in notation) as follows:

$$U \approx L_p \left[\int_\Omega \int_\Omega \bar{m}_{ij} \left(\delta_i \phi_j\right)^{\sigma-1} didj \right]^{\frac{1}{\sigma-1}} \tag{5.1}$$

where L_p is total production labor, Ω is the set of firms in the economy, δ_i and ϕ_i are the fundamental demand and productivity of firm i respectively, and $\bar{m}_{ij} \equiv \sum_{d=0}^{\infty} \left(\frac{\alpha}{\mu}\right)^{d(\sigma-1)} m_{ij}^{(d)}$ is aggregate connectivity between buyer i and seller j as defined by equations (2.19)-(2.21). Now consider, for example, a shock to the fundamental productivities of a set of firms $\hat{\Omega} \subset \Omega$ such that post-shock productivities are $\hat{\phi}_i = \omega \phi_i$ for $i \in \hat{\Omega}$. The first-order welfare effect of this change (around the point $\omega = 1$) can be expressed as:

$$\frac{\omega}{U}\frac{\partial U}{\partial \omega} \approx \frac{L_p^{\hat{\Omega}}}{L_p} + \frac{1}{\sigma-1} \frac{\int_\Omega \int_\Omega \epsilon_{ij}^m \bar{m}_{ij} \left(\delta_i \phi_j\right)^{\sigma-1} didj}{\int_\Omega \int_\Omega \bar{m}_{ij} \left(\delta_i \phi_j\right)^{\sigma-1} didj} + \frac{\omega}{L_p}\frac{\partial L_p}{\partial \omega} \tag{5.2}$$

The first term is the share of production labor employed by firms hit by the shock. The second term captures the response of the production network along the extensive margin, where $\epsilon_{ij}^m \equiv \frac{\omega}{\bar{m}_{ij}} \frac{\partial \bar{m}_{ij}}{\partial \omega}$ is the elasticity of aggregate connectivity $\bar{m}_{ij}$ with respect to the shock. The third term is the elasticity of total production labor with respect to the shock.

Now note that if the network of firm-to-firm relationships and the associated relationship costs are held fixed, then the second and third terms on the right-hand side of (5.2) are zero, and the first-order effect of the shock is simply equal to the share of production labor employed by firms that receive the labor productivity shock (as implied by the theorem of Hulten (1978)). The quantitative issue at hand can therefore be thought of as decomposing the effects of firm-level productivity shocks on business cycle fluctuations into the exogenous network and endogenous network components.[25]

The outline of the counterfactual exercise that I implement in order to examine this is as follows. First, the model is simulated under the baseline parameter specification

[25] Note that the Hulten approximation deviates from the true change in welfare for three reasons: (i) it is a first-order approximation; (ii) the first-order approximation is valid only in the efficient benchmark of the model; and (iii) production networks are endogenous.

determined by the structural estimation procedure as described in section 4.2. By construction, the estimated model matches US business cycle fluctuations in gross output exactly. In addition to values for aggregate variables, the model simulation also yields a baseline production network for each period. To quantify the importance of production network adjustment for the business cycle, I then compute values of gross output and welfare under counterfactual scenarios in which the production network is constrained to remain unchanged over a given number of periods. When the extensive margin adjustment of the production network is constrained, values of gross output and welfare under these counterfactual scenarios are always strictly smaller than under the baseline simulation. I then attribute these differentials to the role of endogenous production network adjustment in the business cycle.

5.1 Baseline simulation

To perform the numerical simulations, the structural model is first solved under the baseline parameter specification over the same set of periods $t \in \{0, \cdots, T\}$ as observed in the data. This yields solutions for aggregate variables such as total output, welfare, and the firm network productivity and quality functions. I then draw values for firm productivities $\phi_{i,t}$ and relationship costs $\xi_{ij,t}$ for a set of artificial firms Ω^{sim}, according to the stochastic processes estimated in sections 4.2.1 and 4.2.2.[26] Given the simulated values of $\phi_{i,t}$, I then determine the corresponding values of the firm network characteristics $\Phi_{i,t}$ and $\Delta_{i,t}$ from the structural solution of the model. Together with the simulated values of $\xi_{ij,t}$, this allows the simulated network of relationships $m_t \equiv \{m_{ij,t}\}_{i,j \in \Omega^{sim}}$ to be constructed based on the relationship activation criterion specified by equation (3.7).

5.2 Counterfactual production networks

To quantify the importance of production network adjustment for aggregate quantities such as output and welfare, I compute counterfactual business cycles as follows. For a given number of lags $K \geq 1$, define the K-year constrained business cycle as the sequence of static market equilibria (see definition 1) for dates $t \in \{K, \cdots T\}$, where the equilibrium at date t is computed assuming that (i) the matching function

[26] The number of simulated firms is set at $10,000$, which is on the order of the number of actual firms observed in the data.

is given by m_{t-k} instead of m_t, and (ii) the total labor cost of relationships $L_{f,t}$ is that implied by the network m_{t-k} and the stochastic process for relationship costs, (3.3) and (3.5). These counterfactual business cycles can be interpreted as the dynamic market equilibrium that would be observed if the set of firm-to-firm relationships in each period was exogenously held fixed over the previous K periods instead of being allowed to adjust at the frequency that is actually observed. For the K-year constrained business cycle, I then compute the differences in outcomes such as real gross output and welfare relative to the baseline simulation, and interpret these differentials as the endogenous network components of the respective variables.

Figures 7 and 8 show the values of real gross output and welfare respectively over these counterfactual business cycles for three different lags, $K \in \{1, 2, 3\}$, as well as the deviation in each variable under the counterfactual simulation relative to the baseline.[27] From these results, we learn the following.

First, when the production network is constrained to remain fixed along the extensive margin, both output and welfare are strictly lower in each period than when firms are allowed to optimally adjust their trading relationships. Note that this is not a strictly mechanical result, given that firms' choices of relationships are conditionally inefficient (see proposition 4).

Second, the longer that the production network is constrained to remain fixed, the greater the decline in gross output and welfare. This is intuitive, as fluctuations in firm productivities and relationship costs result in the static network becoming less optimal over time.

Finally, the endogenous network components of real gross output and welfare fluctuations are quantitatively large, especially when considered over longer time horizons. Over horizons of one year ($K = 1$), production network adjustment accounts for around 4% of real gross output and welfare, while over horizons of two and three years, this value increases to around 9% and 16% respectively.

[27] Across all years and lags, the mean standard error for the percentage deviations (computed using the same bootstrapping procedure described in footnote 19) is 1.3% of the respective estimated value. For example, the loss in real gross output in 1980 under the 1-year constrained business cycle is 3.26%, with a standard error of 0.037 percentage points or 1.13% of the estimated value.

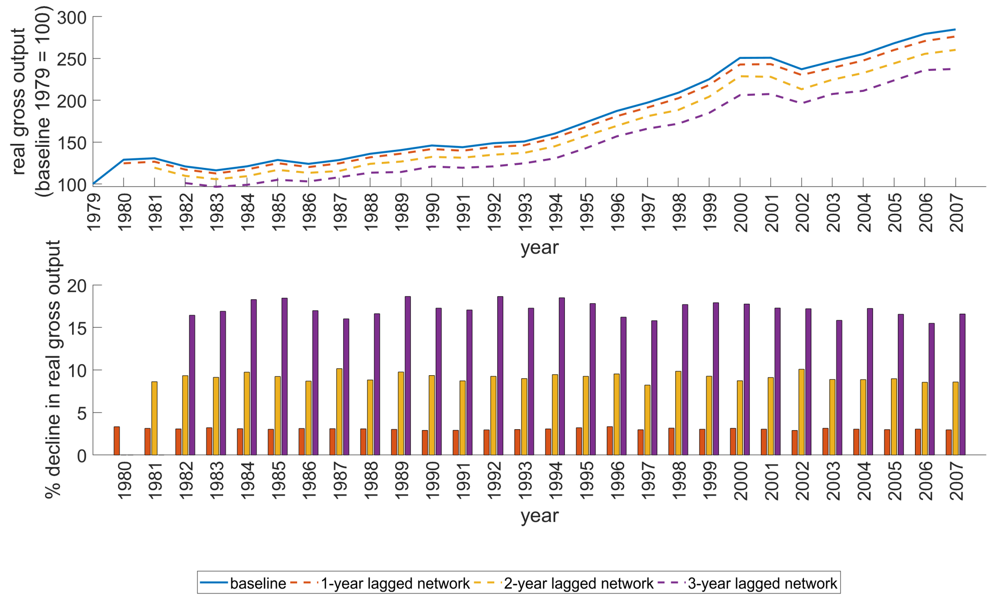

Figure 7: Output over the business cycle under counterfactual production networks

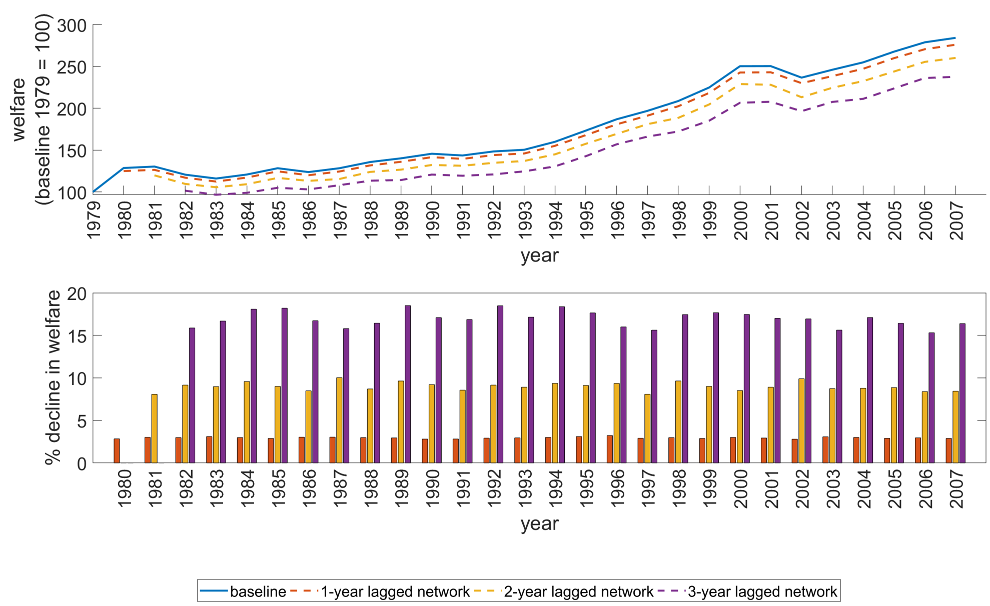

Figure 8: Welfare over the business cycle under counterfactual production networks

5.3 Parameter sensitivity and economic mechanisms

The results of the counterfactual simulations discussed above provide evidence that endogenous adjustment of the production network is quantitatively important for the US business cycle. There are three reasons why this might be so. First, adjustment of buyer-seller relationships might be an important margin of firm response to firm-level productivity shocks. Second, adjustment of buyer-seller relationships might be important as a response to fluctuations in the costs of maintaining these relationships. Finally, adjustment of the production network might be quantitatively important because of the extent to which the formation of new supply relationships enables firms to reduce production costs. To further examine these mechanisms, I now study the sensitivity of the quantitative results in section 5.2 to different values of the model's parameters.

5.3.1 Firm productivity dynamics

To assess the importance of production network adjustment as a response to firm-level productivity shocks, I simulate the same baseline and counterfactual business cycles as in sections 5.1 and 5.2, but holding idiosyncratic firm-level productivity constant across time ($\rho_{\phi,t} = 1$ and $s_{\phi,t} = 0$ for all t). Note that in this case, the production network under the baseline simulation changes over time only in response to aggregate productivity shocks (variation of $m_{\phi,t}$) and relationship-specific cost shocks.

Figure 9 shows the results of these simulations. Note that when idiosyncratic firm productivity is held fixed, the endogenous network components of welfare are smaller than in the baseline counterfactual, but are still of comparable magnitude. For example, the average welfare loss from fixed production networks in the 1-year constrained business cycle is around 3%, compared with 4% in the baseline. This suggests that while firms may adjust buyer-seller relationships in response to idiosyncratic firm-level productivity shocks, these shocks are not the main driver of production network adjustment over the business cycle.

5.3.2 Relationship cost dynamics

Similarly, to assess the importance of production network adjustment as a response to relationship-specific cost shocks, I simulate the same baseline and counterfactual

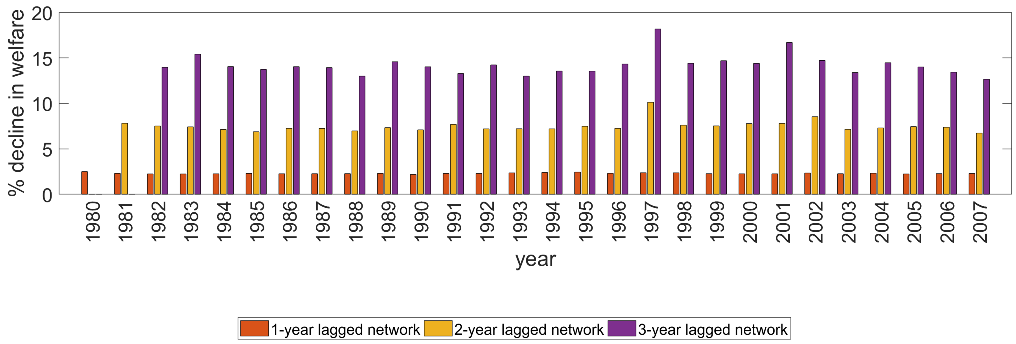

Figure 9: Welfare loss without network adjustment under fixed idiosyncratic firm productivity

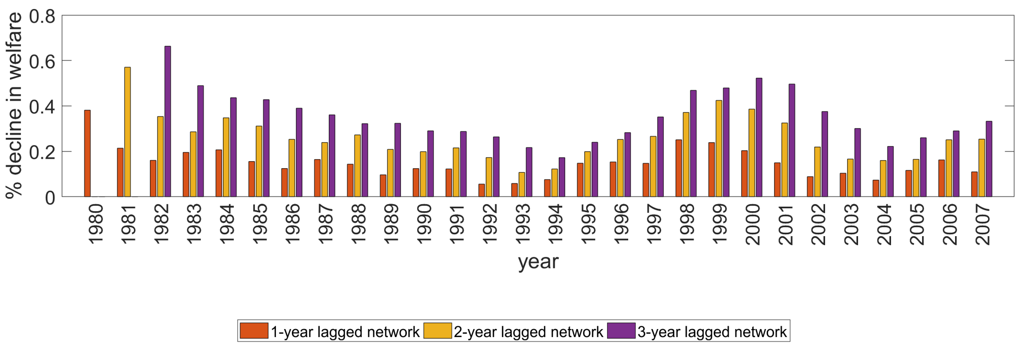

Figure 10: Welfare loss without network adjustment under fixed relationship costs

business cycles as in sections 5.1 and 5.2, but now holding relationship costs constant across time ($\rho_\xi = 1$ and $s_\xi = 0$). Note that in this case, the production network under the baseline simulation changes over time only in response to firm-level productivity shocks.

Figure 5.3.2 shows the results of these simulations. Note that when relationship costs are held fixed, the endogenous network components of welfare are still positive, but are substantially smaller than in the baseline counterfactual. For example, the average welfare loss from fixed production networks in the 1-year constrained business cycle is around 0.2%, compared with 4% in the baseline. This suggests that the extensive margin adjustment of the production network is important for aggregate fluctuations primarily because of idiosyncratic shocks at the relationship level. In other words, accounting for the endogeneity of the production network along the business cycle appears to be important mainly because of dynamics at the relationship-level rather than the firm-level.

5.3.3 The elasticity of substitution across products

Finally, I assess whether production network adjustment matters because of the extent to which new supply relationships enable firms to reduce production costs. Recall that in the model, the elasticity of substitution σ governs the degree to which firm production technologies exhibit "love of variety" - with smaller values of σ, intermediate inputs are more differentiated, and hence firms face greater incentives for relationship formation. Since σ was not estimated directly from data, I now examine the sensitivity of the main quantitative results to different values of this parameter.

Figure 11 shows the endogenous network components of welfare for values of $\sigma = 2$ and $\sigma = 4$, in contrast with the baseline results under $\sigma = 3$.[28] Note that endogenous network adjustment becomes more important for welfare at a *smaller* value of σ, which is as expected. For example, the average welfare loss in the 1-year constrained business cycle is around 5% under $\sigma = 2$, which is slightly larger than the average value of 4% in the baseline with $\sigma = 3$. Furthermore, even at a *larger* value of $\sigma = 4$, the endogenous network components of welfare are of comparable magnitude to the baseline results - for example, 2.5% on average in the 1-year constrained business cycle. In sum, the conclusion that endogenous production network adjustment is quantitatively important for the business cycle is robust to allowing for different values of the elasticity of substitution across inputs.

6 Conclusion

This paper offers a new model of buyer-seller network formation between heterogeneous firms in a computationally tractable framework that is amenable for use in empirical research. The application of the theory to the study of US business cycles then provides evidence that this margin of endogenous production network adjustment is quantitatively important, primarily as a response to idiosyncratic shocks at the relationship-level.

The issues addressed in this paper also provide scope for future research, with two areas in particular warranting further investigation. First, given that the market equilibrium of the model is shown to be inefficient under monopolistic competition, a pertinent theoretical question is how to allow for alternative rules for splitting the

[28] In computing the endogenous network components of welfare under $\sigma = 2$ and $\sigma = 4$, the other structural parameters of the model are also simultaneously re-estimated.

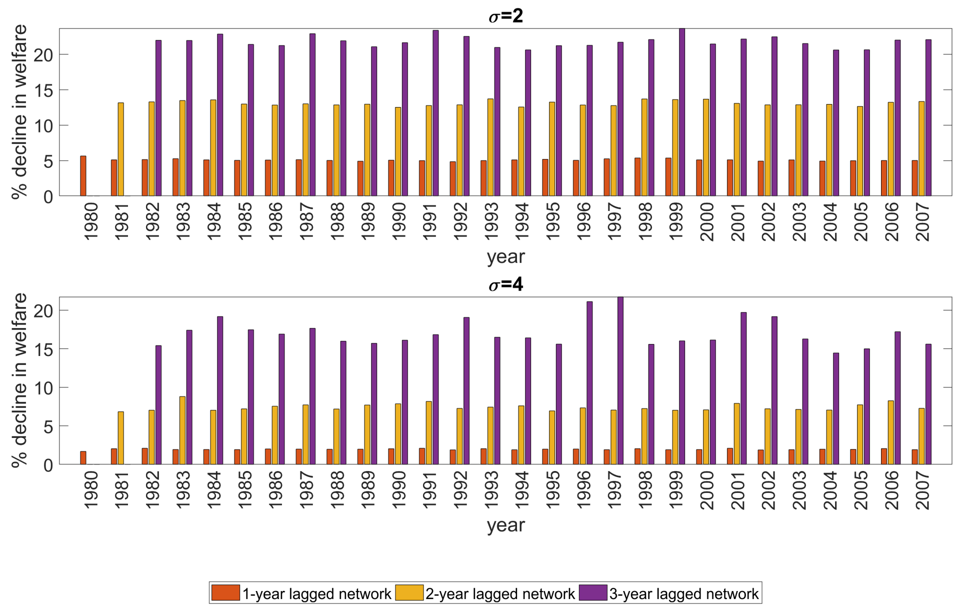

Figure 11: Welfare loss without network adjustment under $\sigma = 2$ and $\sigma = 4$

surplus of a buyer-seller relationship while still preserving computational tractability of the model. For example, since markups in the model are constant across relationships, the theory in its current form cannot be used to address questions of how competition in input markets are shaped by the structure of production networks. Modeling the endogeneity of production network relationships along both the extensive margin and markup margin in a computationally-tractable framework remains an open challenge for the literature.

Second, the modeling of relationship costs in this paper is a reduced-form approach towards capturing the idea that various frictions may impede the formation and maintenance of buyer-seller relationships. Understanding the microfoundations of these frictions requires further work and would likely yield new insights. For example, if these costs have to do with the availability of information about potential buyers and sellers in the market, then accounting for endogeneity of the frictions themselves becomes an important issue, since information propagates through the network in a way that likely depends on its structure and dynamics. Again, microeconomic evidence on the specific nature of these relationship costs remains a promising area for future research.

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