

Master's Thesis (Academic Year 2023)

Link Management For Quantum Network

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Abstract of Bachelor's Thesis - Academic Year 20xx

Link Management For Quantum Network
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I can't write English.

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# Chapter 1

## Introduction

1.1 Background

1.2 Research Contribution

1.3 Thesis Structure

# Chapter 2

## Background

### 2.1 Quantum State

#### 2.1.1 Quantum Bit

A classical bit has two different states, which are 0 and 1. Instead, those of a quantum bit (or **qubit** in short) are  $|0\rangle$  and  $|1\rangle$ , each of which can be described as a vector. For example

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The state of a single qubit  $|\psi\rangle$  can be described as follows.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad (\alpha, \beta \in \mathbb{C}, |\alpha|^2 + |\beta|^2 = 1)$$

. After the operation called measurement, the quantum state would be collapsed into either 0 or 1. The measurement probability of 0 is  $|\alpha|^2$  and that of 1 is  $|\beta|^2$ . In other words, a single qubit can take both states probabilistically at the same time. For instance, a qubit can be

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \tag{1}$$

which can be 50% 0 and 50% 1.

#### 2.1.2 Bloch sphere

Because  $|\alpha|^2 + |\beta|^2 = 1$ , the notation of a single qubit state can be represented like this.

$$|\psi\rangle = e^{i\gamma} \left( \cos \frac{\theta}{2} + e^{i\phi} \sin \frac{\theta}{2} \right) (\gamma, \phi, \theta \in \mathbb{R}) \tag{2.1}$$



Because  $e^{i\gamma}$  is just a global state, it can be ignored and the same state can be rewritten like this.

$$|\psi\rangle = \cos \frac{\theta}{2} + e^{i\phi} \sin \frac{\theta}{2} (\phi, \theta \in \mathbb{C}) \quad (2.2)$$

Because the equation above has two parameters, any pure single qubit state can be considered as a point on the surface and its geometric representation is called **Bloch sphere**.

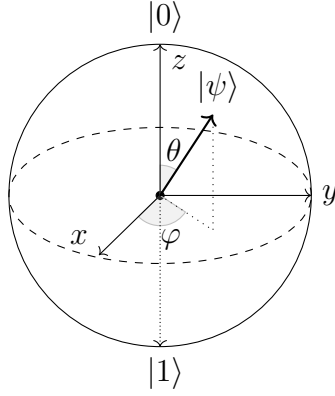


Figure 2.1: Bloch Sphere

### 2.1.3 Multi-Qubit State

The quantum state for multi-qubits is a **tensor product** of a state vector of each qubit. The general notation of two qubit state is

$$|\psi\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes (\gamma|0\rangle + \delta|1\rangle) \quad (2.3)$$

$$= \alpha\gamma|00\rangle + \alpha\delta|01\rangle + \beta\gamma|10\rangle + \beta\delta|11\rangle \quad (2.4)$$

$$(\alpha, \beta, \gamma, \delta \in \mathbb{C}, |\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1) \quad (2.5)$$

For example, the state  $|00\rangle$  is equal to

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (2.6)$$

However, some quantum states such as

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle \quad (2.7)$$

cannot be decomposed into quantum state of each qubit. These special quantum states are called **entangled** states.

## 2.2 Quantum Operations

### 2.2.1 I gate

I gate is equal to the 2x2 identity matrix, which is

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (2.8)$$

For example,

$$I|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle \quad (2.9)$$

$$I|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle \quad (2.10)$$

### 2.2.2 X Gate

**X gate**

X gate flips the logical value of a qubit.

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad (2.11)$$

For example,

$$X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle \quad (2.12)$$

$$X|1\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle \quad (2.13)$$

### 2.2.3 Y gate

Y gate flips the logical value of a qubit and add an imaginary number.

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad (2.14)$$

For example,

$$Y|0\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix} = i|1\rangle \quad (2.15)$$

$$Y|1\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -i \\ 0 \end{bmatrix} = -i|0\rangle \quad (2.16)$$

### 2.2.4 Z Gate

Z gate flips the phase of  $|1\rangle$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad (2.17)$$

For example,

$$Z|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle \quad (2.18)$$

$$Z|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} = -|1\rangle \quad (2.19)$$

### 2.2.5 H Gate

H gate creates superposition.

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (2.20)$$

For example,

$$H|0\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \quad (2.21)$$

$$H|1\rangle = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \quad (2.22)$$

### 2.2.6 CNOT Gate

A CNOT gate involves two qubits, one is called **controlled qubit** and the other is called **target qubit**. If the controlled qubit is 1, the bit value of the target qubit is flipped.

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (2.23)$$

For example,

$$CNOT_{0,1}|10\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = |11\rangle \quad (2.24)$$

$$CNOT_{0,1}|11\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = |10\rangle \quad (2.25)$$

### 2.2.7 Measurement

Quantum measurement can be described by using a group of measurement operators  $\{M_m\}$  ( $m$  is the measurement result that is expected to get). If the quantum state before measurement is  $|\psi\rangle$ , the measurement probability of value  $m$  is

$$p(m) = \langle\psi|M_m^\dagger M_m|\psi\rangle$$

The quantum state after the measurement is

$$\frac{M_m|\psi\rangle}{\sqrt{\langle\psi|M_m^\dagger M_m|\psi\rangle}}$$

The measurement operators satisfy the completeness equation

$$\sum_m M_m^\dagger M_m = I$$

Also, the sum of the measurement probability of each possible measurement outcome is equal to one.

$$\sum_m p(m) = \langle\psi|\sum_m M_m^\dagger M_m|\psi\rangle = 1$$

## 2.3 Quantum Circuit

Here is the example of a quantum circuit.

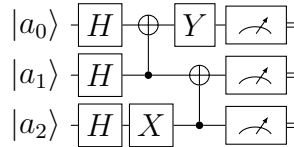


Figure 2.2: A example of quantum circuit

Each horizontal line represents each qubit and the square boxes that contain alphabets mean single quantum gates. The sign which involves a vertical line means a CNOT gate, and the box on the most right side indicates measurement.

## 2.4 Quantum Entanglement

Quantum entanglement is a special type of quantum state that cannot be described in the form of tensor product of the state of each particle.

### 2.4.1 Bell Pair

The entangled states between two qubits are called bell pairs, and each of four states has a special notation.

$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} \quad (2.26)$$

$$|\Phi^-\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}} \quad (2.27)$$

$$|\Psi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}} \quad (2.28)$$

$$|\Psi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}} \quad (2.29)$$

### 2.4.2 Multipartite Entanglement

There are cases that more than two qubits are entangled and that state is called Greenberger–Horne–Zeilinger state or GHZ state.

Here is the braket notation of the GHZ state that involves three qubits.

$$|GHZ\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}} \quad (2.30)$$

In the general case, the braket notation of the GHZ state of N qubits is the following.

$$|GHZ\rangle = \frac{|0\rangle^{\otimes N} + |1\rangle^{\otimes N}}{\sqrt{2}} \quad (2.31)$$

### 2.4.3 Bell State Measurement

Bell state measurement is a special type of quantum measurement that determines which bell pair the given two qubit entangled state is.

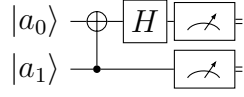


Figure 2.3: Quantum circuit for bell state measurement

Measurement results	Bell state
00	$ \Phi^+\rangle$
01	$ \Phi^-\rangle$
10	$ \Psi^+\rangle$
11	$ \Psi^-\rangle$

Table 2.1: A table of correspondence between measurement result and Bell pair

## 2.4.4 Quantum Teleportation

Unlike classical communication, quantum states cannot be just copied and transmit to other nodes due to the no-cloning theorem, which forbids duplication of any quantum state. However, a method called quantum teleportation was proposed, which overcomes the restriction and allows sender to transmit single qubit state to a distant location.

This method requires both the single qubit state and a new Bell pair, and also the sender have to prepare two qubits and the receiver have to prepare one qubit. After applying a CNOT gate and an H gate in the figure above, the sender have to measure both qubits and send those measurement results over the classical network. After the receiver get those measurement results and apply some quantum gates if the measurement results of corresponding qubits on the sender's side are 1, in order to correct on the quantum state on the receiver's side.

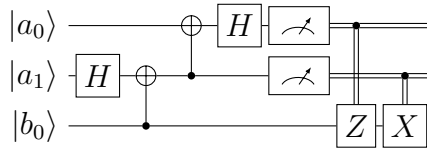


Figure 2.4: Quantum circuit for quantum teleportation

### 2.4.5 Entanglement Swapping

Entanglement swapping is the method to extend quantum entanglement by performing joint measurement on several quantum entanglement. For example, assume Alice has a single qubit, Bob has two qubits, and Charlie has one qubit. Then, there are Bell pairs between Alice's qubit and Bob's first qubit, and Bob's second qubit and Charlie's qubit, respectively. If Bob performs Bell state measurement on both of his qubits, Alice's qubit and Charlie's qubit are eventually entangled, even though they have not interacted with each other. This can be also seen as the teleportation of a Bell pair by sending one of its particles. Here is the figure of quantum circuit to perform entanglement swapping.

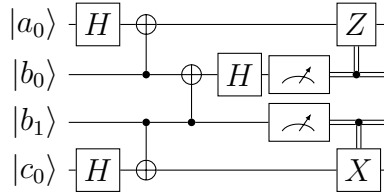


Figure 2.5: Quantum circuit for entanglement swapping

### 2.4.6 Entanglement Purification

Entanglement purification is a scheme to generate a set of quantum entanglements with higher fidelities from a larger set of imperfect quantum entanglements, local quantum operations, and classical communications. This procedure is also called entanglement distillation, or quantum concatenation. This section presents an example of entanglement purification that generates a single bell pair with higher fidelity from two of those with less fidelity.

Assume Alice and Bob are supposed to share  $|\Phi^+\rangle$ , which is one of the Bell pairs. However, the state would be converted to the following mixed state due to the noisy nature of a quantum channel.

$$\rho_{AB} = P_{\Phi^+}|\Phi^+\rangle\langle\Phi^+| + P_{\Phi^-}|\Phi^-\rangle\langle\Phi^-| + P_{\Psi^+}|\Psi^+\rangle\langle\Psi^+| + P_{\Psi^-}|\Psi^-\rangle\langle\Psi^-|$$

$$\sum_{s \in \{\Phi^+, \Phi^-, \Psi^+, \Psi^-\}} P_s = 1$$

Any mixed state can be converted to Werner state by applying Pauli operations and  $\frac{\pi}{2}$  operations, so Alice and Bob can obtain the following state.

$$\rho'_{AB} = F|\Phi^+\rangle\langle\Phi^+| + \frac{1-F}{3}(|\Phi^-\rangle\langle\Phi^-| + |\Psi^+\rangle\langle\Psi^+| + |\Psi^-\rangle\langle\Psi^-|)$$

Two noisy bell pairs are required for entanglement purification, so assume the quantum

state of the entire system can be described as follows.

$$\begin{aligned}
 \rho'_{a_1 b_1} \otimes \rho'_{a_2 b_2} = & F^2 |\Phi^+\rangle |\Phi^+\rangle \langle \Phi^+| \langle \Phi^+| \\
 & + \frac{F(1-F)}{3} (|\Phi^+\rangle |\Phi^-\rangle \langle \Phi^-| \langle \Phi^+| + |\Phi^+\rangle |\Psi^+\rangle \langle \Psi^+| \langle \Phi^+| + |\Phi^+\rangle |\Psi^-\rangle \langle \Psi^-| \langle \Phi^+| \\
 & + |\Phi^-\rangle |\Phi^+\rangle \langle \Phi^+| \langle \Phi^-| + |\Psi^+\rangle |\Phi^+\rangle \langle \Phi^+| \langle \Psi^+| + |\Psi^-\rangle |\Phi^+\rangle \langle \Phi^+| \langle \Psi^-|) \\
 & + \frac{(1-F)^2}{9} (|\Phi^-\rangle |\Phi^-\rangle \langle \Phi^-| \langle \Phi^-| + |\Phi^-\rangle |\Psi^+\rangle \langle \Psi^+| \langle \Phi^-| + |\Phi^-\rangle |\Psi^-\rangle \langle \Psi^-| \langle \Phi^-| \\
 & + |\Psi^+\rangle |\Phi^-\rangle \langle \Phi^-| \langle \Psi^+| + |\Psi^+\rangle |\Psi^+\rangle \langle \Psi^+| \langle \Psi^+| + |\Psi^+\rangle |\Psi^-\rangle \langle \Psi^-| \langle \Psi^+| \\
 & + |\Psi^-\rangle |\Phi^-\rangle \langle \Phi^-| \langle \Psi^-| + |\Psi^-\rangle |\Psi^+\rangle \langle \Psi^+| \langle \Psi^-| + |\Psi^-\rangle |\Psi^-\rangle \langle \Psi^-| \langle \Psi^-|)
 \end{aligned}$$

One of the bell pair  $\rho'_{a_1 b_1}$  is called source bell pair, which may be purified, and the other one  $\rho'_{a_2 b_2}$  is called target bell pair, which is going to be measured. Then, Alice and Bob perform CNOT operations between  $a_1$  and  $a_2$ , and  $b_1$  and  $b_2$ , respectively. The entire quantum state on this point would be as follows.

$$\begin{aligned}
 \rho'_{a_1 b_1} \otimes \rho'_{a_2 b_2} = & F^2 |\Phi^+\rangle |\Phi^+\rangle \langle \Phi^+| \langle \Phi^+| \\
 & + \frac{F(1-F)}{3} (|\Phi^-\rangle |\Phi^-\rangle \langle \Phi^-| \langle \Phi^-| + |\Phi^+\rangle |\Psi^+\rangle \langle \Psi^+| \langle \Phi^+| + |\Phi^-\rangle |\Psi^-\rangle \langle \Psi^-| \langle \Phi^-| \\
 & + |\Phi^-\rangle |\Phi^+\rangle \langle \Phi^+| \langle \Phi^-| + |\Psi^+\rangle |\Psi^+\rangle \langle \Psi^+| \langle \Psi^+| + |\Psi^-\rangle |\Psi^+\rangle \langle \Psi^+| \langle \Psi^-|) \\
 & + \frac{(1-F)^2}{9} (|\Phi^+\rangle |\Phi^-\rangle \langle \Phi^-| \langle \Phi^+| + |\Phi^-\rangle |\Psi^+\rangle \langle \Psi^+| \langle \Phi^-| + |\Phi^+\rangle |\Psi^-\rangle \langle \Psi^-| \langle \Phi^+| \\
 & + |\Psi^+\rangle |\Psi^-\rangle \langle \Psi^-| \langle \Psi^+| + |\Psi^-\rangle |\Phi^+\rangle \langle \Phi^+| \langle \Psi^+| + |\Psi^+\rangle |\Psi^-\rangle \langle \Psi^-| \langle \Psi^+| \\
 & + |\Psi^-\rangle |\Psi^-\rangle \langle \Psi^-| \langle \Psi^-| + |\Psi^-\rangle |\Phi^+\rangle \langle \Phi^+| \langle \Psi^-| + |\Psi^-\rangle |\Phi^-\rangle \langle \Phi^-| \langle \Psi^-|)
 \end{aligned}$$

After that, they measure  $a_2$  and  $b_2$  respectively, which is the qubit on the target bell pair on their side and exchange the measurement results.

If their measurement results match, the purification is successful, while they have to discard the source bell pair and try again if those results do not match.

Here is the quantum state after measuring the target bell pair.

$$\begin{aligned}
 \rho'_{ab} = & \frac{1}{N} \left[ F^2 + \frac{1}{9} (1-F)^2 \right] |\Phi^+\rangle \langle \Phi^+| + \frac{2F(1-F)}{3N} |\Phi^-\rangle \langle \Phi^-| + \frac{2(1-F)^2}{9N} (|\Psi^+\rangle \langle \Psi^+| + |\Psi^-\rangle \langle \Psi^-|) \\
 & (N = F^2 + \frac{2F(1-F)}{3} + \frac{2(1-F)^2}{9})
 \end{aligned}$$

The purification becomes successful if  $F > \frac{1}{2}$



Before applying a CNOT gate	After a applying CNOT gate
$ \Phi^+\rangle \Phi^+\rangle$	$ \Phi^+\rangle \Phi^+\rangle$
$ \Phi^+\rangle \Phi^-\rangle$	$ \Phi^-\rangle \Phi^-\rangle$
$ \Phi^+\rangle \Psi^+\rangle$	$ \Phi^+\rangle \Psi^+\rangle$
$ \Phi^+\rangle \Psi^-\rangle$	$ \Phi^-\rangle \Psi^-\rangle$
$ \Phi^-\rangle \Phi^+\rangle$	$ \Phi^-\rangle \Phi^+\rangle$
$ \Phi^-\rangle \Psi^-\rangle$	$ \Phi^+\rangle \Phi^-\rangle$
$ \Phi^-\rangle \Psi^+\rangle$	$ \Phi^-\rangle \Psi^+\rangle$
$ \Phi^-\rangle \Psi^-\rangle$	$ \Phi^+\rangle \Psi^-\rangle$
$ \Psi^+\rangle \Phi^+\rangle$	$ \Psi^+\rangle \Psi^+\rangle$
$ \Psi^+\rangle \Phi^-\rangle$	$ \Psi^+\rangle \Psi^-\rangle$
$ \Psi^+\rangle \Psi^+\rangle$	$ \Psi^+\rangle \Phi^+\rangle$
$ \Psi^+\rangle \Psi^-\rangle$	$ \Psi^+\rangle \Phi^-\rangle$
$ \Psi^-\rangle \Phi^+\rangle$	$ \Psi^-\rangle \Psi^+\rangle$
$ \Psi^-\rangle \Phi^-\rangle$	$ \Psi^-\rangle \Psi^-\rangle$
$ \Psi^-\rangle \Psi^+\rangle$	$ \Psi^-\rangle \Phi^+\rangle$
$ \Psi^-\rangle \Psi^-\rangle$	$ \Psi^-\rangle \Phi^-\rangle$

Table 2.2: A table of correspondence between Bell pairs before and after applying a CNOT gate

## 2.5 Quantum Networking

### 2.5.1 Quantum Node

### 2.5.2 Quantum Repeater

### 2.5.3 Quantum Link

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### 4.1 Problem Definition

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## Proposal: Link Management For Quantum Network

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7.1.2 Two Node Network With an MIM Link

7.1.3 Two Node Network With an MSM Link

7.1.4 Two Node Network With an MIM Link (Without Timing Negotiation)

# Chapter 8

## Conclusion

### 8.1 Conclusion

### 8.2 Future Works

# Appendix A

## Appendix

### A.1 hoge



# Acknowledgement

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