

Master's Thesis (Academic Year 2023)

Link Management For Quantum Network

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Abstract of Bachelor's Thesis - Academic Year 20xx

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I can't write English.

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Chapter 2

Background

2.1 Quantum State

2.1.1 Quantum Bit

A classical bit has two different states, which are 0 and 1. Instead, those of a quantum bit (or **qubit** in short) are $|0\rangle$ and $|1\rangle$, each of which can be described as a vector. For example

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The state of a single qubit $|\psi\rangle$ can be described as follows.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad (\alpha, \beta \in \mathbb{C}, |\alpha|^2 + |\beta|^2 = 1)$$

. After the operation called measurement, the quantum state would be collapsed into either 0 or 1. The measurement probability of 0 is $|\alpha|^2$ and that of 1 is $|\beta|^2$. In other words, a single qubit can take both states probabilistically at the same time. For instance, a qubit can be

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \tag{1}$$

which can be 50% 0 and 50% 1.

2.1.2 Bloch sphere

Because $|\alpha|^2 + |\beta|^2 = 1$, the notation of a single qubit state can be represented like this.

$$|\psi\rangle = e^{i\gamma} \left(\cos \frac{\theta}{2} + e^{i\phi} \sin \frac{\theta}{2} \right) (\gamma, \phi, \theta \in \mathbb{R}) \tag{2.1}$$

Because $e^{i\gamma}$ is just a global state, it can be ignored and the same state can be rewritten like this.

$$|\psi\rangle = \cos \frac{\theta}{2} + e^{i\phi} \sin \frac{\theta}{2} (\phi, \theta \in \mathbb{C}) \quad (2.2)$$

Because the equation above has two parameters, any pure single qubit state can be considered as a point on the surface and its geometric representation is called **Bloch sphere**.

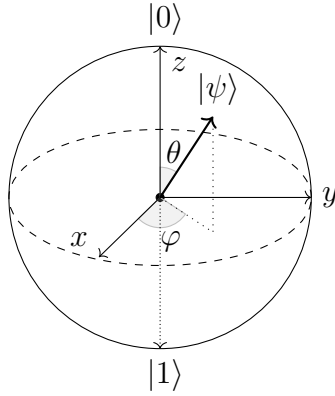


Figure 2.1: Bloch Sphere

2.1.3 Multi-Qubit State

The quantum state for multi-qubits is a **tensor product** of a state vector of each qubit. The general notation of two qubit state is

$$|\psi\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes (\gamma|0\rangle + \delta|1\rangle) \quad (2.3)$$

$$= \alpha\gamma|00\rangle + \alpha\delta|01\rangle + \beta\gamma|10\rangle + \beta\delta|11\rangle \quad (2.4)$$

$$(\alpha, \beta, \gamma, \delta \in \mathbb{C}, |\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1) \quad (2.5)$$

For example, the state $|00\rangle$ is equal to

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (2.6)$$

However, some quantum states such as

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle \quad (2.7)$$

cannot be decomposed into quantum state of each qubit. These special quantum states are called **entangled** states.

2.2 Quantum Operations

2.2.1 I gate

I gate is equal to the 2x2 identity matrix, which is

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (2.8)$$

For example,

$$I|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle \quad (2.9)$$

$$I|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle \quad (2.10)$$

2.2.2 X Gate

X gate

X gate flips the logical value of a qubit.

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad (2.11)$$

For example,

$$X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle \quad (2.12)$$

$$X|1\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle \quad (2.13)$$

2.2.3 Y gate

Y gate flips the logical value of a qubit and add an imaginary number.

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad (2.14)$$

For example,

$$Y|0\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix} = i|1\rangle \quad (2.15)$$

$$Y|1\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -i \\ 0 \end{bmatrix} = -i|0\rangle \quad (2.16)$$

2.2.4 Z Gate

Z gate flips the phase of $|1\rangle$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad (2.17)$$

For example,

$$Z|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle \quad (2.18)$$

$$Z|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} = -|1\rangle \quad (2.19)$$

2.2.5 H Gate

H gate creates superposition.

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (2.20)$$

For example,

$$H|0\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \quad (2.21)$$

$$H|1\rangle = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \quad (2.22)$$

2.2.6 CNOT Gate

A CNOT gate involves two qubits, one is called **controlled qubit** and the other is called **target qubit**. If the controlled qubit is 1, the bit value of the target qubit is flipped.

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (2.23)$$

For example,

$$CNOT_{0,1}|10\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = |11\rangle \quad (2.24)$$

$$CNOT_{0,1}|11\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = |10\rangle \quad (2.25)$$

2.2.7 Measurement

Quantum measurement can be described by using a group of measurement operators $\{M_m\}$ (m is the measurement result that is expected to get). If the quantum state before measurement is $|\psi\rangle$, the measurement probability of value m is

$$p(m) = \langle\psi|M_m^\dagger M_m|\psi\rangle$$

The quantum state after the measurement is

$$\frac{M_m|\psi\rangle}{\sqrt{\langle\psi|M_m^\dagger M_m|\psi\rangle}}$$

The measurement operators satisfy the completeness equation

$$\sum_m M_m^\dagger M_m = I$$

Also, the sum of the measurement probability of each possible measurement outcome is equal to one.

$$\sum_m p(m) = \langle\psi|\sum_m M_m^\dagger M_m|\psi\rangle = 1$$

2.3 Quantum Circuit

2.4 Quantum Entanglement

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4.1 Problem Definition

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Chapter 8

Conclusion

8.1 Conclusion

8.2 Future Works

Appendix A

Appendix

A.1 hoge

Acknowledgement

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