Master's Thesis (Academic Year 2023)

Link Management For Quantum Network

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Abstract of master's Thesis - Academic Year 20xx

Link Management For Quantum Network

Quantum networking is the new paradigm of networking that allows to transfer quantum state and achieve various new applications. RuleSet-based communication protocol is known to be one of the practical communication protocols to establish a scalable quantum network. Ideally, multiplexing and real-time resource management should be realized in order to improve the performance and robustness of the network. However, the protocol to handle multiple connections and allocate of physical links has not been proposed. This thesis proposes the link management protocol for quantum network that involves negotiation to determine a set of RuleSets (which is called a link allocation policy) to execute and the timing of apply the new link allocation policy. It also discusses the implementation of communication setup and teardown based on the proposed protocol and validates the proposed approach by performing a set of network simulations.
Keywords :
1. Quantum Networking, 2. RuleSet-Based Communication Protocol, 3. Networking Protocol,

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Chapter 1

Introduction

1.1 Background

The recent development of quantum technologies such as quantum computing, quantum networking and quantum sensing are expected to provide new capabilities. For example, quantum processors can theoretically simulate quantum systems whose size are intractable even for their classical equivalence. The key to realize these new applications is quantum effect, such as superposition and entanglement, both of which cannot be observed in the classical world.

However, there are two major problems for transmitting quantum data to a distance location, which is required in certain situations such as distributed quantum computing. One is "non-cloning theorem", which is the fact that quantum state cannot be copied. Unlike classical network, it is almost impossible to neither amplify a quantum state or send it forward because the quantum state will be heavily corrupted by the high probability of loss and high error rate. The other problem is that it is so difficult to establish a bell pair between nodes separated by a long distance, again due to a photon will be spoiled by the physical noise and photon loss.

These two problems can be solved by using particular type of nodes called quantum repeaters. Quantum repeaters perform entanglement swapping and purification, each of which extends two neighboring bell pairs to a single longer bell pair, and improves the fidelity of the bell pair, respectively. These operations end up with generating an end-to-end bell pair that can be used by quantum teleportation, which is the protocol to send an arbitrary quantum state to a distant location.

Entanglement swapping and purification involve requires frequent message exchange with neighboring nodes in order to coordinate actions, such as entanglement swapping and purification, with neighboring nodes and those communication slow down the generation of an end-to-end bell pair. However, a communication protocol called RuleSet-based communication protocol solves this problem by distribute an object called RuleSet, which a sequence of operations execute to each node. This feature reduces the amount of unnecessary communication and improves the scalability of the entire network.

1.2 Research Contribution

Multiple connections should be established simultaneously in order to enhance the overall performance and robustness of the entire network and the same thing can be applied to quantum network. However, the previous work only proposes the method to allocate required physical bell pairs and establish a single end-to-end bell pair, in other word, an single connection by consuming those physical resources. This thesis proposes a protocol to realize three important tasks, which are the negotiation about what set of connections are going to be established, the one about when to switch from those in the previous round, and coordinated resource management between two nodes connected by each link. It also discusses the updated procedure of establishing a new connection and tearing down one of the existing connections while several connections are being established by applying the proposed protocol. The approach presented in this thesis is validated by the simulation of RuleSet-based quantum networks under several circumstances.

1.3 Thesis Structure

The structure of this thesis is as follows.

Chapter 2 provides the background knowledge to understand the key concepts readers would encounter throughout this thesis.

Chapter 3 explains the detail of RuleSet-based quantum networking.

Chapter 4 presents the problem that this thesis addresses.

Chapter 5 offers the overview of the link management protocol and the messages required for its negotiation process.

Chapter 6 provides how link management protocol proposed in this thesis will be triggered after the process of connection setup and teardown. It also includes the pseudocode of methods that the node software need to execute and messages outside of the link management protocol.

Chapter 7 presents several scenarios used to validate this protocol.

Chapter 8 offers the conclusion of this thesis and discusses future works.

Chapter 2

Background

2.1 Quantum Physics

This subsection provide the fundamental knowledge of quantum physics, which will make readers feel familiar with the concept and notations that they will encounter throughout this thesis.

2.1.1 Pure State

Pure state is the representation of quantum state of the whole system without the assumption of external noise.

Quantum Bit

A conventional computer uses a bit to represent a basic unit of information, which are 0 and 1. A basic unit of quantum information, on the other hand is called a quantum bit (or **qubit** in short) are $|0\rangle$ and $|1\rangle$, each of which can be described in the form of a vector.

For example

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The state of a single qubit $|\psi\rangle$ can be described as follows.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \, (\alpha, \beta \in \mathbb{C}, |\alpha|^2 + |\beta|^2 = 1)$$

. After the operation called measurement, the quantum state would be collapsed into either 0 or 1. The measurement probability of 0 is $|\alpha|^2$ and that of 1 is $|\beta|^2$. In other words, a single qubit can take both states probabilistically at the same time.

For instance, a qubit can be

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \tag{1}$$

whose measurement probability of 0 and 1 is 50% and 50% respectively.

Bloch Sphere

Because $|\alpha|^2 + |\beta|^2 = 1$, the notation of a single qubit state can be represented like this.

$$|\psi\rangle = e^{i\gamma}(\cos\frac{\theta}{2} + e^{i\phi}\sin\frac{\theta}{2})(\gamma, \phi, \theta \in \mathbb{R}) \tag{2.1}$$

.

Because $e^{i\gamma}$ is just a global state, it can be ignored and the same state can be rewritten like this.

$$|\psi\rangle = \cos\frac{\theta}{2} + e^{i\phi}\sin\frac{\theta}{2}(\phi, \theta \in \mathbb{C}) \tag{2.2}$$

Because the equation above has two parameters, any pure single qubit state can be considered as a point on the surface and its geometric representation is called **Bloch sphere**.

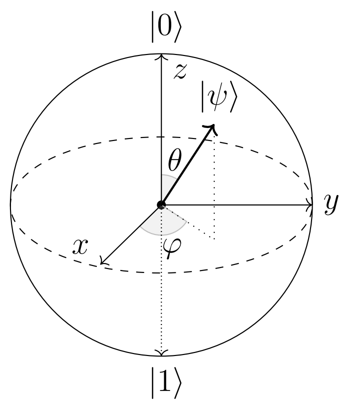

Figure 2.1: Bloch Sphere

Multi-Qubit State

The quantum state for multi-qubits is a **tensor product** of a state vector of each qubit. The general notation of two qubit state is

$$|\psi\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes (\gamma|0\rangle + \delta|1\rangle) \tag{2.3}$$
$$= \alpha\gamma|00\rangle + \alpha\delta|01\rangle + \beta\gamma|10\rangle + \beta\delta|11\rangle \tag{2.4}$$
$$(\alpha, \beta, \gamma, \delta \in \mathbb{C}, |\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1) \tag{2.5}$$

.

For example, the state $|00\rangle$ is equal to

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{2.6}$$

.

However, some quantum states such as

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle \tag{2.7}$$

cannot be decomposed into quantum state of each qubit. These special quantum states are called **entangled** states.

2.1.2 Mixed State

Mixed state is another representation of quantum state in more general cases, such as the presense of physical error. Mixed state is described in the form of a matrix which is called density matrix. Assume quantum system takes one of their collections $\{|\psi_i\rangle\}$ (i is an index) with the probability of p_i.

Mixed State

The density matrix of this system ρ is described by

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i| \tag{2.8}$$

Evolution

The quantum system after applying a unitary operator U is the following.

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i| \xrightarrow{U} \sum_i p_i U|\psi_i\rangle\langle\psi_i|U^\dagger \tag{2.9}$$

Measurement

Suppose one performs measurement on a quantum state $|\psi_i\rangle$ using a measurement operator M_m.

Then, the measurement probability of m is

$$p(m|i) = \langle\psi_i|M_m^\dagger M_m|\psi_i\rangle = tr(M_m^\dagger M_m|\psi_i\rangle\langle\psi_i|) \tag{2.10}$$

The measurement probability of m from the collection of state $\{|\psi_i\rangle\}$ is

$$\begin{aligned} p(m) &= \sum_i p_i p(m|i) \\ &= \sum_i p_i \langle\psi_i|M_m^\dagger M_m|\psi_i\rangle \\ &= \sum_i p_i \operatorname{tr}(M_m^\dagger M_m|\psi_i\rangle\langle\psi_i|) \\ &= \operatorname{tr}(M_m^\dagger M_m \rho) \end{aligned} \tag{2.11}$$

The quantum state after the measuring $|\psi_i\rangle$ is

$$|\psi_i^m\rangle = \frac{M_m|\psi_i^m\rangle}{\sqrt{\langle\psi_i^m|M_m^\dagger M_m|\psi_i^m\rangle}} \tag{2.12}$$

The corresponding density matrix is

$$\rho_m = \sum_i p(i|m)|\psi_i^m\rangle\langle\psi_i^m| = \sum_i p(i|m)\frac{M_m|\psi_i\rangle\langle\psi_i|M_m^\dagger}{\sqrt{\langle\psi_i^m|M_m^\dagger M_m|\psi_i^m\rangle}} \tag{2.13}$$

$$\begin{aligned} p(i|m) &= \frac{p(m,i)}{p(m)} = \frac{p(m|i)p_i}{p(m)} \\ &= \frac{\mathrm{tr}(M_m^\dagger M_m\rho)p_i}{\mathrm{tr}(M_m^\dagger M_m\rho)} \\ &= p_i \end{aligned} \tag{2.14}$$

Therefore, the state can also be described by the equation

$$\begin{aligned} \rho_m &= \sum_i p_i \frac{M_m|\psi_i\rangle\langle\psi_i|M_m^\dagger}{\mathrm{tr}(M_m^\dagger M_m\rho)} \\ &= \frac{M_m\rho M_m^\dagger}{\mathrm{tr}(M_m^\dagger M_m\rho)} \end{aligned} \tag{2.15}$$

2.1.3 Fidelity

Fidelity is one of the distance between two quantum state. the fidelity of quantum state ρ and σ is

$$F(\rho,\sigma) = \mathrm{tr}\sqrt{\rho^{\frac{1}{2}}\sigma\rho^{\frac{1}{2}}} \tag{2.16}$$

For instance, if they commute and diagonal in the same basis like the following,

$$\rho = \sum_i r_i|i\rangle\langle i|, \sigma = \sum_i s_i|i\rangle\langle i| \tag{2.17}$$

The fidelity between these two states would be

$$\begin{aligned} F(\rho,\sigma) &= \mathrm{tr}\sqrt{\sum_i r_i s_i|i\rangle\langle i|} \\ &= \mathrm{tr}(\sum_i \sqrt{r_i s_i}|i\rangle\langle i|) \\ &= \sum_i \sqrt{r_i s_i} \end{aligned} \tag{2.18}$$

The fidelity between a pure state $|\psi\rangle$ and a mixed state ρ is

$$\begin{aligned} F(\psi,\rho) &= \mathrm{tr}\sqrt{\langle\psi|\rho|\psi\rangle|\psi\rangle\langle\psi|} \\ &= \sqrt{\langle\psi|\rho|\psi\rangle} \end{aligned} \tag{2.19}$$

2.2 Quantum Operations

2.2.1 I Gate

I gate is equal to the 2 × 2 identity matrix, which is

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{2.20}$$

.

For example,

$$I|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle \tag{2.21}$$

$$I|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle \tag{2.22}$$

.

2.2.2 X Gate

X gate

X gate flips the logical value of a qubit.

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{2.23}$$

.

For example,

$$X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle \tag{2.24}$$

$$X|1\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle \tag{2.25}$$

.

2.2.3 Y Gate

Y gate flips the logical value of a qubit and add an imaginary number.

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \tag{2.26}$$

.

For example,

$$Y|0\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix} = i|1\rangle \tag{2.27}$$

$$Y|1\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -i \\ 0 \end{bmatrix} = -i|0\rangle \tag{2.28}$$

.

2.2.4 Z Gate

Z gate flips the phase of $|1\rangle$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{2.29}$$

.

For example,

$$Z|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle \tag{2.30}$$

$$Z|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} = -|1\rangle \tag{2.31}$$

.

2.2.5 H Gate

H gate creates superposition.

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{2.32}$$

.

For example,

$$H|0\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \tag{2.33}$$

$$H|1\rangle = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \tag{2.34}$$

.

2.2.6 Rotation Gate

2.2.7 General One Qubit Gate

2.2.8 Controlled-NOT Gate

A CNOT gate involves two qubits, one is called **controlled qubit** and the other is called **target qubit**. If the controlled qubit is 1, the bit value of the target qubit is flipped.

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{2.35}$$

.

For example,

$$CNOT_{0,1}|10\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = |11\rangle \tag{2.36}$$

$$CNOT_{0,1}|11\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = |10\rangle \tag{2.37}$$

.

2.2.9 Measurement

Quantum measurement can be described by using a group of measurement operators $\{M_m\}$ (m is the measurement result that is expected to get). If the quantum state before measurement is $|\psi\rangle$, the measurement probability of value m is

$$p(m) = \langle\psi|M_m^\dagger M_m|\psi\rangle$$

The quantum state after the measurement is

$$\frac{M_m|\psi\rangle}{\sqrt{\langle\psi|M_m^\dagger M_m|\psi\rangle}}$$

The measurement operators satisfy the completeness equation

$$\sum_m M_m^\dagger M_m = I$$

Also, the sum of the measurement probability of each possible measurement outcome is equal to one.

$$\sum_m p(m) = \langle\psi|\sum_m M_m^\dagger M_m|\psi\rangle = 1$$

2.3 Quantum Circuit

Here is the example of a quantum circuit.

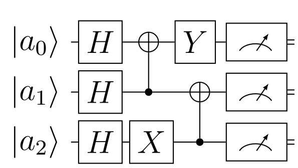

Figure 2.2: A example of quantum circuit

Each horizontal line represents each qubit and the square boxes that contain alphabets mean single quantum gates. The sign which involves a vertical line means a CNOT gate, and the box on the most right side indicates measurement.

2.4 Quantum Entanglement

Quantum entanglement is a special type of quantum state that cannot be described in the form of tensor product of the state of each particle.

2.4.1 Bell Pair

The entangled states between two qubits are called bell pairs, and each of four states has a special notation.

$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} \tag{2.38}$$

$$|\Phi^-\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}} \tag{2.39}$$

$$|\Psi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}} \tag{2.40}$$

$$|\Psi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}} \tag{2.41}$$

.

2.4.2 Multipartite Entanglement

There are cases that more than two qubits are entangled and that state is called Greenberger–Horne–Zeilinger state or GHZ state.

Here is the braket notation of the GHZ state that involves three qubits.

$$|GHZ\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}} \tag{2.42}$$

.

In the general case, the braket notation of the GHZ state of N qubits is the following.

$$|GHZ\rangle = \frac{|0\rangle^{\otimes N} + |1\rangle^{\otimes N}}{\sqrt{2}} \tag{2.43}$$

.

2.4.3 Bell State Measurement

Bell state measurement is a special type of quantum measurement that determines which bell pair the given two qubit entangled state is.

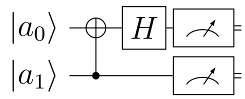

Figure 2.3: Quantum circuit for bell state measurement

Measurement results	Bell state
00	$\|\Phi^+\rangle$
01	$\|\Phi^-\rangle$
10	$\|\Psi^+\rangle$
11	$\|\Psi^-\rangle$

Table 2.1: A table of correspondence between measurement result and Bell pair

2.4.4 Quantum Teleportation

Unlike classical communication, quantum states cannot be just copied and transmit to other nodes due to the no-cloning theorem, which forbids duplication of any quantum state. However, a method called quantum teleportation was proposed, which overcomes the restriction and allows sender to transmit single qubit state to a distant location.

This method requires both the single qubit state and a new Bell pair, and also the sender have to prepare two qubits and the receiver have to prepare one qubit. After applying a CNOT gate and an H gate in the figure above, the sender have to measure both qubits and send those measurement results over the classical network. After the receiver get those measurement results and apply some quantum gates if the measurement results of corresponding qubits on the sender's side are 1, in order to correct on the quantum state on the receiver's side.

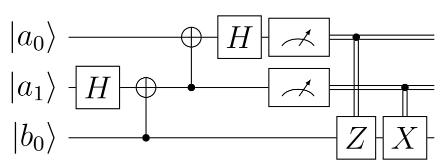

Figure 2.4: Quantum circuit for quantum teleportation

2.4.5 Entanglement Swapping

Entanglement swapping is the method to extend quantum entanglement by performing joint measurement on several quantum entanglement. For example, assume Alice has a single qubit, Bob has two qubits, and Charlie has one qubit. Then, there are Bell pairs between Alice's qubit and Bob's first qubit, and Bob's second qubit and Charlie's qubit, respectively. If Bob performs Bell state measurement on both of his qubits, Alice's qubit and Charlie's qubit are eventually entangled, even though they have not interacted with each other. This can be also seen as the teleporatation of a Bell pair by sending one of its particles. Here is the figure of quantum circuit to perform entanglement swapping.

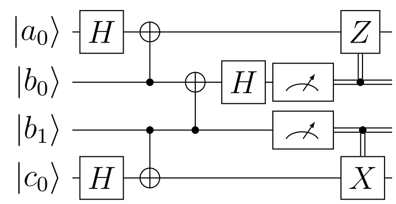

Figure 2.5: Quantum circuit for entanglement swapping

2.4.6 Entanglement Purification

Entanglement purification is a scheme to generate a set of quantum entanglements with higher fidelities from a larger set of imperfect quantum entanglements, local quantum operations, and classical communications. This procedure is also called entanglement distillation, or quantum concatenation. This section presents an example of entanglement purification that generates a single bell pair with higher fidelity from two of those with less fidelity.

Assume Alice and Bob are supposed to share $|\Phi^+\rangle$, which is one of the Bell pairs. However, the state would be converted to the following mixed state due to the noisy nature of a quantum channel.

$$\rho_{AB} = P_{\Phi^+}|\Phi^+\rangle\langle\Phi^+| + P_{\Phi^-}|\Phi^-\rangle\langle\Phi^-| + P_{\Psi^+}|\Psi^+\rangle\langle\Psi^+| + P_{\Psi^-}|\Psi^-\rangle\langle\Psi^-|$$

$$\sum_{s\in\{\Phi^+,\Phi^-,\Psi^+,\Psi^-\}} P_s = 1$$

Any mixed state can be converted to Werner state by applying Pauli operations and $\frac{\pi}{2}$ operations, so Alice and Bob can obtain the following state.

$$\rho'_{AB} = F|\Phi^+\rangle\langle\Phi^+| + \frac{1-F}{3}(|\Phi^-\rangle\langle\Phi^-| + |\Psi^+\rangle\langle\Psi^+| + |\Psi^-\rangle\langle\Psi^-|)$$

Two noisy bell pairs are required for entanglement purification. One of the bell pair $\rho'_{a_1b_1}$ is called source bell pair, which may be purified, and the other one $\rho'_{a_2b_2}$ is called target bell pair, which is going to be measured. Then, Alice and Bob perform CNOT operations between a_1 and a_2, and b_1 and b_2, respectively. After that, they measure a_2

and b_2 respectively, which is the qubit on the target bell pair on their side and exchange the measurement results. If their measurement results match, the purification is successful, while they have to discard the source bell pair and try again if those results do not match.

Here is the quantum state after measuring the target bell pair.

$$\rho'_{ab} = \frac{1}{N}\left[F^2+\frac{1}{9}\left(1-F\right)^2\right]|\Phi^+\rangle\langle\Phi^+|+\frac{2F(1-F)}{3N}|\Phi^-\rangle\langle\Phi^-|+\frac{2(1-F)^2}{9N}(|\Psi^+\rangle\langle\Psi^+|+|\Psi^-\rangle\langle\Psi^-|)$$

$$(N = F^2 + \frac{2F(1-F)}{3} + \frac{2(1-F)^2}{9})$$

The purification becomes successful if F $> \frac{1}{2}$. The readers can refer to more detailed calculation in the AppendixA.1

2.5 Quantum Networking

This section explains the important concepts of quantum networking.

2.5.1 Quantum Node

Quantum nodes are the nodes on a quantum network, which can be one of the following two categories.

1. End node
   (A quantum node that utilizes end-to-end bell pairs to perform a specific task)

2. Quantum repeater
   (An intermediate quantum node that performs entanglement swapping to extend physical bell pairs)

The detail of quantum repeater is discussed in the section 2.5.3

2.5.2 Quantum Link

Quantum link is a physical bell pair that is generated between two neighboring quantum nodes. This subsection introduces three link architectures discussed in.

SenderReceiver

MeetIntheMiddle

MidpointSource

2.5.3 Quantum Repeater

2.5.4 Major Applications of Quantum Networking

Chapter 3

Related Works

3.1 RuleSet-Based Quantum Network

3.2 Quantum Recursive Network Architecture

Chapter 4

Problem Definition

4.1 Problem Definition

Chapter 5

Proposal: Link Management For Quantum Network

5.1 Overview

5.2 Requirements

5.3 Link Allocation Policy

5.4 Link Allocation Policy Negotiation

5.5 Link Allocation Timing Negotiation

5.6 Resource Allocation

5.7 Link Management Finite State Machines

5.8 Link Management Finite State Events

5.9 Type of Messages

5.10 Relationship With Connection Setup

5.11 Relationship With Connection Teardown

Chapter 6

Simulation

6.1 Overview of QuISP (Quantum Internet Simulation Package)

6.2 Major Components

6.3 BSA Controller

6.4 EPPS Controller

6.5 Bell Pair Store

6.6 RuleEngine

6.6.1 Link Allocation Policy Negotiation

6.6.2 Link Allocation Timing Negotiation

6.6.3 Resource Allocation

6.6.4 Connection Teardown

6.7 Connection Manager

6.7.1 LinkAllocationUpdateNotifier

6.7.2 Connection Teardown

Chapter 7

Evaluation

7.1 Experiment

7.1.1 Two Node Network With an MM Link

7.1.2 Two Node Network With an MIM Link

7.1.3 Two Node Network With an MSM Link

7.1.4 Two Node Network With an MIM Link (Without Timing Negotiation)

Chapter 8

Conclusion

8.1 Conclusion

8.2 Future Works

Appendix A

Appendix

A.1 The Entire Calculation To Derive The Bell Pair After Purification

Before applying a CNOT gate	After a applying CNOT gate
$\|\Phi^+\rangle\|\Phi^+\rangle$	$\|\Phi^+\rangle\|\Phi^+\rangle$
$\|\Phi^+\rangle\|\Phi^-\rangle$	$\|\Phi^-\rangle\|\Phi^-\rangle$
$\|\Phi^+\rangle\|\Psi^+\rangle$	$\|\Phi^+\rangle\|\Psi^+\rangle$
$\|\Phi^+\rangle\|\Psi^-\rangle$	$\|\Phi^-\rangle\|\Psi^-\rangle$
$\|\Phi^-\rangle\|\Phi^+\rangle$	$\|\Phi^-\rangle\|\Phi^+\rangle$
$\|\Phi^-\rangle\|\Psi^-\rangle$	$\|\Phi^+\rangle\|\Phi^-\rangle$
$\|\Phi^-\rangle\|\Psi^+\rangle$	$\|\Phi^-\rangle\|\Psi^+\rangle$
$\|\Phi^-\rangle\|\Psi^-\rangle$	$\|\Phi^+\rangle\|\Psi^-\rangle$
$\|\Psi^+\rangle\|\Phi^+\rangle$	$\|\Psi^+\rangle\|\Psi^+\rangle$
$\|\Psi^+\rangle\|\Phi^-\rangle$	$\|\Psi^+\rangle\|\Psi^-\rangle$
$\|\Psi^+\rangle\|\Psi^+\rangle$	$\|\Psi^+\rangle\|\Phi^+\rangle$
$\|\Psi^+\rangle\|\Psi^-\rangle$	$\|\Psi^+\rangle\|\Phi^-\rangle$
$\|\Psi^-\rangle\|\Phi^+\rangle$	$\|\Psi^-\rangle\|\Psi^+\rangle$
$\|\Psi^-\rangle\|\Phi^-\rangle$	$\|\Psi^-\rangle\|\Psi^-\rangle$
$\|\Psi^-\rangle\|\Psi^+\rangle$	$\|\Psi^-\rangle\|\Phi^+\rangle$
$\|\Psi^-\rangle\|\Psi^-\rangle$	$\|\Psi^-\rangle\|\Phi^-\rangle$

Table A.1: A table of correspondence between Bell pairs before and after applying a CNOT gate

Two noisy bell pairs are required for entanglement purification, so assume the quantum state of the entire system can be described as follows.

$$\begin{aligned}\rho'_{a_1b_1} \otimes \rho'_{a_2b_2} &= F^2|\Phi^+\rangle|\Phi^+\rangle\langle\Phi^+|\langle\Phi^+| \\ &+ \frac{F(1-F)}{3}(|\Phi^+\rangle|\Phi^-\rangle\langle\Phi^-|\langle\Phi^+| + |\Phi^+\rangle|\Psi^+\rangle\langle\Psi^+|\langle\Phi^+| + |\Phi^+\rangle|\Psi^-\rangle\langle\Psi^-|\langle\Phi^+| \\ &\quad + |\Phi^-\rangle|\Phi^+\rangle\langle\Phi^+|\langle\Phi^-| + |\Psi^+\rangle|\Phi^+\rangle\langle\Phi^+|\langle\Psi^+| + |\Psi^-\rangle|\Phi^+\rangle\langle\Phi^+|\langle\Psi^-|) \\ &+ \frac{(1-F)^2}{9}(|\Phi^-\rangle|\Phi^-\rangle\langle\Phi^-|\langle\Phi^-| + |\Phi^-\rangle|\Psi^+\rangle\langle\Psi^+|\langle\Phi^-| + |\Phi^-\rangle|\Psi^-\rangle\langle\Psi^-|\langle\Phi^-| \\ &\quad + |\Psi^+\rangle|\Phi^-\rangle\langle\Phi^-|\langle\Psi^+| + |\Psi^+\rangle|\Psi^+\rangle\langle\Psi^+|\langle\Psi^+| + |\Psi^+\rangle|\Psi^-\rangle\langle\Psi^-|\langle\Psi^+| \\ &\quad + |\Psi^-\rangle|\Phi^-\rangle\langle\Psi^-|\langle\Phi^-| + |\Psi^-\rangle|\Psi^+\rangle\langle\Psi^+|\langle\Psi^-| + |\Psi^-\rangle|\Psi^-\rangle\langle\Psi^-|\langle\Psi^-|)\end{aligned}$$

One of the bell pair $\rho'_{a_1b_1}$ is called source bell pair, which may be purified, and the other one $\rho'_{a_2b_2}$ is called target bell pair, which is going to be measured. Then, Alice and Bob perform CNOT operations between a_1 and a_2, and b_1 and b_2, respectively. The entire quantum state on this point would be as follows.

$$\begin{aligned}\rho'_{a_1b_1} \otimes \rho'_{a_2b_2} &= F^2|\Phi^+\rangle|\Phi^+\rangle\langle\Phi^+|\langle\Phi^+| \\ &+ \frac{F(1-F)}{3}(|\Phi^-\rangle|\Phi^-\rangle\langle\Phi^-|\langle\Phi^-| + |\Phi^+\rangle|\Psi^+\rangle\langle\Psi^+|\langle\Phi^+| + |\Phi^-\rangle|\Psi^-\rangle\langle\Psi^-|\langle\Phi^-| \\ &\quad + |\Phi^-\rangle|\Phi^+\rangle\langle\Phi^+|\langle\Phi^-| + |\Psi^+\rangle|\Psi^+\rangle\langle\Psi^+|\langle\Psi^+| + |\Psi^-\rangle|\Psi^+\rangle\langle\Psi^+|\langle\Psi^-|) \\ &+ \frac{(1-F)^2}{9}(|\Phi^+\rangle|\Phi^-\rangle\langle\Phi^-|\langle\Phi^+| + |\Phi^-\rangle|\Psi^+\rangle\langle\Psi^+|\langle\Phi^-| + |\Phi^+\rangle|\Psi^-\rangle\langle\Psi^-|\langle\Phi^+| \\ &\quad + |\Psi^+\rangle|\Psi^-\rangle\langle\Psi^-|\langle\Psi^+| + |\Psi^-\rangle|\Phi^+\rangle\langle\Phi^+|\langle\Psi^+| + |\Psi^+\rangle|\Psi^-\rangle\langle\Psi^-|\langle\Psi^+| \\ &\quad + |\Psi^-\rangle|\Psi^-\rangle\langle\Psi^-|\langle\Psi^-| + |\Psi^-\rangle|\Phi^+\rangle\langle\Phi^+|\langle\Psi^-| + |\Psi^-\rangle|\Phi^-\rangle\langle\Phi^-|\langle\Psi^-|)\end{aligned}$$

Because getting the quantum state after measuring the last two qubits is equivalent to taking the partial trace of the target bell pair, here is the description of the source bell pair after measurement.

$$\rho'_{ab} = \frac{1}{N}\left[F^2+\frac{1}{9}\left(1-F\right)^2\right]|\Phi^+\rangle\langle\Phi^+|+\frac{2F(1-F)}{3N}|\Phi^-\rangle\langle\Phi^-|+\frac{2(1-F)^2}{9N}(|\Psi^+\rangle\langle\Psi^+|+|\Psi^-\rangle\langle\Psi^-|)$$
$$(N = F^2 + \frac{2F(1-F)}{3} + \frac{2(1-F)^2}{9})$$

The purification becomes successful if F $> \frac{1}{2}$

Acknowledgement

Reference

[1] Satoshi Nakamoto. Bitcoin: A peer-to-peer electronic cash system. http://www.cryptovest.co.uk/resources/Bitcoin%20paper%20Original.pdf, 2008.