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Assessing bankruptcy reform in a model with temptation and equilibrium default☆

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ABSTRACT

A life-cycle model with equilibrium default in which agents with and without temptation coexist is constructed to evaluate the 2005 bankruptcy law reform. The calibrated model indicates that the 2005 reform reduces bankruptcies, as seen in the data, and improves welfare, as lower default premia allows better consumption smoothing. A counterfactual reform of changing income garnishment rate is also investigated. Interesting contrasting welfare effects between two types of agents emerge. Agents with temptation prefer a lower garnishment rate as tighter borrowing constraint prevents them from over-borrowing, while those without prefer better consumption smoothing enabled by a higher garnishment rate.

1. Introduction

Preferences that exhibit present bias have become widely used in economics. Based on the success of the models with present bias in replicating various dimensions of borrowing behavior, White (2007) argues that present bias is an important feature in constructing a model of bankruptcies for policy evaluation. In this paper, I construct a novel model in which agents with and without temptation coexist, agents optimally choose whether to default or not, and equilibrium default premium for consumer credit reflects default risk. I use the model to study macroeconomic and welfare implications of bankruptcy law reforms, in particular, the Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA) enacted in 2005 to make bankruptcy law more creditor friendly and prevent borrowers from abusing the lax law and defaulting easily.

This is the first paper that extends the quantitative macroeconomic model with equilibrium default (Livshits et al., 2007 and Chatterjee et al., 2007) by introducing preferences featuring temptation and self-control (Gul and Pesendorfer, 2001, 2004a). Moreover, unlike papers studying macroeconomic implications of the model in which agents are subject to present bias, agents with and without temptation coexist. Using a calibrated model, I can separately analyze the implications of the BAPCPA and other bankruptcy law reforms, with a focus on heterogeneous effects to agents with and without temptation.

The calibrated model implies that the 2005 bankruptcy reform achieves what it is intended for — a reduction in the number of

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bankruptcy filings. The model also indicates expansion of borrowing because agents default less frequently under the reform, and thus a stronger commitment to repay yields lower default premia. In the U.S. after 2005, although it is difficult to separate the effects of the bankruptcy reform and those of the Great Recession, the number of defaults declined, and the charge-off rate seems to have declined, both of which are consistent with the model's predictions. The model predicts an expansion of credit due to the bankruptcy reform, while the U.S. debt-to-income ratio declined after a brief increase until 2007. The latter is most likely the result of the credit tightening after the Great Recession. Regarding welfare, the model implies that the overall effect on social welfare is positive, at 0.29 % of consumption growth. Agents suffer from higher bankruptcy costs and bankruptcy restriction imposed by the 2005 reform. Moreover, agents with temptation suffer from over-borrowing as a result of cheaper credit. However, the welfare gain associated with better consumption smoothing, thanks to lower default premia, dominates the welfare loss. Both agents with and without temptation gain from the reform, although temptation agents gain more as they tend to borrow and default more often.

Other counterfactual bankruptcy policy reforms that also discourage defaulting are also investigated. I find that usury law, which imposes a ceiling on loan interest rates, also results in fewer bankruptcies but yields a small but negative welfare effect. With a binding interest rate ceiling, the market of unsecured loans with high default risk disappears, which implies less default. However, agents who would benefit from taking such loans suffer from the disappearance. While the welfare effects to agents with and without temptation are similar in both the bankruptcy reform in 2005 and the usury law, I find an interesting contrast regarding the welfare effects of changing the income garnishment rate between the two groups of agents. Agents without temptation prefer a higher income garnishment rate because it implies lower loan interest rates and thus better consumption smoothing through cheaper borrowing. On the other hand, agents with temptation prefer a lower garnishment rate because it implies weaker punishment of default and a more strict borrowing limit, which helps them not to over-borrow. These contrasting welfare effects imply that, although the number of defaults declines at the optimal income garnishment rate of zero, the social welfare is non-monotonic and indicate that policy recommendations might be more subtle if we explicitly consider the possibilities that individuals with varying degrees of self-control against temptation coexist.

There is a long history of studies on preferences with present bias, but application to macroeconomics is a recent phenomenon. Building on pioneer studies of Strotz (1956) and Pollak (1968), Laibson (1996, 1997) introduces the hyperbolic-discounting preferences into standard macroeconomic models to investigate the role of present bias. Furthermore, Laibson et al. (2003) show that the hyperbolic-discounting model can explain why the majority of households with credit cards pay interest on the cards even if they have assets as well. On the other hand, Barro (1999) finds the observational equivalence between the neoclassical growth model with hyperbolic-discounting preferences and log utility and the same model with the standard exponential-discounting preferences. Krusell et al. (2010) investigate the optimal long-run capital income taxation in a neoclassical growth model with Gul-Pesendorfer preferences and find that it is negative, to incentivize agents with temptation to save more. İmrohoroğlu et al. (2003) find that unfunded Social Security could be welfare improving in an overlapping-generations model with hyperbolic discounting, by mitigating undersaving. By the same logic, compulsory savings floors can be welfare improving as in Malin (2008). In Nakajima (2012), a relaxed borrowing constraint could imply lower welfare if agents with temptation over-borrow. Akerlof (1991) and O'Donoghue and Rabin (1999, 2001) study cases in which sophisticated agents, who are aware of the time-inconsistent nature of their preferences, and naive agents, who are not, behave differently.

There is a recent development in the literature on the quantitative analysis of default. Athreya (2002) and Chatterjee et al. (2007) study the effects of a means-testing requirement for bankruptcy. Livshits et al. (2007) compare the economy with "fresh start" bankruptcy, which provides a better consumption smoothing across states, and the economy without bankruptcy, which provides a better consumption smoothing over the life cycle. Livshits et al. (2010) explore the causes of the rise in bankruptcies and debt since the 1980s. Narajabad (2012) and Athreya et al. (2012) study the role of the improved information technology used by lenders in explaining the rise in bankruptcies. Li and Sarte (2006) construct a model with bankruptcy under both Chapters 7 and 13. In a recent paper, Benjamin and Mateos-Planas (2013) explicitly analyze the choice between informal default (to stop repaying debt) and formal default (to file for bankruptcy). Mitman (2016) studies the interaction between bankruptcy of unsecured credit and foreclosure of secured credit. Li and White (2009) empirically show that there are interesting interactions between the two. Athreya et al. (2015) investigate the interaction between the 2005 bankruptcy reform and the Great Recession. Compared with the existing literature, the model developed in this paper does not include imperfect information, general equilibrium, multiple assets, choice of default options, or informal default, but none of the existing work investigates the implications of present bias to debt and default.

The remainder of the paper is organized as follows. Section 2 gives an overview of the environment surrounding consumer bankruptcy in the U.S. before bankruptcy reform. Section 3 sets up the model. Section 4 describes calibration. Section 5 comments on the solution method. Section 6 presents the main results, studying various policy reforms that affect borrowing and bankruptcy. Section 7 conducts sensitivity analysis. Section 8 concludes. Appendix A contains detailed information on the data on U.S. credit and default. Appendix B provides more details about calibration, while Appendix C describes the computational algorithm. Appendix D describes the calibration of the alternative models used for sensitivity analysis.

2. Consumer bankruptcy in the U.S.

This section provides an overview of consumer bankruptcy in the U.S. mainly before the BAPCPA was enacted in 2005. When a borrower of unsecured debt fails to repay his debt on schedule, creditors take various measures, such as garnishing labor income, to recover the unrepaid amount.[1] When the borrower files for bankruptcy, these attempts to recover debt are suspended. There are two major types of consumer bankruptcy: Chapter 7 and Chapter 13. Chapter 7, which is also called liquidation, allows debtors to clean up the debt after paying back a part of the existing debt using assets that are non-exempt. A debtor filing for Chapter 7 bankruptcy obtains a "fresh start" in the sense that once the Chapter 7 bankruptcy is in place, there is no future obligation to pay back the written-off debt. Chapter 13 realizes partial reduction of debt and rescheduling of its repayment schedule while allowing the bankrupt to keep their assets. Under Chapter 13, before the 2005 bankruptcy reform, the bankrupt could draw their own repayment plan over three to five years and, upon approval by the judge, reschedule the repayment plan according to the proposed schedule. However, under the BAPCPA, the bankrupt no longer draw the repayment plan themselves. See Section 6.2. The assets at the time of bankruptcy filing

[1] If a borrower stops repaying but does not formally file for bankruptcy, it is called informal default. Although Ausubel and Dawsey (2004) show that it is prevalent, for simplicity, I abstract from informal default in this paper.

need not be used for immediate repayment as in Chapter 7, but the bankrupt have to use their future income for repayment. Once a debtor files for Chapter 7 bankruptcy, that debtor cannot file for Chapter 7 bankruptcy again for six years but can file under Chapter 13. Historically, the proportion of Chapter 7 bankruptcies remains stable at about 70 % of total consumer bankruptcies. Moreover, research shows that many who filed for bankruptcy under Chapter 13 ended up also filing for Chapter 7 bankruptcy. Therefore, this paper focuses on Chapter 7 bankruptcy, and the "default" option in the model mimics the Chapter 7 bankruptcy.[2]

Fig. 1 shows data related to bankruptcies and debt in the U.S.[3] Panel (a) shows the percentage of total bankruptcy filings and Chapter 7 bankruptcy filings over the number of households in the U.S. from 1980 to 2014. There are four notable features: First, the proportion of Chapter 7 bankruptcy filings over total number of bankruptcies has remained stable at about 70 %. Second, the number of bankruptcy filings increased dramatically from 1980 to the early 2000s; the number of Chapter 7 bankruptcy filings increased more than fivefold, from 213,983 in 1980 to 1,117,766 in 2004. Third, there was a significant spike of bankruptcies in 2005, followed by a plunge in 2006. This is because of the enactment of the BAPCPA, which is the focus of this paper. The BAPCPA, which made filing for bankruptcy (especially Chapter 7 bankruptcy) more difficult, became effective in fall 2005, and a large number of debtors rushed to file before the new law took effect. The dip in 2006 was a rebound from that rush to file for Chapter 7 bankruptcy. Finally, the number seems to be rising again after the dip in 2006, but because this period coincides with the Great Recession, it is impossible to tell at which level the number of bankruptcy filings stabilizes. Indeed, after the end of the Great Recession, the number of Chapter 7 bankruptcy filings has been declining, approaching the low level observed in 2006.

The main concern behind the BAPCPA was the notion that many people were abusing the bankruptcy law, which was considered debtor-friendly. Naturally, the reform is intended to transform the bankruptcy scheme from a debtor-friendly one, in which the cost of defaulting is low and anyone can file for bankruptcy, to a more creditor-friendly one, in which the cost of defaulting is high and defaulting is available only to low-income borrowers. More details about the BAPCPA will be provided later when I use the models to study the implications of the reform in Section 6.2.

Behind the dramatic increase in the number of bankruptcy filings was the increase in consumer credit, which is shown in Panel (b) of Fig. 1. Panel (b) shows gross consumer credit balance over disposable personal income in the U.S. from 1980 to 2014. The ratio steadily increased from 3 % in 1980 to 9 % in the late 1990s. It remained at that level before declining when the Great Recession started. Panel (c) shows the changes in the charge-off rate, which is the ratio of loss and the balance of credit card loans extended, among all commercial banks. The ratio had been fluctuating around a slightly positive trend since 1985, before shooting up at the onset of the Great Recession. The rate has since come down to the level before the Great Recession. Finally, Panel (d) shows the movement of the average real interest rate on credit card loans from 1995 to 2014.[4] The rate was on a downward trend between 1995 and 2004, which coincided with the general decline in real interest rates, before increasing during the Great Recession. After the recession, the interest rate rose between 2012 and 2014.

3. Model

The key features of the model are overlapping generations, equilibrium default, and preferences featuring temptation and self-control. Livshits et al. (2007) combine overlapping generations and equilibrium default, while Nakajima (2012) introduces preferences with temptation and self-control into an overlapping-generations model. Moreover, agents with and without temptation coexist in the model that I develop in this paper. Naturally, a policy that doesn't distinguish the two types has different effects on the two types of agents. To the best of my knowledge, this is the first time that not only a model with the three features is constructed but also agents with and without temptation coexist. Finally, I introduce the preferences featuring temptation and self-control, developed by Gul and Pesendorfer (2001), in the same way as Krusell et al. (2010) do to the standard growth model. The finite-horizon model with Gul-Pesendorfer preferences that they construct includes the hyperbolic-discounting model as a special case. I use this special case since estimates for the preference parameter that controls the degree of present bias are available for the hyperbolic-discounting model.

3.1. Demographics

Time is discrete. The economy is populated by I overlapping generations of agents. Each generation is populated by a mass of measure-zero agents, who are born at age 1 and live up to age I. Agents who die are replaced by the same measure of newborns, which makes the total measure of agents constant over time. Agents work up to age I_R (workers) and retire after age I_R (retirees). I_R is a parameter, implying that retirement is mandatory.

3.2. Preferences

There are $j = 1, 2, \ldots, J$ types of agents, with different preference parameters. This setup allows the coexistence of agents with and without temptation in the model. The proportion of type-j agents is ϕ_j, with $\sum_{\forall j}\phi_j = 1$. The preference type of an agent does not change. I discuss the preference types when the agent's problem is formulated in Section 3.5. The period utility function takes the form of $u\left(\frac{c_i}{\nu_i}\right)$, where $u(.)$ is assumed to be strictly increasing and strictly concave; ν_i is the size of a household of age-i in equivalent scale units.[5]

3.3. Endowment

Agents are born with zero assets. Working agents receive labor income e each period. The labor income takes the form of $e(i,p) = e_i p$, where e_i captures the average life-cycle profile of labor income, is common across all age-i workers, and is zero for retired agents. p is the persistent shock to labor income and is assumed to follow a first-order Markov process with the transition probability $\pi^p_{i,p,p'}$.[6] After retirement, an agent receives Social Security benefits $b(i,p)$. The amount of benefits does not change with age. An agent also faces shocks to compulsory expenditure $x \geq 0$. $\pi^x_{i,x}$ represents the probability that an age-i agent faces a compulsory expenditure of amount x; x is independently and identically distributed, as in Livshits et al. (2007).

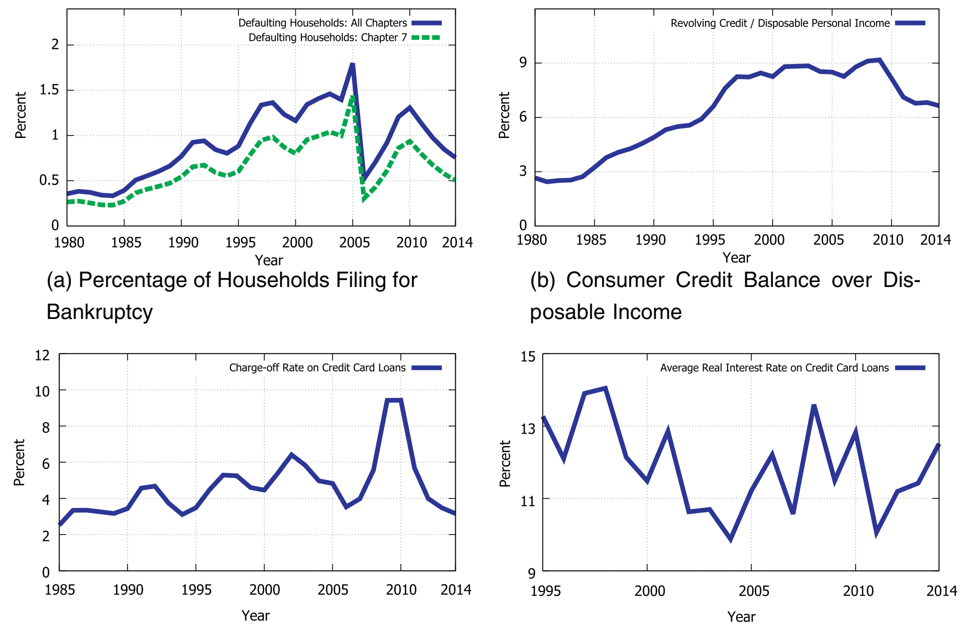
[2] Li and Sarte (2006) study an environment in which debtors can choose between Chapter 7 and Chapter 13 bankruptcy.

[3] Appendix A contains details on the data used to construct Fig. 1.

[4] The nominal average credit card interest rate reported by the Board of Governors of Federal Reserve System (FRB, G.19) is deflated using the Consumer Price Index for all urban consumers (CPI-U).

[5] Changes in household size over the life cycle are found to be important in accounting for the hump-shaped life-cycle profile of consumption (Attanasio and Weber, 1995).

[6] i is an argument of the Markov transition probability to accommodate the case in which the agent is retired and p no longer changes.



(a) Percentage of Households Filing for Bankruptcy

(b) Consumer Credit Balance over Disposable Income

(c) Charge-Off Rate for Credit Card Loans

(d) Average Interest Rate on Credit Card Loans

Fig. 1. Bankruptcy and debt in the U.S.

3.4. Bankruptcy

Agents have an option to default on their debt or bills associated with expenditure shocks. The default option is modeled as in Chatterjee et al. (2007) and Livshits et al. (2007) and resembles in procedure and consequences a Chapter 7 bankruptcy filing — in particular, *before* the reform of the bankruptcy law in 2005, since the main experiment of the paper is to introduce a stylized bankruptcy reform into the model.

Suppose an agent has debt (equivalently, a negative asset position) or receives an expenditure shock from which the asset position becomes negative, and the agent decides to default on the debt. The followings happen:

1. The defaulting agent has to pay for a fixed cost of filing, ξ.
2. The debt and the expenditure shock (think of a hospital bill) are wiped out, and the agent does not have an obligation to pay back the debt or the hospital bill in the future.
3. The agent cannot save during the current period. If the agent tries to save, the savings will be completely garnished by the lender.
4. Proportion η of the current labor income is garnished by the lender. This is also intended to capture the effort of the agent to repay until finally deciding to default within a period (a year). The Social Security benefit is not subject to this garnishment.
5. The credit history of the agent turns from good ($h = 0$) to bad ($h = 1$).
6. While the credit history is bad ($h = 1$), the agent is excluded from the loan market. In other words, the borrowing limit is zero.
7. With probability λ, the agent's bad credit history is wiped out, or h turns from 1 to 0. After that, there is no longer a negative consequence of the past default.

The benefit of defaulting is to get away from debt or an expenditure shock. In this sense, the default option is a means of partial insurance. The costs are (i) monetary cost of filing, (ii) income garnishment in the period of default, (iii) inability to save in the period of default (due to asset garnishment), and (iv) temporary exclusion from the loan market; (i) and (ii) are different since (ii) is received by credit card companies and thus affects (lowers) the interest rate of loans, while (i) does not directly affect the loan interest rate. Agents in debt or with an expenditure shock weigh the benefits and the costs of defaulting, and they default if it is optimal to do so or if there is no other option. The former is called *voluntary default*, and the latter is called *involuntary default*. It is possible that an agent with a bad credit history cannot have a positive consumption when hit by an expenditure shock. Only in this case (involuntary default) is default by agents with a bad credit history allowed. In other words, an agent with a bad credit history ($h = 1$) cannot choose voluntary default. In reality, a record of default remains on the credit record of an agent for 10 years. I use stochastic recovery of the credit status to reduce the size of the state space.[7] For notational convenience, I use $\pi_0^h = \lambda$ and $\pi_1^h = 1-\lambda$, which are the probabilities that a bad credit history is wiped out and not wiped out, respectively. I assume that the stochastic recovery does not start until the period after defaulting, but this timing assumption is not crucial. In Section 7.5, I show that the main results of the paper are robust to changing the timing assumption, in which stochastic recovery starts immediately in the period of defaulting.

3.5. Agent's problem

For a clean notation, I start by defining a recursive problem of an agent with an arbitrary discount factor, d. Using this problem with an arbitrary d, I will define the general problem, which includes preferences featuring temptation and self-control.

The individual state variables are (j, i, h, p, x, a), where j is preference type, i is age, h is credit history, p is the persistent labor income shock, x is the compulsory expenditure shock, and a is asset position.

[7] Thanks to the stochastic recovery, I only need to have $h \in \{0, 1\}$ instead of having 11 different possibilities of h in the case one period is one year.

Given a discount factor d, an agent with a good credit history ($h = 0$) chooses whether or not to default. Formally:

$$V^*(j,i,0,p,x,a;d) = \max\left\{V^*_{\text{non}}(j,i,0,p,x,a;d), V^*_{\text{def}}(j,i,0,p,x,a;d)\right\}, \quad (1)$$

where $V^*_{\text{non}}(j,i,0,p,x,a;d)$ and $V^*_{\text{def}}(j,i,0,p,x,a;d)$ are values conditional on not defaulting and defaulting, respectively. The Bellman equation for an agent with a good credit history ($h = 0$), conditional on not defaulting, is as follows:

$$V^*_{\text{non}}(j,i,0,p,x,a;d) = \begin{cases} -\infty & \text{if } B(j,i,0,p,x,a) = \emptyset \\ \max_{a' \in B(j,i,0,p,x,a)} \left\{u\left(\frac{c}{\nu_i}\right) + d\mathbb{E}V(j,i+1,0,p',x',a')\right\} & \text{if } B(j,i,0,p,x,a) \neq \emptyset \end{cases} \quad (2)$$

subject to:

$$c + a'q(j,i,0,p,x,a') + x = e(i,p) + b(i,p) + a, \quad (3)$$

where $\mathbb{E}$ is an expectation operator, taken with respect to (p', x'). $B(.)$ characterizes the budget set. For an agent with a good credit history ($h = 0$), $B(.)$ is defined as follows:

$$B(j,i,0,p,x,a) = \{a' \in \mathbb{R} | c + a'q(j,i,0,p,x,a') + x = e(i,p) + b(i,p) + a, c \geq 0\}. \quad (4)$$

The first case in Eq. (2) takes care of the case in which the budget set is empty. Since the utility from not defaulting is negative infinity, while the utility from filing is finite, the agent ends up defaulting involuntarily. Notice three things. First, an arbitrary discount factor d is used here. Second, the optimal value characterized by Eq. (1), $V^*(.)$, is different from the future value in the same equation, $V(.)$. $V(.)$ is defined when I describe the problem featuring temptation and self-control. Third, $q(j,i,h,p,x,a')$ denotes the discount price of bonds and depends on the type of agent as well as the amount saved ($a' \geq 0$) or borrowed ($a' < 0$). $q(.)$ depends on the individual type of borrower because I allow credit card companies to adjust the price of loans reflecting perfectly the risk associated with each loan. I will come back to the determination of $q(.)$ in Section 3.6.[8]

Given a discount factor d, the Bellman equation for an agent, conditional on defaulting, is defined below. Notice that this Bellman equation is valid regardless of the current credit status ($\forall h$), because the benefits and the costs of default are the same regardless of the current credit status:

$$V^*_{\text{def}}(j,i,h,p,x,a;d) = u\left(\frac{c}{\nu_i}\right) + d\,\mathbb{E}V(j,i+1,1,p',x',0) \quad (5)$$

$$c = \max\left\{e(i,p)(1-\eta) + b(i,p) - \xi, \bar{c}\right\}. \quad (6)$$

Notice the following five differences from the previous case. First, the existing debt (a) and the expenditure shock (x) are wiped out from the budget constraint (6) as a result of default. Second, on the other hand, the agent has to pay for the default cost ξ, and the fraction η of the current labor income is garnished. Third, the optimal saving level is $a' = 0$, since any assets above 0 would be garnished by assumption, and the defaulting agent cannot borrow. Fourth, there is a consumption floor $\bar{c}$, in the same way as in Hubbard et al. (1995). The consumption floor is used to prevent a (low-income) defaulting agent to consume a negative amount after paying the default cost ξ. Fifth, the credit history of the agent turns bad ($h' = 1$).

Finally, given a discount factor d, the problem of an agent with a bad credit history ($h = 1$) is defined as follows:

$$V^*(j,i,1,p,x,a;d) = \begin{cases} V^*_{\text{def}}(j,i,1,p,x,a;d) & \text{if } B(j,i,1,p,x,a) = \emptyset \\ \max_{a' \in B(j,i,0,p,x,a)} \left\{u\left(\frac{c}{\nu_i}\right) + d \quad \mathbb{E}V(j,i+1,\tilde{h}',p',x',a')\right\} & \text{if } B(j,i,1,p,x,a) \neq \emptyset \end{cases} \quad (7)$$

subject to the budget constraint (3). $\mathbb{E}$ is an expectation operator, taken with respect to $(\tilde{h}', p', x')$.[9] $B(.)$ characterizes the budget set, as follows:

$$B(j,i,1,p,x,a) = \{a' \in \mathbb{R}^+ | c + a'q(j,i,1,p,x,a') + x = e(i,p) + b(i,p) + a, c \geq 0\}. \quad (8)$$

Notice the following three differences from the problem of an agent with a good credit history. First, the agent can default only when the budget set is empty (i.e., involuntary default). In other words, there is no choice with respect to default for an agent with bad credit history. Second, the agent with bad credit history is excluded from the credit market (i.e., $a' \in \mathbb{R}^+$). Third, bad credit history will be wiped out with a probability $\pi^h_0 = \lambda$ and will remain with probability $\pi^h_1 = 1 - \lambda$. This is contained in the expectation operator $\mathbb{E}$.

We are ready to define the problem with temptation and self-control. First, denote the value conditional on a discount factor d, a default decision h', and a saving decision a' as $\tilde{V}(j,i,h,p,x,a,h',a';d)$. Obviously, $V^*(j,i,h,p,x,a;d)$, which is the optimal value conditional on a discount factor d, is $\tilde{V}(j,i,h,p,x,a,h',a';d)$ associated with the optimal default and saving decision. Now, the problem of an agent with preferences featuring temptation and self-control can be defined as follows:

$$V(j,i,h,p,x,a) = \max_{h',a'} \Big\{\tilde{V}(j,i,h,p,x,a,h',a';\delta_j) + \gamma_j\left(\tilde{V}(j,i,h,p,x,a,h',a';\beta_j\delta_j) - V^*(j,i,h,p,x,a;\beta_j\delta_j)\right)\Big\}, \quad (9)$$

where $h' = g_h(j,i,h,p,x,a) \in \{0,1\}$ is the associated optimal default rule, and $a' = g_a(j,i,h,p,x,a)$ is the associated optimal saving rule. The first part in the maximand, $\tilde{V}(.;\delta_j)$, is called *self-control utility*, while the part in the maximand multiplied by γ_j, $\left(\tilde{V}(,;\beta_j\delta_j) - V^*(.;\beta_j\delta_j)\right)$ is called *temptation utility*. To understand Eq. (9), let's assume $\gamma_j = 0$ for now. In this case, the temptation utility drops off from the maximand, and the problem becomes standard: maximizing only the self-control utility using the discount factor δ_j. This situation is when the agent can exert perfect self-control and is not affected by the temptation to consume or borrow more. In other words, when $\gamma_j = 0$, the problem collapses back to the exponential-discounting model with the discount factor δ_j. For this reason, δ_j is called the *self-control discount factor*. Another special case is $\beta_j = 1$. When $\beta_j = 1$, even if the temptation utility is

[8] In the baseline case, I assume that credit card companies can observe all individual state variables and use them to price the debt. However, this might not be the case in reality. In one of the sensitivity exercises, I study the case in which credit card companies can observe a subset of individual state variables. See Section 7.

[9] Credit status in the next period has a tilde ($\tilde{h}'$) to distinguish the future credit history that changes stochastically from the default choice h'.

present ($\gamma_j > 0$), the problem collapses to the standard exponential-discounting model with a sole self-control discount factor δ_j. This is because, when the pair (h', a') is chosen to maximize $\tilde{V}(.; \delta_j)$, the temptation utility is also maximized as well and takes the value of zero.

On the other hand, when $\gamma_j > 0$ and $\beta_j \in [0, 1)$, the agent's optimization problem includes two considerations, corresponding to the two parts in Eq. (9). First, the agent still benefits by maximizing the self-control utility as before. Second, at the same time, the agent suffers from deviating from the optimal decision associated with the discount factor $\beta_j\delta_j$. Remember again, $V^*(.; \beta_j\delta_j)$ is the optimal value associated with the discount factor $\beta_j\delta_j$. When the agent chooses (h', a') that are different from the optimal pair associated with $V^*(.; \beta_j\delta_j)$, the agent suffers a negative temptation utility, which is multiplied by γ_j. In this sense, γ_j represents the *strength of the temptation*, and β_j is called *temptation discount factor*. When γ_j is larger, the agent is more strongly tempted to choose (h', a') that are closer to the optimal pair under the discount factor $\beta_j\delta_j$ and make the utility loss from the temptation utility smaller. In an extreme case in which $\gamma_j \to \infty$, it becomes optimal for an agent to minimize the utility loss from the temptation utility by choosing (h', a') that are optimal under the discount factor $\beta_j\delta_j$. This special case is shown to be equivalent to the hyperbolic-discounting preferences with the *short-term discount factor* β_j and the *long-term discount factor* δ_j, and estimates of β_j are available for the hyperbolic-discounting model. See Krusell et al. (2010) and Nakajima (2012) for a discussion about the equivalence. Notice that when $\gamma_j \to \infty$, Eq. (9) is simplified as follows:

$$V(j, i, h, p, x, a) = \tilde{V}(j, i, h, p, x, a, h', a'; \delta_j), \tag{10}$$

where $h' = g_h(j, i, h, p, x, a)$ and $a' = g_a(j, i, h, p, x, a)$ are the optimal decision rules associated with the value $V^*(j, i, h, p, x, a; \beta_j\delta_j)$, which maximizes the temptation utility. In other words, when an agent completely succumbs to temptation, the agent makes the optimal decision by discounting the future with a discount factor $\beta_j\delta_j$. However, the actual value of the agent is evaluated only with the discount factor δ_j. One way to interpret this is that agents make a choice of (h', a') using the discount factor $\beta_j\delta_j$, affected by temptation (represented by β_j), but what really maximizes agents' utility is to make a choice using the discount factor δ_j. With this interpretation, some form of paternalism could be justified.

3.6. Credit card companies

The only assets available in the model are one-period discount bonds. This is a common assumption in the literature, used by Chatterjee et al. (2007) and Livshits et al. (2007). I also assume that retired agents cannot borrow, following Livshits et al. (2007). The saving interest rate is fixed at r. Since the only financial assets available in the model are discount bonds issued by agents, the bond price of the saving agents in equilibrium is $q(j, i, h, p, x, a' \geq 0) = 1/(1 + r)$. Notice that this is the only bond price for agents with bad credit history because they are excluded from the loan market (i.e., $a' \geq 0$). When an agent borrows, there is a transaction cost of making loans, ι. If there is no default premium, the borrowing interest rate is $r + \iota$ and the price of discount bonds issued by an agent who does not default is $1/(1 + r + \iota)$. However, since agents cannot commit to repaying the debt, a default premium is added to loan interest rates depending on the likelihood of default. The unsecured loans are provided by a competitive credit sector that consists of a large number of credit card companies with free entry. Credit card companies can target agents of one particular type with one particular level of debt. Since the credit sector is competitive, free entry is assumed, and each credit card company can target one specific level of debt, it is impossible in equilibrium to *cross-subsidize*. If a credit card company could offer agents of one type an interest rate implying a negative profit while offering agents of another type an interest rate implying a positive profit, it could make a positive total profit in sum. However, this cannot happen in equilibrium because there is always an incentive for another credit card company to offer a lower interest rate for agents of the second type and steal the profitable customers. In equilibrium, any loans to any type of agents and any level of debt make zero profit.

Suppose that a credit card company makes loans to type-$(j, i, 0, p, x)$ agents who borrow a' each.[10] Remember that the current asset position of the agents, a, does not matter for pricing loans. By making loans to a mass of agents of the same type, the credit card company can exploit the law of large numbers and insure away the idiosyncratic default risks, even if the individual loans are defaultable. In other words, the credit sector provides a partial insurance by pooling risk of default across agents of the same type. Now, assume the credit card company makes loans to measure m agents of the same type. The zero-profit condition associated with the loans made to type-$(j, i, 0, p, x)$ agents whose measure is m and who borrow a' each can be expressed as follows:

$$\begin{aligned} m(-a')\mathbb{E}\mathbb{1}_{g_h(j,i+1,0,p',x',a')=0} + m\mathbb{E}\mathbb{1}_{g_h(j,i+1,0,p',x',a')=1}\eta e(i+1, p')\frac{-a'}{x' - a'} \\ = m(-a'q(j, i, 0, p, x, a'))(1 + r + \iota), \end{aligned} \tag{11}$$

where $\mathbb{1}$ is an indicator function that takes the value of one (zero) if the logical statement attached to it is true (false). $\mathbb{E}$ is an expectation operator and is taken with respect to (p', x'). The two terms on the left-hand side represent the total income from the loans. In particular, if an agent repays the loan ($g_h(.) = 0$), the credit card company receives the amount $-a$. If an agent defaults on its loan, $\eta e(i + 1, p')$ is garnished, but the garnished amount is shared proportionally between the issuer of the bill x' and the credit card company that extended the loan of amount $-a'$. The right-hand side is the total cost of the loans. Specifically, the discount value of a loan $-a'q(.)$ is the principal, and the credit card company has to pay for the interest and the transaction cost $r + \iota$. By solving Eq. (11) for $q(j, i, 0, p, x, a')$, one can obtain the following formula for the equilibrium discount price of loans:

$$q(j, i, 0, p, x, a') = \frac{\mathbb{E}\left\{\mathbb{1}_{g_h(j,i+1,0,p',x',a')=0} + \mathbb{1}_{g_h(j,i+1,0,p',x',a')=1}\frac{\eta e(i+1,p')}{x'-a'}\right\}}{1 + r + \iota}. \tag{12}$$

Finally, I assume there is a maximum limit on the interest rate charged by credit card companies, which is denoted by $\bar{r}$. Since the price of the bond $q(.)$ is used instead of interest rate $r(.)$ for loans, the upper bound of the interest rate $\bar{r}$ is converted into the lower bound of the bond price by $\underline{q} = \frac{1}{1+\bar{r}}$. In the U.S., since the *Marquette* decision in 1978, which basically eliminated the usury law, nationally operating credit card companies are no longer subject to the usury law of the states in which they operate.[11] In other words, currently, there is no effective limit on the interest rate. Therefore, I will set $\bar{r}$ at a level that is virtually non-binding in the baseline calibration and later, in Section 6.3, investigate macroeconomic and welfare implications of introducing a binding interest rate ceiling.

[10] Notice that $h = 0$. I only need to consider the case $h = 0$ because agents with bad credit history ($h = 1$) cannot borrow.

[11] See Supreme Court decision on *Marquette National Bank of Minneapolis v. First of Omaha Service Corp.*

To better understand the pricing of unsecured loans, let's look at some of the special cases. If the default probability is zero, the price of loans will be

$$q(j,i,0,p,x,a') = \frac{1}{1+r+\iota}. \tag{13}$$

If all agents default on the debt in the next period, the price of loans will be

$$q(j,i,0,p,x,a') = \frac{\mathbb{E}\frac{\eta e(i+1,p')}{x'-a'}}{1+r+\iota}. \tag{14}$$

Consider the special case in which there is no garnishment (i.e., $\eta = 0$). If the loan is defaulted with probability one, $q(j,i,0,p,x,a') = 0$. This is because, when $\eta = 0$, credit card companies cannot receive anything from the borrowers. In this case, if $q(j,i,0,p,x,a')$ is monotonically increasing with respect to a', one can define $\underline{a}(j,i,0,p,x)$, which satisfies

$$\underline{a}(j,i,0,p,x) = \max\left\{a' | q(j,i,0,p,x,a') = 0\right\}, \tag{15}$$

where $\underline{a}(j,i,0,p,x)$ is the endogenous borrowing limit for agents of type $(j,i,0,p,x)$. For an agent with a bad credit history, $\underline{a}(j,i,1,p,x) = 0$. By construction, the constraint is less strict than the not-too-tight borrowing constraint of Alvarez and Jermann (2000). This is because the not-too-tight borrowing constraint is associated with no default in equilibrium, while the constraint here allows default in equilibrium. See Chatterjee et al. (2007) for further characterization of the equilibrium loan price function.

Finally, let me make three remarks. First, although the bond price function $q(.)$ takes (j,i,h,p,x,a') as arguments, this is intended for completeness. Actually, $q(.)$ does not depend on x because x is assumed to be i.i.d. In other words, knowing x today does not tell us anything about the default probability of a loan that is extended today and could be defaulted on in the next period. Second, an implicit assumption is that credit card companies can observe and use all the information about individual types. In reality, some information cannot be observe precisely, but this is probably a good approximation of the highly sophisticated credit sector of the current U.S. economy. In Section 7, an alternative model in which credit card companies cannot observe some of the agent types, and thus the price of discount loans does not depend on some of the agent types, is studied. Finally, the assumption that only one-period discount bonds are available is common in the literature, but it is probably less innocuous in the case with temptation preferences. When agents with preferences featuring temptation and self-control can restrict future borrowing, they might want to trade bonds for more than one period ahead. Basically, multi-period bonds could be used as a commitment device against over-borrowing in the future. By assuming that only one-period bonds are traded, such a possibility is assumed away.

3.7. Equilibrium

I define the steady-state recursive equilibrium, in which the type distribution of agents is time invariant, while individual states of agents change over time.[12] Let $\mathbf{M}$ be the space of the individual state. $(j,i,h,p,x,a) \in \mathbf{M}$. Let $\mathbb{M}$ be the Borel σ-*algebra* generated by $\mathbf{M}$ and μ a probability measure defined over $\mathbb{M}$. I will use a probability space $(\mathbf{M},\mathbb{M},\mu)$ to represent a type distribution of agents.

[12] In this sense, this equilibrium concept is also referred to as stationary equilibrium.

Definition 1 (Steady-state recursive equilibrium)**.** A steady–state recursive equilibrium consists of loan pricing function $q(j,i,h,p,x,a')$, value function $V(j,i,h,p,x,a)$, optimal decision rules $g_a(j,i,h,p,x,a)$ and $g_h(j,i,h,p,x,a)$, and the type distribution μ, such that:

1 Given the loan price function, $V(j,i,h,p,x,a)$ is a solution to the agent's optimization problem defined in Section 3.5, and $g_a(j,i,h,p,x,a)$ and $g_h(j,i,h,p,x,a)$ are the associated optimal decision rules.
2 Loan price function $q(j,i,h,p,x,a')$ satisfies the zero-profit conditions for all types. Specifically, the loan price function is characterized by Eq. (12).
3 Measure of agents μ is time invariant and consistent with the demographic transition, the stochastic process of shocks, and the optimal decision rules.

4. Calibration

The baseline model is calibrated to capture salient characteristics of the U.S. economy, especially in terms of debt and default, around 2000. When available, I use the U.S. data during 1999–2004. I choose the period up to 2004 because the main experiment is to introduce a stylized version of the 2005 bankruptcy law reform into the calibrated baseline model and analyze its macroeconomic and welfare implications. Table 1 summarizes the parameter values.

4.1. Demographics

One period is set as one year in the model. Age 1 in the model corresponds to the actual age of 20. I is set at 54, as in

Table 1
Summary of calibration.

Parameter	Value	Remark
I	54	Maximum age (corresponding to 73 years old)
I_R	45	Last working age (corresponding to 64 years old)
σ	2.000	Coefficient of relative risk aversion
$\{\nu_i\}$	Fig. B.1	Household size in family equivalence scale
$\phi_1 = \phi_2$	0.5000	Equal measure for both preference types
γ_1	0.0000	$j = 1$ has no temptation
γ_2	∞	$j = 2$ succumbs to temptation
$\beta = \beta_1 = \beta_2$	0.7000	Temptation discount factor
$\delta = \delta_1 = \delta_2$	0.9121	Self-control discount factor. Calibrated to match $D/Y = 8.6\%$
$\{e_i\}$	Fig. B.2	Average labor income profile. Following Gourinchas and Parker (2002)
ρ_p	0.9900	Persistence of earnings shock. Based on Storesletten et al. (2004)
σ_p^2	0.0277	Variance of earnings shock. Based on Storesletten et al. (2004)
σ_0^2	0.2403	Initial variance of earnings shock. Based on Storesletten et al. (2004)
ψ_e	0.2000	Parameter for social security benefits. From Livshits et al. (2010)
ψ_p	0.3500	Parameter for social security benefits. From Livshits et al. (2010)
π_i^x	See text	Probability of expenditure shocks
x_i	See text	Magnitude of expenditure shocks
λ	0.1000	On average, 10 years of exclusion from loan market upon default
ξ	0.0254	Cost of a bankruptcy filing is \$921
η	0.3358	Garnishment ratio. Calibrated to match number of bankruptcies = 0.94%
r	0.0200	Annual real risk-free interest rate
ι	0.0400	Transaction cost of loans
$\bar{r}$	1.0000	Non-binding ceiling for interest rate in the baseline
$\bar{c}$	0.0307	Consumption floor is \$1111

Livshits et al. (2007), meaning that the maximum actual age is 73. I_R is set at 45, implying that the agents become retired at the actual age of 65.

4.2. Preferences

The utility function takes the following constant relative risk aversion (CRRA) functional form:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}. \tag{16}$$

σ is set at 2.0, which is the commonly used value in macroeconomics, and the baseline choice of Laibson et al. (2007). The household size in equivalent scale units, $\{\nu_i\}$, is constructed using the average household size in the 2006 Current Population Survey (CPS), converted into equivalence scale units following Fernández-Villaverde and Krueger (2007). Fig. B.1 in Appendix B shows the resulting $\{\nu_i\}$.

I assume $J = 2$, i.e., two preference types for agents. $j = 1$ and $j = 2$ represent agents without temptation and those with temptation, respectively. There are equal measures of the two types ($\phi_1 = \phi_2 = 0.5$), since there is no strong prior for the distribution of the preference types.

The rest of the preference parameters can be different for each preference type j. I assume that discount factors β_j and δ_j are the same between the two types and use $\beta = \beta_1 = \beta_2$ and $\delta = \delta_1 = \delta_2$ below. However, the strength of temptation is different. In particular, agents without temptation ($j = 1$) have $\gamma_1 = 0$, by definition, while agents with temptation ($j = 2$) have $\gamma_2 = \infty$. $\gamma_2 = \infty$ implies that the preferences with temptation are equivalent to quasi-hyperbolic-discounting preferences, and thus available estimates for β with quasi-hyperbolic discounting can be utilized. For the baseline calibration, I set $\beta = 0.70$. The temptation discount factor, or the long-term discount factor in the language of hyperbolic discounting, of 0.7 corresponds to the discount rate of 40 %, which is the point estimate obtained by Laibson et al. (2007) with the hyperbolic-discounting model. Angeletos et al. (2001) argue that $\beta = 0.7$ corresponds to the one-year discount factor typically measured in laboratory experiments. By assumption, $\beta = \beta_1 = \beta_2 = 0.7$, but β_1 does not matter since $\gamma_1 = 0$ implies that the temptation discount factor does not affect decision of type-$j = 1$ agents. Finally, for the self-control discount factor, or long-term discount factor in the language of hyperbolic-discounting, δ are calibrated, jointly with another parameter, to match the aggregate debt-to-income ratio, which is 8.6 % during 1999–2004. See Section 4.6. For a sensitivity analysis in Section 7, I use $\beta = 0.6667$, which corresponds to the 50 % annual discount rate.

4.3. Endowment

The average life-cycle earnings profile $\{e_i\}_{i=1}^{I}$ is taken from the estimates of Gourinchas and Parker (2002). Fig. B.2 in Appendix B shows $\{e_i\}_{i=1}^{I}$ used in the model. Since mandatory retirement at the model age is I_R, $e_i = 0$ for $i > I_R$. The three parameters that characterize the stochastic process of earnings shock, $(\rho_p, \sigma_p^2, \sigma_0^2)$ are calibrated to match three statistics of the life-cycle profile of earnings variance, documented by Storesletten et al. (2004). According to them, (i) earnings variance at age 22 is 0.27, (ii) earnings variance at age 60 is 0.89, and (iii) earnings variance increases almost linearly over the life cycle. With $\rho_p = 0.99$, $\sigma_p^2 = 0.0277$, and $\sigma_0^p = 0.2403$, the model replicates the empirical life-cycle profile of earnings variance. Fig. B.3 in Appendix B shows the earnings variance generated by the calibrated model. For the Social Security benefits, I use the same formula as Livshits et al. (2010), which is the sum of $\psi_e = 0.2$ of the average labor income of the economy and $\psi_p = 0.35$ of the persistent component of the individual labor income just before retirement ($i = I_R$).[13]

Distribution of compulsory expenditure shocks is constructed using distribution of out-of-pocket (OOP) expenditure in the Medical Expenditure Panel Study (MEPS), the 2004 wave. In particular, for each age i, I construct the distribution of OOP medical expenditures across all individuals. Then I construct seven bins: the bottom 99.4 %, 99.4 to 99.5 %, 99.5 to 99.6 %, 99.6 to 99.7 %, 99.7 to 99.8 %, 99.8 to 99.9 %, and the top 0.1 %. These percentages are used to construct the probability of each expenditure shock. For each bin, except for the bottom one, I compute mean medical expenditure and assign the mean as the magnitude of the shock. As for the bottom bin, I assume that the compulsory expenditure is zero. Finally, I smooth out the magnitude of expenditure shock using a quadratic function with respect to age. Fig. B.4 in Appendix B shows the life-cycle profile of the magnitude of expenditure shocks for the top 0.1 % and 99.4th to 99.5th percentile.

4.4. Bankruptcy

There are four parameters associated with default: λ, the average length of punishment; ξ, the filing cost of defaulting; η, the amount of labor income garnished during the period of filing; and $\bar{c}$, consumption floor when defaulting. λ is set at 0.1, implying that, on average, defaulters cannot obtain new debt for 10 years after defaulting. This average punishment period corresponds to a 10-year period during which a bankruptcy filing stays on a person's credit record, in accordance with the Fair Credit Reporting Act. According to U.S. Government Accountability Office (2008), the average cost of filing for Chapter 7 bankruptcy was $921 before the BAPCPA was introduced in 2005. $\xi = 0.025$, meaning 2.5 % of the average annual labor income, is obtained by converting $921 into the unit in the model. η is calibrated such that the number of defaults in the model matches the same number in the U.S. economy (0.94 % of households per year during 1999–2004). However, notice that the parameter will be calibrated jointly. I will come back to the calibration of η in Section 4.6. $\bar{c}$ is set at $1111, based on the average monthly benefit per person under the U.S. food stamp program. It is intentionally chosen to be low, just to prevent non-positive consumption for defaulters.

4.5. Credit card companies

The interest rate is set at 2 % per year ($r = 0.02$). This is the average of real interest rates between 1995 and 2004. The real interest rate is constructed as the difference between the market yield on the one-year U.S. Treasury and the inflation rate of personal consumption expenditures (PCE). Neely and Rapach (2008) compute the U.S. average real interest rate between 1989:Q4 and 2007:Q2 to be 1.82 %. The interest rate is calibrated to be 4 % by Livshits et al. (2007). However, as shown by Neely and Rapach (2008), the real interest rate has shifted down significantly since the 1980s. Estimates for the transaction costs of making loans vary. Livshits et al. (2007) use 2.6 %. In a recent paper, Agarwal et al. (2016) compute the average transaction costs to be 4.9 %, using panel data on U.S. credit card accounts.[14] Finally, the average real interest rate of credit card loans during 1999–2004 is 11.3 %, while the charge-off rate during the same period is 5.3 %. Together with the cost of fund (interest rate) of 2.0 %, these imply transaction costs of 4.0 %.

[13] Although the amount of Social Security benefits depends on the average labor income throughout the working life in the U.S., keeping the average labor income is a burden in terms of computation. Using only the labor income shock in the last period helps reducing the computational burden significantly. Besides, because of the high persistence of labor income shocks, it is probably not a bad approximation.

[14] The total transaction costs consist of reward and fraud expenses of 1.4 % and operational costs of 3.5 %.

Therefore, I set $\iota = 0.04$. In Section 7, I investigate implications of $\iota = 0.026$. Davis et al. (2006) report the existence of the wedge between the saving and borrowing rates, and they argue that the wedge is important for their life-cycle model to replicate the observed pattern of equity holding. The upper bound of the lending interest rate is set at 100 % ($\bar{r} = 1.0$) so that it is virtually not binding in the baseline model. I will lower $\bar{r}$ to investigate the effects of the usury law in Section 6.3.

4.6. Simultaneously calibrated parameters

As mentioned, there are two parameters, δ and η, which cannot be pinned down independently from the model. I calibrate the two parameters such that two closely related targets — the aggregate debt-to-income ratio of 8.6 % and the proportion of defaulters each year at 0.94 % — are achieved in the steady-state equilibrium of the model. Notice that, to find such parameter values, it is necessary to run the model many times while trying different combinations of (δ, η). Basically, this is a simulated method of moments with exact identification. In the baseline model with the temptation discount factor of 0.7, δ is calibrated to be 0.9121. This is lower than 0.9588, the point estimate of Laibson et al. (2007). However, Laibson et al. (2007) assume that all agents have hyperbolic-discounting preferences with the same discount factors. The garnishment parameter η is calibrated to be 0.3358 for the baseline model. This value is between the federal limit of the garnishment ratio (25 %) and the calibrated value of Livshits et al. (2007) (35.5 %).[15]

5. Computation

Since the model cannot be solved analytically, numerical methods are employed. I solve the individual agent's problem using backward induction, starting from the last period of life, with discretized state space. Details about the solution algorithm can be found in Appendix C, but one feature of the model is worth pointing out. The equilibrium price of loans, $q(j, i, h, p, x, a')$, is solved simultaneously with the agent's optimization problem. Once the optimal decision rules for age-i agents are obtained, the price of debt for age-$i-1$ agents, $q(j, i-1, h, p, x, a')$, can be computed, using the optimal default policy $g_h(j, i, h, p, x, a)$. $q(j, i-1, h, p, x, a')$ in turn is used to solve the optimization problem of agents of age $i-1$. In short, there is no need to use iteration to find an equilibrium loan price $q(j, i, h, p, x, a')$ as in Chatterjee et al. (2007), which is a model with infinitely-lived agents and an option to default.

6. Results

This section presents the main results. Section 6.1 presents the properties of the calibrated baseline model. Section 6.2 investigates the macroeconomic and welfare implications of the 2005 bankruptcy law reform. Sections 6.3 and 6.4 analyze implications of introducing the usury law and changing the income garnishment ratio, respectively.

6.1. Properties of the baseline model

Before analyzing bankruptcy policy reforms, let us investigate properties of the baseline model. Table 2 presents aggregate statistics of the baseline model (second column), together with the comparable U.S. data (first column). The table also shows the aggregate statistics for agents without and with temptation (third and fourth columns). The first two rows contain calibration targets. The model successfully replicates the aggregate debt-to-income ratio of 8.6 % and the proportion of defaulters of 0.94 %.

The bottom three rows contain statistics that are not targeted. First, the model successfully replicates the proportion in debt. A total of 41.7 % of agents are in debt in the model. This is close to the empirical counterpart of 45.0 %, which is the proportion of households with credit card debt, according to 2001 Survey of Consumer Finances (SCF).[16] However, there is a caveat. As carefully discussed by Livshits et al. (2010), most households in the data have debt and assets simultaneously. I compare the proportion with debt in the model to the proportion with credit card debt because this definition of debt is consistent with the target of the aggregate amount of debt. Alternatively, the debt can be defined as the proportion of households who have more gross debt (including secured debt such as mortgages) than gross assets (including housing assets). Using this definition, the proportion in debt is only 7.3 %. This assumption loses majority of the credit card borrowers, but it is consistent with the model if all the assets are confiscated upon defaulting. In reality, however, as Livshits et al. (2010) argue, many assets are exempt from confiscation when a debtor defaults. Moreover, debtors might be able to spend down assets before having their assets seized.

The charge-off rate in the model (3.8 %) is lower than the empirical counterpart (5.3 %), although the model replicates the observed number of Chapter 7 bankruptcies. There are two main reasons for this discrepancy. First, the model does not have Chapter 13 bankruptcy or informal default. The former accounts for about 30 % of consumer bankruptcies, although the amount defaulted under Chapter 13 bankruptcy might be typically smaller than that under Chapter 7 bankruptcy, and individuals filing for Chapter 13 bankruptcy often end up filing for Chapter 7 bankruptcy eventually. As for the latter, Ausubel and Dawsey (2004) report that 50.7 % of charge-offs occurred without bankruptcy filing in their data set. By abstracting from the two alternative sources of default, the model underestimates the amount of debt defaulted and thus the aggregate charge-off rate. The second reason is that I assume that agents default without borrowing when they default due to an expenditure shock in the model. Therefore, defaults due to an expenditure shock are not priced into the default premium or charge-off rate.[17] Finally, the average credit card interest rate in the model is 10.2 %, which is slightly lower than in the data, which is 11.3 %. The difference is close to the difference in the charge-off rate discussed earlier.

The third and fourth rows of Table 2 report the reasons for defaulting in the model. Overall, 0.22 % of all agents, or 23 % of total defaults, occur after a compulsory expenditure shock, while 0.72 %, or 77 % of total defaults, occur without an expenditure shock, meaning default occurs as a result of a negative income shock. In the data, Chakravarty and Rhee (1999) classify the reasons for bankruptcy filings into five categories using the Panel Study of Income Dynamics (PSID) for the period 1984–1995.[18] According to their calculation, 46.5 % of total filers cite marital disruption, health care, or lawsuits and harassment as the primary reasons for bankruptcy. The remaining 53.5 % of filers cite job loss or credit misuse. The former can be reasonably compared with defaults due to a compulsory expenditure shock, while the latter can be compared with defaults due to a negative income shock. In the model, the proportion of filers due to a compulsory expenditure shock (23 %) is

[15] Livshits et al. (2007) calibrate η by fixing the discount factor and using η to match the debt-to-income ratio.

[16] Laibson et al. (2007) report 67.8 % using the 1995 and 1998 waves of the SCF, but they only look at a subset of households headed by someone with a high school degree but not a college degree.

[17] This is less of a problem if agents who want to default due to an expenditure shock have to borrow at first, at a high fixed roll-over rate. This is the assumption of Livshits et al. (2007). They use 20 % roll-over rate.

[18] Chatterjee et al. (2007) summarize their results in Table 1.

Table 2
Debt and bankruptcies: baseline model vs. data (Percent).

	U.S.	Baseline model		
	1999–2004[a]	All agents	Agents without temptation	Agents with temptation
Total debt over income	8.6	8.6	1.8	15.5
Total bankruptcies	0.94	0.94	0.19	1.70
Due to expenditure shock	–	0.22	0.19	0.25
Due to income shock only	–	0.72	0.00	1.44
Proportion in debt	45.0	41.7	27.1	56.3
Charge-off rate	5.3	3.8	0.4	4.2
Avg loan interest rate	11.3	10.2	6.4	10.6

[a] See Appendix A for the details of the data. Proportion in debt is the proportion with positive credit card debt in the Survey of Consumer Finances, average of 1998, 2001, and 2004 waves.

lower than the empirical counterpart (46.5 %), but it is also reasonable to think that part of expenditures by those filers in the data are not compulsory. Therefore, the fact that the proportion of filings due to a compulsory expenditure shock is lower in the model is not a serious problem.

Moreover, having a relatively small fraction of defaults due to an expenditure shock helps the model generating a higher average credit card interest rate because of the way a default due to an expenditure shock is modeled. In the model, agents can and do file for bankruptcy *without borrowing* because they can and do default on their expenditure shock right away. Therefore, default risks due to expenditure shocks are not directly priced into the credit card interest rate. If the model is calibrated in a way that a larger fraction of defaults occur due to a compulsory expenditure shock, the average default premium would be lower, and higher transaction costs of making loans are needed for the model to replicate a high average credit card interest rate.

One problem of the calibrated baseline model is that the model cannot replicate the total amount of assets or its distribution in the economy. For example, Laibson et al. (2007) report that the average asset-to-income ratio among households headed by individuals in their 50 s is 2.60. The corresponding number in the baseline model is 0.52. Technically, this is because the discount factor is used to match the debt-to-income ratio rather than the aggregate amount of savings. However, there are missing features that would enable the model to match the aggregate savings in the data. For example, permanent differences in earnings (due to differences in educational attainment), heterogeneity in the discount factor (Krusell and Smith, 1998), existence of agents with extremely high productivity (Castañeda et al., 2003), which captures entrepreneurs, and multiple assets, especially housing, would help bring the model closer to the data in this regard.

The last two columns of Table 2 compare the statistics among agents without temptation (third column) and those with temptation (fourth column) in the baseline model. Since both types of agents share the same value for the self-control discount factor (δ), but only the agents with temptation are affected by temptation associated with the discount factor β, it is not surprising that agents with temptation borrow more, and, as a result, default more often. Agents with temptation borrow at a higher interest rate on average because they have a lower discount factor. The numbers in Table 2 are consistent with the intuition: 56.3 % of agents with temptation are in debt, and their average debt is 15.5 % of income, while only 27.1 % of agents without temptation are in debt, and their average debt-to-income ratio is 1.8 %. Every year, 1.70 % of agents with temptation default, while the default rate is 0.19 % among agents without temptation. Interestingly, almost all the defaults among agents without temptation are due to expenditure shocks, while only 15 % of defaults among agents with temptation are due to expenditure shocks. The remaining 85 % default due to a series of bad income shocks. The difference highlights the important distinction between agents with and without temptation. It is considered a puzzle as to why agents without temptation, that is, agents with the standard exponential-discounting preferences, borrow at an interest rate as high as 10 % per year when the discount factor (rate) is reasonably high (low). For agents with temptation, this is not a puzzle since those agents discount the immediate future at a very low discount factor, such as the 0.70 temptation discount factor used in the baseline calibration. Not surprisingly, the charge-off rate is higher for loans to agents with temptation (4.2 %) compared with the charge-off rate for loans to agents without temptation (0.4 %). Naturally, the average credit card interest is higher for agents with temptation (10.6 %) than for those without (6.4 %).

Fig. 2 compares life-cycle profiles of the two types of agents in the model. Panel (a) shows life-cycle profiles of non-financial income (labor income and Social Security benefits) and consumption of agents without and with temptation. Notice that the two types of agents have the same non-financial income. Both types of agents save to smooth out consumption over the life cycle. However, since agents with temptation tend to save less during working years, their consumption declines more sharply after retirement. Panel (b) compares life-cycle profiles of mean asset holdings. As discussed already, agents without temptation save more or borrow less than those with temptation, on average, over the life cycle. Agents with temptation also borrow more at the beginning of their life cycle to consume more while young. Panel (c), which shows life-cycle profiles of the proportion of agents in debt, presents a consistent picture. The proportion is consistently higher among agents with temptation. Finally, not surprisingly, the proportion of defaulting agents is higher for all ages among agents with temptation. In particular, since agents without temptation default almost exclusively due to a compulsory expenditure shock, their default profile tracks the probabilities of large expenditure shocks over the life cycle.

6.2. *Assessing the 2005 bankruptcy reform*

This section investigates the effects of the BAPCPA by introducing its stylized version into the baseline model, which is calibrated to the pre-reform U.S. economy. Section 6.2.1 presents the positive implications of introducing the BAPCPA, while Section 6.2.2 discusses welfare implications. Section 6.2.3 looks at the transition dynamics after the introduction of the bankruptcy law reform. Finally, Section 6.2.4 explores possibility of improving the BAPCPA.

6.2.1. *Positive implications of the BAPCPA*

According to White (2007), the main elements of the BAPCPA, which was enacted in 2005, are the following three:

(1) **Means testing**. If a debtor's household income over the past six months prior to the filing is over the median income of the state in which he or she lives, the borrower cannot file

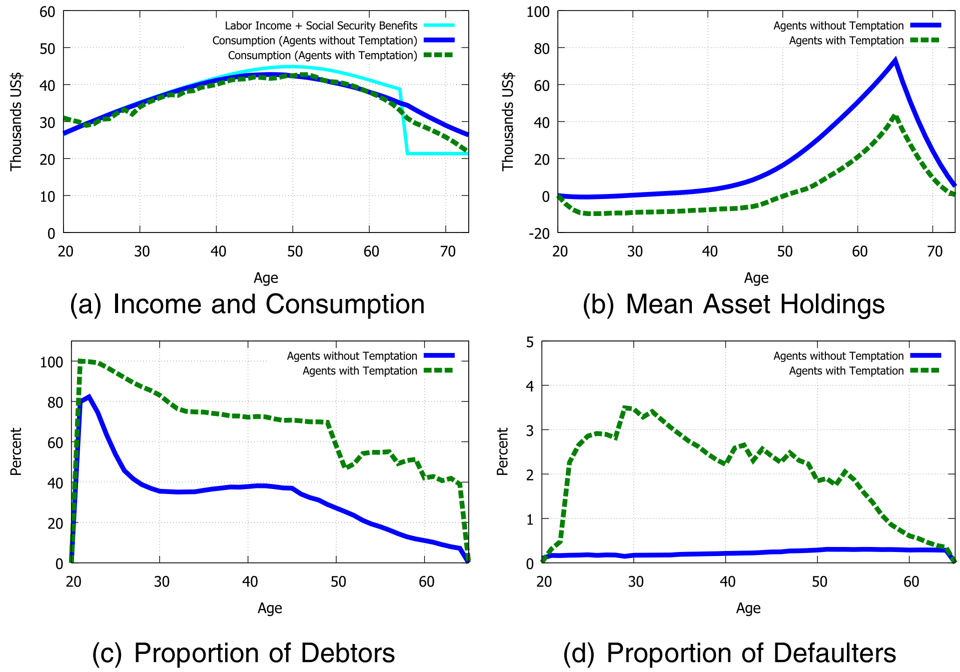


Fig. 2. Life-cycle profiles of the baseline model.

for Chapter 7 bankruptcy and can only file for Chapter 13 bankruptcy.

(2) **Higher cost of filing**. For his debt to be discharged, a debtor is required to take credit counseling, complete a financial management course, and submit detailed financial information that has to be certified by a lawyer. The typical cost of filing for Chapter 7 bankruptcy is raised from $921 to $1477, according to the U.S. Government Accountability Office (2008).[19]

(3) **Repayment schedule**. A debtor filing for a Chapter 13 bankruptcy can no longer propose a repayment schedule. Instead, the law determines how much a filer has to pay back.

Since the Chapter 13 is not modeled, I focus on the effects of (1) means testing and (2) the higher filing cost. As for the means testing, a borrower cannot default if his current income is above the median income of the model economy.[20] The exception is when the budget set is empty. As for the higher cost of defaulting, I raise ξ from $921 (converted into the model unit) to $1477. I also implement experiments in which only one of the two components of the BAPCPA is enacted to evaluate separately the effects of each component. I investigate the aggregate effects of the bankruptcy reform as well as heterogeneous effects for agents with and without temptation.

I also compare the model predictions with the observed changes in the U.S. However, the numbers for the U.S. have to be examined carefully, especially when the U.S. numbers are compared with the numbers in the steady state of the model. Because the year 2005 saw a surge in bankruptcy filings before the BAPCPA became effective and 2006 observed a rebound from the spike in 2005, I will not use the data in 2005 or 2006. On the other hand, since the Great Recession started at the end of 2007, it is difficult to disentangle the effects from the BAPCPA and the cyclical effects from the Great Recession, especially when the economy seems to be still on its recovery from the recession, as seen in Fig. 1. To deal with the issue, I compare the model predictions with three sets of data. First, I use only the data in 2007. Second, I also use the data in 2014, which are the latest available. Finally, I use the average between 2007 and 2014. Admittedly, neither is perfect, but this is the best among the feasible options.

Table 3 compares the U.S. data (top panel) with the effects of the BAPCPA implied by the baseline model (second panel). The third and fourth panels show the changes induced by the BAPCPA among agents without and with temptation, respectively. Let us start by comparing the first two panels. In response to introduction of the BAPCPA, the baseline model implies a large decline in the number of bankruptcies, from 0.94 to 0.42 % (first column). In the sense that the BAPCPA aims to reduce the number of bankruptcy filings, the model implies that the reform is successful. The number of bankruptcy filings also declined in the data. The number of bankruptcies dropped from 0.94 % during 1999–2004 to 0.43 in 2007 before temporarily going up during the Great Recession. In 2014, the number is 0.50. The average between 2007 and 2014 is 0.67. Considering that the number of bankruptcies is still on a downward trajectory in 2014 (see Fig. 1), the model's prediction regarding the number of bankruptcies is consistent with data.

Both of the main components of the BAPCPA contributed to the decline, but the contribution of higher default costs is larger as can be seen in the bottom two lines in the second panel. The proportion of agents filing for bankruptcy increases slightly to 0.92 % if only means testing is introduced, while the number drops to 0.49 % if only the higher default cost is introduced. This is because higher-income agents, whom the means testing mainly affects, borrow and default less from the beginning, while default costs affect more

[19] The total cost of Chapter 7 bankruptcy consists of filing fees of $209 and attorney fees of $712 before the BAPCPA. With the introduction of BAPCPA, the two types of costs went up to $299 and $1078, respectively. In addition, filers usually have to pay for $100 for credit counseling.

[20] In the actual law, what matters is the median income of the state in which the debtor resides. However, since there is no across-state income heterogeneity in the model, I use the overall median income in the model for the means testing.

Table 3
Effects of the 2005 bankruptcy reform[a].

	% default	% in debt	D/Y %	Charge-off %	Avg r %	Welfare %
U.S. Economy						
Avg of 1999–2004	0.94	45.0	8.6	5.3	11.3	–
2007	0.43	46.0	8.8	4.0	10.6	–
2014	0.50	38.2	6.6	3.2	12.5	–
Avg of 2007–2014	0.67	41.2	7.8	5.3	11.7	–
Baseline model: all agents						
Baseline	0.94	41.7	8.6	3.8	10.2	–
BAPCPA	0.42	47.4	13.0	2.4	8.6	+0.29
Means testing	0.92	42.6	16.8	4.5	11.0	−0.01
Default cost	0.49	46.8	10.2	1.7	7.9	+0.25
Baseline model: agents without temptation						
Baseline	0.19	27.1	1.8	0.4	6.4	–
BAPCPA	0.16	27.9	2.3	0.7	6.7	+0.04
Means testing	0.18	27.3	1.9	1.0	7.1	−0.00
Default cost	0.17	27.8	2.2	0.4	6.4	+0.04
Baseline model: agents with temptation						
Baseline	1.70	56.3	15.5	4.2	10.6	–
BAPCPA	0.68	67.0	23.8	2.6	8.8	+0.54
Means testing	1.66	57.9	31.9	4.7	11.2	−0.02
Default cost	0.82	65.8	18.2	1.9	8.1	+0.46

[a] The six columns show the proportion defaulting, proportion in debt, debt-to-income ratio, charge-off rate, average loan interest rate, and welfare gain from policy reform, represented as the rate of permanent increase in consumption. See Appendix A for the details of the data. The data for the proportion in debt are from the proportion of households with positive credit card debt, taken computed from the various waves of the Survey of Consumer Finances. *Average of 1999–2004* is the average of 1998, 2001, and 2004. The data for *2014* is taken from the 2013 wave. *Average of 2007–2014* is the average of 2007, 2010, and 2013 values.

strongly lower-income agents, who tend to borrow and default more frequently.

Since both elements of the BAPCPA lower defaults, which implies lower default premium in equilibrium, the BAPCPA induces an increase in debt in the model (second column). The debt-to-income ratio increases from 8.6 % in the baseline model to 13.0 % under the BAPCPA. Both means testing (16.8 % if implemented alone) and higher costs (10.2 %) raise the balance of debt. In the data, however, although the debt-to-income ratio increased in 2007, from 8.6 % to 8.8 %, which is consistent with the prediction of the model, it declined to 7.8 % during 2007–2014. The latter is most likely due to the effect of tightening the lending standard after the Great Recession. Proportion in debt (third column) also increases in the model, from 41.7 % in the baseline model to 47.4 % under the BAPCPA. In the data, again, the proportion in debt went up to from 45.0 % to 46.0 % in 2007 but declined since then.

The overall charge-off rate declines from the baseline level of 3.8 to 2.4 % under the BAPCPA (fourth column), consistent with the declining default rate. The charge-off rate in the data also declined, from 5.3 % during 1999–2004 to 4.0 % in 2007 and 3.2 % in 2014. However, not surprisingly, the charge-off rate rose during the Great Recession, making the average charge-off rate during 2007–2014 the same level as before BAPCPA. In the model, it is interesting that the effect to the charge-off rate is different between the two elements of the BAPCPA. When default costs go up, the charge-off rate declines significantly, from 3.8 to 1.7 %. On the other hand, the charge-off rate rises to 4.5 % when means testing alone is introduced. This is because relatively higher-income agents borrow more instead of defaulting as a result of the means testing, which pushes up the overall charge-off rate. This explanation is consistent with the larger response of the debt-to-income ratio when means testing alone is introduced.

Finally, the average loan interest rate declines from 10.2 % to 8.6 % when the BAPCPA is introduced in the model (fifth column). In the data, although the average loan interest rate declined from 11.3 % during 1999–2004 to 10.6 % in 2007, which is consistent with the model's prediction, the average loan interest rate was higher at 11.7 % during 2007–2014. Therefore, if the data during 2007–2014 are compared with the model prediction, the data moved into the opposite direction, although the data are affected by the Great Recession and possibly higher costs for financial institutions in general. Consistent with the changes in the charge-off rate, the average loan interest rate increases, to 11.0 %, in response to introducing means testing alone, while it declines to 7.9 % when default costs alone are changed.

Fig. 3 confirms the response of borrowing interest rates to the BAPCPA. The figure compares the loan rate schedules under the baseline model and the alternative model with the BAPCPA, for agents with and without temptation. The loan rate schedules for age-30 agents with the median income shock and zero expenditure shock are shown. Clearly, for both agents with and without temptation, the BAPCPA pushes the loan rate schedule to the right and lowers the loan interest rate, since the BAPCPA makes defaulting more difficult, either by charging a higher cost of defaulting or imposing a means-testing requirement.[21]

The third and fourth panels separately look at the changes induced by the BAPCPA, for agents without and with temptation in the baseline model. In terms of defaults and debt, both types of agents respond similarly. The number of defaults declines in response to both components of the BAPCPA, while the debt-to-income ratio as well as the proportion in debt rises. There are interesting differences in terms of the charge-off rate and the average loan interest rate. For agents without temptation, both the charge-off rate and the average loan rate increase with the introduction of the BAPCPA, while they decline for agents with temptation. The differences are due to the composition of defaulters. Agents without temptation default almost exclusively due to a large expenditure shock. Therefore, higher default costs do not deter them from defaulting. It is easy to see that neither the charge-off rate nor the average interest rate among agents without temptation responds to higher default costs. On the other hand, means testing deters some agents from defaulting and forces them to borrow instead, making the charge-off rate and the average loan rate higher. Among

[21] One interesting feature of Fig. 3 is that agents without temptation are charged higher interest rates controlling the individual characteristics and amount of debt. Since age-30 agents without temptation tend to be saving more than agents without, they value less the future option to borrow, making them more likely to default on their debt.

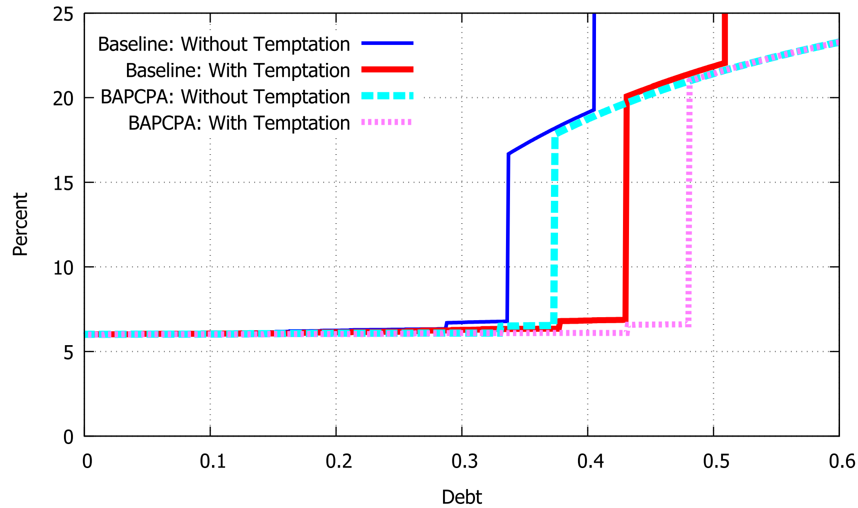


Fig. 3. Loan interest rate schedule: baseline model and under BAPCPA.

the agents with temptation, higher default costs deter many agents from defaulting, making the charge-off rate and the average loan rate lower. Although, similar to agents without temptation, means testing raises both the charge-off rate and the average loan rate, this effect is dominated by the effect with higher default costs, since more lower-income agents among them default, and they are the ones affected by higher default costs.

6.2.2. Welfare implications of the BAPCPA

Social welfare is evaluated as the ex-ante expected lifetime utility. Formally, social welfare $\mathbb{E}V$ is defined as

$$\mathbb{E}V = \sum_j \sum_p \sum_x \phi_j \pi_p^0 \pi_x^x \, V(j, 1, 0, p, x, 0), \tag{17}$$

where π_p^0 and π_x^x denote the initial distribution of the persistent income shock p and the distribution of the expenditure shock x, respectively. ϕ_j is the measure of type-j agents. Also notice that an agent is born into the model economy with the initial age ($i = 1$), a good credit history ($h = 0$), and zero assets ($a = 0$). This definition of social welfare is the standard in the life-cycle model with heterogeneous agents in evaluating the welfare effects of policy changes (e.g., Conesa et al., 2009). I also use welfare of type-j agents ($\mathbb{E}V_j$) and welfare of agents in different income quintiles, which are defined similarly. The difference in welfare between the two economies is measured by consumption equivalent variation (CEV), which is the percentage change in consumption every period and every contingency, which would make the welfare under the alternative economy the same as in the baseline economy.

Remember that the model with temptation and self-control has the same behavioral implications as the hyperbolic-discounting model. Although there could be differences in how to conduct welfare analysis under the two formulations, both formulations support the use of $V(.)$ as the basis of welfare analysis. As for the preferences featuring temptation and self-control, as argued by Gul and Pesendorfer (2004b), the preference of an agent is dynamically consistent. Therefore, the welfare of an agent can be defined in a straightforward manner using $V(.)$. On the other hand, since a hyperbolic-discounting agent is dynamically inconsistent and the same agents in different time periods are considered as different selves, there is no naturally accepted notion of agent's welfare. However, O'Donoghue and Rabin (2001) advocate conducting welfare analysis by discounting continuation payoffs exponentially. This basically means the use of $V(.)$ as well.[22]

The last column of Table 3 shows welfare effects by moving from the baseline economy without the BAPCPA to the economy with it. The second panel indicates that, even though the BAPCPA makes it either more costly or difficult to file for bankruptcy and lowers the number of bankruptcy filings as a result, the policy reform causes welfare gain equivalent to 0.29 % of consumption growth on average among all agents. Interestingly, higher default costs induce the majority of the welfare gain (0.25 % of consumption growth), while means testing alone induces a tiny (0.01 %) welfare loss. With higher default costs, significantly smaller number of agents default (number of defaults declines from 0.94 to 0.49 %), but stronger commitment to repay, induced by harsher punishment, allows agents to borrow more and smooth consumption better. The proportion of agents in debt rises from 41.7 to 46.8 %, and the debt-to-income ratio increases from 8.6 to 10.2 %. Why does means testing induce a welfare loss? Means testing only deters defaulting of higher-income agents, and thus only borrowing by higher-income agents is relaxed as a result. Therefore, the benefit from a stronger commitment is limited, while higher-income agents who are hit by a large expenditure shock are prohibited from defaulting, which induces a slightly larger welfare loss.

The third and fourth panels of Table 3 show that the welfare effects are qualitatively similar but quantitatively different for both types of agents. For agents without temptation, the welfare gain from the BAPCPA is 0.04 % of consumption growth, while the welfare gain is 0.54 % for agents with temptation. Agents with temptation benefit significantly more than those without temptation because the former tend to borrow a lot more than the latter and thus benefit significantly more from a stronger commitment to repay and a resulting lower default premium when borrowing. The result suggests that such welfare gain dominates the welfare loss from the higher cost or restriction associated with default and that from over-borrowing, both of which are larger among agents with temptation. For both types of agents, means testing yields a small welfare loss, but higher default costs yield a larger welfare gain, making the overall welfare

[22] I thank an anonymous referee for pointing this out.

effect of the BAPCPA positive. This similarity is mildly surprising, since, as emphasized by Nakajima (2012), there is an additional negative welfare effect for agents with temptation. When the cost of borrowing becomes cheaper with the BAPCPA, agents might over-borrow, which implies a welfare loss. The simulation result indicates that the positive welfare effect from a lower cost of borrowing and subsequent better consumption smoothing dominates the negative welfare effect of induced over-borrowing.

Fig. 4 shows the different dimension of the heterogeneous effects of the bankruptcy reform. The figure shows the welfare effect of implementing the BAPCPA for agents with different income quintiles at birth.[23] Let me make three remarks. First, the welfare effect is small for agents without temptation (dotted line) because they borrow and default less. Second, agents with lower initial income tend to gain more from the reform (bold dark line). This is because agents with lower initial income are more likely to borrow and thus benefit from the lower default premium under the BAPCPA. On the other hand, agents in the top income quintile lose from the reform because they default less, and when they want to, they are more likely to be constrained by the means-testing requirement. Finally, even among those who gain from the reform, the welfare gain is not monotonic in income level. Agents in the lowest quintile do not gain from the reform as much as those in the second and the third quintile because they tend to default more and thus suffer from the higher default cost under the BAPCPA.

6.2.3. Transition dynamics

Fig. 5 shows the transition dynamics of the model after the BAPCPA is introduced in 2005. It is assumed that the economy is in its initial steady state in 2004, and suddenly the bankruptcy reform is introduced in 2005. There is no other change after 2005. The four panels of Fig. 5 present (a) the percentage of agents filing for bankruptcy, (b) the aggregate debt-to-income ratio, (c) the charge-off rate of loans, and (d) the average loan interest rate. Each of the four panels of Fig. 5 corresponds to each of the panels in Fig. 1, respectively.

Although the Great Recession and the fact that the BAPCPA was known before it was enacted make the comparison between the model and the data since 2005 difficult, the model's predictions are consistent with the data in many dimensions. According to the model, the number of bankruptcy drops immediately in 2005 and continues to decline gradually after 2005. In the data, the number of bankruptcy drops as well, but in 2006, after the BAPCPA was introduced took effect in late 2005. There was a spike up in 2005, since the introduction of BAPCPA was announced in advance. On the other hand, since it is assumed that the reform is a complete surprise in the model experiment, the model does not generate the rush. In the data, the number of bankruptcy seems to be approaching the level comparable to the level in 2006, but there was another spike between 2006 and 2014, since there was a deep recession. Although the announcement prior to implementation of the BAPCPA and the Great Recession make comparing the model and the data not straightforward, the transition path generated by the model can be seen generally consistent if the data had not been affected by the two. The aggregate debt-to-income ratio gradually increases and converges to the new steady-state level in the model. In the data, although there is an increase in 2006 right after the introduction of the BAPCPA, the debt-to-income ratio declined since 2009 with the Great Recession. The charge-off rate and the average loan interest rate in the model decline sharply upon the introduction of the reform and more or less remains at the levels after the initial drop. In the data, the average interest rate has been jumpy and did not show any significant change after the introduction of the BAPCPA. On the other hand, the charge-off rate was lower after the BAPCPA was introduced, except for the sharp spike during the Great Recession.

6.2.4. Adjusting the BAPCPA

In this section, I implement a preliminary investigation as to whether there is a way to improve the BAPCPA. In particular, I change (1) the level of threshold income for means testing and (2) the cost of bankruptcy filing, independently.

Table 4 summarizes the results. In the top panel, I change only the threshold income levels for the means testing from the baseline model while keeping the default cost at the baseline level of \$921. The means-testing thresholds in Table 4 are shown as percentage of median income. The baseline model corresponds to the threshold income level of infinity because the threshold is never binding with it. As the threshold level is lowered, the number of bankruptcy filings declines, since more and more agents cannot satisfy the means-testing requirement. At the same time, since fewer agents are eligible for defaulting, loan interest rates decline, and the amount of debt and the proportion in debt increase. When the threshold level is set at zero, only involuntary defaults are allowed, and the loan interest rate becomes risk-free loan rate of 6.0 %. The debt-to-income ratio goes up from the baseline level of 8.6 % to 13.8 %, and the proportion in debt increases from 41.7 to 49.9 %. As shown in the last column, social welfare generally increases as the means-testing threshold is lowered, the number of default declines, and the loan interest rates lower. When the means-testing threshold is set at zero, the size of the social welfare gain is a 0.50 % increase in consumption, which is sizably larger than that of the BAPCPA (0.29 %). The positive welfare effect of being able to borrow at a lower rate dominates the negative effects of not being able to default when income is higher and over-borrowing in the case of agents with temptation.

In the bottom panel, I start from the baseline model and change the default cost to various levels. The means-testing requirement is not introduced. Similar to the results in the first panel, lower default costs, which encourage defaulting and thus reduce borrowing, are associated with a negative welfare effect. When the default cost is lowered to zero, about twice as many agents (1.87 % compared with 0.94 % in the baseline model) default, but the debt-to-income ratio declines from 8.6 % to 7.1 %. The average loan rate rises from 10.2 % to 15.7 %. The social welfare loss is equivalent to 0.11 % of flow consumption in this case. On the other hand, higher default costs, which make defaulting less frequent and thus lower borrowing interest rates and make borrowing cheaper, induce a welfare gain. When the default costs are doubled from the level of the BAPCPA (\$2954), the number of defaults declines to 0.21 %, but the average borrowing interest rate shifts down to 6.6 %, while the debt-to-income ratio rises to 12.5 %. The welfare gain associated with the high default cost is equivalent to consumption growth of 0.40 %. However, interestingly, the welfare effect is not monotonic in the default cost because a higher default cost, which is a waste, hurts the welfare of defaulters. As can be seen in the bottom line of Table 4, when the default cost is tripled (\$4431), the welfare gain declines to 0.27 % of consumption growth, although the number of default declines further to 0.11 %, and the debt-to-income ratio reaches 13.0 %. This is why lowering the means-testing requirement induces a higher welfare gain than increasing the default costs as long as the default cost is a waste, although welfare gains are achieved through very similar channels.

6.3. Assessing usury law

Until the early 1980s in the U.S., banks and other lending institutions were subject to limits on the interest rates they could charge. This usury law was imposed by the state in which each loan was

[23] Because of the way the income shock is discretized, each income quintile does not exactly contain 0.20 of agents but close. For example, the "middle quintile" contains 0.219 of agents.

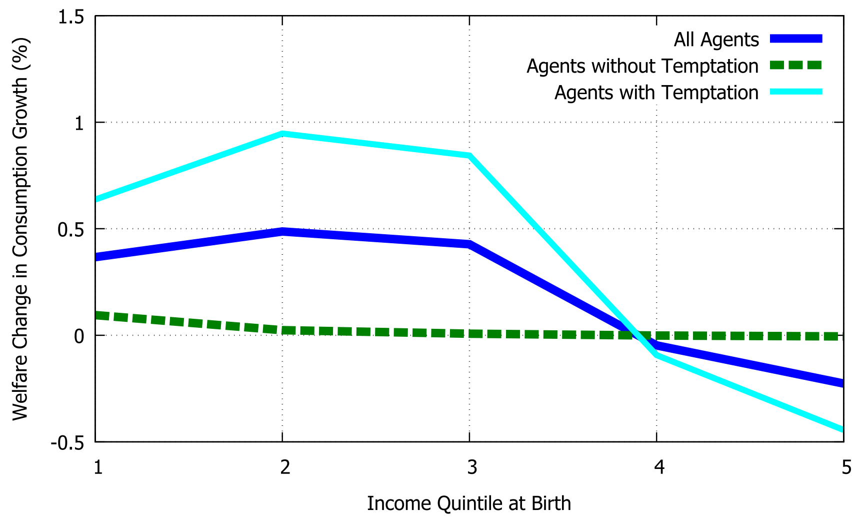


Fig. 4. Welfare effects of the BAPCPA for agents in each income quintile.

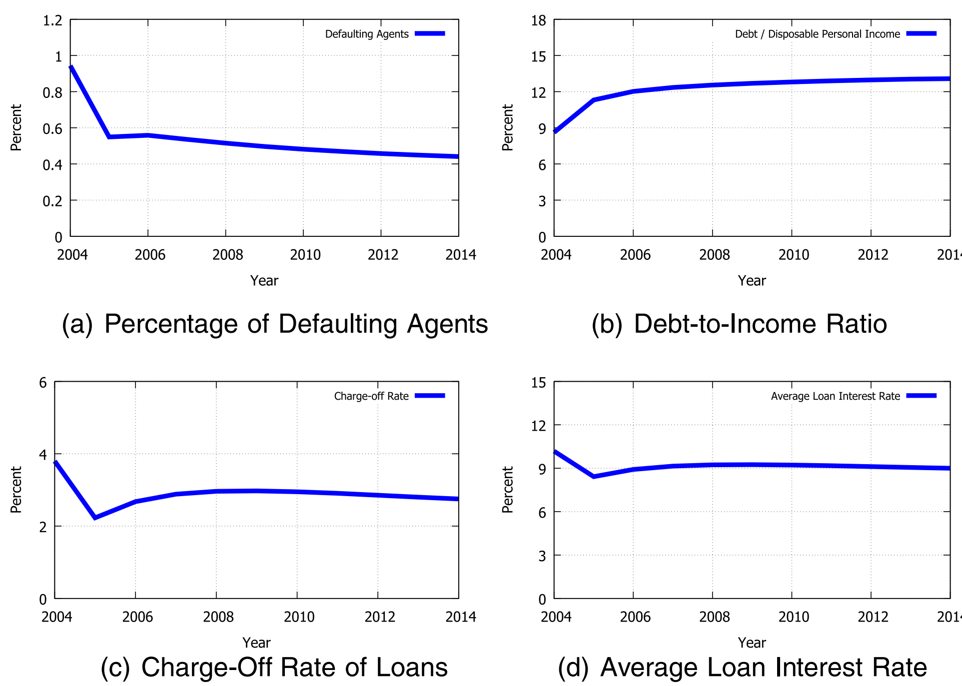


Fig. 5. Transition dynamics after introducing the BAPCPA.

made. However, the *Marquette* decision in 1978 and the Depository Institutions Deregulation and Monetary Control Act virtually freed banks and lending institutions of interest rate limits and allowed them to charge any rate they chose.[24] In the U.S., there is currently no effective upper bound for the loan interest rate that financial institutions can charge, but it is reasonable to think that a usury law, by discouraging loans to risky borrowers who require high default premia, could also help achieve the same goal as the recent bankruptcy law reform — reducing the number of bankruptcies.[25] What are the effects of this usury law? How are the effects of this law different between agents with and without temptation? To answer these questions, I introduce a usury law with various interest rate limits to the baseline model.

Table 5 summarizes the results. The first panel shows the overall effects of introducing various upper bounds for the loan interest rate. The second and third panels show the effects on agents without and

[24] Practically, financial institutions can locate the headquarters in a state with a high or no interest rate ceiling and extend loans to borrowers in other states with a lower interest rate ceiling.

[25] For example, in Japan, a law that prohibits loans with an interest rate higher than 20 % per year was implemented in 2007. Indeed, the 20 % upper bound existed even before the reform, but it wasn't virtually effective, and only the 29.2 % limit was valid. The reform was intended to discourage *predatory* lending.

Table 4
Adjusting the BAPCPA[a].

	% default	% in debt	D/Y %	Charge-off %	Avg r %	Welfare %
Changing means-testing threshold						
0%	0.02	49.9	13.8	0.0	6.0	+0.50
50%	0.47	45.4	12.7	2.0	8.2	+0.10
100% (BAPCPA)	0.92	42.6	16.8	4.5	11.0	−0.01
∞% (Baseline)	0.94	41.7	8.6	3.8	10.2	–
Changing default cost						
$0	1.87	32.8	7.1	8.4	15.7	−0.11
$921 (Baseline)	0.94	41.7	8.6	3.8	10.2	–
$1477 (BAPCPA)	0.49	46.8	10.2	1.7	7.9	+0.25
$2954 (double)	0.21	49.1	12.5	0.6	6.6	+0.40
$4431 (triple)	0.11	48.8	13.0	0.2	6.2	+0.27

[a] The six columns show the proportion defaulting, proportion in debt, debt-to-income ratio, charge-off rate, average borrowing rate, and welfare gain from policy reform, represented as the growth rate of consumption.

Table 5
Effects of usury law[a].

	% default	% in debt	D/Y %	Charge-off %	Avg r %	Welfare %
Baseline model: all agents						
Baseline	0.94	41.7	8.6	3.8	10.2	–
Usury law: 20%	0.66	44.9	9.9	2.2	8.3	−0.00
Usury law: 10%	0.37	47.4	11.6	0.6	6.6	−0.01
Usury law: 7%	0.36	47.3	11.6	0.6	6.6	−0.02
Baseline model: agents without temptation						
Baseline	0.19	27.1	1.8	0.4	6.4	–
Usury law: 20%	0.19	27.1	1.8	0.4	6.4	–
Usury law: 10%	0.19	27.1	1.8	0.4	6.4	–
Usury law: 7%	0.19	27.1	1.8	0.4	6.4	−0.00
Baseline model: agents with temptation						
Baseline	1.70	56.3	15.5	4.2	10.6	–
Usury law: 20%	1.14	62.7	18.1	2.3	8.5	−0.01
Usury law: 10%	0.54	67.6	21.4	0.6	6.6	−0.02
Usury law: 7%	0.54	67.5	21.5	0.6	6.6	−0.04

[a] The six columns show the proportion defaulting, proportion in debt, debt-to-income ratio, charge-off rate, average borrowing rate, and welfare gain from policy reform, represented as the growth rate of consumption.

with temptation, respectively. The first panel shows that macroeconomic effects of a usury law, which are somewhat counterintuitive. The debt-to-income ratio as well as the proportion in debt increase when loans whose interest rate is above a certain threshold is prohibited. Why? First of all, usury law makes loans with a higher interest rate unavailable for agents. Fig. 6 shows what happens to the interest rate schedule for age-30 agents with median income and zero expenditure shock when the interest ceiling is set at 10 %. The figure shows that usury law basically eliminates markets for loans whose interest rate is above the interest rate limit. Therefore, the average interest rate declines, from 10.2 to 6.6 % in the case of 7.0 % interest rate ceiling. However, since agents cannot take loans with a high interest, which reflects ex-post high default, the number of bankruptcies declines. The number of defaults goes down to 0.36 % instead of 0.94 % in the model without such ceiling.[26] Agents who do not default remain non-delinquent and keep borrowing, which increases borrowing. In the end, the second effect dominates the first, and the level of indebtedness increases in response to usury law. If the interest rate ceiling is set at 7.0 %, the aggregate debt-to-income ratio goes up from 8.6 to 11.6 %, and the proportion in debt rises from 41.7 to 47.3 %. The second and the third panels show that most actions occur with agents with temptation. Since the probability of default among agents without temptation is already low, they are hardly affected by usury law.

Although a tight interest rate ceiling achieves similar macroeconomic effects as the BAPCPA — a lower number of defaults and expanded credit — the welfare effect of such interest ceiling is negative, albeit small. The overall welfare effect when the interest rate ceiling is set at 7.0 % is 0.02 % of consumption loss. Since only agents with temptation are affected, the welfare loss for agents with temptation is 0.04 %. The difference in the welfare effects between the BAPCPA and usury law is that the interest rate schedule declines with the former, because the BAPCPA virtually implies stronger commitment to replay, while the interest rate schedule itself is not directly affected with usury law. Instead, usury law only makes loans with a high interest rate unavailable to agents.

6.4. *Assessing the effect of changing the level of income garnishment*

Another way to discourage bankruptcy, albeit less directly than the BAPCPA, is to allow lenders to garnish more earnings upon default. Currently, federal law sets an upper bound on how much a creditor can garnish its debtors' wages per week: It is either 25 % of wages or the wage amount exceeding 30 times the federal minimum wage, whichever is smaller.[27][28] By raising the amount that creditors can garnish, the government could discourage defaulting and make loans more creditor friendly. This section explores the implications

[26] If the interest rate ceiling is set at 6 %, which is the risk-free loan rate, there are virtually no loans in the economy. This is because borrowers with any amount of debt have a small probability of default, because of a large expenditure shock. Therefore, the loan interest rate never becomes the risk-free level in equilibrium.

[27] States can set a lower limit for garnishment, providing better protection for debtors. For example, in Massachusetts, the garnishment limit is typically 15 % of wages.

[28] In the model, the upper bound of the amount of garnishment associated with the minimum wage is abstracted.

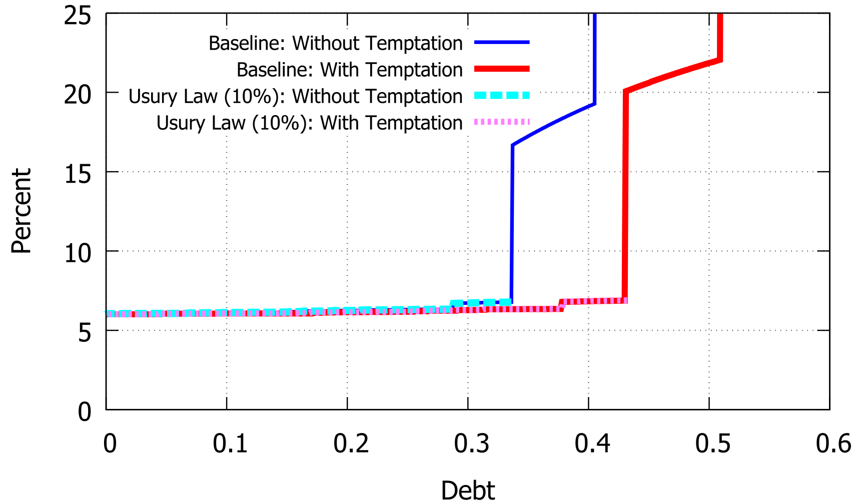


Fig. 6. Loan interest rate: baseline model and under usury law of 10%.

of changing the income garnishment ratio using the model. I also discuss the *optimal* level of income garnishment.

Let me make two remarks before showing the results. First, the word *optimal* is used in the specific sense that I change η without changing other elements of the bankruptcy law and call η optimal when the social welfare $\mathbb{E}V$ defined in Eq. (17) is maximized. The social welfare is ex-ante expected lifetime utility. I leave the problem of designing the optimal bankruptcy law in a less restricted policy space for future research. Second, the general equilibrium effect is not considered here. Nakajima (2012) considers the general equilibrium effect when the optimal level of the borrowing constraint is investigated. As expected, the general equilibrium effect lowers (increases) welfare when the borrowing constraint is relaxed (tightened), because of capital decumulation (accumulation).

Fig. 7 shows the effects of changing the income garnishment rate η. The figures show how (a) social welfare, (b) number of defaults, (c) aggregate debt-to-income ratio, (d) proportion of agents in debt, (e) charge-off rate, and (f) average loan interest rate change with η (x-axis). Remember that the baseline garnishment rate η is 0.34. For all figures, the effects to all agents and effects to agents without and with temptation are shown.

Let's start by looking at the effects to agents without temptation. Their welfare keeps increasing in the income garnishment rate until around $\eta = 2/3$, after which the welfare remains more or less flat (Fig. 7(a)). There are two welfare effects behind this profile. First, when the garnishment rate is increased, lenders can offer lower interest rates for any type because their loss from default (after taking into account the garnished amount) would be smaller. With lower interest rates, agents can borrow more to better smooth out consumption fluctuations. Fig. 7(c) shows that the debt-to-income ratio increases in η up to around $\eta = 2/3$, and Fig. 7(d) shows the proportion in debt increases in η as well. The welfare gain becomes flat at around $\eta = 2/3$ because the debt-to-income ratio plateaus. At that point, agents stop defaulting except for involuntarily defaulting due to a large expenditure shock, and the interest rate schedule stops shifting lower. Even if more agents are in debt, the number of defaults declines in η, which is consistent with a stronger discouragement from defaulting when η is raised (Fig. 7(b)). The charge-off rate (Fig. 7(e)) and the average interest rate (Fig. 7(f)) are little affected because agents without temptation, with their high discount factor, do not borrow when a relatively high default premium is charged. But it does not mean that they do not benefit from lower interest rates; they still can borrow more because of the relaxed borrowing constraint implied by the lower interest rates. The second welfare effect is that a higher garnishment rate means a higher punishment for defaulting, which implies that welfare declines in the garnishment rate. The fact that welfare is increasing with η implies that the first effect dominates the second.

The welfare effect for agents with temptation turns out to be non-monotonic as seen in Fig. 7(a). When the income garnishment rate is raised from the baseline level of 0.34, the effects are similar to those for agents without temptation. A higher garnishment rate discourages default (Fig. 7(b)), while the debt-to-income ratio (Fig. 7(c)) and proportion in debt (Fig. 7(d)) increase. The charge-off rate (Fig. 7(e)) and the average loan interest rate (Fig. 7(f)) decline with lower defaults. Lower interest rates allow agents to smooth out consumption better, which implies welfare gain, but the welfare gain plateaus when the number of defaults stops declining and the debt-to-income ratio stops increasing. What is different from the welfare effects to agents without temptation is when the garnishment rate is lowered from its baseline level. As in the case with agents with temptation, the debt-to-income ratio keeps falling when η is lowered, which implies welfare loss. At the same time, up to $\eta = 0.20$, number of defaults increases, as the punishment for defaulting weakens. Consistently, the charge-off rate and the average interest rate rise. Between the baseline level of $\eta = 0.34$ and $\eta = 0.20$, welfare increases when η is lowered because of the two positive welfare effects dominating the negative effect of worse consumption smoothing. First, as in the case with agents without temptation, lower punishment for defaulting generates welfare gains, which is especially large for those who default more often. Second, a higher cost of borrowing benefits them by preventing them from over-borrowing. This effect is unique for agents who suffer from temptation. Indeed, when η is further lowered from 0.20, social welfare keeps increasing, although both the number of defaults and the debt-to-income ratio keep declining. This implies that the welfare gain for agents with temptation from not being able to over-borrow is strong. In other words, a source of the welfare gain when η is lowered and loan interest rates are higher is that agents with temptation enjoy stronger *discipline* not to over-borrow. One way to prove that this is actually the case is to look at the welfare effect when η is fixed at the baseline level of 0.34, but the loan interest rate schedule is changed to what is consistent with a lower η. In this experiment, the benefit from weaker default punishment is eliminated. Indeed, even without changing η, it turns out that agents with temptation benefit having higher loan interest rate schedule, which implies stronger discipline not to over-borrow. For example, if the interest rate schedule associated with $\eta = 0.0$ is used without actually changing η from its

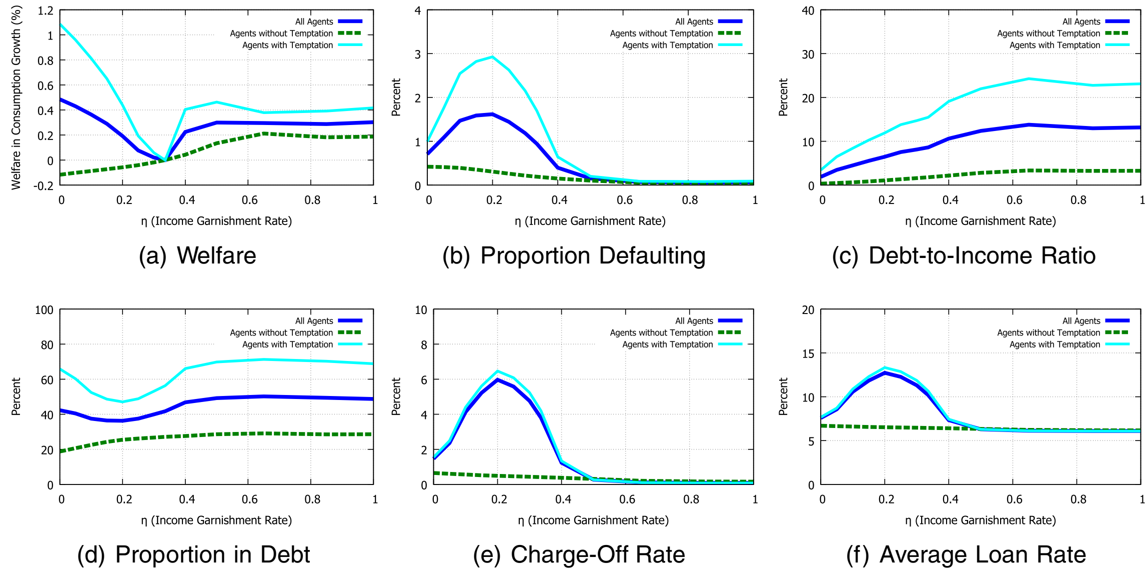


Fig. 7. Effects of changing income garnishment rate in the baseline model.

baseline value, agents with temptation gain by 0.75 %, while, not surprisingly, agents without temptation lose by 0.44 %. It is interesting that because of the different kinds of welfare gain when η is either raised or lowered from its baseline level of 0.34, not only the welfare is non-monotonic in η but also the baseline level of η is where the social welfare is at its lowest.

What is interesting is the contrast of the welfare effects of changing the income garnishment rate between agents with and without temptation. For agents with temptation, social welfare improves whether the income garnishment rate is either raised or lowered. However, while the welfare gain of raising η to one is equivalent to 0.42 % of consumption growth, the welfare gain of lowering η to zero is larger at 1.09 %. For agents with temptation, the welfare gain from being prohibited from over-borrowing is stronger than other effects, and thus the social welfare is maximized when η is brought to zero. On the other hand, for agents without temptation, the social welfare is increasing in η up to around $\eta = 2/3$, and then welfare remains more or less flat in η. The social welfare is maximized at $\eta = 0.59$ with 0.23 %. Since the welfare gain for agents with temptation is larger, the overall optimal level of the income garnishment rate is zero, where the gain in social welfare is equivalent to 0.49 % of consumption growth. However, interestingly, there is a significant contrast between the two groups. The optimal level of $\eta = 0$ benefits agents with temptation significantly (1.09 %), while agents without temptation lose by 0.12 % of consumption. When η is raised, both types of agents gain, but the average welfare gain is not as large as the one under $\eta = 0$.

7. Sensitivity analysis

This section investigates sensitivity of the main results of the paper. Because of space restrictions, only the effects of introducing the BAPCPA in the model are presented. I investigate (i) the model with naive agents, (ii) the model with a lower (higher) temptation discount factor (rate), (iii) the model with information frictions, (iv) the model with a bequest motive, and (v) the model with different parameter values. Table 6 summarizes all the results, including

Table 6
Sensitivity analysis: effects of the 2005 bankruptcy reform.[a]

	Pre-BAPCPA			Post-BAPCPA			
	Default	D/Y	Avg r	Default	D/Y	Avg r	Welfare
1. Baseline model	0.94	8.6	10.2	0.42	13.0	8.6	+0.29
2. Naive agents	0.94	8.6	8.5	0.57	13.2	9.4	+0.22
3. $\beta = 0.6667$	0.95	8.6	10.5	0.44	13.1	8.6	+0.30
4. Information frictions	0.94	8.6	9.1	0.79	18.5	10.6	+0.01
5. Bequest motive	0.94	8.6	10.0	0.42	11.8	8.8	+0.27
6. $\psi_e = 0.55, \psi_p = 0$	0.94	8.7	10.1	0.39	11.0	8.4	+0.28
7. $\psi_e = 0, \psi_p = 0.55$	0.95	8.7	10.6	0.32	9.8	8.2	+0.35
8. $\sigma = 3.0$	0.94	8.6	9.5	0.59	14.0	9.5	+0.59
9. $\iota = 0.026$	0.94	8.6	8.5	0.38	10.7	7.3	+0.30
10. $\bar{c} = 0$	0.94	8.6	10.2	0.43	12.8	8.7	+0.27
11. $\lambda = 0.20$	0.94	8.6	10.0	0.41	11.5	8.5	+0.28
12. Immediate recovery	0.94	8.6	10.1	0.41	11.8	8.5	+0.30
13. Proportional ξ	0.94	8.6	9.8	0.74	11.8	9.5	+0.00
14. Double p_x	0.94	8.6	9.5	0.59	13.6	8.7	+0.01
15. Half x	0.94	8.6	10.6	0.33	11.6	8.4	+0.37
16. $\rho_p = 0.94$	0.95	8.6	9.3	0.80	11.8	12.4	+0.56
17. Half σ_p^2	0.94	8.6	8.9	0.61	14.2	9.5	−0.13

[a] *Default*, *D/Y*, and *avg r* denote the number of default, aggregate debt-to-income ratio, and the average loan interest rate, in percent, respectively. *Welfare* denotes the welfare gain from introducing the BAPCPA, represented as the permanent percentage increase in consumption.

the results of the baseline case (Line 1). In all cases, I recalibrate parameters δ and η in the model without the BAPCPA while leaving all other parameters unchanged, so that all models replicate the number of defaults of 0.94 % and the debt-to-income ratio of 8.6 %. Appendix D provides the calibrated parameter values for all economies studies in this section.

7.1. Model with naive agents

Agents in the baseline model presented above are called "sophisticated" because they correctly see themselves as subject to temptation in the future when making decisions. In the language of hyperbolic discounting, agents are aware that they are time inconsistent, that is, they correctly expect their future selves to also use hyperbolic discounting. The alternative assumption is what is called "naive." Naive agents are not aware that they are time inconsistent, or they do not recognize they will suffer temptation in the future. In this sensitivity experiment, I assume agents with temptation are naive. Notice that agents without temptation are not affected.

Line 2 of Table 6 shows the results with naive agents. The effects of the BAPCPA to debt and defaults are similar between the baseline model with sophisticated agents and the one with naive agents. With the BAPCPA, the number of defaults declines to 0.57 % instead of 0.42 %, while the debt-to-income ratio increases from 8.6 to 13.2 % instead of 13.0 %. However, the average loan interest rate moves in the opposite directions. The average rate rises from 8.5 to 9.4 % in the model with naive agents, while the average rate declines from 10.2 to 8.6 % in the baseline model. The size of the welfare gain of introducing the BAPCPA is just slightly smaller, at 0.22 % in the model with naive agents compared with 0.29 % in the baseline model.

Akerlof (1991) and O'Donoghue and Rabin (1999, 2001) study cases in which sophisticated and naive agents behave in very different ways, while Angeletos et al. (2001) find that the two types of agents produce similar results in their estimated life-cycle model. It is not surprising that the two types of agents generate similar results in the current experiment because the current model shares a lot with the latter.

7.2. Model with stronger present bias

In the next experiment, I change the temptation (short-term) discount factor β from its baseline value of 0.70 to 0.6667. This change is equivalent to raising the temptation discount rate from the baseline value of 40 % annually to 50 %. Line 3 of Table 6 shows the results. Basically, the effects of introducing the BAPCPA are virtually the same as in the baseline model.

7.3. Model with information frictions

It is assumed in the baseline model that all the characteristics of agents are observed by lenders, which allows lenders to tailor default premium precisely. However, some characteristics, such as preference types, might be more difficult to observe.[29] In the experiment summarized in Line 4 of Table 6, I assume that lenders cannot observe the preference type b but can observe and use all the other characteristics of borrowers, namely (i, p, a'), to adjust default premium offered to each borrower.[30] Because of a technical difficulty, I assume that lenders use the proportion of agents with and without temptation in debt in the baseline model regardless of the debt level to calculate the bond price for agents of type (i, p, a'). Ideally, lenders take the distribution of different types of agents given (i, p, a') and compute the bond price schedule accordingly, and the distribution of borrowers is realized as an outcome of optimal decisions of agents in equilibrium. However, it turns out that an equilibrium cannot be obtained consistently, since agents of good types try to separate themselves from the pooling and enjoy lower interest rates on loans. As a result, credit card companies do not make zero profit on their loans in this experiment.

Line 4 of Table 6 shows the effects of the BAPCPA in the economy with such information frictions are different from those of the baseline model. The number of defaults declines, but less than the baseline, from 0.94 to 0.79 %. This is because the response of indebtedness is significantly stronger. The debt-to-income ratio rises from 8.6 to 18.5 %. The large increase in agents in debt pushes up the number of defaults, even though the BAPCPA discourages defaults for each debtor. The average interest rate rises, from 9.1 to 10.6 %. The welfare effect of introducing the BAPCPA is still positive but significantly smaller at 0.01 % of consumption growth. Part of the differences are probably due to the negative effect of over-borrowing by agents with temptation.

7.4. Model with bequest motive

Next, I introduce a warm-glow bequest motive to the model. Since agents are assumed to live up to age I with certainty, it is assumed that agents gain utility from leaving assets in age-I. I use the following functional form used by De Nardi et al. (2010):

$$\tilde{u}(a) = \zeta_0 \frac{(\zeta_1 + a)^{1-\sigma}}{1-\sigma}, \tag{18}$$

where ζ_0 represents the strength of the bequest motive and ζ_1 represents the curvature of the motive. When ζ_1 is larger, the bequest becomes more luxurious goods. De Nardi et al. (2010) also use the same curvature parameter σ as in the period utility function. In calibrating the parameters, I use the estimates of De Nardi et al. (2010) and convert them into the parameter values in the current model.[31] Line 5 of Table 6 shows the results. Basically, the results from the model with bequest motives are similar to those in the baseline model, with a slightly weaker response of the debt-to-income ratio and thus slightly smaller welfare gain.

7.5. Alternative parameter values

Lines 6 to 17 of Table 6 show the results from the models, in each which only one parameter is changed from its baseline value. In Line 6, I assume that the amount of the Social Security benefits is the same across all agents, and it is $\psi_e = 0.55$ of the average labor income. In Line 7, I assume the opposite: The amount of Social Security benefits does not depend on the average labor income and instead depends entirely on the realization of the persistent income shock in the last working age. In Line 8, I raise the coefficient of relative risk aversion to 3.0 from its baseline value of 2.0. In Line 9, I follow Livshits et al. (2010) and assume a lower (2.6 instead of 4 %) transaction costs of making loans. In Line 10, I eliminate the consumption floor. In Line 11, a history of the past default is erased on average five years after default instead of 10. Although credit card companies can retain the history of past default for 10 years, some studies such as Han and Li (2011) find that households can use unsecured credit again less than 10 years after filing for bankruptcy.[32] In Line 12, I assume that agents could recover from bad credit history immediately after a bankruptcy filing with probability λ. In Line 13, I assume

[29] Athreya et al. (2012) argue that an economy in which credit card companies cannot distinguish borrower types and thus pool the borrowers when pricing loans better replicates credit conditions in the U.S. economy in the 1980s.

[30] Since expenditure shocks x are i.i.d. and thus do not contain any information about the probability of default in the next period, they do not affect the bond prices.

[31] See Appendix D of the working paper version of De Nardi et al. (2010) for details on converting their estimated parameter values into the current model.

[32] I thank a referee for pointing out this study.

that the cost of filing for bankruptcy is proportional to the current labor income instead of a fixed cost. I set the parameter representing the proportion (ξ) such that the default cost is the same for agents with mean labor income. In Line 14, I assume that the probabilities of experiencing positive expenditure shocks are twice as large as their baseline values. Since the total number of defaults is the same as in the baseline model, this change yields a relatively larger proportion of bankruptcies due to expenditure shocks rather than income shocks. In Line 15, the size of the expenditure shocks is halved from their baseline values. In Line 16, the persistence parameter of persistent income shocks p is lowered from the baseline value of 0.99 to 0.94. At the same time, the variance is adjusted (to $\sigma_p^2 = 0.102$) so that the cross-sectional variance of income shocks is not affected. In Line 17, the variance of the innovation of the persistent income shocks is halved from its baseline value of 0.0277 to 0.01385.

The findings from the sensitivity analysis are summarized below:

1. With the exceptions of high risk aversion (Line 8), proportional default cost (Line 13), higher probabilities of expenditure shocks (Line 14), lower persistence of income shocks (Line 16), and lower variance of income shocks (Line 17), the effects of the BAPCPA are robust to changing parameter values. The BAPCPA reduces the number of bankruptcies while increases the total amount of loans. The average loan interest rate shifts down because there are less defaults. The overall welfare effect is a small positive.
2. When agents have a higher risk aversion (Line 8), the welfare gain from introducing the BAPCPA is larger, at 0.59 % instead of 0.29 % in the baseline model. Better consumption smoothing due to lower loan interest rates generates a higher welfare gain when agents are more risk averse.
3. Proportional default cost (Line 13) implies no welfare gain from introducing the BAPCPA. Since lower income agents suffer less from defaulting and the number of defaults does not decline as much (from 0.94 to 0.74 %), the loan interest rate does not decline as much as in the baseline model, and thus the gain from better consumption smoothing is smaller.
4. The experiment with larger probabilities of expenditure shocks (Line 14) reveals that the welfare effect of the BAPCPA depends on the relative importance of the two reasons of default. If the relative importance of expenditure shocks is larger, as in Line 14, the negative effect of stronger default punishment dominates, as many agents default without choice after a large expenditure shock. The welfare effect becomes close to zero (+0.01%). It can become negative if the relative importance of expenditure shocks is further increased.
5. With a lower persistence of income shocks (Line 16), loan interest rates depend less on the current realization of income shock, and thus the benefits of lower borrowing interest rates are spread to more agents with different income levels. This yields a larger welfare gain (0.56 %) from introducing the BAPCPA than in the baseline model.
6. With lower variance of income shocks (Line 17), the welfare gain from better consumption smoothing is smaller than in the baseline. Therefore, welfare loss from higher default costs dominates, and the overall welfare effect from introducing the BAPCPA turns out to be negative (−0.13%). This experiment confirms that the composition of reasons of default matters for the evaluation of the bankruptcy law reform.

8. Conclusion

This paper develops a novel model in which some agents suffer from temptation to consume now and others have self-control against such temptation, agents can default on their debt, and the equilibrium loan interest rates reflect the risk of default. I use the calibrated model to evaluate the 2005 bankruptcy law reform, whose aim was to discourage abusers of the debtor-friendly bankruptcy law from filing for bankruptcy. The model indicates that the reform achieves its goal of reducing the number of bankruptcies by about 55 %, and the overall welfare effect is moderate but positive at 0.29 % of consumption growth. The positive welfare effect associated with lower loan interest rates and better consumption smoothing dominates the negative welfare effects due to stronger default punishment and over-borrowing for agents with temptation. Although it is hard to eliminate from the data the effects of the Great Recession, which happened soon after the 2005 reform, I find that the model's implications of the reform are generally consistent with the data. I also investigate other ways to curb the number of defaults, namely, usury law and changing the income garnishment rate. I find that changing the income garnishment rate reveals interesting contrast between agents with and without temptation. Agents with temptation benefit from a lower garnishment rate and associated tight borrowing constraint, which provides discipline against over-borrowing, while agents without temptation prefer a higher garnishment rate, which yields lower loan interest rates and thus better consumption smoothing.

Let me conclude by pointing out three promising directions for future research. First, more micro data should be used to improve estimates of parameter values for different preference types and their distribution. In particular, although I assume no correlation between preference types and other individual characteristics, such as income, these are likely to be correlated. Second, the analysis in the current paper can be expanded to investigate the optimal design of bankruptcy law in a less restricted policy space. Interaction with the tax system or other public programs offering a safety net is an important thing to consider. Finally, macroeconomic policy analysis based on carefully calibrated *behavioral macroeconomic models* such as the one developed in this paper is still limited. However, as more evidence in favor of various behavioral assumptions is accumulated, macroeconomic analysis based on various *behavioral macroeconomic models* should be encouraged.

Appendix A. Appendix

Appendix A includes details about the data related to debt and default, which are used to construct Fig. 1, Table 2, and Table 3. Appendix B contains additional details about calibration. Appendix C presents the computation algorithm of the baseline model. Finally, Appendix D contains additional details of the sensitivity analysis conducted in Section 7.

A.1. Data Appendix

This appendix contains the detailed description of the data used to construct Fig. 1, Table 2, and Table 3. The four panels in Fig. 1 show the number of bankruptcies, the debt-to-income ratio, the charge-off rate, and the average interest rate. The number of bankruptcies is computed by dividing the number of total consumer bankruptcy filings and the Chapter 7 bankruptcy filings by the number of households in respective years. The data on bankruptcy filings are obtained from the U.S. courts. The total number of households is from the U.S. Bureau of the Census.[33] The number of bankruptcy filings can be considered as the upper bound because multiple persons in a single household could file for bankruptcy simultaneously. The debt-to-income ratio is computed by dividing the balance of the revolving

[33] Since the number of households in 2014 is not yet available, I extrapolate the number in 2013 using the growth rate of the number of households between 2012 and 2013.

credit by disposable personal income. The former is constructed by the Federal Reserve Board (FRB, G.19). The revolving credit differs from unsecured credit in the sense that the revolving credit does not capture nonauto nonrevolving credit. However, after constructing the corrected measure of unsecured credit, Livshits et al. (2010) find that the gap between the two measures has been shrinking (see Fig. 3 of their paper). Disposable personal income is obtained from the Bureau of Economic Analysis (BEA). The charge-off rate for all credit card loans is obtained from the FRB (G.19). The average nominal interest rate on credit card loans is also obtained from the FRB (G.19). This is an account-weighted average. The nominal rate is converted into real rate by subtracting the inflation rate of Consumer Price Index for all urban consumers (CPI-U) for respective year.

In constructing the numbers in Table 2, the averages of the data used in Fig. 1 during 1999–2004 are computed. Proportion in debt shown in Table 2 is the proportion of households whose head is between 20 and 99 years old and with positive credit card debt, in respective waves of the Survey of Consumer Finances (SCF). The average of 1998, 2001, and 2004 is taken, but the proportion is stable during the three years; the proportion in debt is 44.3 % in 1998, 44.4 % in 2001, and 46.2 % in 2004. I also compute the proportion of households with negative net worth, from the same waves of the SCF.

To construct numbers in Table 3, the data used in Fig. 1 for the years 2007–2014 are used. In addition, the proportion in debt are constructed using the SCF in the same way as described above. I use the average of 2007, 2010, and 2013 values for the proportion in debt during 20017–2014, and the value in the 2013 SCF for the proportion in debt for 2014. The latter is because the SCF is constructed every three years, and there is no 2014 wave.

Appendix B. Calibration appendix

This appendix contains additional information about calibration. Fig. B.1 shows the household size in family equivalence scale, ν_i, used to discount consumption over the life cycle. Fig. B.2 shows the life-cycle profile of labor productivity, e_i, taken from the estimates of Gourinchas and Parker (2002). Fig. B.3 shows the variance of log labor earnings over the life cycle in the model. The profile, which is almost linearly increasing in age, is consistent with the finding of Storesletten et al. (2004). Fig. B.4 shows the life-cycle profile of the expenditure shocks for the top 0.1 % and 99.4 to 99.5 %, estimated using the Medical Expenditure Panel Study (MEPS) and used in the model simulations.

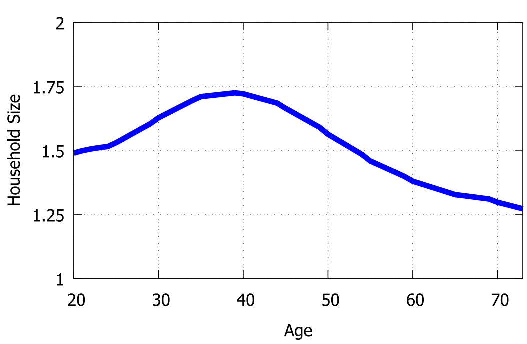


Fig. B.1. Household size in family equivalence scale.

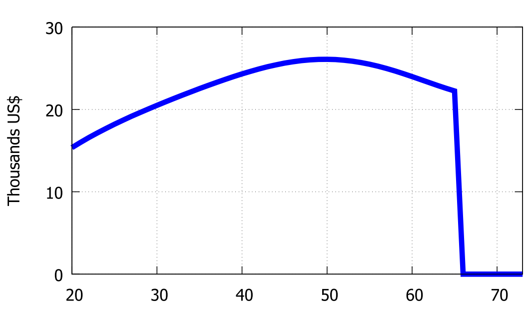


Fig. B.2. Average life-cycle profile of labor income.

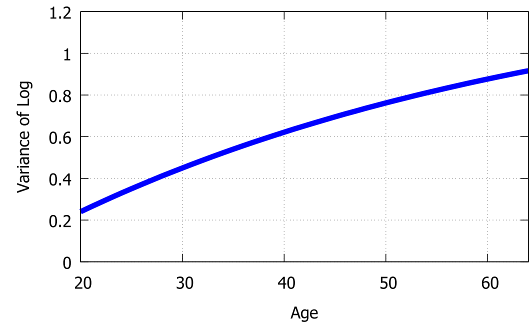


Fig. B.3. Life-cycle profile of log labor income variance.

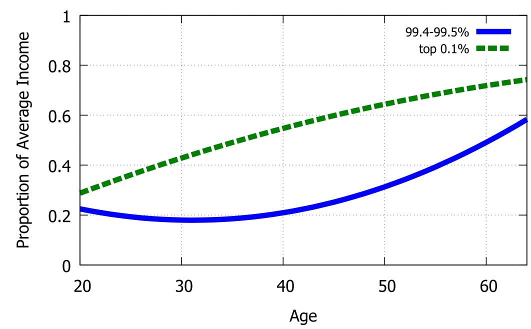


Fig. B.4. Life-cycle profile of compulsory expenditure shocks.

Appendix C. Computational appendix

I describe below the computational algorithm to solve the steady-state equilibrium of the baseline model inhabited by agents with and without temptation.

Algorithm 1 (**Computation algorithm for solving steady-state equilibrium**).

1. Obtain the optimal value function $V(j, i, h, p, x, a)$ and the optimal decision rules $g_h(j, i, h, p, x, a)$ and $g_a(j, i, h, p, x, a)$ by solving the optimization problem backward.
   (a) Start from the problem of age-I agents.
   (b) If $i = I$, set $V(j, i+1, h, p, x, a) = 0$ for all (j, h, p, x, a). In the case $i < I$, $V(j, i+1, h, p, x, a)$ is already obtained in the previous step.
   (c) If $i = I$, set $q(j, i, h, p, x, a') = 0$ for all $(j, h, p, x, a' < 0)$ and $q(j, i, h, p, x, a') = 1/(1+r)$ for all $(j, h, p, x, a' \geq 0)$. If $i < I$, $q(j, i, h, p, x, a')$ is already obtained in the previous step.
   (d) For agents with temptation, the temptation problem is solved first. In case of $h = 0$ (clean credit history), using the discount factor $d = \beta_j \delta_j$, and given $V(j, i+1, h, p, x, a)$ and $q(j, i, h, p, x, a')$, values conditional on non-defaulting and defaulting are obtained from Bellman Eqs. (2) and (5).

The optimal default decision $g_h(j,i,h,p,x,a)$ is characterized by Eq. (1). The optimal saving decision $g_a(j,i,h,p,x,a)$ is the one conditional on not defaulting if $g_h(j,i,h,p,x,a) = 0$ and is zero if $g_h(j,i,h,p,x,a) = 1$. The optimal value of the temptation problem $V^*(j,i,0,p,x,a;\beta_j\delta_j)$ is obtained.

(e) In case of $h = 1$ (bad credit history), using the discount factor $\beta_j\delta_j$, and given $V(j,i+1,h,p,x,a)$ and $q(j,i,h,p,x,a')$, the optimal default decision $g_h(j,i,h,p,x,a)$ and the optimal saving decision $g_a(j,i,h,p,x,a)$ are obtained from Eqs. (7) and (5). Notice that there is no optimal default decision because only involuntary default is allowed. The optimal value of the temptation problem $V^*(j,i,1,p,x,a;\beta_j\delta_j)$ is obtained.

(f) Solve the self-control problem. In general, this step requires solving Eq. (9). However, this step becomes trivial because, by assumption of $\gamma = \infty$, an agent completely succumbs to temptation. Formally, this step only requires updating the value function using Eq. (10).

(g) Once the optimal default decision rule for age-i agents is obtained, the loan price for age-$i-1$, $q(j,i-1,h,p,x,a')$, can be computed using Eq. (12).

(h) If $i > 1$, go back to step (b) and solve the problem of age-$i-1$ agents. If $i = 1$ (initial age), this step is over.

2. Using the obtained optimal decision rules $g_h(j,i,h,p,x,a)$ and $g_a(j,i,h,p,x,a)$, simulate the model forward, starting from the type distribution of age-1 agents.
   (a) Set the type distribution for the newborns, which is exogenously given. In particular, all newborns have $i = 1$, $h = 1$ (no default history), and $a = 0$ (no asset/debt). The initial distributions of j, p, and x are also exogenously given.
   (b) Update the type distribution using the stochastic process for (p,x) and the optimal decision rules $g_h(j,i,h,p,x,a)$ and $g_a(j,i,h,p,x,a)$.
   (c) Keep updating until age I (last age).
3. Once the type distribution of agents is obtained, aggregate data can be computed by aggregating the individual data.

Appendix D. Additional details of sensitivity analysis

Section 7 conducts sensitivity analysis using various alternative models. Notice that endogenously calibrated parameters δ and η are recalibrated for all economies so that all the models yield the same proportion of agents filing for bankruptcy (0.94 %) and the same aggregate debt-to-income ratio (8.6 %). Table D.1 summarizes the calibrated values of (δ,η) in all alternative model economies studied in Section 7. Each row in Table D.1 corresponds to each row in Table 6. Except for the model with naive agents (lower η), the model with $\sigma = 3$ (lower δ and η), and the model with a lower persistence of income shocks (lower δ), calibrated parameter values are similar to their respective values in the baseline model.

Table D.1
Calibration of models for sensitivity analysis.

	δ	η	Remarks
1. Baseline model	0.9121	0.3358	–
2. Naive agents	0.9439	0.1699	Agents do not consider temptation in the future.
3. $\beta = 0.6667$	0.9145	0.3450	Temptation discount rate is 50% instead of 40%.
4. Information frictions	0.9102	0.2974	b is unobservable.
5 .Bequest motive	0.9123	0.3239	With warm-glow bequest motive.
6. $\psi_e = 0.55, \psi_p = 0$	0.9138	0.3336	Uniform pension benefits.
7. $\psi_e = 0, \psi_p = 0.55$	0.9150	0.3211	Pension benefits are proportional to income shock.
8. $\sigma = 3.0$	0.8412	0.2072	Higher risk aversion.
9. $\iota = 0.026$	0.9248	0.3250	Smaller transaction costs as in Livshits et al. (2010).
10. $\bar{c} = 0$	0.9121	0.3358	No consumption floor when defaulting.
11. $\lambda = 0.20$	0.9151	0.3272	Shorter default punishment.
12. Immediate recovery	0.9143	0.3334	History of defaulting immediately erased with prob λ.
13. Proportional ξ	0.9147	0.3841	Default cost is proportional to earnings.
14. Double π^x	0.9120	0.3339	Probability of positive x is doubled.
15. Half x	0.9109	0.3233	Size of expenditure shocks is halved.
16. $\rho_p = 0.93$	0.8452	0.3007	Less persistent income shocks.
17. Half σ_p^2	0.9494	0.2449	Variance of income shocks is halved.

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