

# Doves for the Rich, Hawks for the Poor? Distributional Consequences of Systematic Monetary Policy\*

Nils Gornemann<sup>†</sup>

Keith Kuester <sup>‡</sup>

Makoto Nakajima<sup>§</sup>

July 6, 2021

## Abstract

We build a New Keynesian business-cycle model with rich household heterogeneity. In the model, systematic monetary stabilization policy affects the distribution of income, income risks, and the demand for funds and supply of assets: the demand, because matching frictions render idiosyncratic labor-market risk endogenous; the supply, because markups, adjustment costs, and the tax system mean that the average profitability of firms is endogenous. Disagreement about systematic monetary stabilization policy is pronounced. The wealth-rich or retired tend to favor inflation targeting. The wealth-poor working class, instead, favors unemployment-centric policy. One- and two-agent alternatives can show unanimous disapproval of inflation-centric policy, instead. We highlight how the political support for inflation-centric policy depends on wage setting, the tax system, and the portfolio that households have.

JEL Classification: E12, E21, E24, E32, E52, J64.

Keywords: Monetary Policy, Unemployment, Search and Matching, Heterogeneous Agents, General Equilibrium, Dual Mandate.

---

\*We thank the editor, Gianluca Violante, and three anonymous referees for very valuable comments. We thank seminar participants at numerous conferences and workshops and would also like to thank our discussants Paola Boel, Francesca Carapella, Edouard Challe, Winfried Koeniger, Agnieszka Markiewicz, Leonardo Melosi, Thijs van Rens, Jay Hong, and Arlene Wong. In addition, we thank George Alessandria, Roc Armenter, Adrien Auclert, Christian Bayer, Marcus Hagedorn, Dirk Krueger, Markus Riegler, Jesús Fernández-Villaverde, Morten Ravn, Vincent Sterk, and Víctor Ríos-Rull for their suggestions. The views expressed in this paper are those of the authors. They do not necessarily coincide with the views of the Board of Governors, the Federal Reserve Bank of Philadelphia, or the Federal Reserve System. Kuester gratefully acknowledges support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy – EXC2126/1 – 390838866

<sup>†</sup>Board of Governors of the Federal Reserve System, International Finance Division, Washington, D.C. 20551, email: [nils.m.gornemann@frb.gov](mailto:nils.m.gornemann@frb.gov)

<sup>‡</sup>University of Bonn, Adenauerallee 24-42, 53113 Bonn, Germany, phone: +49-228 73-62195, email: [keith.kuester@uni-bonn.de](mailto:keith.kuester@uni-bonn.de)

<sup>§</sup>(corresponding author) Federal Reserve Bank of Philadelphia, Ten Independence Mall, Philadelphia, PA 19106-1574, email: [makoto.nakajima@phil.frb.org](mailto:makoto.nakajima@phil.frb.org)

# 1 Introduction

Households differ in their wealth and the composition of their sources of income. Different households, thereby, can be exposed to fundamentally different risks and opportunities. Monetary policy, in turn, shapes this profile through its *systematic* response to inflation and unemployment: in the case of the U.S., for example, through the choice of strategies within the confines of the Federal Reserve's dual mandate. We illustrate that accounting for inequality in wealth and sources of income could profoundly affect our view on how much support specific systematic monetary stabilization policies have.<sup>1</sup>

Toward this end, we build a heterogeneous-agent New Keynesian business-cycle model ("HANK," in short) with rich household heterogeneity. The core of the model is "standard" nominal rigidities, search and matching frictions in the labor market, and incomplete financial markets. In this environment, three channels mean that the systematic conduct of monetary stabilization policy can have distributional consequences. First, a more inflation-centric monetary policy not only raises the cyclical nature of unemployment but can also raise *average unemployment*, the exposure to which differs across households. Second, since monetary stabilization policy shapes households' idiosyncratic unemployment risk, it affects the aggregate capital stock and thereby wages and the return to capital through *precautionary savings*. Beyond that, third, the systematic conduct of monetary policy could have the potential to affect the *distribution of income* and, thus, the value of financial assets. One example is precautionary pricing, where in the presence of markup shocks firms choose higher average markups when the central bank seeks to stabilize inflation. Depending on the wage's response, employment alone would fall, the labor share would fall, or both would.

We calibrate the model so as to match key features of the U.S. wealth and income distribution, tax and welfare system, age structure, and the business cycle. We then ask households if they, compared to the baseline, prefer the central bank's policy rate to respond more/less to inflation/unemployment. At the extreme, we look at a natural benchmark: strict inflation targeting. A representative agent ("RANK," henceforth) would disapprove of this policy and would favor a stronger focus on unemployment. In our HANK baseline, instead, 44 percent of households would favor strict inflation targeting. The gains accrue to the wealth-rich and to retirees, whereas the wealth-poor working class loses. A two-agent saver-spender analog (that follows Campbell and Mankiw 1989, "TANK," henceforth) would miss the size of the support for hawkish policy.

More in detail, the HANK model features rich household heterogeneity. Households transition over time between working age and retirement. Working-age households draw labor income, income from financial sources, or unemployment benefits. Search and matching frictions (as in Krusell et al. 2010) render average and cyclical unemployment endogenous to monetary policy (as in Christiano et al. 2016). Wages adjust gradually to shocks. To adequately capture the consumption risk associated with unemployment, we allow for persistent earnings losses upon job loss (following Couch and Placzek 2010, Altonji et al. 2013, Davis and von Wachter (2011)). Next, differences in education imply different exposures to unemployment risk (Cairó and Cajner 2018). Retired households are not exposed to labor-income risk. They receive pensions and supplement them with retirement savings (De Nardi 2004). So as to match further salient features of the heterogeneity in

---

<sup>1</sup>We focus on the welfare consequences of systematic stabilization policy. This sets the paper apart from related work in heterogeneous-household settings with ample heterogeneity, such as McKay and Reis (2016) and Kaplan et al. (2018). Toward the end of the introduction, we provide a detailed overview of how our paper relates to earlier work in the literature.

wealth and income, we allow for transitory idiosyncratic productivity shocks (including temporary transitions to very high income as in Castañeda et al. 2003, Nakajima 2012a) and differences in patience (as in Krusell and Smith 1998, Carroll et al. 2017).<sup>2</sup> Financial markets are incomplete and households save through a mutual fund. This fund exposes households to the effects that systematic monetary policy has on the value of assets.

In our baseline calibration, the strength of the *average unemployment channel* is comparable across the different model environments (HANK/RANK/TANK). What sets the HANK economy apart from its RANK and TANK counterparts, instead, is that households in HANK self-insure against idiosyncratic risk. This gives a role to the interplay of the demand for funds and the supply of assets, a central mechanism of Aiyagari (1994)-type economies.

If hawkish monetary policy raises households' idiosyncratic labor-income risk, households exposed to this should increase *precautionary savings* and, thus, the demand for funds. Consistent with this, we find that a move to inflation-centric policy raises the net worth held by the bottom half of the wealth distribution. Still, upon a move toward inflation-centric policy, the aggregate capital stock falls sharply in the HANK baseline instead of rising, and more so than in RANK/TANK.<sup>3</sup> The key to this, instead, lies with the *supply side* of financial assets.<sup>4</sup> In our baseline, hawkish monetary stabilization policy reduces the labor share. This leads to windfall gains to financial capital and raises the supply of financial assets by value. In RANK/TANK this is of little consequence because the aggregate demand for funds adjusts elastically to supply. In HANK, instead, households save for a reason, making the demand for funds inelastic to changes in returns.<sup>5</sup> With the demand for funds being less elastic than in RANK/TANK, and holding government debt constant (we assume a balanced budget throughout), the supply of assets has to fall to clear the asset market. A fall in the capital stock achieves this.

Through the after-tax *income distribution*, the HANK economy can give rise to distributional concerns beyond productive efficiency. The exact trade-offs in turn depend on particular assumptions, most prominently regarding the wage setting protocol and fiscal policy. The search and matching model allows for a wide range of surplus-sharing rules. This matters because wages allocate changes in average activity between labor and financial capital. Our baseline features a wage rule that is consistent with balanced growth in that wages in the long run move one-to-one with economic activity. If monetary stabilization policy is neutral on employment, changes in aggregate income are shared equally among labor and capital. Instead, if average employment falls, the average labor share falls, too. In the model, retirees are exposed to the windfall gains to capital through retirement savings, but are not exposed to labor income. This is shown in their welfare gains from strict inflation targeting (equivalent to 0.3 percent of lifetime consumption). The aggregate consequences as regards the labor share and the real interest rate (which, respectively, fall by

---

<sup>2</sup>The literature finds that household savings are only in part driven by precautionary motives, for example, Hurst et al. (2010); and a still smaller share will be driven by cyclical risk. For the quantitative results of the paper, it matters that households have reasons to save beyond business-cycle risk. Necessarily we cannot model these reasons in much detail.

<sup>3</sup>A move toward strict inflation targeting, for example, reduces the capital stock by roughly 2 percent in HANK, and only by about 0.4 percent in RANK/TANK.

<sup>4</sup>That a change in business-cycle characteristics can affect the aggregate capital stock and welfare through the demand for funds is well-known and discussed, for example, in Krusell et al. (2009). Our paper adds to this a supply-side channel.

<sup>5</sup>Many of the reasons to save that we entertain are not related to the business cycle in the first place (aging, bequests, fluctuations in residual income risk); see Footnote 2.

0.1 percentage point and rise by 5bps annualized) are small, instead, and might easily go unnoticed.

Rather than providing a final verdict on potential winners and losers from a monetary stabilization policy, the current paper has a much more modest aim. We seek to highlight that systematic monetary stabilization policy, through affecting the distribution of average incomes and income risks, could have sizable distributional consequences. We show that assumptions matter for both the size and the sign of the gains from monetary stabilization policies. When we assume, for example, that wages are set such that the long-run labor share remains constant by design, employment falls by more. But so does the return to capital. Households, then, share more evenly in the fall of productive activity. The pattern of losses is also affected. Poor households would lose least from hawkish monetary policy (we keep the welfare and progressive tax system in place), the middle class most.

The rest of the paper is organized as follows. Next, we review the literature. Section 2 introduces the model. Section 3 highlights the calibration and business-cycle implications of household heterogeneity. Section 4 discusses the welfare effects of a switch to a different systematic monetary policy. The same section discusses optimal simple monetary rules for different segments of the population. So as to corroborate the results and mechanisms, Section 5 provides sensitivity analysis, namely, with regard to wage setting, with regard to fiscal policy, and with regard to the structure of household portfolios. A final section concludes. An extensive (online) appendix provides further details.

## Relation to the literature

One can think about the distributional effects of monetary policy in different ways. One stream of the literature considers the distributional effects of surprise inflation. Doepke and Schneider (2006b) document that differences in portfolios negatively expose wealth-rich retirees to surprise inflation, whereas the young mortgaged middle class gains from surprise inflation. Doepke and Schneider (2006a), Meh et al. (2010), and Sterk and Tenreyro (2018) focus on modeling the aggregate effects of such wealth redistributions under flexible prices. Our paper, instead, thinks about *systematic* monetary stabilization policy rather than one-sided shocks. To make this clear, we fix the inflation target throughout. Instead, the disagreement among households in our setting comes from exposure to the real effects of systematic monetary policy: the valuation gains, the labor-market response, and fiscal effects.

Another stream of the literature considers the distributional effects of one-time shocks to the policy rate: “monetary shocks.” This literature has seen strong growth in recent years, so we provide a selective overview only. This “HANK” literature emphasizes, in different guises, that the effect of inequality on monetary transmission in the aggregate crucially depends on how the shock affects households along the distribution of marginal propensities to consume (MPC, henceforth). This renders the response of labor and financial after-tax income and income risks central. Kaplan et al. (2018) highlight the importance of disposable income (shaped by fiscal policy) as opposed to intertemporal substitution.<sup>6</sup> Several papers provide insights into environments that do not have self-insurance in equilibrium (zero-liquidity). Ravn and Sterk (forthcoming) emphasize that countercyclical income risk makes the natural rate of interest fall in recessions, which deepens recessions if monetary policy does not adjust. Bilbiie (2020) shows that if the income of high-MPC households is procyclical, this provides further amplification. Acharya and Dogra (2020) provide an

---

<sup>6</sup>In their case, a large share of households tend to be wealthy, but invested in an illiquid asset. Our model’s average MPC is of a size comparable to theirs, but it originates from impatience rather than modeling liquidity.

important perspective in a tractable environment with CARA utility. Broer et al. (2019) show that real wage rigidity, which we have, makes profits procyclical even in a New Keynesian model without capital. This dampens wealth effects on labor supply (from which we abstract), and in their case ensures a reasonable monetary transmission channel. Relative to all these papers, what sets our contribution apart is that we study *systematic* monetary stabilization policy rather than the effect of monetary shocks. We do so in an environment in which valuation effects can be important, and we show that households' relative exposure to these can cause quantitatively meaningful disagreement about systematic stabilization policy.

Throughout, we compare the positive and normative implications of our HANK economy to RANK and TANK analogs. On the positive side, Debortoli and Gal (2017) conclude that TANK approximates the positive implications of a simpler HANK well, once HANK and TANK are calibrated to have comparable shares of borrowing-constrained/spender households. When we follow this strategy, TANK misses the sizable policy support for inflation targeting. Liquidity constraints are the wrong calibration target for our exercise. What matters for a household's evaluation of systematic monetary policy is the relative exposure to valuation gains and labor-market risk.<sup>7</sup>

A growing literature is concerned with optimal monetary policy in an incomplete-market setting. In a zero-liquidity economy, Challe (2020) argues that interest rates should be more accommodative in recessions since the desire to do precautionary saving reduces the natural rate of interest. Berger et al. (2019) look at a zero-liquidity environment in which layoffs have permanent scarring effects on human capital. They find that monetary policy should primarily focus on stabilizing unemployment. Our model also allows for long-term earnings losses. Still, households are not uniformly in favor of unemployment stabilization. One reason for this may be that our modeling of earnings losses is more rudimentary, and earnings losses are not permanent; another difference may originate from the fact that households in our model can self-insure in the first place. Another common thread in this literature is that monetary policy can provide consumption insurance *ex post*. Acharya et al. (2020) provide closed-form intuition for a HANK economy with CARA utility and self-insurance, but absent borrowing constraints. With countercyclical income risk, in response to productivity shocks they find that monetary policy should stabilize output more. This reduces the spread of consumption inequality in a recession. Comparably, Bhandari et al. (forthcoming) show the optimal monetary response to positive price markup shocks. In their setting, these shocks are distributed from labor earnings to dividends. If monetary policy is accommodative for a short time, it provides income insurance, and partly undoes the rise in price markups. Two dimensions set our work apart from the aforementioned papers. First, we look at a discrete set of simple, systematic monetary policy rules rather than a response that separates between shocks in real time. Second, we conduct our welfare assessment explicitly *not* under the veil of ignorance. Rather, we ask households at their current state of wealth, income, and employment – and thus, at their current exposure to systematic monetary policy changes – what policy they would prefer.

Our paper stands on the shoulders of a large and influential stream of research that emphasizes the inflation-unemployment trade-off in the New Keynesian model with search and matching frictions. Prominent examples in this literature are Faia (2009), Blanchard and Galí (2010), and Ravenna and Walsh (2011). This literature stresses that deviations from the Hosios (1990) condition, such as those caused by workers' "excessive" bargaining power or wage rigidities, can lead to

---

<sup>7</sup>Other important papers that discuss how inequality affects monetary transmission or the business cycle are McKay and Reis (2016), Auclert (2019), McKay et al. (2016), and Bayer et al. (2020), to name but a few.

the inefficient amplification of employment responses, which can induce monetary policy to deviate from price stability in spite of price adjustment costs (including when shocks are productivity shocks). Ravenna and Walsh (2012) show that such deviations can lead to quantitatively meaningful welfare gains, as they do in our setting. With Sala et al. (2008), who present an estimated New Keynesian model with search and matching frictions for the U.S. economy, our calibration shares the idea that most of the inflation-unemployment stabilization trade-off arises from markup shocks. The RANK counterpart to our model, indeed, has monetary policy balance inflation and unemployment variability. What sets our work apart is that we focus on the potential for disagreement about monetary stabilization policy, when conditioning on a household’s current idiosyncratic state. We show that the disagreement can be large and that heterogeneity in savings plays an important role in this disagreement.

In terms of technique, we extend the perturbation method developed by Reiter (2009) and Reiter (2010a) to compute a second-order approximation with a parameterized law of motion for the distribution of households. The technique allows us to explicitly control the policy counterfactuals so that the average inflation rate remains constant. And it allows us to compute the transition path toward the new stochastic steady state.

## 2 Model

There is a unit mass of infinitely lived households. Households receive labor income, social security transfers, and financial income. Idiosyncratic employment risk fluctuates due to Mortensen and Pissarides (1994) search and matching frictions. The prices of goods are sticky. The central bank can, therefore, influence real activity and the distribution of employment risk over the business cycle. A household that loses its job faces the risk of persistent earnings losses. Households save to self-insure against idiosyncratic and aggregate risk, and they save for retirement, modeled as a transitory state such that the household no longer works but receives retirement benefits.

### 2.1 States

The model is defined in recursive form. The economy inherits from the previous period the aggregate capital stock,  $K_{-1}$ , and last period’s level of wages, investment, and the central bank’s policy rate,  $w_{-1}, i_{-1}, R_{-1}$ . Next, the economy inherits the type distribution of households from the previous period,  $\mu_{-1}$ . Let  $\zeta$  be the vector of aggregate shocks.

For the decisions of firms and households during the period, the notation entertains two different state vectors. A tilde marks the time after aggregate shocks have been realized, but before employment-related transitions (separations, hiring, and earnings losses) have occurred. Let  $\tilde{X} = (K_{-1}, w_{-1}, i_{-1}, R_{-1}, \zeta, \tilde{\mu})$  denote the state of the economy at that time.  $X = (K_{-1}, w_{-1}, i_{-1}, R_{-1}, \zeta, \mu)$ , in turn, marks the state of the economy once employment-related transitions have occurred. This is the state of the economy on which production and consumption decisions are based.

#### 2.1.1 Shocks

Vector  $\zeta := (\zeta_I, \zeta_R, \zeta_{TFP}, \zeta_w, \zeta_P)$  collects the five aggregate business-cycle shocks.  $\zeta_I$  is a shock to the marginal efficiency of investment and  $\zeta_R$  a monetary policy (interest-rate) shock.  $\zeta_{TFP}$  is a productivity shock,  $\zeta_w$  a wage markup shock, and  $\zeta_P$  a price markup shock. These shocks are the most common business cycle shocks.<sup>8</sup> Each shock follows an AR(1)-process with normally

---

<sup>8</sup>Christiano et al. (2016) identify the first three. Smets and Wouters (2007), in addition, identify a “wage-markup” shock and a “price-markup” shock as important.

distributed innovation

$$\log(\zeta'_j / \bar{\zeta}_j) = \rho_{\zeta_j} \log(\zeta_j / \bar{\zeta}_j) + \epsilon'_{\zeta_j}, \epsilon_{\zeta_j} \stackrel{\text{iid}}{\sim} N(0, \sigma_{\zeta_j}^2), \rho_{\zeta_j} \in [0, 1].$$

Here and in the following, a bar over a variable refers to the variable's value in the non-stochastic steady state, and a prime marks the next period.

### 2.1.2 Individual states

Household heterogeneity can be summarized by six transitory idiosyncratic states  $(n, a, l, e, b, s)$ . The first three  $(n, a, l)$  are affected by the business cycle, and so are endogenous to monetary policy. The last three  $(e, b, s)$ , instead, evolve independently of the business cycle.  $n \in \{0, 1\}$  denotes the household's employment state,  $a \in [0, 1]$  marks the household's holdings of shares of a representative mutual fund, and  $l \in \{0, 1\}$  the household's earnings-loss state.  $e \in \{e_L, e_H\}$  marks the household's education level (high or low).  $b \in \{0, 1\}$  marks the household's impatience.  $s \in \mathcal{S}$  marks an exogenous component of a household's current labor productivity ("skills").

Transitory labor productivity  $s$  follows a first-order Markov process with  $s \in \mathcal{S} = \{s_0, s_1, s_2, s_3\}$ .  $\pi_S(s, s')$  denotes the probability of a transition from  $s$  to  $s'$ . Skill state  $s_0 = 0$  is associated with retirement: the household does not work but receives retirement benefits. If  $s \in \mathcal{S}_+ := \mathcal{S} \setminus s_0$ , the household is in the labor force and  $s$  captures differences in productivity after conditioning on education and the household's employment history. The household draws a fresh  $s$  at the beginning of every period. This is the case regardless of the current employment status. The probability of retiring is the same for each skill state  $s \in \mathcal{S}_+$ , so that  $\pi_S(s, s_0) = \pi_{s_0}$  for each  $s \in \mathcal{S}_+$ . Each period, a retired household returns to the labor force (is "born") with probability  $\pi_S(s_0, s')$ ,  $s' \in \mathcal{S}_+$ .

A household draws the education level each time it transitions from  $s = s_0$  to  $s \in \mathcal{S}_+$  (that is, at the beginning of a household's working life), and only then.  $\pi_E(e, e')$  marks the probability of moving from education level  $e$  to education level  $e'$ . We allow for a correlation between  $e$  and  $e'$  so as to capture intergenerational persistence in income. The risk of a lower education status upon birth means that highly educated retired households have an incentive to retain savings.

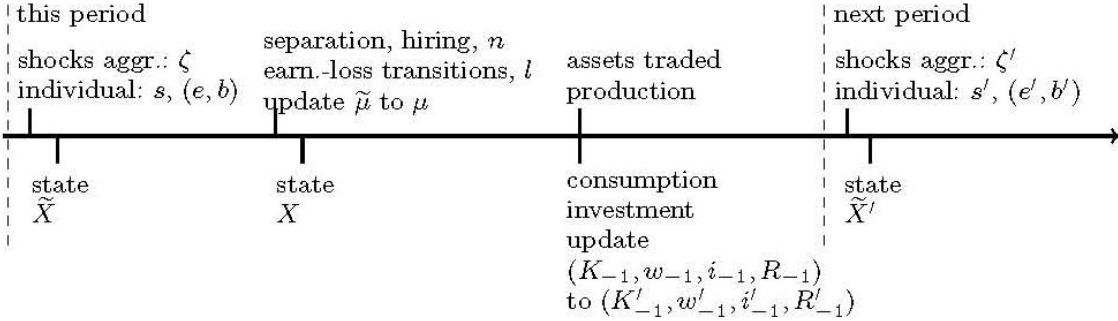
Let  $\beta(e, b)$  mark the household's time discount factor. Time preferences depend on education and the impatience state  $b$ . A household draws  $b$  every time that the education level is drawn (and only then).  $\pi_{\Delta_\beta}(b)$  marks the probability of drawing impatience state  $b$ . Conditional on education, with probability  $\pi_{\Delta_\beta}(0)$  the household will have time preference  $\beta(e, b) = \beta_e$  (with  $\beta_e \in \{\beta_{e_L}, \beta_{e_H}\}$ ); otherwise the household has time preference  $\beta_e - \Delta_\beta$  ( $b = 1$ ). We assume that both education groups have the same share of rather impatient households, and the same gap in patience  $\Delta_\beta$ .<sup>9</sup>

It remains to specify the evolution of the endogenous individual states  $(n, a, l)$ . Share holdings  $a$  are determined by the savings behavior of the household (the household's optimization problem is described in Section 2.3). For households in the labor force, the evolution of the employment state  $n$  is governed by the search and matching structure of the model, also described in Section 2.3.  $l$  captures an earnings loss. When the household is employed, its idiosyncratic productivity is given by the product  $e \cdot s \cdot (1 - \varrho l)$ . Parameter  $\varrho$  in  $[0, 1]$  measures the size of the earnings loss ( $l = 1$ ). We are agnostic about the microeconomic source of the loss of earnings, be it a temporary loss of skills or temporarily poorer match quality. The earnings-loss state evolves with the household's employment history.  $\pi_L^{uem}(1)$  is the probability of suffering an earnings loss when moving from

---

<sup>9</sup>Time-discount factors depend on education. We use this to match the wealth distribution by education. Heterogeneity in discount factors within an education group is used to match the low net worth of the poor.

**Figure 1: Timing of decisions**



unemployment to employment.  $\pi_L^{emp}(l, l')$  is the probability of the earnings loss state changing from  $l$  to  $l'$  if the household enters the period employed. Households change their earnings-loss state after employment transitions have occurred.

The mass of households that are born, by construction, equals that of retiring households. After having drawn the education state, the newborn household draws states  $n$  and  $l$  such that the mass of households of type  $(n, l, e)$  is not affected by transitions to and from retirement. Section 2.3.3 provides details.

Let  $\mu(n, a, l, e, b, s)$  mark the type distribution of households at the time that production takes place, that is, after all idiosyncratic transitions have taken place.  $\mu$  has support on  $\mathcal{M} := \{0, 1\} \times [0, 1] \times \{0, 1\} \times \{e_L, e_H\} \times \{0, 1\} \times \mathcal{S}$ .<sup>10</sup>

### 2.1.3 Employment transitions

We assume that job-finding rates  $f(\tilde{X})$  are the same for *all* unemployed households. Flow rates into unemployment, instead, depend on education. To accommodate this in a parsimonious way, we proceed as follows. Hiring decisions in the model will be made by firms. The common-to-all job-finding rate will fluctuate over the business cycle. Let  $\lambda(e)$  be the (constant) probability that a firm and household separate. We split this rate in two:  $\lambda(e) = \lambda_x(e) + \lambda_n(e)$ . At rate  $\lambda_x(e)$ , households flow directly into the unemployment pool for the period. At rate  $\lambda_n(e)$  the household can search for a job in the same period. If successful, the household will not go through an unemployment spell. Otherwise, the household will be unemployed. By choosing  $\lambda_x(e)$  and  $\lambda_n(e)$ , we can control the cyclical fluctuation of the risk of becoming unemployed.<sup>11</sup>

## 2.2 Timing

The timing is shown in Figure 1. At the beginning of each period, households draw new skills  $s$ . If a household is born (a transition from  $s_0$  to some  $s \in \mathcal{S}_+$ ), the household draws an education level from  $\pi_E(e, e')$  and time preferences  $b \in \{0, 1\}$ . A household that is born is randomly assigned to states  $n$  and  $l$  in such a way that it replaces a retiring household with these employment and

<sup>10</sup>Only some combinations of idiosyncratic states are admissible. We consider all retired households ( $s = s_0$ ) as unemployed ( $n = 0$ ). Only the employed ( $n = 1$ ) can be subject to skill loss ( $l = 1$ ).

<sup>11</sup>There is an alternative large-firm interpretation of our setup. Namely, one may think of the  $\lambda_n(e)$ -type separations as including cases in which firms have to expend costs in order to make an existing match fit a changing job profile.

earnings-loss characteristics and the same education level.<sup>12</sup> Aggregate shocks are drawn. The tilde marks the time at the beginning of the period after all those shocks have been realized, but before employment transitions (separations and hiring) have occurred. Denote by  $\tilde{\mu}$  the corresponding type distribution at that point in time. Let  $\tilde{X} = (K_{-1}, w_{-1}, i_{-1}, R_{-1}, \zeta, \tilde{\mu})$  denote the corresponding state of the economy. Before production takes place, firms separate from a household of education  $e$  with probability  $\lambda_x(e) + \lambda_n(e)$ . Thereafter, the employed with earnings losses shed those with probability  $\pi_L^{emp}(1, 0)$ . Then, firms post vacancies. A share  $\lambda_n(e)/(\lambda_x(e) + \lambda_n(e))$  of the separated households of education level  $e$  search for a new job in the same period, as do the unemployed. All other separations flow directly into the unemployment pool for the period. Matching takes place. Households hired out of unemployment face the earnings-loss probability  $\pi_L^{uem}(1)$ . Accounting for the employment transitions, and subsequent transitions in the earnings-loss state, the aggregate state becomes  $X = (K_{-1}, w_{-1}, i_{-1}, R_{-1}, \zeta, \mu)$ , where  $\mu$  marks the type distribution at the time of production. Then the remaining decisions are made and firms produce.

## 2.3 Households' problems

Household preferences are time-separable with education- and shock-dependent time-discount factor  $\beta(e, b) \in (0, 1)$ . Households derive utility from consumption,  $c$ . Period felicity is given by  $u(c) = c^{1-\sigma}/(1 - \sigma)$ ,  $\sigma > 0$ . In addition, retired households derive utility from leaving bequests to the newborn upon “death.” The utility from leaving a bequest of  $a$  shares worth  $p_a(X)a$ , conditional on death, is  $\gamma_1 \cdot (p_a(X)a + \gamma_2)^{1-\sigma}/(1 - \sigma)$ , where  $\gamma_1, \gamma_2 \geq 0$ . The approach and functional form for this warm-glow utility of bequest follow De Nardi (2004).<sup>13</sup> Government consumption enters household preferences in an additively separable way. Since it is held constant throughout the paper, we do not model this part of preferences. We first describe the problem of a household that is employed after the employment transitions have taken place. Thereafter, we describe the problem of an unemployed household. Last, we describe the problem of a household that is retired.

### 2.3.1 Employed households

Let  $W(X, n, a, l, e, b, s)$  be the value of a household at the time of production. The employed household's Bellman equation ( $n = 1$ ,  $s \in \mathcal{S}_+$ ) is given by

$$\begin{aligned} W(X, 1, a, l, e, b, s) = & \max_{c, a' \geq 0} \left\{ u(c) + \pi_{s_0} \beta(e, b) \mathbb{E}_\zeta [W(X', 0, a', 0, e, b, s_0)] \right. \\ & + \sum_{s' \in \mathcal{S}_+} \pi_S(s, s') \beta(e, b) \mathbb{E}_\zeta \left[ [1 - \lambda_x(e) - \lambda_n(e)(1 - f(\tilde{X}'))] \sum_{l'} \pi_L^{emp}(l, l') W(X', 1, a', l', e, b, s') \right. \\ & \left. \left. + [\lambda_x(e) + \lambda_n(e)(1 - f(\tilde{X}'))] W(X', 0, a', 0, e, b, s') \right] \right\} \\ \text{s.t. } & (1 + \tau_c)c + p_a(X)a' = [p_a(X) + d_a(X)]a \\ & + w(X)es(1 - l\varrho) [1 - \tau_{RET} - \tau_{UI} - \tau(X, w(X)es(1 - l\varrho))] . \end{aligned}$$

The household chooses consumption and non-negative share holdings. On the right-hand side of the Bellman equation appear period felicity and the continuation values. Next period, the household will enter retirement with probability  $\pi_{s_0}$ , carrying with it its asset holdings and education

<sup>12</sup>We assume that households begin their working life at age 25. The assumptions above ensure that the households at that age have reasonable employment rates.

<sup>13</sup> $\gamma_1$  can be thought to control the strength of the bequest motive, while  $\gamma_2$  determines how much of a luxury good giving a bequest is.

status, the household's value being  $W(X', 0, a', 0, e, b, s_0)$  (first row).<sup>14</sup> Otherwise, the household will remain in the labor force at newly drawn skill state  $s'$  (second row). The expectation operator  $\mathbb{E}_\zeta$  marks expectations formed with regard to aggregate shocks. Conditional on not retiring next period, with probability  $1 - \lambda_x(e) - \lambda_n(e)(1 - f(\tilde{X}'))$ , the household will be employed. The household draws a new idiosyncratic earnings-loss state  $l'$ , with  $\pi_L^{emp}(l, l')$  marking the transition probability of the earnings-loss state for an employed household. The household's value then is  $W(X', 1, a', l', e, b, s')$ . Otherwise, the household will move into unemployment (third row), with associated value  $W(X', 0, a', 0, e, b, s')$ .

As per the budget constraint, the household buys consumption goods,  $c$ , pays consumption tax  $\tau_c$ , and purchases shares at cost  $p_a(X)a'$ . On the income side, the household has the cum-dividend value of shares brought into the period (first row) and labor earnings,  $w(X)es(1-l\varrho)$ , net of taxation (second row).  $w(X)$  is the real wage per efficiency unit of labor and  $\varrho \in [0, 1]$  is the loss of earnings associated with the earnings-loss state. Three types of taxes are applied to earnings: social security taxes,  $\tau_{RET}$ , and unemployment-insurance taxes,  $\tau_{UI}$ , as well as a progressive labor-income tax ( $\tau(X, .)$ ).

### 2.3.2 Unemployed households

The unemployed household's Bellman equation ( $n = 0, s \in \mathcal{S}_+$ ) is given by

$$W(X, 0, a, 0, e, b, s) = \max_{c, a' \geq 0} \left\{ u(c) + \pi_{s_0} \beta(e, b) \mathbb{E}_\zeta [W(X', 0, a', 0, e, b, s_0)] \right. \\ \left. + \sum_{s' \in \mathcal{S}_+} \pi_S(s, s') \beta(e, b) \mathbb{E}_\zeta \left[ f(\tilde{X}') \sum_{l'} \pi_L^{uem}(l') W(X', 1, a', l', e, b, s') \right. \right. \\ \left. \left. + (1 - f(\tilde{X}')) W(X', 0, a', 0, e, b, s') \right] \right\}$$

$$\text{s.t. } (1 + \tau_c)c + p_a(X)a' = [p_a(X) + d_a(X)]a + b_{UI}(es)[1 - \tau(X, b_{UI}(es))].$$

With probability  $\pi_{s_0}$ , the household moves into retirement (first row). Otherwise, next period, the household will move into employment with state-dependent probability  $f(\tilde{X}')$  (second row). Upon reemployment, with probability  $\pi_L^{uem}(1)$ , the household will suffer an earnings loss, or else no earnings loss. If the household does not find a new job, it will stay unemployed next period (last row). As per the budget constraint, instead of labor earnings the unemployed household receives unemployment benefits,  $b_{UI}(es)$ . They are assumed to depend on the household's earnings capacity. This is meant to capture, in a parsimonious way, that benefits depend on past earnings.

---

<sup>14</sup>In terms of notation all households that enter retirement or unemployment are moved to the no-earnings-loss state  $l = 0$ . This is without consequence: Retired households do not have labor income; in addition, unemployment insurance benefits do not depend on the earnings-loss state. Unemployed households redraw the earnings-loss state upon moving to employment. The transition of the aggregate and idiosyncratic states that are not affected by the household's decisions is described in Section 2.1.

### 2.3.3 Retired households

The retired household's Bellman equation ( $s = s_0$ ) is given by

$$\begin{aligned}
W(X, 0, a, 0, e, b, s_0) = \max_{c, a' \geq 0} & \left\{ u(c) + \pi_S(s_0, s_0) \beta(e, b) \mathbb{E}_\zeta [W(X', 0, a', 0, e, b, s_0)] \right. \\
& + (1 - \pi_S(s_0, s_0)) \mathbb{E}_\zeta [\gamma_1(p_a(X')a + \gamma_2)^{1-\sigma} / (1 - \sigma)] \\
& + \beta(e, b) \sum_{s' \in \mathcal{S}_+} \sum_{e'} \sum_{b'} \sum_l \pi_S(s_0, s') \pi_E(e, e') \pi_{\Delta_\beta}(b') \Pr(n = 1, l | X, e') \\
& \mathbb{E}_\zeta \left[ [1 - \lambda_x(e') - \lambda_n(e')(1 - f(\tilde{X}'))] \sum_{l'} \pi_L^{emp}(l, l') W(X', 1, a', l', e', b', s') \right. \\
& \quad \left. + [\lambda_x(e') + \lambda_n(e')(1 - f(\tilde{X}'))] W(X', 0, a', 0, e', b', s') \right] \\
& + \beta(e, b) \sum_{s' \in \mathcal{S}_+} \sum_{e'} \sum_{b'} \pi_S(s_0, s') \pi_E(e, e') \pi_{\Delta_\beta}(b') \Pr(n = 0 | X, e') \\
& \mathbb{E}_\zeta \left[ f(\tilde{X}') [\pi_L^{uem}(1) W(X', 1, x', 1, e', b', s') + \pi_L^{uem}(0) W(X', 1, x', 0, e', b', s')] \right. \\
& \quad \left. \left. + [1 - f(\tilde{X}')] W(X', 0, a', 0, e', b', s') \right] \right\} \\
\text{s.t. } & (1 + \tau_c)c + p_a(X)a' = [p_a(X) + d_a(X)]a + b_{RET}(e)[1 - \tau(X, b_{RET}(e))].
\end{aligned}$$

The first row describes that next period the household will stay in retirement with probability  $\pi_S(s_0, s_0)$ . The following rows concern a household that is born (joins the labor force out of retirement). Upon "death," the household receives utility from a bequest (second row). The newly born household then draws new idiosyncratic skills,  $s$ , and also redraws the education level. In terms of employment and earnings-loss status ( $n, l$ ), we assume that the newborn household randomly replaces a retiring household. This is reflected by the terms  $\Pr(n, l | X, e')$ . These transitions are captured in the third row. The fourth and fifth rows concern a household that is employed after birth. As before, that household may remain employed or lose the job during the next period. The final three rows concern a household that is born unemployed at the beginning of next period. The household may find a new job and become employed or may not find a job. The budget constraint is the same as for the unemployed, but features retirement benefits  $b_{RET}(e)$  instead of unemployment benefit payments.

In order to define the transition probabilities  $\Pr$ , let  $N(X, l, e)$  mark the mass of employed households with earnings-loss state  $l$  and education level  $e$ , and let  $U(X, e)$  mark the mass of households of the same education level that are unemployed, all measured during the production stage of this period.<sup>15</sup> Then, for any  $l \in \{0, 1\}, e' \in \{e_L, e_H\}$

$$\Pr(n = 1, l | X, e') := \frac{N(X, l, e')}{\sum_l N(X, l, e') + U(X, e')},$$

and

$$\Pr(n = 0 | X, e') := \frac{U(X, e')}{\sum_l N(X, l, e') + U(X, e')}.$$

---

<sup>15</sup> So that  $N(X, l, e) = \sum_{s \in \mathcal{S}_+} \int_x d\mu(1, x, l, e, s)$  and  $U(X, e) := \sum_{s \in \mathcal{S}_+} \int_x d\mu(0, x, 0, e, s)$ .

## 2.4 Non-financial firms

Non-financial firms are owned by competitive mutual funds. The funds discount the future using discount factor  $Q(X, X')$ . The funds and the discount factor are described in Section 2.5. There is a unit mass of producers of differentiated intermediate goods, indexed by  $j \in [0, 1]$ . These are subject to nominal rigidities. Intermediate goods are both used directly in the production of final consumption and investment goods, and expended as costs for adjusting prices and employment. Since all firms in the economy are owned by the household sector (through shares in the mutual funds), all profits flow to households.

### 2.4.1 Final goods

There is a representative competitive final goods firm that transforms differentiated intermediate goods into homogeneous final goods. Let  $X_p := (X, \eta_p)$  be state  $X$  augmented by the distribution  $\eta_p$  of last period's prices,  $P_{j,-1}$ , across differentiated goods firms.<sup>16</sup> Final goods can be used for personal consumption expenditures, government consumption, and physical investment. The firm solves

$$\begin{aligned} & \max_{y_f, (y_{f,j})_{j \in [0,1]}} (1 - \tau_d) \left( P(X_p) y_f - \int_0^1 P_j(X_p) y_{f,j} dj \right) \\ \text{s.t. } & y_f = \left( \int_0^1 y_{f,j}^{\frac{\vartheta \cdot \exp\{\zeta_P\} - 1}{\vartheta \cdot \exp\{\zeta_P\}}} dj \right)^{\frac{\vartheta \cdot \exp\{\zeta_P\}}{\vartheta \cdot \exp\{\zeta_P\} - 1}}, \end{aligned}$$

where parameter  $\vartheta > 1$  marks the elasticity of demand.  $\zeta_P$  is a shock to the elasticity of demand that directly affects price-setting firms' markups (a “price-markup shock”).  $y_f$  marks output of final goods.  $P_j(X_p)$  marks the price of differentiated input  $j$  and  $y_{f,j}$  the quantity demanded of that input by final goods firms.  $P(X_p)$  is the consumer price index.

### 2.4.2 Intermediate inputs

For the sake of exposition, we assume that different activities are conducted by different firms. Next to final goods firms, there are firms that produce intermediate inputs: homogeneous labor services, capital services, and adjustment services, as well as differentiated intermediate goods that are used in final good production. A setting in which the producers of differentiated intermediate goods make all the related decisions would be isomorphic; see Appendix A.

**Differentiated goods producers.** There is a unit mass of producers of differentiated goods. Producer  $j \in [0, 1]$  produces type  $j$  of the good. Differentiated goods are sold in monopolistically competitive markets. Producers face Rotemberg (1982) quadratic price adjustment costs. Dividends are taxed at a fixed rate  $\tau_d$ . The value of the producer of variety  $j$  (after taxes) is

$$\begin{aligned} J_D(X_p; j) = & \max_{P_j, \ell_j, k_j} (1 - \tau_d) \left( y_j(X, P, P_j) \left( \frac{P_j}{P(X_p)} \right) - r(X) k_j - h(X) \ell_j - \Xi \right. \\ & \left. - \frac{\psi}{2} \left( \frac{P_j}{P_{j,-1}} - \bar{\Pi} \right)^2 y(X) \right) + \mathbb{E}_\zeta [Q(X, X') J_D(X'_p; j)] \end{aligned}$$

---

<sup>16</sup>In equilibrium, all differentiated goods producers will set the same price. Therefore, in equilibrium,  $X$  describes the state of the economy. Anticipating this, in much of the exposition we use  $X$  to index the state of the economy, rather than  $X_p$ . We use  $X_p$  whenever necessary for clarity.

$$\text{s.t. } y_j(X, P_j, P(X_p)) = \zeta_{TFP} k_j^\theta \ell_j^{1-\theta}, \quad (1)$$

$$y_j(X, P_j, P(X_p)) = \left( \frac{P_j(X_p)}{P(X_p)} \right)^{-\vartheta \cdot \exp\{\zeta_P\}} y(X). \quad (2)$$

After setting price  $P_j$ , producer  $j$  faces demand  $y_j(X, P_j, P(X_p))$ , where  $P(X_p)$  marks the aggregate price level. In order to meet demand, producer  $j \in [0, 1]$  rents capital and labor services  $k_j$  and  $\ell_j$  at the competitive rates  $r(X)$  and  $h(X)$  (first line).  $\Xi > 0$  is a fixed cost of production. Price adjustment is costly. In order to adjust the price by more or less than the steady-state inflation rate,  $\bar{\Pi}$ , the producer has to buy adjustment services (second line). Parameter  $\psi > 0$  indexes the extent of nominal rigidities. In terms of constraints, equation (1) is the production function of differentiated good  $j$ , with  $\theta \in (0, 1)$ . Constraint (2) is the demand function, where  $y(X)$  is total demand for differentiated goods. In equilibrium, all differentiated goods producers face the same marginal costs and will, therefore, set the same price and choose the same amount of labor and capital inputs, so that  $k_j$  and  $\ell_j$  will be identical for all firms  $j$ .

**Labor services.** Labor services are homogeneous. They are intermediated by employment agencies, which operate under constant returns to scale. The value of a household to the employment agency depends on the household's characteristics  $(l, e, s)$ . It is given by

$$\begin{aligned} J_L(X, l, e, s) &= (1 - \tau_d)[h(X) - w(X)] \cdot es(1 - \varrho l) \\ &\quad + \sum_{s' \in \mathcal{S}_+} \pi_S(s, s' | s' \neq s_0) (1 - \lambda_x(e) - \lambda_n(e)) \mathbb{E}_\zeta [Q(X, X') \sum_{l'} \pi_L^{emp}(l, l') J_L(X, l', e, s')]. \end{aligned}$$

A household with characteristics  $l, e, s$  produces  $es(1 - \varrho l)$  units of labor services, which the agency sells at competitive price  $h(X)$  to producers of differentiated goods. Per efficiency unit of labor, the agency pays a real wage of  $w(X)$ . The remaining lines concern the continuation value. The household has transitions in temporary skills  $s$ . A household leaving the agency into retirement will immediately be replaced by a “newborn” household of the same payoff-relevant characteristics for the firm. At the same time, the household may separate into unemployment with probability  $\lambda_x(e) + \lambda_n(e)$ . If not, the household remains with the agency at next period's production stage, has an earnings-loss transition from  $l$  to  $l'$ , and provides value  $J_L(X, l', e, s')$  to the agency.

After separations have occurred, and before production, employment agencies can recruit new households. Let  $V(\tilde{X})$  be the aggregate number of vacancies posted and  $M(\tilde{X}, V)$  the mass of new matches. The job-filling probability is identical for all vacancies, and given by  $q(\tilde{X}) = \frac{M(\tilde{X}, V(\tilde{X}))}{V(\tilde{X})}$ . Letting  $\kappa(\tilde{X})/q(\tilde{X})$  be the average cost per hire, the free-entry condition for recruiting is given by

$$\begin{aligned} &\sum_{e, s \in \mathcal{S}_+} \pi_S(s | s \in S_+) \frac{U(\tilde{X}, e)}{\sum_e [U(\tilde{X}, e) + \lambda_n(e) \sum_l N(\tilde{X}, l, e)]} \sum_{l'} \pi_L^{uem}(l') J_L(X, l', e, s) \\ &\quad + \sum_{e, l, s \in \mathcal{S}_+} \pi_S(s | s \in S_+) \frac{\lambda_n(e) N(\tilde{X}, l, e)}{\sum_e [U(\tilde{X}, e) + \lambda_n(e) \sum_l N(\tilde{X}, l, e)]} \sum_{l'} \pi_L^{emp}(l, l') J_L(X, l', e, s) \\ &\quad = (1 - \tau_d) \kappa(\tilde{X}) / q(\tilde{X}). \end{aligned}$$

In equilibrium, recruiting will occur until the expected gain of a hire (left-hand side) equals the average after-tax cost per hire. The gain is given by the expected value of a household to the

employment agency, accounting for the distribution of household characteristics in the pool of households searching for employment, and their subsequent earnings-loss transitions. The pool of searching households is composed of the unemployed and of those households that were separated from their firm in the same period and they look for new employment in the same period.

Recruiting requires purchasing adjustment services. Following Christiano et al. (2016), we shall assume that there are two components to the cost of recruiting: a cost per hired household and a cost of posting a vacancy:

$$\kappa(\tilde{X}) := (\kappa_H \cdot q(\tilde{X}) + \kappa_v) \cdot \left( \frac{M(\tilde{X}, V(\tilde{X})) / \left( \sum_{l,e} N(\tilde{X}, l, e) \right)}{\bar{M}/\bar{N}} \right)^2.$$

Here  $\bar{M}$  and  $\bar{N}$  mark steady-state values of matches and employment.  $\kappa_H$  marks the steady-state cost upon hiring.  $\kappa_V$  marks the steady-state cost for posting a vacancy. Both of these costs fluctuate with the hiring rate in the economy as reflected by the quadratic term.<sup>17</sup>

Matches emerge according to the following matching function (see den Haan et al. (2000)), which links the mass of households searching for a job to the mass of vacancies:

$$M(\tilde{X}, V(\tilde{X})) = \frac{\left( \sum_e \left[ U(\tilde{X}, e) + \lambda_n(e) \sum_l N(\tilde{X}, l, e) \right] \right) V(\tilde{X})}{\left( \left( \sum_e \left[ U(\tilde{X}, e) + \lambda_n(e) \sum_l N(\tilde{X}, l, e) \right] \right)^\alpha + V(\tilde{X})^\alpha \right)^{\frac{1}{\alpha}}},$$

with  $\alpha > 0$ . Searching households have the job-finding rate

$$f(\tilde{X}) = \frac{M(\tilde{X}, V(\tilde{X}))}{\sum_e \left[ U(\tilde{X}, e) + \lambda_n(e) \sum_l N(\tilde{X}, l, e) \right]}.$$

A wide range of wages is bilaterally efficient. We postulate that the wage evolves according to a wage rule that allows for wage rigidity. In particular, the wage evolves according to

$$\log(w(X)/\bar{w}) = \phi_w \log(w_{-1}(X)/\bar{w}) + (1 - \phi_w) \log\left(\frac{y(X)}{\bar{y}}\right) + \zeta_w. \quad (3)$$

This rule has the potential to amplify the effect of business-cycle shocks on unemployment and to propagate the shocks over time; see Blanchard and Galí (2010) and the literature overview in Rogerson and Shimer (2011). Above,  $\bar{w}$  is the steady-state wage level. Parameter  $\phi_w \in [0, 1]$  governs wage rigidities over time, and how much the wage reacts to economic activity. Last, there is the wage-markup shock.

**Capital services.** There is a representative producer of homogeneous “capital services.” The value of the producer is

$$\begin{aligned} J_K(X, k_{-1}, i_{-1}) &= \max_{v,i,k} (1 - \tau_d)(r(X)k_{-1}v - i) + \mathbb{E}_\zeta [Q(X, X')J_K(X', k, i)] \\ \text{s.t. } k &= [1 - \delta(v)] \cdot k_{-1} + \zeta_I \cdot [1 - \Gamma(i/i_{-1})]i. \end{aligned}$$

---

<sup>17</sup>Translated to a multi-household setup, this means that the marginal costs per hire are convex in the hiring rate, as in Gertler and Trigari (2009) and Yashiv (2000). This leads to a more drawn out response of vacancies in response to shocks.

Capital services are the product of the capital stock,  $K$ , and the utilization rate of capital,  $v$ . Depreciation of capital depends on utilization as in Greenwood et al. (1988).

$$\delta(v) = \delta_0 + \delta_1 v^{\delta_2}, \quad \delta_1 > 0, \delta_2 > 1.$$

The extent to which outlays for investment today,  $i'$ , result in new capital,  $k'$ , depends on the marginal efficiency of investment,  $\zeta_I$ , and on the past level of investment.<sup>18</sup> The transformation function that governs how investment is transformed into physical capital is given by

$$\Gamma\left(\frac{i}{i_{-1}}\right) = \phi_K/2 \left(\frac{i}{i_{-1}} - 1\right)^2, \quad \phi_K \geq 0.$$

This form of investment adjustment costs is customary in the New Keynesian literature, and follows Christiano et al. (2005). Parameter  $\phi_K$  indexes the ability of the economy to generate new capital (aggregate savings) at short horizons.

**Adjustment services.** The activity of recruiting and of adjusting prices requires homogeneous adjustment services. The competitive representative adjustment-services firm solves

$$\begin{aligned} \max_{y_a, (y_{a,j})_{j \in [0,1]}} & (1 - \tau_d) \left( P(X_p) y_a - \int_0^1 P_j(X_p) y_{a,j} dj \right) \\ \text{s.t. } & y_a = \left( \int_0^1 y_{a,j}^{\frac{\vartheta \exp\{\zeta_{P,t}\}-1}{\vartheta \exp\{\zeta_{P,t}\}}} dj \right)^{\frac{\vartheta \exp\{\zeta_{P,t}\}}{\vartheta \exp\{\zeta_{P,t}\}-1}}, \end{aligned}$$

where  $y_a$  are total adjustment services produced and  $y_{a,j}$  is demand for differentiated good  $j$  by the adjustment-services firm. Appendix B provides the first-order conditions related to the firms' problems.

## 2.5 Financial firms

Households can own claims to firms' cash flows only indirectly, through holding shares in representative mutual funds that cater equally to all households. In equilibrium, all the funds hold the same relative portfolio shares. It remains to fix the stochastic discount factor that the funds apply and endow onto the firms. With incomplete financial markets, the stochastic discount factor is not necessarily unique. For tractability, we assume that the funds discount the future using

$$Q(X, X') = \frac{p_a(X)}{p_a(X') + d_a(X')}.$$

This discount factor is consistent with the fund-holding households' Euler equations by construction. Next to this, it can be constructed by the mutual fund from market information.<sup>19</sup> The way that households' demand for savings will affect investment decisions by firms, then, is as follows. If aggregate demand for savings rises temporarily, for a given dividend stream the market-clearing price of shares,  $p_a(X)$ , rises. By the above discount factor, this induces the mutual fund and the

---

<sup>18</sup>Note that  $i'$  will be measurable with respect to  $X$ .

<sup>19</sup>In the absence of aggregate risk or in a model solution with certainty equivalence, this discount factor would simply equal the real interest rate.

firms it owns to value future cash flow more. In turn, this induces a rise in investment. Appendix C discusses this choice in more detail.

We use the cashless limit assumption (Woodford, 1998), by which the central bank controls the nominal gross rate of return  $R(X)$  on the risk-free nominal bonds that the funds trade. Letting  $\Pi(X)$  denote the gross rate of inflation, the mutual funds' optimal decisions yield a standard Euler equation (for the mutual fund rather than a household)

$$1 = \mathbb{E}_\zeta \left[ Q(X, X') \frac{R(X)}{\Pi(X')} \right].$$

The mutual fund distributes to the households all income that is not reinvested, after paying taxes to the government. After-tax dividends are given by

$$d_a(X) = (1 - \tau_d) \left( y_f(X) - i(X) - \int_{\mathcal{M}} w(X) s e(1 - \varrho l) \mathbb{1}_{n=1} d\mu \right),$$

where  $\mathbb{1}$  marks the indicator function, meaning  $\mathbb{1}_{n=1}$  marks employment of the household.

## 2.6 Central bank and fiscal authority

The central bank sets the gross nominal interest rate according to Taylor rule

$$\log \left( \frac{R(X)}{\bar{R}} \right) = \phi_R \log \left( \frac{R_{-1}}{\bar{R}} \right) + (1 - \phi_R) \left[ \phi_\Pi \log \left( \frac{\Pi(X)}{\bar{\Pi}} \right) - \phi_u \left( \frac{U(X) - \bar{U}}{\pi_S(\mathcal{S}_+)} \right) \right] + \log \zeta_R. \quad (4)$$

The first term on the right-hand side reflects interest persistence, with  $\phi_R \in [0, 1]$  ( $R_{-1}$  is the rate set in the previous period). Interest persistence apart, the central bank raises the nominal rate above its steady-state level  $\bar{R}$  whenever inflation exceeds the inflation target of  $\bar{\Pi}$  ( $\phi_\Pi > 1$ ) or the unemployment rate is lower than its steady-state value (parameter  $\phi_u \geq 0$ ).<sup>20</sup>

The fiscal authority is bound by a balanced-budget rule. The government budget constraint is given by

$$\begin{aligned} & \int_{\mathcal{M}} \mathbb{1}_{s \in \mathcal{S}_+} \mathbb{1}_{n=0} b_{UI}(es) d\mu + \int_{\mathcal{M}} \mathbb{1}_{s=s_0} b_{RET}(e) d\mu + g \\ &= \tau_d \frac{d_a(X)}{1 - \tau_d} + \tau_c \int_{\mathcal{M}} c(X, n, a, l, e, s) d\mu \\ &+ \int_{\mathcal{M}} \mathbb{1}_{s \in \mathcal{S}_+} \mathbb{1}_{n=1} (\tau_{UI} + \tau_{RET}) w(X) es(1 - l\varrho) d\mu \\ &+ \int_{\mathcal{M}} \mathbb{1}_{s \in \mathcal{S}_+} \mathbb{1}_{n=1} \tau(X, w(X) es(1 - l\varrho)) [w(X) es(1 - l\varrho)] d\mu. \\ &+ \int_{\mathcal{M}} \mathbb{1}_{s \in \mathcal{S}_+} \mathbb{1}_{n=0} \tau(X, b_{UI}(es)) b_{UI}(es) d\mu. \\ &+ \int_{\mathcal{M}} \mathbb{1}_{s=s_0} \tau(X, b_{RET}(e)) b_{RET}(e) d\mu. \end{aligned}$$

---

<sup>20</sup>In terms of notation,  $U(X) := \sum_e U(X, e)$  is the mass of unemployed households at the production stage, and  $\pi_S(\mathcal{S}_+)$  is the mass of households in the labor force (that is, not retired), so that  $U(X)/\pi_S(\mathcal{S}_+)$  is the unemployment rate.

The fiscal authority spends on unemployment and retirement benefits, and government consumption expenditures,  $g$  (first line). These expenditures are financed through a tax on dividends ( $d_a(X)/(1-\tau_d)$  marks dividends pre-tax) and consumption ( $c(X, n, a, l, e, s)$  marks the consumption policy of households), second line. In addition, there are unemployment insurance and social security taxes on earnings (third line), and progressive income taxes on earnings, unemployment benefits, and retirement benefits.<sup>21</sup>

## 2.7 Market clearing and equilibrium

Our notion of equilibrium is fairly standard; we collect the full definition in Appendix D, including the law of motion for the distribution. Here we only state the market-clearing conditions. Market clearing for final goods requires that all final output be used for personal consumption, investment, or government consumption:

$$y_f(X) = \int_{\mathcal{M}} c(X, n, a, l, e, s) d\mu + i(X) + g.$$

Total demand for differentiated goods is given by

$$y(X) = y_f(X) + y_a(X).$$

The market for differentiated goods clears if demand equals production (using symmetry in both price setting and demand for each differentiated good  $j$ ), so

$$y(X) = \zeta_{TFP} k_j^\theta \ell_j^{1-\theta}$$

with  $k_j$  and  $\ell_j$  identical for all  $j \in [0, 1]$ . The market for adjustment services clears if all such services are used for adjusting prices or employment or as fixed costs,

$$y_a(X) = \frac{\psi}{2} (\Pi(X) - \bar{\Pi})^2 y(X) + \kappa(\tilde{X}) V(\tilde{X}) + \Xi.$$

The market for labor services clears if all labor services supplied are used in the production of differentiated goods,

$$\int_{\mathcal{M}} se(1 - \varrho l) \mathbb{1}_{n=1} d\mu = \int_0^1 \ell_j dj,$$

The market for capital services clears if

$$v(X) K_{-1}(X) = \int_0^1 k_j dj.$$

Normalizing the supply of shares to unity, and mark with  $a(X, n, a, l, e, s)$  the savings policies of households, the market for shares in the mutual fund clears if

$$\int_{\mathcal{M}} a(X, n, a, l, e, s) d\mu = 1.$$

---

<sup>21</sup>The model has non-Ricardian households. Fiscal policy, therefore, shapes the equilibrium allocations. We consider the balanced-budget rule to be transparent. At the same time, this is but one set of fiscal rules. It prevents us from examining interesting dimensions of government policy, such as active debt management policies or, more fundamentally, tax smoothing.

Last, the bond market clears if inside bonds are in zero net supply.

Throughout the paper, we will compare results for the HANK model shown above to a simple two-agent saver-spender analogue (the TANK model), which Appendix E describes, and the corresponding representative-agent version (RANK).

### 3 Stylized facts and calibration

We calibrate the HANK model (and the RANK/TANK variants) to the U.S., one period being a quarter. The calibration sample is 1984Q1 to 2008Q3. It covers the Great Moderation and stops right before the zero lower bound on nominal interest rates becomes binding. The solution method is a version of the method developed by Reiter (2009) and Reiter (2010a), described in detail in Appendix F. We use splines to approximate households' decision rules along their asset dimension and approximate the distribution of households as a histogram on the product of a household's skill, education, employment, and a grid on the wealth distribution. All agents use this function to construct their forecasts about the evolution of the economy. We start by documenting stylized facts about income, wealth, and employment risk that we wish the model to replicate. Then, we discuss the calibration of the model.

#### 3.1 Households' source of income and unemployment risk

This section documents that U.S. households' sources of income differ starkly by net worth and that those households that tend to rely most on labor income also tend to have the most volatile employment pattern. Table 1 reports the share of income derived from different sources, by age and percentiles of net worth. All data are from the 2004 Survey of Consumer Finances (SCF), the last wave before the financial crisis. The table shows this split of income sources for the two stylized

**Table 1: Data. Income sources by net worth (percent of total income)**

		Percentile of net worth						
ages 25-65		0-20	20-40	40-60	60-80	80-95	Top 5	Top 1
Labor income		90.4	93.2	90.0	86.5	76.5	58.3	51.6
Financial income		1.2	3.5	8.0	12.1	23.0	40.6	47.6
Transfers		8.4	3.3	2.1	1.5	0.5	0.9	0.8
ages 66-99		0-20	20-40	40-60	60-80	80-95	Top 5	Top 1
Financial income		2.8	6.8	22.8	33.5	50.5	78.5	89.1
Transfers		97.2	93.2	77.2	66.5	49.5	21.5	10.9

*Notes:* Based on SCF 2004. Households with heads ages 25 to 65 and households with heads ages 66 to 99. All entries in percent. Share of annual income coming from labor income, financial income, social security, and transfers other than social security (such as unemployment benefits). For the block with households ages 25-65, we exclude households receiving social security income. For this age group transfers reported here are transfers other than social security. For the block with households ages 66-99, the measure of annual income excludes labor income. Transfers are the sum of social security and other transfers. For the exact definitions, see Appendix G.

age groups that we will have in the model. Working-age households are defined to be aged 25 to 65 years, retired households are ages 66 and over. The first block reports sources of income and

wealth for what we define as working-age households (household heads aged 25-65 with no social security income). The table splits income into three sources: labor income (including a share of 60 percent of the income derived from actively managed businesses), financial income, and transfers (transfers other than social security income, since we exclude working-age households that draw social security income). Earnings are the dominant source of income for all but the wealth-richest working-age households.

Financial income, instead, becomes notably more important for older households (the second block of the table). In keeping with our modeling, the composition of income for the retired focuses only on financial income and transfers (the shares of income reported exclude any remaining labor income). Transfers (primarily social security) are the dominant source of income for the wealth-poorest households of retirement age. Already for the median-wealth old household, however, financial income makes up roughly a quarter of income. For the wealth-richest 5 percent of older households, financial income accounts for 78 percent of income. Retirees are more exposed to changes in financial wealth than households of working age.

At the same time, working-age households are exposed to unemployment risk, exposure to which is unevenly distributed across the population, see Cairó and Cajner (2018) and Elsby et al. (2010). For calibrating the model, we are interested in quarterly flow rates into and out of employment for

**Table 2: Data. Moments of (Un)employment and Labor-Market Flow Rates**

Variable	edu	std	corr	AR	mean
Unemployment rate	nclg	0.63	-0.83	0.97	5.33
	clg	0.33	-0.81	0.97	2.36
Flow rate unempl. → employ.	all	4.06	0.81	0.97	82.37
Flow rate employ. → unempl.	nclg	0.31	-0.87	0.96	4.60
	clg	0.15	-0.77	0.93	1.92

*Notes:* The table reports labor-market moments in the data. Second moments are based on detrended data. The trend is an HP-trend with weight 1,600 and derived on a sample from 1977Q1 to 2015Q4. The moments reported here refer to the detrended data from 1984Q1 to 2008Q3. The column labeled “edu” specifies the sample (“all” = all workers, “nclg” = no college degree, “clg” = college degree). Thereafter, “std.” reports the standard deviation of each series; “corr” shows the correlation of the series with GDP. The next column (“AR”) shows the first-order autocorrelation of the series. The final column shows the mean of the unfiltered series.

the working-age population. Following the methodology of Cairó and Cajner (2018), we compute these from the Current Population Survey. Appendix H provides details. We split the population into two education groups. The low-education group comprises workers with less education than a completed college degree. The high-education group is composed of workers with a college degree or higher educational attainment. Table 2 reports first and second moments of the resulting labor-market series. Unemployment rates are about twice as high and volatile for the low-educated as for the high-educated. The flow rate into unemployment, too, differs notably by education. For the low-educated it is on average about twice as high as the flow rate for the high-educated. And it is about twice as volatile as well. What this means is that the low-educated are exposed to both average and cyclical unemployment risk to a larger extent. In line with the findings in Cairó

and Cajner (2018), the flow rates into employment of the two groups, instead, are very similar; Appendix H documents this. Hence, we report and model only a job-finding rate that is common to all education levels.

### 3.2 Calibration

In calibrating the model, wherever possible we choose parameters based on direct outside evidence or based on targets for the steady state. Unless mentioned otherwise, these targets are to be met exactly. We calibrate the shock processes with a view toward the business-cycle properties of the model.

#### 3.2.1 Preferences, skills, and education

Table 3 reports the calibration of parameters pertaining to the household problem. The coefficient of relative risk aversion is set to  $\sigma = 2.5$ , a value within the typical range in the literature; see, for example, Blundell et al. (2016). We assume that the mass of patient and impatient households is equal, so that  $\pi_{\Delta_\beta}(0) = \pi_{\Delta_\beta}(1) = 0.5$ . In order to pin down time and bequest preferences, we need five targets for the steady state so as to jointly determine  $(\beta_{e_L}, \beta_{e_H}, \Delta_\beta, \gamma_1, \gamma_2)$ . We target an aggregate post-tax real rate of return of 3.2 percent, which is the value we inferred from the SCF; see Appendix G. Next to this, we target a wealth share of the low-educated of 30 percent, a wealth share for the poorest 20 percent of the working-age population of close to zero, and a wealth share of 5.25 percent for the poorest 50 percent of the retired; all taken from the SCF. Last, we minimize the distance of the wealth Lorenz curve for working-age households in the SCF and the steady state of the model.<sup>22</sup> Taken together, this gives  $\beta_{e_L} = 0.974$ ,  $\beta_{e_H} = 0.984$ ,  $\pi_{\Delta_\beta} = 0.5$ ,  $\Delta\beta = 0.11$ ,  $\gamma_1 = 3182$ , and  $\gamma_2 = 6.1$ .<sup>23</sup> The labor productivity of the low-educated is set to  $e_L = 1$ , by way of normalization. We fix  $e_H = 1.5$  to match the college premium as in Mukoyama and Sahin (2006).

Next, two targets determine the two free parameters of the transition matrix of education levels upon birth. First, of working-age heads of households in the SCF 60 percent have low education by our definition, Second, we target an intergenerational elasticity of incomes of about 0.5, in the mid-range of what the literature finds, for example, Solon (1992) and Mazumder (2005). This implies  $\pi_E(e_L, e_L) = 0.8$  and  $\pi_E(e_H, e_H) = 0.7$ . As regards earnings losses, Couch and Placzek (2010) report that earnings losses upon displacement are 30 percent, Altonji et al. (2013) report an initial drop of 20 percent; We set  $\varrho = 0.25$  to match the midpoint. Couch and Placzek (2010) report that earnings losses still run at 13-15 percent six years after displacement. We set  $\pi_L^{emp}(1, 0) = 0.025$  to match a loss of 14 percent after that time. Comparable estimates of earnings losses are in Davis and von Wachter (2011) and the literature reviewed in Berger et al. (2019). While employed, households can shed an earnings loss, but cannot acquire one, so  $\pi_L^{emp}(0, 1) = 0$ . We set the probability of acquiring an earnings loss when leaving unemployment to  $\pi_L^{uem}(1) = 1 - \pi_L^{emp}(1, 0)$ . This makes sure that a household is not more likely to shed an earnings loss through a spell of unemployment than in employment.

Turning next to skills,  $s$ , we entertain four skill states.  $s_0$  marks retirement.  $s_1$  is the lowest skill level during working age, and  $s_2$  is a medium skill level.  $s_3$  is used to capture vastly more

---

<sup>22</sup>To be more precise we minimize  $\sum i \in \{5, 10, \dots, 95\} \left( \frac{\max(L_i^D, 0) - L_i^M}{\max(L_i^D, 0.001)} \right)^2$ . Here,  $L_i^D$  is the wealth share of the lower  $i$  percent in the SCF and  $L_i^M$  is the corresponding model quantity. As the model has a strict borrowing limit at zero, we replace negative shares in the data with zero as shown in the formula. See Appendix I.1 for the fit.

<sup>23</sup>These values imply that 50 percent of low-educated households have a subjective discount factor of 0.86 and 50 percent of high-educated households have a subjective discount factor of 0.874 at any point in time.

**Table 3: Preferences, Education, Earnings Losses. Targets and Parameterization**

Parameter	Value	Target
Preferences		
$\sigma$	2.5	Blundell et al. (2016).
$\pi_{\Delta\beta}(0)$	0.50	Equal mass of patient and impatient.
$\beta_{e_L}$	0.974	Low-educated hold 30% of aggregate net worth, SCF.
$\beta_{e_H}$	0.984	Real rate of interest of 3.2% p.a.
$\Delta\beta$	0.110	Wealth share poorest 20% of working-age, SCF.
$\gamma_1$	3182	Wealth share of the poorest 50% of retirees, SCF.
$\gamma_2$	6.1	Minimize distance wealth Lorenz curve working-age, SCF.
Education		
$e_L$	1	Normalized to unity.
$e_H$	1.5	College wage premium, Mukoyama and Sahin (2006).
$\pi_E(e_L, e_L)$	0.8	Share of low-educated, SCF.
$\pi_E(e_H, e_H)$	0.7	Intergen. elasticity of income of 0.5.
Earnings losses		
$\varrho$	0.25	Initial loss, Couch/Placzek (2010), Altonji <i>et al.</i> (2013)
$\pi_L^{emp}(1, 0)$	0.025	Loss of 14% after six years, Couch/Placzek (2010).
$\pi_L^{emp}(0, 1)$	0	Cannot acquire earnings loss while employed.
$\pi_L^{uem}(1)$	0.975	$\pi_L^{uem}(1) = 1 - \pi_L^{emp}(1, 0)$ .

*Notes:* Calibrated parameters for preferences, education, and earnings losses.

The main text provides further details.

productive households, the “super-skilled,” as in Castañeda et al. (2003). Skills follow a first-order Markov process. For the skills, we have three sets of targets.

The first set of targets concerns the life-cycle transitions. We target an average working life of 40 years and average length of retirement of 12 years. The latter is in line with the average age of households of retirement age in the SCF. Transitions into retirement are assumed to be independent of a working-age worker’s skill level. Upon entering working age, workers draw a skill level according to the ergodic distribution of skills. The second set of targets involves the transitions between skill states for working-age households. We follow Nakajima (2012b) and assume that 1 percent of the working-age population is super-skilled, and that the probability of remaining super-skilled (if not retiring) is 0.975, the probability of drawing the highest skill state  $s_3$  being independent of the current skill state  $s_1$  or  $s_2$ . We assume that the probability of moving to the lower-skill states from  $s_3$  is based on the ergodic distribution. We do so with an eye toward keeping the distribution of households by skill  $s$  constant over time. Last, we assume that low- and medium-skill households have the same mass in the ergodic distribution of skills. This imposes symmetry on the transitions between  $s_1$  and  $s_2$ . The transitions between  $s_1$  and  $s_2$  are based on Floden and Lindé (2001). The authors estimate the persistence of residual earnings after removing age, education, measurement error, and time fixed effects. The third set of targets concerns the level of skills. We normalize the average skill of working-age workers to 1. We obtain the gap between skill levels  $s_1$  and  $s_2$  by targeting the standard deviation of residual earnings from Floden and Lindé (2001). Last, we choose skill level  $s_3$  so as to replicate the dispersion of wealth of the working-age population in the SCF, as measured by the Gini index. Appendix I.2 provides the targets in table form and lists how

many restrictions each delivers. We meet the targets exactly. Table 4 provides the skill levels and the transition matrix of skills that result.

**Table 4: Skills. Parameterization.**

Level	Transition probabilities, $\pi_S(s, s')$				
	$s'_0$	$s'_1$	$s'_2$	$s'_3$	
$s_0$	0	0.9792	0.0103	0.0103	0.0002
$s_1$	0.490	0.0063	0.9812	0.0122	0.0003
$s_2$	1.301	0.0063	0.0122	0.9812	0.0003
$s_3$	11.375	0.0063	0.0124	0.0124	0.9689

*Notes:* Levels of idiosyncratic productivity (left), transition probabilities of skills per quarter (right).  $s_0$  : retirement,  $s_1$  : lowest skill group,  $s_3$  : highest skill group. Rounding means that rows may not sum to 1. See the text for the targets.

### 3.2.2 Firms and production

Table 5 shows the parameterization of the production sector. We target a capital depreciation rate of 1.5 percent per quarter, a unitary capacity utilization rate in steady state, and a curvature of the depreciation rate in utilization of  $\delta_2 = 1.33$ ; see, for example, Comin and Gertler (2006). Together with our target for the real rate of return, this gives  $\delta_0 = -0.022$  and  $\delta_1 = 0.0172$ . As to the investment adjustment costs, we assume that  $\phi_K = 10$ , the mid-point of the range of estimates in Christiano et al. (2016).

For the labor services, we calibrate wage persistence to  $\phi_w = 0.837$ , the estimate in Barattieri et al. (2014) for job stayers. We determine  $\lambda_x(e_L)$ ,  $\lambda_x(e_H)$ ,  $\lambda_n(e_L)$ , and  $\lambda_n(e_H)$  as follows. Throughout, we target a steady-state job-finding rate as implied by the sample averages of Table 2. Using this with the two education groups' average unemployment rates and their relative standard deviations of the flow rate into unemployment and into employment in the table, gives us estimates of the share of exogenous separations for the two groups,  $\lambda_x(e)/[\lambda_x(e) + \lambda_n(e)(1 - f)]$ . These shares serve as two of our targets. Next, we target the relative unemployment rates of the two groups as in Table 2. In addition, we decided to scale average unemployment rates to the average value for the whole economy. We target an economy-wide unemployment rate of 6 percent; which is the average value for workers of all ages during our sample period.<sup>24</sup> The four targets lead to  $\lambda_x(e_L)$ ,  $\lambda_x(e_H)$ ,  $\lambda_n(e_L) = 0.076$ , and  $\lambda_n(e_H) = 0.037$ .

Conditional on a target for the labor income share of 66 percent (used below), we obtain the remaining labor-market parameters  $\bar{w}$ ,  $\alpha$ ,  $\kappa_V$ , and  $\kappa_H$  jointly by matching the target for the job-finding rate and three additional targets. Namely, we target a steady-state job-filling rate of  $q = 0.71$  as in den Haan et al. (2000). Next, we target that the total cost per hire amounts to 50 percent of a quarterly wage, in line with a broad concept of hiring costs; see Silva and Toledo (2009). And, following Christiano et al. (2016), we target that 94 percent of these costs are paid upon making a

---

<sup>24</sup>These two targets scale the unemployment rates for each education group reported in Table 2 in proportion so that they are 7.7 and 3.4 percent, respectively.

successful hire.<sup>25</sup> This gives us matching function parameter  $\alpha = 2.63$ , the steady-state wage per efficiency unit of labor  $\bar{w} = 0.898$ , and parameters  $\kappa_V = 0.014$  and  $\kappa_H = 0.310$ .

Next, for the differentiated goods, we set  $\psi = 179.11$  such that the Phillips curve's slope is in line with a Calvo stickiness of 0.85, the estimate of Galí and Gertler (1999). We set the demand elasticity to a value of  $\vartheta = 6$ , implying a 20 percent markup over marginal costs. We target a steady-state investment-GDP ratio of 0.18. Together with the above-mentioned target for the labor share, this gives  $\theta = 0.2836$  and implies fixed costs of  $\Xi = 0.130$ .<sup>26</sup> The remaining parameters listed

**Table 5: Production Sector. Parameterization and Targets**

Parameter	Value	Target
Capital services		
$\delta_0$	-0.0022	depreciation rate of 1.5% per quarter, NIPA.
$\delta_1$	0.0172	unitary utilization in steady state.
$\delta_2$	1.33	Comin and Gertler (2006).
$\phi_K$	10	mid-point of estimates in Christiano et al. (2016).
Labor services		
$\phi_w$	0.837	Barattieri et al. (2014) for job stayers.
$\lambda_x(e_L)$	0.048	69.5% of separations for $e_L$ exogenous; see text.
$\lambda_x(e_H)$	0.019	65.7% of separations for $e_H$ exogenous; see text.
$\lambda_n(e_L)$	0.116	rel. unempl. rate $e_H$ and $e_L$ as in Table 2.
$\lambda_n(e_H)$	0.074	economy-wide average unempl. rate of 6%; sample average.
$\bar{w}$	0.898	st.-st. job-finding rate, $f = 0.82$ .
$\alpha$	2.63	st.-st. job-filling rate $q = 0.71$ , den Haan et al. (2000).
$\kappa_V$	0.014	share fixed hiring costs 94%, Christiano et al. (2016).
$\kappa_H$	0.310	total hire cost 50% of qtrly wage, Silva and Toledo (2009).
Differentiated goods		
$\psi$	179.11	slope of Phillips curve as in Galí and Gertler (1999).
$\theta$	0.284	investment-GDP ratio of 0.18.
$\vartheta$	6	20% markup.
$\Xi$	0.13	labor-income share of 0.66.
Implied steady-state values used as parameters		
$\bar{y}$	1.16	implied steady-state level of production $y$ .
$\bar{M}$	0.092	implied steady-state value of matches $M$ .
$\bar{N}$	0.737	implied steady-state value of employment, $N(\tilde{X})$ .
$\bar{N}$	0.737	implied steady-state value of employment, $N$ .

*Notes:* Calibration for capital services, labor services, differentiated goods, and parameters that are related to steady-state values. The main text provides further details.

in Table 5 refer to the steady-state values implied by the calibration that are used elsewhere in the model.

<sup>25</sup>These are joint targets. The wage, in particular, has to be consistent with the targeted job-finding rate. For given parameter values, changing the wage would change the steady-state job-finding rate and, therefore, unemployment.

<sup>26</sup>The implied ratio of capital to quarterly GDP is 12. The ratio of *ex-dividend*, after-tax wealth to quarterly GDP implied by the calibration is 10.3.

### 3.2.3 Central bank and fiscal authority

Table 6 shows the parameterization for the central bank and the fiscal authority. Interest-rate persistence is set to  $\phi_R = 0.8$ , a conventional value. The responses to inflation and unemployment,  $\phi_{\Pi} = 1.5$  and  $\phi_u = 0.15$ , are based on Taylor (1993).<sup>27</sup>  $\bar{\Pi} = 1.005$  implies a steady-state inflation rate of 2 percent annualized, in line with the Federal Reserve System's inflation objective. The unemployment target is  $\bar{U} = 0.0462$ . Since 77 percent of households in the calibration are of working age this is line with a steady-state unemployment rate of 6 percent.  $\bar{R}$  is set to the steady-state interest rate consistent with an annual after-tax real interest rate of 3.2 percent; a target used earlier.

**Table 6: Central bank and fiscal authority. Parameterization and Targets.**

Parameter	Value	Target
Central bank		
$\phi_R$	0.8	Christiano et al. (2016).
$\phi_{\Pi}$	1.5	Taylor (1993)
$\phi_u$	0.15	Taylor (1993).
$\bar{\Pi}$	1.005	inflation target 2% p.a.
$\bar{U}$	0.0462	steady-state level of unemployment rate of 6%.
$\bar{R}$	1.013	in line with annual real rate of 3.2% p.a.
Fiscal authority – expenditures		
$g$	0.19	NIPA, share of government spending in GDP.
$b_{UI}$	0.5	based in Graves (2020); see text.
$b_{RET}(e_L)$	0.32	Huggett and Parra (2010).
$b_{RET}(e_H)$	0.46	Huggett and Parra (2010).
Fiscal authority – revenues		
$\tau_{RET} \cdot 100$	13.2	balances social security system in steady state.
$\tau_{UI} \cdot 100$	1.5	balances UI system in steady state.
$\tau_c \cdot 100$	7	NIPA, as in Fernández-Villaverde et al. (2015).
$\tau_d \cdot 100$	36	NIPA, as in Fernández-Villaverde et al. (2015).
$\tau_0$	0.182	Guner et al. (2014).
$\tau_1$	3.044	Guner et al. (2014).
$\tau_2$	1.496	Guner et al. (2014).

*Notes:* The table shows the calibrated parameters for the monetary and fiscal authority. The main text explains the calibration targets.

Government consumption is constant, and set to 19 percent of steady-state GDP, the average value in the data. We model unemployment benefits as  $b_{UI}(es) = \min(\bar{b}_{UI} \cdot e \cdot s \cdot \bar{w}, \bar{b}_{UI} \cdot \text{steady-state average economy-wide earnings})$ . For unemployment benefits we set a replacement rate of 50 percent with a cap at two thirds of average earnings based on the summary in Graves (2020); the values are close to the ones reported in Shimer (2005) and

<sup>27</sup>Taylor (1993) has a response of annualized interest rates to the log output gap of 0.5. Regressing the CBO's measure of the output gap on unemployment, and realizing that the Taylor rule here is specified for quarterly interest rates, we arrive at the value for  $\phi_u$ .

Chetty (2008).<sup>28</sup> Next, we discuss the social security system. Huggett and Parra (2010) model retirement benefits as a piecewise linear function of past earnings. In the current paper, we cannot condition payments on the entire history of past earnings. Rather, we index retirement benefits to the education level of the household, which serves as a rough guide to lifetime earnings. Using the replacement schedule reported in Huggett and Parra (2010), we arrive at a replacement rate of 47 percent for the low-education group, resulting in  $b_{RET}(e_L) = 0.47 \cdot \bar{L}(e_L) \cdot \bar{w} = 0.32$ , where  $\bar{L}(e)$  denotes the average productivity of a worker of education  $e$  in the steady state. For the high-education group, instead, the replacement rate is 41 percent, meaning  $b_{RET}(e_H) = 0.41 \cdot \bar{L}(e_H) \cdot \bar{w} = 0.46$ . Social security taxes and UI taxes are set to balance their respective scheme in the steady state. The choices made here imply steady-state unemployment insurance and social security payroll tax rates of  $\tau_{UI} = 0.015$  and  $\tau_{RET} = 0.13$ . We construct consumption and capital taxes from the National Income and Product Accounts as in Fernández-Villaverde et al. (2015). This gives tax rates on consumption and capital income of  $\tau_c = 0.07$  and  $\tau_d = 0.36$ , respectively. For the functional form of labor-income taxes, we follow Gouveia and Strauss (1994) setting

$$\tau(X, w(X)es(1-l\varrho)) = \tau_{BC}(X) + \tau_0 \left[ 1 - \left( \frac{w \cdot e \cdot s \cdot (1-l\varrho)}{\text{economy-wide avg. earn. in st.-st.}} \right)^{\tau_2} + 1 \right]^{-1/\tau_2}.$$

We follow the estimates of Guner et al. (2014) and set  $\tau_1 = 0.008 \cdot (53,063/1000)^{\tau_2}$ , and  $\tau_2 = 1.496$ .<sup>29</sup>  $\tau_0 = 0.182$  is normalized to balance the budget in the steady state.  $\tau_{BC}(X)$  is zero in the steady state.

### 3.2.4 Shocks

There are five shocks in the calibrated model: shocks to the marginal efficiency of investment, monetary shocks, productivity shocks, wage-markup shocks, and price-markup shocks. For each of these, we have to parameterize the steady-state value, the persistence, and the standard deviation of the innovation. The steady-state values are mere normalizations. We set  $\bar{\zeta}_{TFP} = 0.6920$  such that steady-state GDP is normalized to unity. Last,  $\bar{\zeta}_I = \bar{\zeta}_R = \bar{\zeta}_w = \bar{\zeta}_P = 1$  to normalize the corresponding shocks such that they have zero mean in logs. We set  $\rho_{\zeta_{TFP}} = 0.95$ , so as to match the persistence utilization-adjusted TFP in Fernald (2014). We set the persistence of the wage-markup shock to  $\rho_{\zeta_w} = 0$  (it is propagated through wage persistence). As is customary, the monetary shock is white noise, too,  $\rho_{\zeta_R} = 0$ .

This leaves seven parameters of the shock processes to be calibrated ( $\rho_{\zeta_I}, \rho_{\zeta_P}, \sigma_{\zeta_I}, \sigma_{\zeta_{TFP}}, \sigma_{\zeta_R}, \sigma_{\zeta_w}, \sigma_{\zeta_P}$ ). Conditional on the calibration sketched above, we estimate a linearized version of the representative-household version of the model by maximum likelihood, having six time series as observables: the growth rate of real consumption, the growth rate of real investment, the growth rate of the real wage, the interest rate, the inflation rate, and the unemployment rate. All series are demeaned. Appendix I.5 provides an exact definition of the data source. The sample is 1984Q1 to 2008Q3. We allow for iid measurement error in each of the observation equations, setting the variance of the measurement error equal to 1 percent of the underlying series' unconditional standard deviation. Table 7 summarizes the resulting parameter values for the shocks.

---

<sup>28</sup>In the calibration, the average drop in consumption in the first quarter after becoming unemployed is 11 percent, a value well within the range of estimates in the literature; see, for example, the survey in Chodorow-Reich and Karabarbounis (2016).

<sup>29</sup>Parameters are based on the “only-labor-income” case in Guner et al. (2014) (their Table 12). We re-normalize

**Table 7: Parameters chosen for the shock processes**

	$\bar{\zeta}_x$	$\rho_{\zeta_x}$	$\sigma_{\zeta_x}$
MEI shock, $\zeta_I$	1 <sup>a)</sup>	0.0 <sup>e)</sup>	0.3487 <sup>e)</sup>
TFP shock, $\zeta_{TFP}$	0.6925 <sup>b)</sup>	0.95 <sup>c)</sup>	0.0028 <sup>e)</sup>
Monetary shock, $\zeta_R$	1 <sup>a)</sup>	0 <sup>d)</sup>	0.0020 <sup>e)</sup>
Wage shock, $\zeta_w$	1 <sup>a)</sup>	0 <sup>d)</sup>	0.0073 <sup>e)</sup>
Price-markup shock, $\zeta_P$	1 <sup>a)</sup>	0.8475 <sup>e)</sup>	0.0514 <sup>e)</sup>

*Notes:* Calibrated parameters for the shock processes. <sup>a)</sup>: normalization so log process has unit mean. <sup>b)</sup>: normalizes steady-state GDP to unity. <sup>c)</sup>: based on Fernald (2014). <sup>d)</sup> : customary normalization. <sup>e)</sup>: determined using maximum likelihood (see main text for details).

### 3.3 Parameterization of the RANK/TANK variant

Wherever possible, parameters are identical in HANK/RANK/TANK. The RANK model has a representative family of households of all ages and education levels. The TANK variant is identical to the HANK model, other than that it strips the ability to self-insure from households. Instead, there are two infinitely lived families of savers and spenders, respectively. Each family includes households of different ages and pools all the member households' income. We continue to target a real rate of 3.2 percent per annum, and so set the time-discount factor for the saver family to  $\beta^{saver} = 0.992$  (likewise in RANK, where all households are savers). In keeping with the HANK calibration, spenders in the TANK model have time-discount factor  $\beta^{spend} := \beta^{saver} - \Delta\beta = 0.882$ . We set the mass of spenders in TANK to 15 percent. We choose 15 percent of spenders so as to match the share of households in HANK that hold zero net worth. This strategy is akin to Debortoli and Gal (2017).

### 3.4 Properties of the calibrated model variants

Appendix I.3 shows that the HANK model closely matches the wealth distribution in the U.S. economy. Appendix I.4 reports the distribution of income sources implied by the HANK model (the counterpart to Table 1). Appendix I.5 shows that the standard deviation of consumption is higher in HANK than in RANK/TANK, but somewhat smaller than in the data. Still, the model matches the data remarkably well. Appendix I.6 reports impulse responses for the three variants. In the baseline, the MEI shock works like a demand shock, generating comovement in the GDP aggregates, employment, interest rates, and inflation. The TFP shock, instead, raises output, reduces inflation, and – due to nominal rigidities – reduces employment in the short term. The price-markup and wage shocks work like cost-push shocks, moving output and inflation in opposite directions, and implying positive comovement of employment and output. A monetary shock persistently raises interest rates and reduces output, employment, and inflation. The impulse responses do not show a hump-shaped pattern, though. Crucial elements that bring this about in New Keynesian models are habit persistence in consumption or sticky information, both of which the current model does not consider. See Auclert et al. (2020) for a detailed discussion of the hump-shaped response to monetary shocks in HANK models. Appendix I.8 reports a forecast error variance decomposition.

---

parameter  $\tau_1$  to reflect scaling. \$53063 is the average income in their sample for the year 2000, on which the estimates are based.

The MEI shock accounts for 75 percent of the fluctuations in investment and about half of fluctuations in GDP. The TFP shock accounts for about a third of the variance in GDP. The price- and wage-markup shocks, respectively, account for about 10 and under 2 percent of the variance of output. Appendix I.9 documents marginal propensities to consume for different groups of households. On average households would consume about 33.5 percent of a smaller gift within the course of one year. MPCs differ starkly, but are not exactly unity for any of the groups shown, a difference from the TANK economy. Appendix I.10 documents the corresponding consumption policies by idiosyncratic states. Appendix I.11 discusses the extent to which the model matches Guvenen et al. (2014) in that the cross-sectional skewness of earnings growth is countercyclical.

## 4 A political economy of systematic monetary policy?

We are now in a position to ask what type of systematic monetary stabilization policy different types of households would wish to have. We first show that household net worth and exposure to labor income are important predictors of who wins from inflation-centric policy. Thereafter, we explain the disagreement and contrast the results for the HANK model with the results for RANK/TANK.

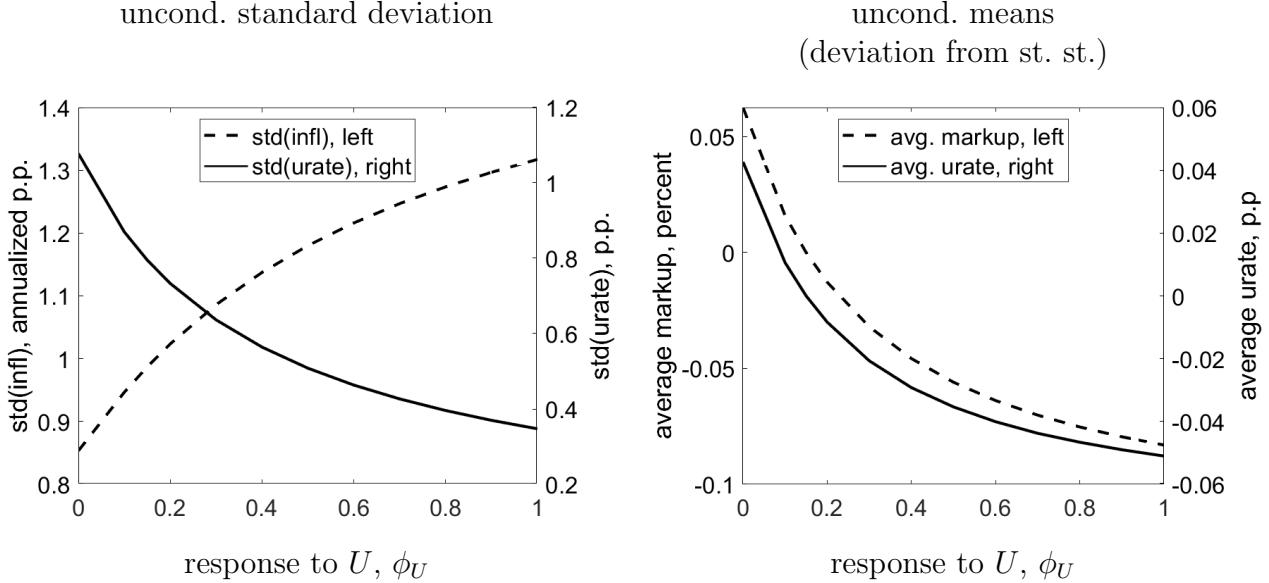
### 4.1 The experiments

We consider an unanticipated, permanent change in the parameters of the monetary policy rule (4). As is customary in exercises of this kind, we abstract from monetary shocks and set  $\sigma_{\zeta_R} = 0$ . The welfare assessments are predicated on the initial aggregate state (including the distribution of households across idiosyncratic states) being the ergodic mean under the calibrated policy rule. We wish to make sure that the results do not arise from a change in the average inflation rate (with commensurate price adjustment costs). Therefore, both in the baseline and when we change policies, we always adjust the Taylor rule's intercept such that the average inflation rate remains at exactly 2 percent annualized; Appendix J describes the algorithm.

### 4.2 Inflation-unemployment trade-off

Households differ in their exposure to inflation-centric monetary policy. Toward this end, Figure 2 shows the inflation-unemployment trade-off that is inherent in the model by varying the response to unemployment in the Taylor rule. The left panel shows the unconditional standard deviation of inflation (annualized percentage point scale, left axis) and the unemployment rate (percentage point scale, right axis). The right panel shows the effect of the same variation on the average unemployment rate (in percentage points). The average inflation rate, by design, is held constant. The panel on the right also shows the effect of stabilization policy on average markups. The more the central bank responds to unemployment, the less volatile is unemployment and the more volatile inflation becomes. The standard deviation of inflation varies between a little over 0.8 p.p. annualized and a little over 1.3 p.p. annualized over the range of parameters shown here (left panel, dashed line, left axis). The standard deviation of unemployment falls from 1.2 percentage points to 0.4 percentage point. A more unemployment-centric monetary policy reduces not only the cyclical of unemployment, but also average unemployment. The presence of such an effect is well-established in the search and matching literature (Jung and Kuester 2011, Hairault et al. 2010). Over the range of parameters shown here, the average unemployment rate varies by about 0.1 percentage point. The right panel documents that systematic monetary policy affects not only average unemployment, but also average markups. Namely, going from  $\phi_U = 0$  to  $\phi_U = 1$ , the average markup of price setters falls by about 0.15 percentage point (in terms of magnitude, think of a fall in the average

**Figure 2: HANK – Inflation-unemployment trade-off, varying  $\phi_U$**



*Notes:* Left panel: unconditional standard deviation of inflation (dashed line, left axis) and the unemployment rate (solid line, right axis). Right panel: unconditional means of the markup and unemployment in deviation from the mean under baseline policy. In each of the panels, the x-axis varies the response to unemployment in the Taylor rule,  $\phi_u$ . The value of  $\phi_u$  in the baseline is 0.15. The monetary shock is set to zero.

markup from 20 percent to 19.85 percent). Most of the movement in average markups is due to the price-markup shocks, suggesting precautionary pricing by firms. To see the logic, consider for example, a negative aggregate shock to price markups (a rise in the elasticity of demand). Such a shock is disinflationary. If monetary policy seeks to stabilize inflation in the face of such a shock (as hawkish policy would do), it has to stimulate demand. This raises marginal costs precisely at a time of low markups. Faced with the risk of attracting demand precisely when their marginal costs are high, firms may precautionarily choose higher average markups to start with. Appendix K shows that these trade-offs are present in the RANK/TANK variants, too. Systematic monetary stabilization policy in our model affects average incomes and their cyclical fluctuations.

### 4.3 Welfare gains and net worth

We now document the welfare effects of systematic stabilization policy in the HANK economy. Table 8 groups the HANK households by their position in the initial wealth distribution at the time of the policy change, reporting the average consumption-equivalent welfare gain in the group. These account for both the long-run effects of the change in policy and the transition path. Disagreement about systematic monetary stabilization policy is pronounced. Households in the lower wealth percentiles (rows “0-20” and “20-40”) favor more accommodative monetary policy. The wealth-richer, instead, favor a stricter focus on inflation. To see this most starkly, focus on the extreme: a change toward a policy of strict inflation targeting (the table’s right-most column). Although the policy raises the average unemployment rate by 0.17 percentage point, support for this policy extends well into the middle class: 43 percent of households would favor moving toward strict inflation targeting. Under this policy, the welfare gains of the wealth-richest 5 percent of households

**Table 8: HANK – Welfare effects of changing monetary stabilization policy**

		Response to unemployment, $\phi_u$								$\Pi = \bar{\Pi}$
		0	0.1	0.15	0.2	0.3	0.4	0.5	0.6	
Consumption-equivalent welfare gain (in percent)										
Wealth percentile	0-20	-0.027	-0.006	—	0.006	0.009	0.010	<b>0.016</b>	0.015	-0.118
	20-40	-0.023	-0.005	—	<b>0.003</b>	0.001	-0.003	-0.001	-0.005	-0.109
	40-60	0.004	<b>0.007</b>	—	0.001	-0.008	-0.020	-0.029	-0.039	0.001
	60-80	0.021	0.014	—	0.001	-0.014	-0.030	-0.047	-0.062	<b>0.112</b>
	80-95	0.024	0.010	—	-0.008	-0.025	-0.039	-0.058	-0.072	<b>0.238</b>
	95+	0.025	0.009	—	-0.008	-0.025	-0.037	-0.052	-0.065	<b>0.255</b>

*Notes:* Welfare effects of a permanent policy change from the baseline policy to a policy that has a different response to unemployment,  $\phi_u$ ; left-most columns. Right-most column: a change toward strict inflation targeting ( $\phi_{\Pi} = \infty$ ). From top to bottom: average lifetime consumption-equivalent welfare gains (in percent of consumption) by wealth percentile. Households are grouped by their position in the wealth distribution at the time of the policy change.

would amount to about a quarter of a percent of lifetime consumption. The losses of the poor run to about half of that.

To have a better idea of the magnitude of these gains and losses, Table 9 reports the endowment one would need (in dollars) to finance the lifetime consumption-equivalent welfare gains reported in Table 8.<sup>30</sup> Financing a comparably sized consumption-equivalent welfare gain requires fewer dollars

**Table 9: HANK – One-time dollar-equivalent gain from policy change – 2004 US\$**

		Response to unemployment, $\phi_u$								$\Pi = \bar{\Pi}$
		0	0.1	0.15	0.2	0.3	0.4	0.5	0.6	
Endowment required (US\$)										
Wealth percentile	0-20	-546	-119	—	114	185	199	<b>327</b>	293	-2,371
	20-40	-506	-108	—	<b>58</b>	38	-45	-4	-67	-2,438
	40-60	100	<b>168</b>	—	28	-196	-474	-674	-906	-30
	60-80	<b>444</b>	312	—	15	-321	-702	-1,095	-1,444	2,573
	80-95	<b>712</b>	291	—	-263	-771	-1,238	-1,825	-2,271	<b>7,649</b>
	95-100	<b>2,889</b>	921	—	-852	-2,477	-3,637	-4,987	-6,095	<b>24,037</b>

*Notes:* This table reports the endowment, valued in 2004 US\$, that is needed to finance the consumption-equivalent welfare gains of each group reported in Table 8. A positive entry is a gain for the household.

for the poor than for the wealth-rich. The dollar stakes are, therefore, highly unequal. Transitioning to a policy of strict inflation targeting would translate into a loss of \$2,400 for a poor household, but a ten-fold gain for the richest 5 percent by net worth.

<sup>30</sup>In the model, we compute the endowment required as a percent of the average quarterly earnings per household in the economy. The table, then, maps these numbers into 2004 US\$ terms, using the average quarterly earnings of working-age households in the SCF. The SCF for 2004 puts the quarterly earnings of working-age households at \$20,675.

Appendix L reports the welfare gains assuming that only one shock is present at a time. If there were only price-markup shocks, 26 percent of households would favor strict inflation targeting; if the MEI shock were the only shock, 77 percent of households would. For wage-markup shocks and the productivity shock, all households favor inflation targeting. This suggests that a failure of divine coincidence as in Faia (2009), Blanchard and Galí (2010), and Ravenna and Walsh (2011) quantitatively is not the central driving force behind the disagreement that we find. Appendix M reports the welfare effects of a one-time monetary shock: all but the wealth-richest 1 percent of households dislike the shock. The important take-away of the current section is that households may disagree not only about monetary shocks, but even more so about systematic (rule-based) monetary stabilization policy.

#### 4.4 Welfare gains and exposure to labor income

This section shows that a household's relative exposure to the labor-income effects and the financial effects matters for its assessment of monetary stabilization policy. The first block of Table 10

**Table 10: HANK – One-time dollar-equivalent gain by dimension of heterogeneity**

		Response to unemployment, $\phi_u$							$\Pi = \bar{\Pi}$
		0	0.1	0.15	0.2	0.3	0.4	0.5	
skills	$s_0$ (retired)	1,672	665	—	-276	-1,087	-1,807	-2,528	<b>7,815</b>
	$s_1$ (low)	-284	-24	—	<b>38</b>	-28	-134	-206	-162
	$s_2$ (middle)	-337	-12	—	<b>35</b>	-72	-232	-343	<b>166</b>
	$s_3$ (super)	2,048	753	—	-742	-2,279	-3,548	-4,786	<b>19,636</b>
empl., loss	$n = 0$ (unemp.)	-360	-30	—	<b>54</b>	-6	-128	-185	-905
	$n = 1, l = 1$	-318	-20	—	<b>41</b>	-37	-169	-253	-127
	$n = 1, l = 0$	-220	<b>10</b>	—	1	-149	-321	-468	<b>992</b>

*Notes:* Same as Table 9, but sorting the population by residual skill (retired, low skill, medium skill, super-skill) or current employment status (unemployed, employed with skill loss, employed without skill loss). Average dollar-equivalent gains for each group (2004 US\$).

groups households by the transitory skill state,  $s$ . Retirees (skill state  $s_0$ ) account for roughly 23 percent of households. As a group they hold a quarter of the economy's net worth, and the average retiree derives a notable share of income from financial sources (compare Table 1 in the main text and Appendix I.3). Besides, they are completely insulated from the direct effects of monetary stabilization policy on labor income. For them, what matters is that monetary stabilization policy can affect the value of their savings and the amount of taxes they pay. Retirees are among the strongest beneficiaries of a move toward strict inflation targeting with a gain equivalent to roughly \$7,800 (last column, row  $s_0$ ). In terms of the working-age population, the average household of normal skill ( $s_1$  and  $s_2$ ), instead, is close to indifferent to a policy change. The super-skilled, instead, tend to accumulate wealth, and on average firmly favor inflation-centric policy. Appendix N shows the decomposition for the idiosyncratic states. Among the working-age population, it shows, for example, that the currently unemployed and employed households that suffer from an earnings loss tend to favor more dovish policy, while the rest favor more hawkish policy. The next

section shows that there is disagreement not only for the specific policies considered here, but also with regard to optimal monetary policy.

#### 4.5 Optimal simple rules

This section chooses the unemployment response  $\phi_u$  and the inflation response  $\phi_{\Pi}$  in the Taylor rule such that they maximize the *ex-ante* utilitarian welfare of a subset of the population.<sup>31</sup> The rules can, thus, accommodate a desire for more stabilization in general while at the same time stabilizing inflation more than unemployment, or *vice versa*.

For three wealth percentiles, Table 11 shows the implied optimal rule, the support, and the effect that the rules have on average unemployment and the business cycle. The left column shows results for a rule that is optimal for the bottom 20 percent by wealth, the middle column that of the central wealth percentiles, and the right column that of the wealthiest 5 percent of households. Appendix O shows the gains and losses for all rules on the grid.

**Table 11: HANK – (Dis-)agreement about Optimal Simple Policies**

		Optimal for wealth percentile		
		0-20	40-60	95-100
Wealth percentile	Consumption-equivalent welfare gain (in percent)			
	0-20	0.028	-0.027	-0.174
	20-40	0.016	-0.011	-0.153
	40-60	-0.019	0.037	-0.040
	60-80	-0.035	0.082	0.068
	80-95	-0.028	0.115	0.177
	95+	-0.021	0.110	0.192
Share in favor		56	56	38
Sum 2004 US\$		-296	1,463	504
Std	$\Pi$	1.16	0.71	0.28
	u rate	0.43	0.92	1.55
Mean	u rate	-0.04	0.04	0.15
Parameters of the optimal rule				
	$\phi_{\Pi}$	2.25	3.25	7.75
	$\phi_u$	1.00	0.25	0.00

*Notes:* Changing policy to an optimal rule for a specific wealth percentile. From top to bottom: consumption-equivalent welfare gains by wealth percentile, share of households in favor of the change, and average dollar-equivalent gain for all households. Continued from top to bottom: standard deviation of inflation (ann. pp) and the unemployment rate (in percentage points). The numbers reported are the raw standard deviations. Next: change in the average unemployment rate (in percentage points). Last: parameters of the optimal simple rule.

Focus on the bottom block of rows in the table first, which report the optimal coefficients of the

<sup>31</sup>We keep conditioning on the initial state being the non-stochastic steady state. The grid points we allow are  $\phi_{\Pi} \in \{1.25, 1.5, \dots, 8\}$  and  $\phi_u \in \{0, 0.25, \dots, 1.5\}$ . Parameter  $\phi_R = 0.8$  as in the calibrated baseline.

rules. All groups of households would favor a policy that is more responsive to the business cycle than the baseline. As regards the balance between inflation and unemployment, however, there is a strong wealth gradient. The wealth-poor favor dovish policy (left column); the wealth-rich favor hawkish policy (right column). A utilitarian planner for the entire population would choose  $\phi_{\Pi} = 5.5$  and  $\phi_u = 0.75$  (result not reported in the table). Next focus on the middle block of rows. In line with the inflation-unemployment stabilization trade-off in the model, the unemployment rate is notably more volatile for the policy favored by the wealth-rich than the wealth-poor (the standard deviation of unemployment is 1.57 percentage points and 0.42 percentage point, respectively). Similarly, average unemployment is 0.2 percentage point higher in the policy favored by the wealth-rich.

Two results are noteworthy. First, rather different policies could attract support by a majority of households. To see this, focus on the top block of rows. We see that 56 percent of households each favor the policies for the wealth-poor and the middle-class (the particular similarity in numbers is a coincidence) over the *status quo* in spite of these having fundamentally different distributional implications: the policy for the middle-class hurts the wealth-poor and benefits the wealth-rich. *Vice versa* the policy for the wealth-poor hurts the wealth-rich. The second result that is noteworthy is the spread in consumption-equivalent welfare gains and losses across households. This spread is about 0.05 percentage point for the policy targeted at the wealth-poor, and 0.37 percentage point, almost an order of magnitude larger, for a transition to a policy targeted at the wealth-rich.

## 4.6 Explaining the disagreement

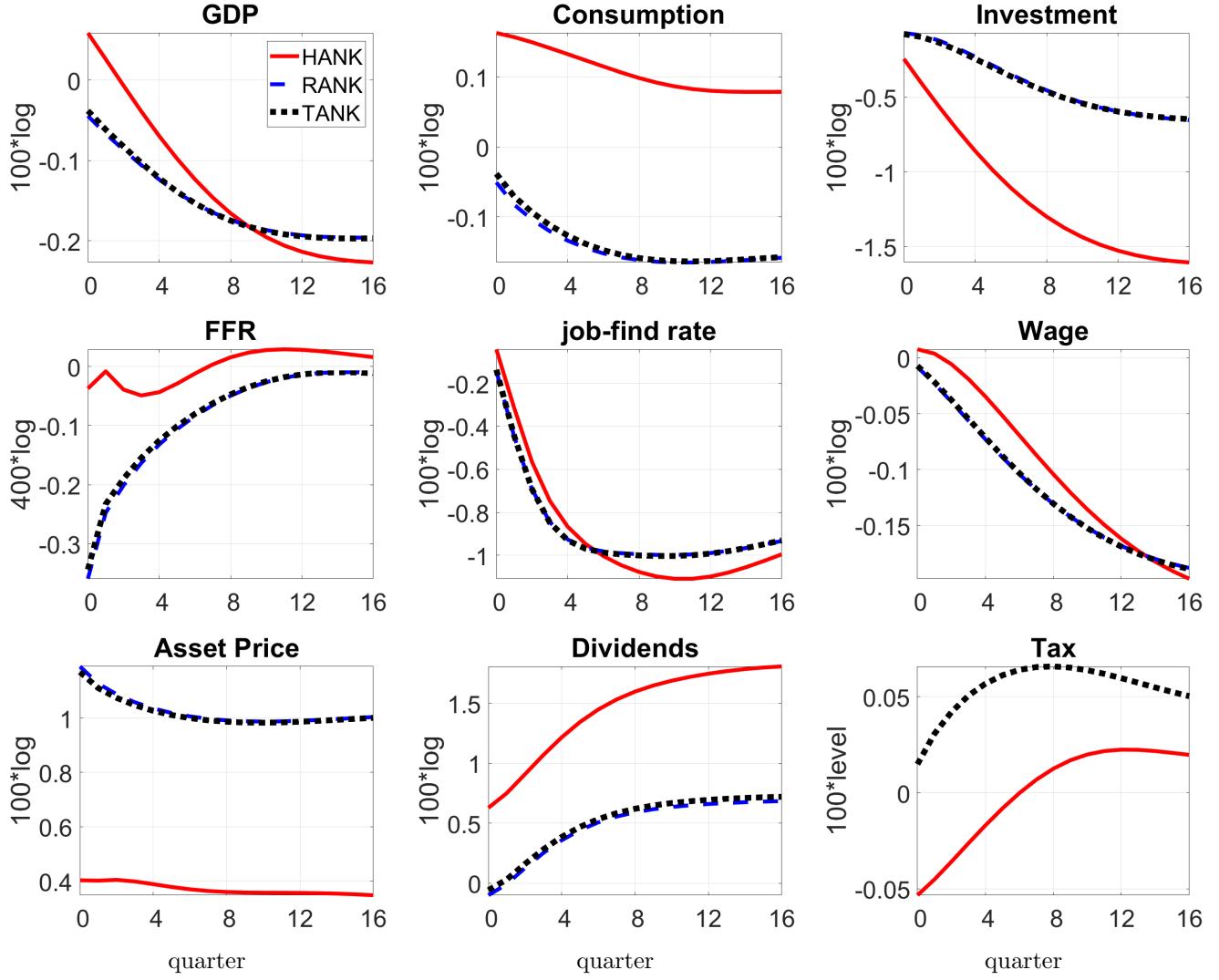
Households in the HANK economy strongly disagree about systematic monetary stabilization policy. In order to illustrate the channels at work, throughout this section we focus on one of the polar cases: a transition toward strict inflation targeting.

### 4.6.1 The transition path

All of the welfare assessments that we show take into account the transition path. The current section shows why: a change in systematic stabilization policy sets in motion pronounced transition dynamics. Abstracting from this transition would, therefore, lead to erroneous welfare assessments for the HANK model (for comparison, Appendix P shows the welfare assessments that focus on the long run only). Figure 3 shows the first four years (16 quarters) of the average transition path after the new policy is implemented. Each panel shows the difference between the expected path after strict inflation targeting is introduced and the expected path in the baseline. Appendix Q reports the algorithm employed. Appendix R presents the same transitions over a longer horizon (125 years). The panels plot the transition path in the HANK economy as solid red lines, and in the RANK and TANK variants (the virtually indistinguishable dashed blue and dotted black lines).

Upon implementing inflation targeting, all three economies see valuation gains for owners of shares: In the HANK economy, the asset price jumps by 0.4 percent (bottom left panel, solid red line). Since in HANK asset holders are neither infinitely lived, nor necessarily patient, many owners of shares have high marginal propensities to consume (compare Appendix I.9). Even for the average owner of shares (using wealth weights to average), the marginal propensity to consume out of wealth over the course of a year is about 8.5 percent compared to only 3.1 percent in RANK/TANK. The windfall gains to wealth are, therefore, effective for aggregate consumption demand in HANK. In HANK consumption demand supports real activity (first row, left) and wages (second row, right). Dividends rise. Since dividends are taxed and there is an initial expansion in incomes, in the short run, income taxes fall in HANK, further supporting consumption demand. In RANK/TANK,

Figure 3: Transition toward policy of  $\Pi = \bar{\Pi}$



*Notes:* Short-run expected transition path after strict inflation-targeting is introduced. Quarter 0 is the quarter of the policy change. HANK: solid red, RANK: dashed blue, TANK: dotted black. In terms of scale: “ $100^*\log$ ” means percent deviation from the baseline path. “ $400^*\log$ ” means annualized percentage points. “ $100^*\text{level}$ ” means p.p. change in levels.

instead, real activity falls and taxes rise. This is so in spite of a sharper rise in the asset price than in RANK/TANK (bottom-left panel).<sup>32</sup> Nevertheless, in the RANK/TANK economies this is not met by a consumption boom. This is reasonable. In both RANK and TANK, the valuation gains do not accrue to any household with a higher marginal propensity to consume. All households either are not exposed to the valuation gains of assets, because they do not hold shares (the spenders), or are infinitely lived families. What is also noteworthy is the fall in investment on the transition path (top right panel) that is much more pronounced in HANK than in RANK/TANK.

What sets the HANK economy apart from RANK/TANK is that households are exposed to risk

<sup>32</sup>The sharper rise in asset prices in RANK/TANK is in line with more monetary accommodation on impact and persistently lower real rates than in HANK. Appendix S shows this.

and can self-insure against that. Namely, households seek to self-insure against income fluctuations that are life-cycle related (retirement), purely idiosyncratic (skill shocks), or business-cycle related (the risk of persistent unemployment, for example). This strengthens two channels in HANK that are muted in RANK and TANK. We turn to these next.

#### 4.6.2 Precautionary savings

To the extent that monetary policy increases idiosyncratic risk by stabilizing inflation at the expense of employment, in HANK working-age households would be expected to increase their precautionary savings. As the demand for funds rises, this would put upward pressure on the asset price and increase the mutual funds' investment in the capital stock. Appendix I.10 shows the households' consumption policies as a function of net worth for households in different idiosyncratic states. Upon moving toward inflation targeting, the consumption policies, for a given level of savings shift downward (toward higher savings). And consistent with a precautionary savings channel, they shift downward by more for working-age households than for retirees. Section 4.6.3 will revisit this.

Still, the precautionary savings channel likely is not the dominant explanation for the differences between the HANK and RANK/TANK economies shown above. The reason is simple: the effect on investment in Figure 3 goes in the wrong direction, relative to RANK and TANK. The capital stock falls faster and by more in the HANK economy.

#### 4.6.3 Valuation gains and the supply of capital

Instead, we emphasize that a change in systematic monetary policy in our calibration affects the income distribution, the cash flow of firms, and the discounting of cash flows. A policy change can, therefore, induce *valuation gains* or losses on financial assets, giving rise to the second channel that is specific to HANK. To the extent that systematic monetary stabilization policy increases, say, the profitability of firms, it raises the value of financial assets. The corresponding increase in the effective supply of assets is inconsequential in RANK and TANK, where the demand for funds is rather interest-elastic. In HANK, instead, the demand for funds is not, because funds serve a purpose. Namely, households purchase funds so as to insure consumption against fluctuations in income, over both the life and business cycle. Therefore, if the value of assets rises, for a given stock of capital and employment, all else equal the economy can provide the same degree of insurance with less productive capacity. This puts downward pressure on the price of assets and upward pressure on the discount factor that firms apply, until productive capacity has fallen sufficiently so as to realign the supply of assets with the demand for funds.<sup>33</sup> At the same time, there are second-round effects that affect the demand for funds. Namely, a fall in the capital stock reduces the wages that workers receive and, thus, it reduces permanent income for wage earners, and with it their savings.

Table 12 reports the long-run effect that systematic monetary stabilization policy has on average unemployment, capital, and wages across the model variants (HANK/RANK/TANK). The first line focuses on the unemployment rate. In line with an average-employment effect running through unemployment variability, the central bank's focus on inflation raises the average unemployment rate (by 0.17 percentage point). This effect, though, is strikingly similar across model variants. Differences in the strength of the average employment channel, therefore, do not seem to be a

---

<sup>33</sup>We abstract from government debt. This is not innocuous. In particular, suppose that there was government debt. Then, a government debt management policy could counteract the rise in the supply of assets on the transition path. Since government debt is net worth for individual households, we would conjecture that a policy that reduces debt after the change in systematic monetary policy would cushion the fall in the capital stock.

**Table 12: Change in long run, strict inflation targeting**

	HANK	RANK	TANK
$E(\text{urate})$ (p.p)	0.17	0.17	0.17
$E(\log(k))$ (%)	-2.12	-0.36	-0.37
$E(\log(w))$ (%)	-0.39	-0.23	-0.23

*Notes:* Change of average unemployment rate (p.p.), long-run average capital stock and wage (in percent) induced by a change to inflation targeting. Negative numbers mean that unemployment, capital, or the wage falls.

candidate explanation for the different response of the three model variants to the policy change.

The next row, instead, shows that the average capital stock shows sharply different effects in HANK and RANK/TANK. In all the variants, a fall in employment reduces the marginal product of capital, and so would be expected to go hand in hand with a fall in the capital stock. The fall in capital is much more pronounced in HANK, however, than in RANK/TANK. A move toward strict inflation targeting makes the long-run average capital stock in HANK fall by 2.1 percent. In the RANK and TANK economies, instead, the same policy change would induce a fall in the capital stock of only about a fifth of this magnitude. With this, economic activity in the long run shrinks more in HANK than in RANK/TANK, and so do wages (last row).<sup>34</sup>

#### 4.6.4 Policy and the distribution of net worth

The interplay of the channels affects the wealth distribution. Table 13 documents the long-run

**Table 13: HANK – Effect of systematic monetary policy on net worth of groups**

		Response to unemployment, $\phi_u$								
		0	0.1	0.15	0.2	0.3	0.4	0.5	0.6	$\Pi = \bar{\Pi}$
		Savings change at long-run mean								
Wealth percentile	0-20	-0.02	0.00	—	0.00	0.01	0.01	0.01	0.01	-0.09
	20-40	0.01	0.00	—	0.00	-0.01	-0.01	-0.01	-0.01	<b>0.10</b>
	40-60	<b>0.23</b>	0.05	—	-0.03	-0.09	-0.12	-0.12	-0.13	<b>1.21</b>
	60-80	-0.11	-0.02	—	0.01	0.03	<b>0.04</b>	0.02	0.03	-0.70
	80-95	-0.32	-0.08	—	0.06	0.15	0.20	0.25	<b>0.28</b>	-1.54
	95-100	<b>-7.24</b>	<b>-1.79</b>	—	1.48	3.50	4.97	6.39	<b>7.20</b>	-31.30
	Total	-0.39	-0.10	—	0.08	0.19	0.26	0.34	0.38	-1.69

*Notes:* By wealth percentile. Entries are expressed as per-household multiples of per-household quarterly GDP. An entry of -1.54 means that wealth per household of the respective group falls by an amount equivalent to 1.54 percent of quarterly per-household GDP. By way of reference, the per-household net worth of the respective groups (from top row to bottom row) are: 0.01 (0-20), 0.43 (20-40), 2.38 (40-60), 9.02 (60-80), 26.46(80-95), 154.21 (95-100) and 14.05 (for the total) times per-household quarterly GDP. Net worth here is defined as  $p(X) \cdot a$ , that is, by *ex-dividend* net worth.

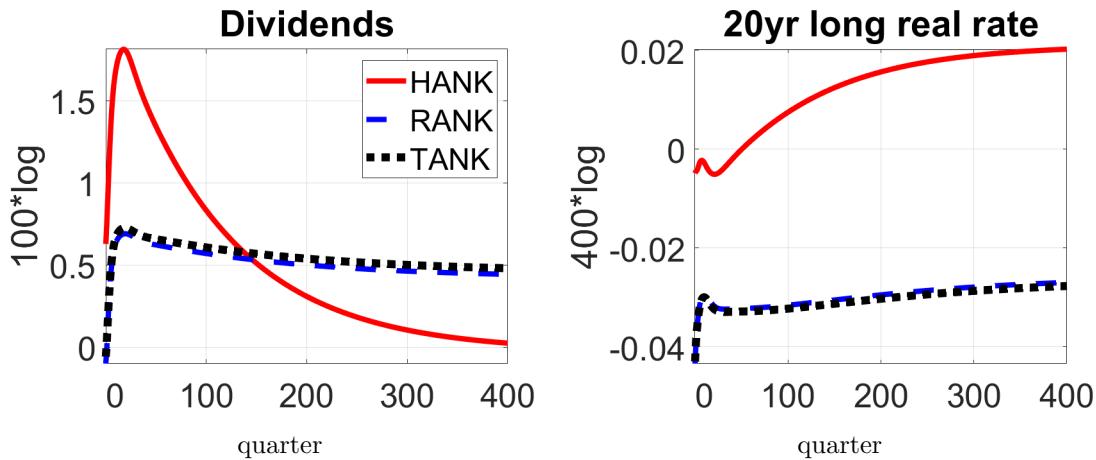
<sup>34</sup>Note that the average capital stock and employment do not need to move exactly in lock-step since the model features a capacity-utilization decision.

effect of a shift in systematic monetary policy on the market value of *ex-dividend* net worth held by different segments of the wealth distribution that then prevails. In order to gain an idea of how much they shape the aggregate, the numbers are multiples of steady-state quarterly GDP per household. As monetary policy becomes more inflation-centric, for the wealth-poorest 20 percent, net worth hardly changes. They save little to start with and their average earnings fall, whereas, by design, the generosity of social insurance does not. Households around the median of the wealth distribution, instead, increase their net worth. Under strict inflation targeting, their net worth rises by about 1 percent of steady-state per-household GDP, equivalent to half a percent of an increase in the group's net worth. This is in line with the group's rising exposure to employment risk and occurs in spite of a fall in labor income. Households at the top of the wealth distribution, instead, have lower net worth than under the baseline policy. The net worth of the 5 percent wealthiest households falls by about 0.2 percent, commensurate (in levels) with a fall in ex-dividend net worth by a third of per-household GDP.

#### 4.6.5 The real rate of interest

Figure 3 shows dividends and the long real rate along the transition path, for a longer horizon. Namely, shown is a long horizon of 400 quarters (100 years). Dividends in RANK and TANK

**Figure 4: Transition induced by policy change to  $\Pi = \bar{\Pi} - 125$  years**



*Notes:* Same as Figure 3, but for a longer horizon.

rise by about half a percent in the long run (left panel, blue dashed and black dotted lines that overlap). In HANK dividends per share rise on the transition path, but eventually return to about the level they had prior to the policy change. The right panel plots a putative long real rate of interest (for a 20-year real bond). In the long run, the effect of a move toward inflation targeting in HANK is to raise the average real rate by 2 bps (annualized). In HANK/RANK, instead, the average real rate falls. The difference in the real rate between HANK and RANK/TANK is 4.7 bps annualized.<sup>35</sup> Note that, on purpose, we have used the term “real rate” rather than the term “natural rate.” Monetary policy in all three variants affects allocations and the real rate only

<sup>35</sup>The mechanics are reminiscent of Krusell et al. (2009), who find in a real business-cycle model that removing cyclical fluctuations reduces the capital stock and raises the real rate of interest, which significantly raises the welfare of the wealth richest. Here, the gains to the rich, instead, arise when business-cycle volatility *increases*.

when there are nominal rigidities. The natural (flex-price) rate of interest, thus, is not affected by systematic monetary policy in either variant. Instead, the real rate of interest is.

#### 4.7 Do the RANK and TANK variants capture the trade-offs?

We have built a HANK business-cycle model with substantial heterogeneity. A central element of that model is the households' ability to save. At the same time, this makes solving the model computationally involved. So the question arises of whether simpler models would be an equally adequate guide to the welfare consequences of systematic monetary stabilization policy. Toward this end, this section explores the welfare assessments provided by the RANK and TANK variant.

Appendix E describes the TANK economy in more detail. Appendices I.5 through I.8 show that in terms of fluctuations alone, the three economies for the baseline policy provide a rather similar view at first glance. Namely, second moments in the HANK and TANK/RANK economies are comparable. Similarly, the impulse responses to shocks in the TANK/RANK economies differ somewhat from HANK, but not fundamentally so. The same is true of (first-order) forecast error variance decompositions. And also the unemployment-inflation trade-off is comparable in the three model variants (Figure 2 and Appendix K).

In spite of this, as we show next, neither the RANK nor the TANK variant captures the policy trade-offs in HANK, adding to the differences in the mean dynamics that we highlighted earlier. Table 14 provides consumption-equivalent welfare gains for the RANK and TANK economies.

**Table 14: RANK/TANK – Consumption-equivalent welfare gains from changing policy**

		Response to unemployment, $\phi_u$								$\Pi = \bar{\Pi}$
		0.0	0.1	0.15	0.2	0.4	0.6	0.8	1.0	
TANK	saver	-0.044	-0.009	—	0.006	<b>0.016</b>	0.015	0.011	0.007	-0.195
	spender	-0.068	-0.016	—	0.012	0.041	0.053	0.059	<b>0.063</b>	-0.279
RANK		-0.046	-0.010	—	0.007	0.020	<b>0.021</b>	0.019	0.016	-0.215

*Notes:* Same as Table 8, but for RANK and TANK. For the latter, welfare is reported for the saver family and the spender family. Boldface marks the maximum welfare gains.

Compare these to the corresponding numbers for the HANK model in Table 8. The most striking observation here is that in TANK, disagreement is mild at best, whereas in HANK 43 percent of households supported inflation targeting. In RANK and TANK alike, all households dislike this policy (the right-most column). In the presence of markup shocks inflation targeting is socially costly.<sup>36</sup> Also for smaller policy changes all households in the two models agree on the direction, preferring a monetary policy that is *more* responsive to unemployment than in the baseline.<sup>37</sup>

One may then wonder if the TANK model can be *made* to provide guidance similar to that of the HANK economy, through a judicious choice of calibration strategy. Qualitatively it can, if one calibrates wealth to be more concentrated, but not too concentrated. Table 15 reports the welfare and long-run effects in TANK, when doing so by increasing the calibrated mass of spenders. Then, as wealth is more concentrated, the TANK model does feature heterogeneity in

<sup>36</sup> Appendix T.1 provides the consumption-equivalent welfare gains for spenders when spender households do not pool incomes across idiosyncratic labor-market state, education, and age. Still, there is no disagreement about policy.

<sup>37</sup> A utilitarian planner that would choose optimal simple policies in RANK and TANK would pick, respectively,  $\phi_{\Pi} = 1.59$  and  $\phi_u = 0.59$  and  $\phi_{\Pi} = 1.71$  and  $\phi_u = 0.66$  (numbers not reported in the table).

**Table 15: TANK – Welfare gains for SAVERS by share of spenders**

Share of spenders	Response to unemployment, $\phi_u$								$\Pi = \bar{\Pi}$
	0.0	0.1	0.15	0.2	0.3	0.4	0.5	0.6	
50	-0.033	-0.006	—	0.003	<b>0.003</b>	0.000	-0.004	-0.010	-0.112
70	-0.022	-0.002	—	-0.001	-0.008	-0.017	-0.028	-0.037	-0.003
75	-0.018	0.000	—	-0.003	-0.012	-0.024	-0.036	-0.048	<b>0.041</b>
80	-0.013	0.002	—	-0.004	-0.017	-0.032	-0.047	-0.060	<b>0.099</b>

*Notes:* TANK model. Consumption equivalent welfare gains for saver households. Share of spenders varies between 50 percent (first row) and 80 percent (last row). Otherwise, the exercise is analogous to Table 14.

policy assessments. Spenders always dislike inflation targeting (see Appendix T.2). Once wealth is sufficiently concentrated, however, *savers* begin to approve of inflation targeting. Note that this requires a mass of spenders beyond 70 percent of the population, however, so that the share of households favoring this policy is at most 30 percent. This falls considerably short of the 43 percent of households that support moving toward inflation targeting in the HANK baseline. Accounting for the relative exposure that households have to the employment effects and financial effects that systematic monetary stabilization policy may have, thus, is important for determining support for the policies.

## 5 Sensitivity analysis

The paper is concerned with the distributional effects of systematic monetary policy when households have different sources of income and, therefore, different exposure to a policy change. In our model, this runs through windfall gains to owners of capital, falling average labor income, and different exposures to a rise in average labor-market risk. This section probes the wage rule, the role of household portfolios, fiscal policy, and the role of price adjustment costs. All of these dimensions are important for the distributional effects of systematic monetary stabilization policy. In order to keep the dimensionality limited, we report results for a move toward strict inflation targeting only.

Table 16 reports results of the sensitivity checks we run. The first column repeats the results for the baseline. The remaining columns report results for the sensitivity checks (to be described in detail below). For each scenario, we report the welfare gains in the HANK model by wealth percentile and the share of households in favor of the policy change from the baseline policy rule. This is the first set of rows. The second set of rows reports results for the corresponding TANK variant: the welfare gains for saver and spender households (at the baseline calibration of a share of spenders by 0.15 percent), and the rise in the average unemployment rate (in p.p.) that the change to inflation targeting brings. In all but one scenario, the share of households in favor of inflation targeting is on the order of 29 to 51 percent in HANK, while the TANK model (in our baseline calibration) indicates losses for both savers and spenders.

### 5.1 The wage rule

Wages allocate the surplus in the employment-services sector. This matters both for the business cycle (Shimer, 2005 and Hagedorn and Manovskii, 2008) and the long run. To the extent that average aggregate economic activity is affected by monetary stabilization policy, as it is in the current model environment, the wage rule determines how much of this is passed on to the wage

**Table 16: HANK – Sensitivity analysis**

		Wage rule			Lever.		Balanced	Price	
		Basel.	Nash	High Share	Flex	portf.	taxation	adj. cost	
Wealth percent.	0-20	-0.12	-0.07	-0.08	-0.41	-0.08	-0.13	-0.10	-0.09
	20-40	-0.11	-0.04	-0.13	-0.54	-0.05	-0.11	-0.10	-0.10
	40-60	0.00	0.05	-0.03	-0.65	0.04	0.04	0.02	-0.05
	60-80	0.11	0.12	-0.02	-0.80	0.15	0.12	0.13	0.02
	80-95	0.24	0.23	0.07	-0.75	0.26	0.23	0.29	0.13
	95-100	0.26	0.23	0.09	-0.63	0.26	0.22	0.30	0.16
	In favor	0.43	0.48	0.29	0	0.50	0.51	0.46	0.38
TANK	saver	-0.20	-0.15	-0.49	-1.22	-0.10	-0.20	-0.19	-0.21
	spender	-0.28	-0.21	-0.52	-0.74	-0.17	-0.28	-0.26	-0.20
	$\Delta E(urate)$	0.17	0.14	0.40	1.06	0.48	0.17	0.17	0.17

*Notes:* First block: HANK economy. Consumption-equivalent welfare gains (in percent) by wealth percentile, and share of households in favor of a change toward strict inflation targeting. Second block: consumption-equivalent welfare gains for savers and spenders in TANK, and change in the average unemployment rate. Scenarios described in the main text.

(and potentially the labor share) or to employment. If adjustment is through the wage, share holders favor stabilizing inflation at the expense of employment. If adjustment goes through employment, instead, the marginal product of capital falls, hurting the corporate sector as well. To show the role that the wage response plays in assigning winners and losers of monetary stabilization policy, we have run several counterfactuals. Throughout, unless noted otherwise, the values of namesake parameters are the same as under the wage rule we used for the baseline HANK model.

First, we let wages in the long run emerge as implied by the Nash-bargaining protocol. The wage rule that would prevail in the simple search-and-matching analog with a risk-neutral household, through surplus sharing, would lead the wage to respond to the price of labor services and to market tightness. Adding wage rigidity to this gives the following wage rule:

$$w(X) - \bar{w}(X) = \omega + \phi_w (w_1(X) - \bar{w}) + (1 - \phi_w)\eta \left[ h(X) + \beta \mathbb{E}_\zeta \kappa(\tilde{X}') \frac{f(\tilde{X}')}{q(\tilde{X}')} \right] + \zeta_w.$$

We choose the new parameter  $\omega$  here to have the same average unemployment rate as under the baseline wage rule if policy were to follow the baseline's monetary rule. In addition, we choose a bargaining-power parameter of  $\eta = 0.5$ , a customary value. Not only does the model with the Nash wage rule show second moments similar to the baseline (not shown here), but also the welfare implications are rather similar to the baseline; see column "Nash" of Table 16. Indeed, still more households would support strict inflation targeting, a support that the TANK model would miss. Second, we highlight how the policy assessment would be affected if wages in the long run do not fall in lock-step with economic activity, but remain high. We assume that the wage moves according to:

$$\log(w(X)/\bar{w}) = \phi_w \log(w_{-1}(X)/\bar{w}) + \frac{1}{2} \left[ (1 - \phi_w) \cdot \log \left( \frac{y(X)}{\bar{y}} \right) \right] + \zeta_w.$$

Under this scenario (labeled “High” in Table 16), unemployment rises by about twice as much as with the baseline wage rule. The support for inflation targeting shrinks, but at 29 percent of households remains sizable. The TANK model again would miss this support. Third, we choose a wage rule that is designed to explicitly make sure that under all circumstances the long-run labor share can never be affected by monetary policy. The measured labor share in the long run is given by  $\frac{w(X) \int_M se(1-\varrho l) \mathbb{1}_{n=1} d\mu}{GDP(X)}$ , so we entertain the following wage rule:

$$\log(w(X)/\bar{w}) = \phi_w \log(w_{-1}(X)/\bar{w}) + (1 - \phi_w) \cdot \log \left( \frac{GDP(X)}{\int_M se(1 - \varrho l) \mathbb{1}_{n=1} d\mu} \right) + \zeta_w.$$

The column “Share” shows the results. With this wage rule, average unemployment rises strongly upon a move to inflation targeting (the unemployment rate rises by 1.06 percentage points). All segments of the population then agree that inflation targeting is not a preferable policy, the middle class now being the biggest losers from a change toward inflation-centric monetary policy. Fourth, we abstract from wage rigidity, setting  $\phi_w = 0$  in the baseline wage rule, column “Flex” in the Table. The support for hawkish monetary policy rises to slightly above 50% of households.<sup>38</sup>

In sum, in our model systematic monetary stabilization policy affects economic activity in the short and the long run. It is central, then, to form a view of how the wage-setting process distributes the gains and losses.

## 5.2 Household portfolio and exposure to financial gains

An important literature has shown that monetary shocks in part propagate through the heterogeneity of household portfolios; see, for example, Cloyne et al. (2019). The current paper is concerned with systematic monetary stabilization policy, rather than monetary shocks. Still, the portfolio structure will matter for two reasons at least: first, because it determines the exposure that different households have to the gains and losses from a change in systematic monetary policy; second, because systematic monetary policy determines the response of incomes following economic shocks. To the extent that monetary policy allows inflation to fall in a recession, for example, this provides a windfall gain to holders of nominal assets, providing them with additional insurance; see Bhandari et al. (forthcoming). The current section seeks to illustrate the role of portfolios through a simple counterfactual. Namely, we assume that household portfolios now are composed of two assets: short-term nominal bonds and shares of a mutual fund. The mutual fund is the counterparty for bond holdings. A household can be short or long in bonds. We then assign the portfolio weights in bonds and shares that emerge, by age, education, and net worth, from the 2004 SCF. Appendix U provides details. We wish to emphasize that the mapping is coarse. In the data, we assign non-nominal assets, including housing, to the share component. For the bond counterpart, we disregard maturity.<sup>39</sup> The column labeled “Leveraged portfolio” in Table 16 shows the results.

---

<sup>38</sup>We have also run a counterfactual with the Nash rule above, but flexible wages. Then, the support for strict inflation targeting rises further to 88%. Again, however, the effect on average unemployment (an increase of 0.9 percentage point) is large.

<sup>39</sup>Many other dimensions of the portfolio structure will likely matter: how liquid a household’s assets are, for example, or how households can finance leverage. For example, in a model in which a riskless bond is used for precautionary savings (as in Kaplan et al. (2018)) the demand for funds may be channeled there rather than into capital. More generally, it will also matter if the valuation gains accrue equally to all real assets. One may wonder, in particular, about the implicit assumption here that housing wealth moves in lock-step with business wealth. A more detailed analysis of such spillovers is beyond the scope of the current paper, however. The more the valuation

Namely, the support for inflation targeting rises to above 51 percent of households. This is because those in the working-age middle class tend to hold highly levered portfolios. Even though their *net worth* is small, they now receive a larger share of the financial windfall gains that inflation targeting assigns to owners of shares. The portfolios of retirees are more nominal on the asset side to start with. They, therefore, benefit less than in the baseline. At the same time, they were so solidly in favor of inflation targeting in the baseline that they remain so now. This channel, too, would be missing in the TANK model, where the composition of portfolios does not play a role.

### 5.3 Monetary stabilization policy and the tax system

In the HANK baseline, the gains from unemployment stabilization accrue to the factor labor, the gains from inflation stabilization to the owners of shares. The move toward more inflation-centric policy on average leads to lower employment and wages. If taxes were kept constant, the government would have lower tax revenue and higher expenditure (for unemployment benefits, in particular). In the baseline, therefore, labor-income taxes rise to balance the budget. This burdens labor twice, through lower income and higher taxes. By design, dividend taxes, instead, were kept constant before. In a sensitivity check, we have made the financing more balanced, having both labor and dividend taxes move to finance the government budget. In particular, we assume that they move in lock-step: whenever the labor tax rate rises by 1 percentage point, so will the tax on dividends. The column labeled “Balanced taxation” of Table 16 reports the results. A more balanced financing spreads the gains from hawkish monetary stabilization policy more widely and the assessment is slightly more favorable to inflation targeting than in the baseline.

### 5.4 How important are the costs of price adjustment themselves?

We have kept an important question until the end of the paper, namely, the role of price adjustment costs. A long literature in monetary economics discusses the costs of inflation variability and whether they fall on households or firms. The baseline assigns price adjustment costs to owners of firms. In order to show how sensitive the results are to the distribution across society of these costs, scenario “Price adjustment cost” in Table 16 assumes that price adjustment costs affect firms’ policies at the margin, but that these costs do not enter the firms’ profits. Instead, we assume that the government reimburses the firm sector for the price adjustment costs in a lump-sum fashion, with the expenses financed through labor-income taxes. This means that the nominal rigidities continue to affect economic outcomes, but that the direct costs of price fluctuations are borne by all households, each in proportion to its non-financial income. Doing this, the direct gains from inflation stabilization policy no longer accrue to financial capital. The support for strict inflation targeting shrinks somewhat, but at 38 percent of households remains sizable. We conclude that the costs of price adjustment themselves are not essential for our finding of disagreement.

## 6 Conclusions

Monetary policy affects aggregate economic activity, the distribution of income, and income risks that households face. To assess the distributional effects of the systematic conduct of monetary policy, we have built a New Keynesian heterogeneous-agent DSGE model that features asset-market incompleteness, heterogeneity in preferences, skills, and age, a frictional labor market, and sticky prices. The model was calibrated to the U.S. in tranquil times.

---

gains in equilibrium are concentrated only on the asset classes held by the top of the wealth distribution, the lower we suppose will be the support for a change toward hawkish policy.

The main finding is that households may strongly disagree as to how monetary policy should *systematically* respond to the business cycle. That disagreement can be traced to households' relative exposure to labor income and wealth. The reason was that the gains from stabilizing inflation and the costs of doing so were not evenly distributed across different households. If the central bank stabilizes inflation, it raises average markups. To the extent that wages fall with economic activity, as they do in the baseline, corporate profits are stabilized at the expense of labor income. Thus, stabilizing inflation may lead to winners and losers. We document that, in our model, this channel dominates the effect of precautionary savings on capital accumulation that arises from higher employment risk. The households that gain from inflation stabilization are the wealth-rich (for whom labor income is a small part of lifetime wealth) and retirees (who tend to have assets, but are not exposed to labor income). The wealth-poorest households (those who draw most or all of their income from labor) would be willing to forgo up to 0.12 percent of their lifetime consumption to avoid a move to strict inflation targeting. The wealth-richest 5 percent of households, instead, would gain the equivalent of about 0.25 percent. To finance these changes in consumption, in 2004 US\$, the wealth-poorest would need to be compensated by \$2,400. The wealth-richest 5 percent, instead, would gain, and at \$24,000 an order of magnitude more. Nominal redistribution does not play a role in these results. The results emerge with real assets only and when fixing the average inflation rate at 2 percent p.a. throughout.

Our results are, of course, neither model-free nor independent of the assumptions we make. The way in which society splits the surplus from employment matters in particular. In the baseline, the labor share falls with a move toward inflation-centric policy, a result that also emerges when we assume Nash-bargained wages. At the same time, we also showed that systematic monetary stabilization policy may be assessed more equally across the population, when wages are set to keep the labor share constant. The conundrum is that all the wage rules we look at are potential equilibrium wages. Our paper, thus, points to a need for evidence on how the valuation of assets and the labor share move with systematic monetary policy. We also discussed how the results depend on the prevailing sources of shocks and how the tax system shapes the support for an inflation-focused monetary policy. Last, we discussed that – to the extent that systematic monetary stabilization policy has a bearing on the income distribution – household portfolios can play an important role in distributing any gains and losses. The current paper certainly is not meant to be an indictment of monetary stabilization policy as it is. Rather, we hope to highlight that the choice of systematic monetary stabilization policy may not be entirely innocuous, be it for aggregate activity or the cross-section of households. We hope that future work will clarify that link and also the quantitative importance of the channels highlighted here.

## References

- ACHARYA, S., E. CHALLE, AND K. DOGRA (2020): “Optimal Monetary Policy According to HANK,” CEPR Discussion Papers 14429.
- ACHARYA, S. AND K. DOGRA (2020): “Understanding HANK: Insights From a PRANK,” *Econometrica*, 88, 1113–1158.
- AHN, S., G. KAPLAN, B. MOLL, T. WINBERRY, AND C. WOLF (2017): “When Inequality Matters for Macro and Macro Matters for Inequality,” in *NBER Macroeconomics Annual 2017, volume 32*, University of Chicago Press.
- AIYAGARI, S. R. (1994): “Uninsured Idiosyncratic Risk and Aggregate Saving,” *Quarterly Journal of Economics*, 109, 659–84.
- ALTONJI, J. G., A. A. SMITH, AND I. VIDANGOS (2013): “Modeling Earnings Dynamics,” *Econometrica*, 81, 1395–1454.
- ANDREASEN, M. M., J. FERNANDEZ-VILLAVERDE, AND J. F. RUBIO-RAMREZ (2018): “The Pruned State-Space System for Non-Linear DSGE Models: Theory and Empirical Applications,” *Review of Economic Studies*, 85, 1–49.
- AUCLERT, A. (2019): “Monetary Policy and the Redistribution Channel,” *American Economic Review*, 109, 2333–2367.
- AUCLERT, A., M. ROGNLIE, AND L. STRAUB (2020): “Micro Jumps, Macro Humps: Monetary Policy and Business Cycles in an Estimated HANK Model,” Working Paper 26647, National Bureau of Economic Research.
- BARATTIERI, A., S. BASU, AND P. GOTTSCHALK (2014): “Some Evidence on the Importance of Sticky Wages,” *American Economic Journal: Macroeconomics*, 6, 70–101.
- BARNICHON, R. (2010): “Building a Composite Help-Wanted Index,” *Economics Letters*, 109, 175–178.
- BAYER, C., B. BORN, AND R. LUETTICKE (2020): “Shocks, Frictions, and Inequality in US Business Cycles,” CEPR Discussion Papers 14364.
- BAYER, C., L. PHAM-DAO, R. LUETTICKE, AND V. TJADEN (2019): “Precautionary Savings, Illiquid Assets, and the Aggregate Consequences of Shocks to Household Income Risk,” *Econometrica*, 87, 255–290.
- BERGER, D., I. DEW-BECKER, L. D. W. SCHMIDT, AND Y. TAKAHASHI (2019): “Layoff Risk, the Welfare Cost of Business Cycles, and Monetary Policy,” mimeo, Northwestern University.
- BHANDARI, A., D. EVANS, M. GOLOSOV, AND T. J. SARGENT (forthcoming): “Inequality, Business Cycles, and Monetary-Fiscal Policy,” *Econometrica*.
- BILBIIE, F. O. (2020): “The New Keynesian Cross,” *Journal of Monetary Economics*, 114, 90 – 108.
- BLANCHARD, O. AND J. GALÍ (2010): “Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment,” *American Economic Journal: Macroeconomics*, 2, 1–30.
- BLUNDELL, R., L. PISTAFERRI, AND I. SAPORTA-EKSTEN (2016): “Consumption Inequality and Family Labor Supply,” *American Economic Review*, 106, 387–435.
- BROER, T., N.-J. HARBO HANSEN, P. KRUSELL, AND E. BERG (2019): “The New Keynesian Transmission Mechanism: A Heterogeneous-Agent Perspective,” *Review of Economic Studies*, 87, 77–101.
- CAIRÓ, I. AND T. CAJNER (2018): “Human Capital and Unemployment Dynamics: Why More Educated Workers Enjoy Greater Employment Stability,” *Economic Journal*, 128, 652–682.

- CAMPBELL, J. Y. AND N. G. MANKIW (1989): "Consumption, Income and Interest Rates: Reinterpreting the Time Series Evidence," in *NBER Macroeconomics Annual 1989, Volume 4*, National Bureau of Economic Research, 185–246.
- CARCELES-POVEDA, E. AND D. COEN-PIRANI (2010): "Owning Capital or Being Shareholders: An Equivalence Result with Incomplete Markets," *Review of Economic Dynamics*, 13, 537 – 558.
- CARROLL, C., J. SLACALEK, K. TOKUOKA, AND M. N. WHITE (2017): "The Distribution of Wealth and the Marginal Propensity to Consume," *Quantitative Economics*, 8, 977–1020.
- CASTAÑEDA, A., J. DÍAZ-GIMÉNEZ, AND J.-V. RÍOS-RULL (2003): "Accounting for the U.S. Earnings and Wealth Inequality," *Journal of Political Economy*, 111, 818–857.
- CHALLE, E. (2020): "Uninsured Unemployment Risk and Optimal Monetary Policy in a Zero-Liquidity Economy," *American Economic Journal: Macroeconomics*, 12, 241–83.
- CHETTY, R. (2008): "Moral Hazard Versus Liquidity and Optimal Unemployment Insurance," *Journal of Political Economy*, 116, 173–234.
- CHODOROW-REICH, G. AND L. KARABARBOUNIS (2016): "The Cyclicalities of the Opportunity Cost of Employment," *Journal of Political Economy*, 124, 1563–1618.
- CHRISTIANO, L. J., M. EICHENBAUM, AND C. L. EVANS (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113, 1–45.
- CHRISTIANO, L. J., M. S. EICHENBAUM, AND M. TRABANDT (2016): "Unemployment and Business Cycles," *Econometrica*, 84, 1523–1569.
- CLOYNE, J., C. FERREIRA, AND P. SURICO (2019): "Monetary Policy when Households have Debt: New Evidence on the Transmission Mechanism," *Review of Economic Studies*, 87, 102–129.
- COMIN, D. AND M. GERTLER (2006): "Medium-Term Business Cycles," *American Economic Review*, 96, 523–551.
- COUCH, K. A. AND D. W. PLACZEK (2010): "Earnings Losses of Displaced Workers Revisited," *American Economic Review*, 100, 572–89.
- DAVIS, S. J. AND T. VON WACHTER (2011): "Recessions and the Costs of Job Loss," *Brookings Papers on Economic Activity*, 1–552.
- DE NARDI, M. (2004): "Wealth Inequality and Intergenerational Links," *Review of Economic Studies*, 71, 743–768.
- DEBORTOLI, D. AND J. GAL (2017): "Monetary Policy with Heterogeneous Agents: Insights from TANK Models," Economics Working Papers 1686, Universitat Pompeu Fabra.
- DEN HAAN, W., P. RENDAHL, AND M. RIEGLER (2017): "Unemployment (Fears) and Deflationary Spirals," *Journal of the European Economic Association*, 16, 1281–1349.
- DEN HAAN, W. J., G. RAMEY, AND J. WATSON (2000): "Job Destruction and Propagation of Shocks," *American Economic Review*, 90, 482–498.
- DOEPKE, M. AND M. SCHNEIDER (2006a): "Aggregate Implications of Wealth Redistribution: The Case of Inflation," *Journal of the European Economic Association*, 4, 493–502.
- (2006b): "Inflation and the Redistribution of Nominal Wealth," *Journal of Political Economy*, 114, 1069–1097.
- ELSBY, M. W. L., B. HOBIJN, AND A. SAHIN (2010): "The Labor Market in the Great Recession," *Brookings Papers on Economic Activity*, 41, 1–69.
- FAIA, E. (2009): "Ramsey Monetary Policy with Labor Market Frictions," *Journal of Monetary Economics*, 56, 570–581.
- FERNALD, J. (2014): "A Quarterly, Utilization-Adjusted Series on Total Factor Productivity," Federal Reserve Bank of San Francisco.

- FERNÁNDEZ-VILLAVERDE, J., K. KUESTER, P. GUERRÓN-QUINTANA, AND J. RUBIO-RAMÍREZ (2015): “Fiscal Volatility Shocks and Economic Activity,” *American Economic Review*, 105, 3352–84.
- FLODEN, M. AND J. LINDÉ (2001): “Idiosyncratic Risk in the United States and Sweden: Is There a Role for Government Insurance?” *Review of Economic Dynamics*, 4, 406–437.
- GALÍ, J. AND M. GERTLER (1999): “Inflation Dynamics: A Structural Econometric Analysis,” *Journal of Monetary Economics*, 44, 195–222.
- GERTLER, M. AND A. TRIGARI (2009): “Unemployment Fluctuations with Staggered Nash Wage Bargaining,” *Journal of Political Economy*, 117, 38–86.
- GOUVEIA, M. AND R. P. STRAUSS (1994): “Effective Federal Individual Income Tax Functions: An Explanatory Empirical Analysis,” *National Tax Journal*, 47, 317–339.
- GRAVES, S. (2020): “Does Unemployment Risk Affect Business Cycle Dynamics?” International Finance Discussion Papers 1298. Board of Governors of the Federal Reserve System.
- GREENWOOD, J., Z. HERCOWITZ, AND G. W. HUFFMAN (1988): “Investment, Capacity Utilization, and the Real Business Cycle,” *American Economic Review*, 78, 402–417.
- GUNER, N., R. KAYGUSUZ, AND G. VENTURA (2014): “Income Taxation of U.S. Households: Facts and Parametric Estimates,” *Review of Economic Dynamics*, 17, 559–581.
- GUVENEN, F., S. OZKAN, AND J. SONG (2014): “The Nature of Countercyclical Income Risk,” *Journal of Political Economy*, 122, 621–660.
- HAGEDORN, M. AND I. MANOVSKII (2008): “The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited,” *American Economic Review*, 98, 1692–1706.
- HAIRAUT, J.-O., F. LANGOT, AND S. OSOTIMEHIN (2010): “Matching Frictions, Unemployment Dynamics and the Cost of Business Cycles,” *Review of Economic Dynamics*, 13, 759–779.
- HOSIOS, A. J. (1990): “On the Efficiency of Matching and Related Models of Search and Unemployment,” *Review of Economic Studies*, 57, 279–298.
- HUGGETT, M. AND J. C. PARRA (2010): “How Well Does the U.S. Social Insurance System Provide Social Insurance?” *Journal of Political Economy*, 118, 76–112.
- HURST, E., A. LUSARDI, A. KENNICKELL, AND F. TORRALBA (2010): “The Importance of Business Owners in Assessing the Size of Precautionary Savings,” *Review of Economics and Statistics*, 92, 61–69.
- JUNG, P. AND K. KUESTER (2011): “The (Un)importance of Unemployment Fluctuations for the Welfare Cost of Business Cycles,” *Journal of Economic Dynamics and Control*, 35, 1744–1768.
- KAPLAN, G., B. MOLL, AND G. L. VIOLANTE (2018): “Monetary Policy According to HANK,” *American Economic Review*, 108, 697–743.
- KRUSELL, P., T. MUKOYAMA, A. SAHIN, AND J. ANTHONY A. SMITH (2009): “Revisiting the Welfare Effects of Eliminating Business Cycles,” *Review of Economic Dynamics*, 12, 393–402.
- KRUSELL, P., A. SAHIN, AND T. MUKOYAMA (2010): “Labour-Market Matching with Precautionary Savings and Aggregate Fluctuations,” *Review of Economic Studies*, 77, 1477–1507.
- KRUSELL, P. AND A. SMITH (1998): “Income and Wealth Heterogeneity in the Macroeconomy,” *Journal of Political Economy*, 106, 867–896.
- MAZUMDER, B. (2005): “Fortunate Sons: New Estimates of Intergenerational Mobility in the United States Using Social Security Earnings Data,” *Review of Economics and Statistics*, 87, 235–255.
- MCKAY, A., E. NAKAMURA, AND J. STEINSSON (2016): “The Power of Forward Guidance Revisited,” *American Economic Review*, 106, 3133–58.

- MCKAY, A. AND R. REIS (2016): “The Role of Automatic Stabilizers in the U.S. Business Cycle,” *Econometrica*, 84, 141–194.
- MEH, C. A., J.-V. RÍOS-RULL, AND Y. TERAJIMA (2010): “Aggregate and Welfare Effects of Redistribution of Wealth Under Inflation and Price-Level Targeting,” *Journal of Monetary Economics*, 57, 637–652.
- MORTENSEN, D. T. AND C. A. PISSARIDES (1994): “Job Creation and Job Destruction in the Theory of Unemployment,” *Review of Economic Studies*, 61, 397–415.
- MUKOYAMA, T. AND A. SAHIN (2006): “Costs of Business Cycles for Unskilled Workers,” *Journal of Monetary Economics*, 53, 2179 – 2193.
- NAKAJIMA, M. (2012a): “Business Cycles in the Equilibrium Model of Labor Market Search and Self-Insurance,” *International Economic Review*, 53, 399–432.
- (2012b): “Rising Indebtedness and Temptation: A Welfare Analysis,” *Quantitative Economics*, 3, 257–288.
- RAVENNA, F. AND C. E. WALSH (2011): “Welfare-Based Optimal Monetary Policy with Unemployment and Sticky Prices: A Linear-Quadratic Framework,” *American Economic Journal: Macroeconomics*, 3, 130–62.
- (2012): “Monetary Policy and Labor Market Frictions: A Tax Interpretation,” *Journal of Monetary Economics*, 59, 180 – 195.
- RAVN, M. O. AND V. STERK (forthcoming): “Macroeconomic Fluctuations with HANK and SAM: An Analytical Approach,” *Journal of the European Economic Association*.
- REITER, M. (2009): “Solving Heterogeneous-Agent Models by Projection and Perturbation,” *Journal of Economic Dynamics and Control*, 33, 649–665.
- (2010a): “Approximate and Almost-Exact Aggregation in Dynamic Stochastic Heterogeneous-Agent Models,” Economics Series 258, Institute for Advanced Studies.
- (2010b): “Solving the Incomplete Markets Model with Aggregate Uncertainty by Backward Induction,” *Journal of Economic Dynamics and Control*, 34, 28–35.
- ROGERSON, R. AND R. SHIMER (2011): *Search in Macroeconomic Models of the Labor Market*, Elsevier, vol. 4 of *Handbook of Labor Economics*, chap. 7, 619–700.
- ROTEMBERG, J. J. (1982): “Sticky Prices in the United States,” *Journal of Political Economy*, 90, 1187–1211.
- SALA, L., U. SÖDERSTRÖM, AND A. TRIGARI (2008): “Monetary Policy Under Uncertainty in an Estimated Model with Labor Market Frictions,” *Journal of Monetary Economics*, 55, 983–1006.
- SCHMITT-GROHÉ, S. AND M. URIBE (2004): “Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function,” *Journal of Economic Dynamics and Control*, 28, 755–775.
- SHIMER, R. (2005): “The Cyclical Behavior of Equilibrium Unemployment and Vacancies,” *American Economic Review*, 95, 25–49.
- SILVA, J. I. AND M. TOLEDO (2009): “Labor Turnover Costs and the Cyclical Behavior of Vacancies and Unemployment,” *Macroeconomic Dynamics*, 13, 76–96.
- SMETS, F. AND R. WOUTERS (2007): “Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach,” *American Economic Review*, 97, 586–606.
- SOLON, G. (1992): “Intergenerational Income Mobility in the United States,” *American Economic Review*, 82, 393–408.
- STERK, V. AND S. TENREYRO (2018): “The Transmission of Monetary Policy Through Redistributions and Durable Purchases,” *Journal of Monetary Economics*, 99, 124 – 137.

- TAYLOR, J. B. (1993): “Discretion Versus Policy Rules in Practice,” *Carnegie-Rochester Conference Series on Public Policy*, 39, 195–214.
- WINBERRY, T. (2018): “A Method for Solving and Estimating Heterogeneous Agent Macro Models,” *Quantitative Economics*, 9, 1123–1151.
- WOODFORD, M. (1998): “Doing Without Money: Controlling Inflation in a Post-Monetary World,” *Review of Economic Dynamics*, 1, 173–219.
- YASHIV, E. (2000): “The Determinants of Equilibrium Unemployment,” *American Economic Review*, 90, 1297–1322.

## ———— Online Appendix ——

### A Single-firm setup

This appendix provides the one-firm setup alluded to in Section 2.4.2 of the main text. We first write out the optimization problem of this firm. Then we compare the resulting optimality conditions to the ones collected in Appendix B that characterize the behavior of firms in the model in the main text. By showing that they are equal, we conclude that the two setups are equivalent.

The setup is as described in the general text, but instead of renting labor and capital services from other firms, each intermediate good producer owns its capital stock, chooses utilization rates, and hires new workers. It still sells its good as a monopolist to a final good producer, who is described in Subsection 2.4.1.<sup>A1</sup> We denote the value function of the firm by  $J^F$ . To distinguish the individual choices and state variables of a firm from aggregate equilibrium quantities, we index them by  $j$ . The firm chooses its own utilization rate for capital  $v^j$ , vacancy postings  $V^j$ , investment spending  $i^j$ , and the price of its good  $P^j$  each period. It makes these decisions given the following state variables: the firm's capital stock at the beginning of the period,  $K^j$ , its investment in the last period  $i_{-1}^j$ , the firm's mass of workers of different skill  $s$ , skill-loss state  $l$ , and education  $e$  it starts the period with, denoted by  $N^j(s, l, e)$ ,<sup>A2</sup> and the last period price,  $P_{-1}^j$ , it charged. In addition,  $\tilde{X}$  is the current aggregate state of the economy.<sup>A3</sup>

The producer's optimization problem is:

$$\begin{aligned} & J_F(K_{-1}^j, i_{-1}^j, (N_{-1}^j(s, l, e))_{\forall(s,l,e)}, P_{-1}^j, \tilde{X}) \\ &= \max_{v^j, V^j, i^j, P^j} (1 - \tau^d) \left( y_j \left( \frac{P^j}{P(X)} \right) - i^j - \kappa(\tilde{X})V^j - \sum_{s,l,e} es(1 - \rho l)w(X)N^j(s, l, e) \right. \\ &\quad \left. - \frac{\psi}{2} \left( \frac{P^j}{P_{-1}^j} - \bar{\Pi} \right)^2 y(X) + \mathbb{E}_\zeta \left[ Q(X, X') J_F((K^j)', i^j, (N^j(s, l, e))_{\forall(s,l,e)}, P^j, \tilde{X}') \right] \right) \end{aligned}$$

---

<sup>A1</sup>For the argument in Section 2.4.2 of the main text the final good producer is irrelevant, as it makes no profits or intertemporal choice.

<sup>A2</sup> $s$  is understood to exclude the retired here.

<sup>A3</sup>In the following, some aggregate variables will depend on  $\tilde{X}$ , while others will depend on  $X$ . As  $X$  is a deterministic function of  $\tilde{X}$  we will follow the convention in the main text, with the silent understanding that  $X \equiv X(\tilde{X})$ . In addition, given our assumption on such transitions we omit the transitions between retirement and re-birth in the law of motion for the worker types.

$$\begin{aligned}
\text{s.t. } y_j &= \left( \frac{P^j}{P(X)} \right)^{-\vartheta \exp(\zeta_P)} y(X) \\
y_j &= \zeta_{TFP} (K^j v^j)^\theta \left( \sum_{s,l,e} es(1 - \rho l) N^j(s, l, e) \right)^{1-\theta} \\
(K^j)' &= (1 - \delta(v^j)) K^j + \zeta_I (1 - \Gamma(i^j / i_{-1}^j)) i^j \\
N^j(s, l, e) &= \sum_{s_{-1} \in \mathcal{S}_+} \sum_{l_{-1}} \pi_S(s_{-1}, s | s \neq s_0) \pi_L^{emp}(l_{-1}, l) (1 - \lambda_x(e) - \lambda_n(e)) N_{-1}^j(s_{-1}, l_{-1}, e) \\
&+ q(\tilde{X}) V^j \frac{U(\tilde{X}, s, e) \pi_L^{unemp}(l) + \lambda_n(e) N(\tilde{X}, s, l, e)}{\sum_{s', e'} [U(\tilde{X}, s', e') + \lambda_n(e') \sum_{l'} N(\tilde{X}, s', l', e')]} \forall(s, e).
\end{aligned}$$

Here, we have assumed that the firm takes the aggregate job-filling rate by group and the cost of posting a vacancy as given. This means, for example, that the quadratic adjustment costs are assumed to be an aggregate effect in line with our assumptions in the main text.

Having described the firm's optimization problem, we now collect the conditions characterizing an interior solution of this problem, by combining first-order conditions, constraints, and envelope conditions. To simplify notation, we suppress the dependence on  $X$ .<sup>A4</sup> While we substitute out  $y^j$  using the demand equation, we need to add Lagrange multipliers for the other constraints. We denote those by  $\nu^y$ ,  $\nu^k$ , and  $\nu^n(s, l, e)$ . We arrive at the following equations.

$$\begin{aligned}
(\text{Optimal } P): 0 &= (1 - \tau^d) \left( (1 - \vartheta \exp(\zeta_P)) \frac{y}{P} \left( \frac{P^j}{P} \right)^{-\vartheta \exp(\zeta_P)} - \psi \left( \frac{P^j}{P_{-1}^j} - \bar{\Pi} \right) \frac{y}{P_{-1}^j} \right) \\
&+ \vartheta \exp(\zeta_P) \frac{y}{P} \left( \frac{P^j}{P} \right)^{-\vartheta \exp(\zeta_P)-1} \nu^y + (1 - \tau^d) \mathbb{E}_\zeta Q \psi \left( \frac{P_{+1}^j}{P^j} - \bar{\Pi} \right) \frac{P_{+1}^j y_{+1}}{(P^j)^2}
\end{aligned}$$

$$(\text{Optimal } v): \nu^y \theta \frac{y^j}{v^j} = \nu^k \delta'(v^j) K^j$$

$$(\text{Optimal } V): (1 - \tau^d) \kappa = q \sum_{s,l,e} \nu^n(s, l, e) \frac{U(s, e) \pi_L^{unemp}(l) + \lambda_n(e) N(s, l, e)}{\sum_{s', e'} [U(s', e') + \lambda_n(e') \sum_{l'} N(s', l', e')]} \quad \text{---}$$

$$\begin{aligned}
(\text{Optimal } I): 0 &= -(1 - \tau^d) + \nu^k (\zeta_I (1 - \Gamma(i^j / i_{-1}^j)) - \zeta_I \Gamma'(i^j / i_{-1}^j) i^j / i_{-1}^j) \\
&+ \mathbb{E}_\zeta Q \nu_{+1}^k \zeta_I \Gamma'(i_{+1}^j / i^j) (i_{+1}^j / i^j)^2
\end{aligned}$$

$$\begin{aligned}
(\text{Optimal } N): \forall(s, l, e) \nu^n(s, l, e) &= (1 - \tau^d) es(1 - \rho l) w + \nu^y (1 - \theta) \frac{y^j es(1 - \rho l)}{\sum_{s,l,e} es(1 - \rho l) N^j(s, l, e)} \\
&+ \mathbb{E}_\zeta Q \sum_{s \in \mathcal{S}_+} \sum_l \pi_S(s, s_{+1} | s_{+1} \neq s_0) \pi_L^{emp}(l, l_{+1}) (1 - \lambda_x(e) - \lambda_n(e)) \nu_{+1}^n(s_{+1}, l_{+1}, e)
\end{aligned}$$

---

<sup>A4</sup>The superscript  $j$  still allows us to distinguish between variables at the firm and aggregate levels.

$$(\text{Optimal } K): \nu^k = \mathbb{E}_\zeta \nu_{+1}^y \theta \frac{y_{+1}^j}{(K^j)'} + \mathbb{E}_\zeta \nu_{+1}^k (1 - \delta(v_{+1}^j))$$

$$(\text{Production}): \left( \frac{P^j}{P} \right)^{-\vartheta \exp(\zeta_P)} y = \zeta_{TFP} (K^j v^j)^\theta \left( \sum_{s,l,e} e s (1 - \rho l) N^j(s, l, e) \right)^{1-\theta}$$

$$(\text{LoM } K): (K^j)' = (1 - \delta(v^j)) K^j + \zeta_I (1 - \Gamma(i^j / i_{-1}^j)) i^j$$

$$\begin{aligned} (\text{LoM } N): & \sum_{s_{-1} \in \mathcal{S}_+} \sum_{l_{-1}} \pi_S(s_{-1}, s | s \neq s_0) \pi_L^{emp}(l_{-1}, l) (1 - \lambda_x(e) - \lambda_n(e)) N_{-1}^j(s_{-1}, l_{-1}, e) \\ & + q V^j \frac{U(s, e) \pi_L^{unemp}(l) + \lambda_n(e) N(s, l, e)}{\sum_{s', e'} [U(s', e') + \lambda_n(e') \sum_{l'} N(s', l', e')]} = N^j(s, l, e) \quad \forall (s, l, e). \end{aligned}$$

We focus on a symmetric equilibrium where all firms had the same initial conditions and where the initial distribution of skills equals the ergodic distribution. Given a unique solution to the above equations this also implies that all firms make the same choices. Imposing the definition of inflation, and  $U(s, e) = \pi_S(s | s \in \mathcal{S}_+) U(e)$ ,  $N(s, l, e) = \pi_S(s | s \in \mathcal{S}_+) N(l, e)$  we arrive at the system of equations below.

$$\begin{aligned} (\text{Optimal } P): & 0 = (1 - \tau^d) ((1 - \vartheta \exp(\zeta_P)) y - \psi (\Pi - \bar{\Pi}) y \Pi) \\ & + \vartheta \exp(\zeta_P) y \nu^y + (1 - \tau^d) \mathbb{E}_\zeta Q \psi (\Pi_{+1} - \bar{\Pi}) y_{+1} \Pi_{+1} \end{aligned}$$

$$(\text{Optimal } v): \nu^y \theta \frac{y}{v} = \nu^k \delta'(v) K$$

$$(\text{Optimal } V): (1 - \tau^d) \kappa = q \sum_{s,l,e} \nu^n(s, l, e) \frac{\pi_S(s | s \in \mathcal{S}_+) U(e) \pi_L^{unemp}(l) + \lambda_n(e) \pi_S(s | s \in \mathcal{S}_+) N(l, e)}{\sum_{s',e'} [\pi_S(s' | s' \in \mathcal{S}_+) U(e') + \lambda_n(e') \sum_{l'} \pi_S(s' | s' \in \mathcal{S}_+) N(l', e')]} \kappa$$

$$\begin{aligned} (\text{Optimal } I): & 0 = -(1 - \tau^d) + \nu^k (\zeta_I (1 - \Gamma(i/i_{-1})) - \zeta_I \Gamma'(i/i_{-1}) i/i_{-1}) \\ & + \mathbb{E}_\zeta Q \nu_{+1}^k \zeta_I \Gamma'(i_{+1}/i) (i_{+1}/i)^2 \end{aligned}$$

$$\begin{aligned} (\text{Optimal } N): \forall (s, l, e) \nu^n(s, l, e) = & (1 - \tau^d) e s (1 - \rho l) w + \nu^y (1 - \theta) \frac{y e s (1 - \rho l)}{\pi_S(s | s \in \mathcal{S}_+) N(l, e)} \\ & + \mathbb{E}_\zeta Q \sum_{s \in \mathcal{S}_+} \sum_l \pi_S(s, s_{+1} | s_{+1} \neq s_0) \pi_L^{emp}(l, l_{+1}) (1 - \lambda_x(e) - \lambda_n(e)) \nu_{+1}^n(s_{+1}, l_{+1}, e) \end{aligned}$$

$$(\text{Optimal } K): \nu^k = \mathbb{E}_\zeta \nu_{+1}^y \theta \frac{y_{+1}}{K'} + \mathbb{E}_\zeta \nu_{+1}^k (1 - \delta(v_{+1}))$$

$$(\text{Production}): y = \zeta_{TFP} (Kv)^\theta \left( \sum_{s,l,e} es(1 - \rho l) \pi_S(s|s \in \mathcal{S}_+) N(l, e) \right)^{1-\theta}$$

$$(\text{LoM } K): K' = (1 - \delta(v))K + \zeta_I (1 - \Gamma(i/i_{-1}))i$$

$$\begin{aligned} (\text{LoM } N): & \sum_{s_{-1} \in \mathcal{S}_+} \sum_{l_{-1}} \pi_S(s_{-1}, s|s \neq s_0) \pi_L^{emp}(l_{-1}, l) (1 - \lambda_x(e) - \lambda_n(e)) \pi_S(s_{-1}|s_{-1} \in \mathcal{S}_+) N_{-1}(l_{-1}, e) \\ & + qV^j \frac{\pi_S(s|s \in \mathcal{S}_+) U(e) \pi_L^{unemp}(l) + \lambda_n(e) \pi_S(s|s \in \mathcal{S}_+) N(l, e)}{\sum_{s',e'} [\pi_S(s'|s' \in \mathcal{S}_+) U(e') + \lambda_n(e') \sum_{l'} \pi_S(s'|s' \in \mathcal{S}_+) N(l', e')]} \\ & = \pi_S(s|s \in \mathcal{S}_+) N(l, e) \quad \forall (s, l, e). \end{aligned}$$

We are now in a position to compare the equations here to the ones for the model description in the main text. Obviously, comparing the last two equations, labeled (LoM  $K$ ) and (LoM  $N$ ), to the main text we see that the laws of motion for capital and various employment groups are the same. Given the market clearing conditions for labor and capital services, we also see that the expressions for the production functions are the same between the main text and the model in this appendix. Therefore, it remains to compare the optimality conditions. To simplify this we define the following new variables:  $\hat{J}(s, l, e) := \nu^n(s, l, e)$ ,  $\hat{r} = \frac{\theta y}{Kv(1-\tau^d)}$ ,  $\hat{h} = \frac{(1-\theta)y\nu^y}{(1-\tau^d)\sum_{s,l,e} es(1-\rho l)\pi_S(s|s \in \mathcal{S}_+)N(l,e)}$ . Substituting them into our optimality conditions we arrive at the expressions below:

$$\begin{aligned} (\text{Optimal } P): 0 &= (1 - \tau^d) ((1 - \vartheta \exp(\zeta_P))y - \psi(\Pi - \bar{\Pi})y\Pi) \\ &+ \vartheta \exp(\zeta_P)y\nu^y + (1 - \tau^d)\mathbb{E}_\zeta Q\psi(\Pi_{+1} - \bar{\Pi})y_{+1}\Pi_{+1} \end{aligned}$$

$$(\text{Optimal } v): \nu^y \hat{r} (1 - \tau^d) K = \nu^k \delta'(v) K$$

$$(\text{Optimal } V): (1 - \tau^d)\kappa = q \sum_{s,l,e} \hat{J}(s, l, e) \frac{\pi_S(s|s \in \mathcal{S}_+) U(e) \pi_L^{unemp}(l) + \lambda_n(e) \pi_S(s|s \in \mathcal{S}_+) N(l, e)}{\sum_{s',e'} [\pi_S(s'|s' \in \mathcal{S}_+) U(e') + \lambda_n(e') \sum_{l'} \pi_S(s'|s' \in \mathcal{S}_+) N(l', e')]} \frac{(1-\theta)y\nu^y}{(1-\tau^d)\sum_{s,l,e} es(1-\rho l)\pi_S(s|s \in \mathcal{S}_+)N(l,e)}$$

$$\begin{aligned} (\text{Optimal } I): 0 &= -(1 - \tau^d) + \nu^k (\zeta_I (1 - \Gamma(i/i_{-1})) - \zeta_I \Gamma'(i/i_{-1})i/i_{-1}) \\ &+ \mathbb{E}_\zeta Q\nu_{+1}^k \zeta_I \Gamma'(i_{+1}/i) (i_{+1}/i)^2 \end{aligned}$$

$$\begin{aligned}
(\text{Optimal } N): \forall(s, l, e) \quad & \hat{J}(s, l, e) = es(1 - \rho l)(\hat{h} - w)(1 - \tau^d) \\
& + \mathbb{E}_\zeta Q \sum_{s \in \mathcal{S}_+} \sum_l \pi_S(s, s_{+1} | s_{+1} \neq s_0) \pi_L^{emp}(l, l_{+1}) (1 - \lambda_x(e) - \lambda_n(e)) \hat{J}_{+1}(s_{+1}, l_{+1}, e)
\end{aligned}$$

$$(\text{Optimal } K): \nu^k = \mathbb{E}_\zeta \nu_{+1}^y \hat{r}_{+1} v_{+1} (1 - \tau^d) + \mathbb{E}_\zeta \nu_{+1}^k (1 - \delta(v_{+1})).$$

Now, we can compare (Optimal  $N$ ) and (Optimal  $V$ ) to the definition of the value of a match and the free-entry condition for labor service producers in the main text and see that they are identical once we equate  $\mu$ s and  $\nu$ s and hatted variables with their un-hatted cousins. We can conclude their equivalence. The same applies if we compare the remaining optimality conditions here with the ones for the differentiated goods producers and capital services producers in Appendix B. So we can conclude that the equations describing the firm behavior conditional on a discount factor  $Q$  are the same and imply the same allocations, proving the equivalence of the two setups in terms of aggregates.

## B The firms' optimization problems

This appendix collects the equations characterizing the solution to the optimization problem of all the firms in the model, after taking first-order conditions, applying envelope conditions, and simplifying. We describe the resulting equations collected under the respective firm's name. We suppress the dependence on  $X$  to keep the notation simple.

### B.1 Final goods and adjustment services

Since all differentiated goods producers in equilibrium will set the same price, final goods firms and adjustment services firms have isomorphic demand functions. So  $y = y_f + y_a$ .

### B.2 Differentiated goods producers

The problem of the differentiated goods producer is characterized by the following set of equations, in which  $\mu^y$  is the multiplier on the production function.

First, there is the optimality condition for inflation, where  $\Pi = \frac{P}{P_{-1}}$ :

$$\psi\Pi(\Pi - \bar{\Pi}) = \psi\mathbb{E}_\zeta [Q(X, X')\Pi'(\Pi' - \bar{\Pi})y'/y] + [\vartheta\exp\{\zeta_P\}mc - (\vartheta\exp\{\zeta_P\} - 1)].$$

Where the optimality conditions for inputs imply that marginal costs,  $mc$ , are given by

$$mc = \left(\frac{1}{\theta}\right)^\theta \left(\frac{1}{1-\theta}\right)^{1-\theta} \frac{r^\theta h^{1-\theta}}{\zeta_{TFP}}.$$

The optimality conditions for capital and labor services input imply

$$\frac{\theta}{1-\theta} \frac{\ell}{k} = \frac{r}{h}.$$

Finally, there is the production function:

$$y = \zeta_{TFP}(k)^\theta (\ell)^{1-\theta}.$$

### B.3 Labor services producers

The problem of employment agencies does not involve any decision beyond the one contained in the free-entry condition. Therefore, the relevant equations are already given in the main text. When solving the model numerically, we make use of its structure to simplify. The only decision of the employment agency that is influenced by the value of various types of matches is the vacancy posting condition. Job-finding and separation rates do not depend on a household's idiosyncratic skills  $s$ . Therefore, the share of households of skill  $s$  in each education-employment status subgroup follows the constant ergodic distribution of skills. For the free-entry condition, it is, therefore, enough to track the expected value of  $\tilde{J}_L$  with respect to  $s$ . Define  $\tilde{J}_L(X, l, e) = \sum_{s \in \mathcal{S}_+} \pi_S(s|s \in \mathcal{S}_+) J_L(X, l, e, s)$ . Using that  $\sum_{s \in \mathcal{S}_+} \pi_S(s|s \in \mathcal{S}_+) = 1$  (the calibration assumption that average skills are equal to 1) and using that  $\pi_S(s|s \in \mathcal{S}_+)$  is ergodic, we obtain that

$$\begin{aligned} \tilde{J}_L(X, l, e) = & (1 - \tau_d)[h(X) - w(X)] \cdot e(1 - \varrho l) \\ & + \sum_{l'} \pi_L^{emp}(l, l') \cdot \mathbb{E}_\zeta [Q(X, X')(1 - \lambda_x(e) - \lambda_n(e)) \tilde{J}_L(X, l', e)]. \end{aligned}$$

With this, one can re-write the free-entry condition as

$$\begin{aligned} & \sum_e \sum_l [U(\tilde{X}, e) \pi_L^{uemp}(l) + \lambda_n(e) N(\tilde{X}, l, e)] \tilde{J}_L(X, l, e) \left[ \sum_e [U(\tilde{X}, e) + \lambda_n(e) \sum_l N(\tilde{X}, l, e)] \right]^{-1} \\ &= (1 - \tau_d) \kappa(\tilde{X}) / q(\tilde{X}). \end{aligned}$$

#### B.4 Capital services producers

Denoting the Lagrange multiplier on the law of motion for capital by  $\mu_k$  we obtain three optimality conditions for the producer of capital services. First, there is the intra-temporal condition for utilization:

$$(1 - \tau^d)r = \mu_k \delta_1 \delta_2 v^{\delta_2 - 1}.$$

Second, we get an Euler equation for investment:

$$(1 - \tau^d) = \mu_k \zeta_I \left[ 1 - \frac{\phi_K}{2} \left( \frac{i}{i_{-1}} - 1 \right)^2 - \phi_K \left( \frac{i}{i_{-1}} - 1 \right) \frac{i}{i_{-1}} \right] + \mathbb{E}_\zeta \left[ Q(X, X') \mu'_k \zeta'_I \phi_K \left( \frac{i'}{i} - 1 \right) (i'/i)^2 \right].$$

Third, we get an optimality condition for capital:

$$\mu_k = (1 - \tau^d) \mathbb{E}_\zeta [Q(X, X') r' v'] + \mathbb{E}_\zeta [Q(X, X') \mu'_k (1 - \delta_1(v')^{\delta_2})].$$

For completeness we repeat the law of motion for capital:

$$K' = (1 - \delta_1(v_t)^{\delta_2}) K + \zeta_I \left( 1 - \frac{\phi_K}{2} \left( \frac{i}{i_{-1}} - 1 \right)^2 \right) i.$$

## C Discount factor

In this appendix, we describe the modeling of the mutual funds' stochastic discount factor  $Q$ . The discount factor governs how the mutual funds evaluate profits across time and states of nature. If asset markets were complete, or if there would be a representative household, there would be a unique stochastic discount factor. In our model, instead, asset markets are incomplete. There are, therefore, several possible choices for the discount factor, and different households may not agree on the dynamic decisions that the firms should make.<sup>A5</sup> Different choices of the discount factor could have different implications for the behavior of the model.<sup>A6</sup> That said, one restriction on the discount factor appears to be natural and easy to implement. The starting point is that for all households that hold shares in the mutual fund, the consumption Euler equation takes the form<sup>A7</sup>

$$\mathbb{E}_{X',S'|X,S} \left[ \beta(S') \frac{u'(c(X', S'))}{u'(c(X, S))} (p_a(X') + d_a(X')) \right] = p_a(X).$$

Here  $S$  summarizes the idiosyncratic states of the household and  $X$  denotes the aggregate state as in the main text. The expectation operator  $\mathbb{E}$  incorporates both the idiosyncratic and aggregate transition probabilities.  $\beta(\cdot)$  and  $c(\cdot)$  are the household's time discount factor and the household's consumption function, respectively. This equation says that in equilibrium all households holding shares have to agree on the valuation of the mutual fund. Therefore, there is a discount factor  $Q$  such that

$$\mathbb{E}_{X'|X} [Q(X, X') (p_a(X') + d_a(X))] = p_a(X).$$

Clearly, different discount factors fulfill the restriction, including any weighted average (with non-negative weights) of individual equity holders' discount factors. Evaluating the discount factor  $Q$  starting from a set of weights of households is computationally burdensome, however.<sup>A8</sup> Rather, we use the fact that the asset price is determined by asset-market clearing. Therefore, in equilibrium, the price incorporates time preferences and risk premiums. We postulate that the individual mutual fund  $i$  observes the pricing functions  $p(X)$ ,  $p(X')$  that the market applies to the *other* (representative) mutual funds' cash flows. Further, mutual fund  $i$  observes the dividend policies  $d_a(X)$  of the

<sup>A5</sup>Even under incomplete markets there can be cases in which a natural candidate for the discount factor emerges. In Carceles-Poveda and Coen-Pirani (2010) and Krusell et al. (2010), while households face idiosyncratic risk and incomplete asset markets, the asset structure is rich enough to generate a unique choice of discount factor along the dimensions relevant for the firms' choices. Other papers with incomplete markets and idiosyncratic risk do not need to specify the firm's discount factor in the first place. In Krusell and Smith (1998) the only long-lived asset is capital. Households invest directly in capital. Firms are competitive and have only static rental decisions to make. A similar argument applies in den Haan et al. (2017), where firms make decisions once and for all. Neither paper has sticky prices (and so dynamic decisions by firms). See also Carceles-Poveda and Coen-Pirani (2010) for results regarding investor unanimity with heterogeneous households.

<sup>A6</sup>Had we solved our model using a first-order approximation or along a transition path with perfect foresight, instead, it would have been enough to specify the dynamics of the real interest rate, which then could have been uniquely derived from the households' Euler equation.

<sup>A7</sup>To keep the notation simple we suppress the role of the bequest term here. The same argument applies once we add the additional marginal utility that old agents receive from the value of bequest.

<sup>A8</sup>In an earlier draft of the current paper we used the asset-weighted mean of households' marginal utilities to define the discount factor. Implementing this discount factor in our perturbation solution would require tracking more policy functions, which would have increased the numerical burden substantially. We have verified in a simpler version of our model that up to second-order dynamics and long-run properties were comparable in both approaches.

other (representative) mutual funds. A mutual fund  $i$  then applies to its own cash flows the market discount factor

$$Q_i(X, X') = \frac{p_a(X)}{p_a(X') + d_a(X')}.$$

To repeat, the variables entering  $Q_i$  are the other funds' equity price and the other funds' dividend policies. In equilibrium, due to symmetry, all mutual funds then apply the same discount factor  $Q(X, X') = Q_i(X, X')$ , and that discount factor is consistent with households' decisions.<sup>A9</sup>

---

<sup>A9</sup>As a robustness check, we have also solved the model using the risk-free rate  $R^f(X)$  defined as

$$\frac{1}{R^f(X)} \mathbb{E}_{X'|X} [(p_a(X') + d_a(X'))] = p_a(X)$$

to discount instead of using  $Q$ . Dynamics and long-run moments were almost indistinguishable from the results reported in the main text (detailed results available upon request). This may not be surprising. Up to first order, any discount factor fulfilling the above conditions would lead to the same real interest rate dynamics. Even higher-order approximations would not be expected to change much, since the modeling assumptions we make will not generate large risk premia, or significant dynamics in these, that could drive a wedge between the risk free-rate and other reasonable discount factors.

## D Definition of equilibrium

This appendix spells out the full definition of a recursive equilibrium in our setting.

**Definition** (Recursive Equilibrium). A recursive equilibrium is

a set of value functions  $W(., n, a, l, e, b, s)$ ,  $J_K(.)$ ,  $J_D(.; j)$ ,  $J_L(., l, e, s)$ , a set of private-sector policy functions  $c(., n, a, l, e, b, s)$ ,  $a(., n, a, l, e, b, s)$ ,  $y_f$ ,  $y_{f,j}(.)$ ,  $y_a(.)$ ,  $y_{a,j}(.)$ ,  $d_a(.)$ ,  $v(.)$ ,  $i(.)$ ,  $K(.)$ ,  $l_j(.)$ ,  $k_j(.)$ ,  $V(.)$ ,  $P_j(.)$ ,  $y(.)$ , a set of prices and discount factors  $w(.)$ ,  $p_a(.)$ ,  $h(.)$ ,  $r(.)$ ,  $Q(., .)$ ,  $P(.)$ ,  $\Pi(.)$ , a set of labor-market variables  $Pr(., e)$ ,  $f(.)$ ,  $N(., l, e)$ ,  $U(., e)$ ,  $q(.)$ ,  $N(.)$ ,  $U(.)$ ,  $\kappa(.)$ , a set of government policies  $\tau(., .)$ ,  $R(.)$ , and a set of transition functions  $T(.)$ ,  $\tilde{T}$ , such that  $X = \tilde{T}(\tilde{X})$  and  $\tilde{X}' = T(X)$ , for all aggregate states  $X$ ,  $\tilde{X}$  idiosyncratic states  $n, a, l, e, b, s$ , and firm indexes  $j$  such that

1. (Households' problems) given asset price  $p_a(.)$ , dividends  $d_a(.)$ , wage  $w(.)$ , job-finding rate  $f(.)$ , taxes  $\tau(., .)$ , transition probabilities  $Pr(., e)$ , and transition functions  $T(.)$ , and  $\tilde{T}(.)$ , the value functions  $W(., n, a, l, e, b, s)$  solve the households' Bellman equations in Section 2.3, and  $c(., n, a, l, e, b, s)$  and  $a(., n, a, l, e, b, s)$  are the resulting optimal policy functions for consumption and assets;
2. (Final goods) given  $P(.)$  and  $P_j(.)$ , policy functions  $y_f(.)$  and  $y_{f,j}(.)$  solve the problem of the final goods producers in Section 2.4.1;
3. (Differentiated goods) given demand function  $y_j(.)$ , and given prices  $r(.)$ ,  $h(.)$ ,  $P(.)$ , discount factor  $Q(., .)$ , and transition functions  $T(.)$  and  $\tilde{T}(.)$ ,  $J_D(.; j)$  solves the differentiated goods producers' Bellman equation given in Section 2.4.2, and  $k_j(.)$ ,  $l_j(.)$ , and  $P_j(.)$  are the corresponding optimal policy functions;
4. (Labor services) given prices  $h(.)$ , wage  $w(.)$ , discount factor  $Q(., .)$ , unemployment  $U(., e)$ , employment  $N(., l, e)$  and transition functions  $T(.)$ , and  $\tilde{T}(.)$ ,  $J_L(., l, e, s)$  solves the employment agencies' valuation equation in Section 2.4.2; given  $J_L(., l, e, s)$ ,  $U(., e)$ ,  $N(., l, e)$ , and  $\kappa(.)$ ,  $q(.)$ , solves the free-entry condition in the same section; given  $M(., .)$  and  $q(.)$ ,  $V(.)$  conforms with the definition of the job-filling rate in the same section; given  $M(., .)$  and  $q(.)$ , vacancy posting costs  $\kappa(.)$  follow the form given in the same section; given  $V(.)$ ,  $U(., e)$ , and  $N(., l, e)$ ,  $M(., V(.))$  is given by the matching function spelled out in the same section; given  $V(.)$ ,  $M(., V(.))$ ,  $U(., e)$  and  $N(., l, e)$ ,  $f(.)$ , the job-finding rate  $f(.)$  is as defined in Section 2.4.2;
5. (Capital services) given rental rate  $r(.)$ , discount factor  $Q(., .)$ , and transition functions  $T(.)$  and  $\tilde{T}(.)$ ,  $J_K(., K, i)$  solves the Bellman equation of the representative producer of capital in Section 2.4.2, and  $i(.)$ ,  $K(.)$  and  $v(.)$  are the resulting optimal policy functions for investment, capital, and utilization, respectively;
6. (Adjustment services goods) given  $P(.)$  and  $P_j(.)$ , policy functions  $y_a(.)$  and  $y_{a,j}(.)$  solve the problem of the adjustment-services producers in Section 2.4.2;
7. (Financial firms) given interest rates  $R(X)$ , inflation  $\Pi(X)$ , and transition function  $T(.)$  and  $\tilde{T}(.)$ , the discount factor satisfies the Euler equation in Section 2.5; given final output  $y_f(.)$ ,

investment policy  $i(\cdot)$ , wage  $w(\cdot)$ , dividends are as described in the same section; for given dividends  $d_a(X)$  and the discount factor  $Q(\cdot, \cdot)$ , the asset price  $p_a(\cdot)$  is consistent with the definition of the discount factor in the same section.

8. (Central bank) given inflation  $\Pi(\cdot)$  and unemployment  $U(\cdot)$ , the interest rate  $R(\cdot)$  follows the Taylor rule given in Section 2.6;
9. (Fiscal authority) given dividends  $d_a(\cdot)$ , consumption policies  $c(\cdot, n, a, l, e, b, s)$ , and wage  $w(\cdot)$ ,  $\tau(\cdot, \cdot)$  balances the government budget in every period (Section 2.6);
10. (Wage) given  $y(\cdot)$ , the wage follows the wage rule spelled out in Section 2.4.2;
11. (Birth)  $Pr(\cdot | \cdot, e)$  is consistent with the flows into retirement and new birth described in Section 2.3.3;
12. (Consistency, demand function)  $y_j(\cdot) = y_{f,j}(\cdot) + y_{a,j}(\cdot)$  is the demand for good  $j$ ;
13. (Symmetry) for all  $j$ ,  $P_j(\cdot) = P(\cdot)$ ,  $y_{f,j}(\cdot) = y_f(\cdot)$ ,  $y_{a,j}(\cdot) = y_a(\cdot)$ , and  $y_j(\cdot) = y(\cdot)$ ;
14. (Market clearing, final goods)

$$y_f(\cdot) = \int_{\mathcal{M}} c(\cdot, n, a, l, e, b, s) d\mu(\cdot) + i(\cdot) + g;$$

15. (Market clearing, adjustment services goods)

$$y_a(\cdot) = \frac{\psi}{2} (\Pi(\cdot) - \bar{\Pi})^2 y(\cdot) + \kappa(\cdot) V(\cdot);$$

16. (Market clearing, differentiated goods)  $y(\cdot) = y_f(\cdot) + y_a(\cdot)$ ;
17. (Market clearing, capital)  $\int_0^1 k_j(\cdot) dj = K_{-1}(\cdot) v(\cdot)$ ;
18. (Market clearing, labor services)  $\int_{\mathcal{M}} se(1 - \varrho l) 1_{n=1} d\mu = \int_0^1 l_j dj$ ;
19. (Market clearing, shares)  $\int_{\mathcal{M}} a(X, n, a, l, e, s) dm_u = 1$ ;
20. (Consistency, capital flow)

$$K(\cdot) = [1 - \delta_1 v(\cdot)^{\delta_2}] \cdot K_{-1}(\cdot) + \zeta_I [1 - \phi_K / 2 \left( \frac{i(\cdot)}{i_{-1}(\cdot)} - 1 \right)^2] i(\cdot)$$

21. (Consistency, employment flow) Employment flows have to be consistent with the evolution of  $\mu$  and  $\tilde{\mu}$  and the respective definition of the employment aggregates,  $N, N(\cdot, l, e), U(\cdot, e), U$ ;
22. (Consistency, aggregate state transition)  $T, \tilde{T}$  are consistent with  $K_{-1}(T(X)) = K(X), w_{-1}(T(X)) = w(X), i_{-1}(T(X)) = i(X), R_{-1}(T(X)) = R(X)$  and  $K_{-1}(\tilde{T}(\tilde{X})) = K_{-1}(\tilde{X}), w_{-1}(\tilde{T}(\tilde{X})) = w_{-1}(\tilde{X}), i_{-1}(\tilde{T}(\tilde{X})) = i_{-1}(\tilde{X}), R_{-1}(\tilde{T}(\tilde{X})) = R_{-1}(\tilde{X})$ ;
23. (Consistency, law of motion for type distribution)  $T, \tilde{T}$  are consistent with the law of motion for the distribution (described in Section D.1).

## D.1 Law of motion of distributions $\mu$ , $\tilde{\mu}$

It remains to state the law of motion for  $\mu$  and  $\tilde{\mu}$ . Let  $A$  be a measurable subset of  $[0, 1]$ , the set of feasible asset holdings. We need to describe the updating of  $\tilde{\mu}$  to  $\mu$  and of  $\mu$  to  $\tilde{\mu}'$  for all feasible combinations of  $(n, A, l, e, b, s)$ .

### D.1.1 Transitions from $\tilde{\mu}$ to $\mu$

We start with the transition from  $\tilde{\mu}$  to  $\mu$ , that is from the beginning of the period (after shocks to the exogenous idiosyncratic states  $(e, b, s)$  and the aggregate states have been realized, but before employment transitions of working-age households have occurred and before earnings-loss transitions have materialized) to the end of the period (the production stage).

The retired can neither lose nor find a job, and the earnings-loss state does not matter for their income. Therefore, for  $s = s_0$  we have  $\mu(n, A, l, e, b, s_0) = \tilde{\mu}(n, A, l, e, b, s_0)$ . For  $s \in \mathcal{S}_+$  (working-age households, including those that have just been reborn), we have for  $n = 1$  at the production stage

$$\begin{aligned} \mu(1, A, l, e, b, s) &= \sum_{l' \in \{0, 1\}} (1 - \lambda_x(e) - \lambda_n(e)(1 - f(\tilde{X}))) \pi_L^{emp}(l', l) \tilde{\mu}(1, A, l', e, b, s) \\ &\quad + \sum_{l' \in \{0, 1\}} f(\tilde{X}) \pi_L^{uem}(l) \tilde{\mu}(0, A, 0, e, b, s), \end{aligned}$$

and for  $n = 0$  at the production stage

$$\begin{aligned} \mu(0, A, 0, e, b, s) &= \sum_{l' \in \{0, 1\}} (\lambda_x(e) + \lambda_n(e)(1 - f(\tilde{X}))) \tilde{\mu}(1, A, l', e, b, s) \\ &\quad + (1 - f(\tilde{X})) \tilde{\mu}(0, A, 0, e, b, s). \end{aligned}$$

### D.1.2 Transitions from $\mu$ to $\tilde{\mu}'$

We now turn to the transition from  $\mu$  to  $\tilde{\mu}'$ . We start with households that end up in the labor force next period. For  $s \in \mathcal{S}_+$ , we have

$$\begin{aligned} \tilde{\mu}'(n, A, l, e, b, s) &= \sum_{s' \in \mathcal{S}_+} \pi_S(s', s) \int_{a':a(X, 1, a', l, e, b, s') \in A} d\mu(X, n, a', l, e, b, s') \\ &\quad + \pi_S(s_0, s) \Pr(n, l | X, e) \sum_{e'} \sum_{\hat{b}} \pi_E(e', e) \pi_{\Delta\beta}(b) \int_{a':a(X, 0, a', l, e', \hat{b}, 0) \in A} d\mu(X, 0, a', l, e', \hat{b}, 0). \end{aligned}$$

For transitions into old age, the following rule applies:

$$\begin{aligned} \tilde{\mu}(\tilde{X}', 0, A, 0, e, b, s_0)' &= \sum_l \sum_n \sum_{s' \in \mathcal{S}_+} \pi_S(s', s_0) \int_{a':a(X, n, a', l, e, b, s') \in A} d\mu(X, n, a', l, e, b, s') \\ &\quad + \pi_S(s_0, s_0) \int_{a':a(X, 0, a', 0, e, b, s_0) \in A} d\mu(X, 0, a', 0, e, b, s_0). \end{aligned}$$

## E TANK model variant

This appendix spells out the TANK model variant. There are two groups of households. A mass  $\pi^{\text{saver}}$  of the population are savers. Savers have access to the mutual fund. Savers all have the same discount factor,  $\beta^{\text{saver}}$ . Savers live in a family that pools all incomes of its members. Thus, although savers' incomes depend on education and fluctuate with employment and retirement, their consumption is not exposed to idiosyncratic income risk or to their life-cycle income profile. The remaining households are excluded from asset markets (the spenders). Spenders have discount factor  $\beta^{\text{spend}} < \beta^{\text{saver}}$ . Spender households' incomes directly translate into their consumption. Incomes differ by education, skill-loss, employment status, and retirement status. For both savers and spenders, we abstract from fluctuations of skills  $s$  during working age. We also abstract from a bequest motive. The education status is assumed to be permanent. In order to simplify notation, we no longer explicitly highlight the dependence on aggregate state variables. Rather, a subscript  $t$  is used to index dependence on the period  $t$  state of the economy.

### E.1 Spenders

Spenders have either permanently low or permanently high education. Spenders in each education group can be in one of four idiosyncratic states: they can be unemployed, employed with an earnings loss, employed without an earnings loss, or retired. In order to preserve on notation, here we discuss incomes and welfare of low-educated spenders only. The formulae for high-education spenders are analogous. In the following,  $\pi_{\text{RET}} = \pi_{s_0}$  marks the probability of retiring.  $\pi_{\text{born}}$  marks the probability of leaving retirement.

#### E.1.1 Spenders' consumption

When employed, the spender consumes  $c_{0,t}^L$  if it does not suffer an earnings loss and  $c_{1,t}^L$  if it does. Superscript  $L$  marks low education. Without an earnings loss, the spender consumes:

$$c_{0,t}^{L,\text{spend}}(1 + \tau_c) = w_t e_L [1 - \tau_{\text{RET}} - \tau_{\text{UI}} - \tau_t(w_t e_L)],$$

where  $\tau_t(\cdot)$  is the progressive income tax function. With an earnings loss, the spender consumes:

$$c_{1,t}^{L,\text{spend}}(1 + \tau_c) = w_t e_L (1 - \varrho) [1 - \tau_{\text{RET}} - \tau_{\text{UI}} - \tau_t(w_t e_L (1 - \varrho))].$$

When unemployed, the spender consumes  $c_{U,t}^L$ , where the  $U$  marks the unemployment state:

$$c_{U,t}^{L,\text{spend}}(1 + \tau_c) = b_{\text{UI}}(e_L) [1 - \tau_t(b_{\text{UI}}(e_L))].$$

Last, when retired, the spender household consumes

$$c_{R,t}^{L,\text{spend}}(1 + \tau_c) = b_{\text{RET}}(e_L) [1 - \tau(b_{\text{RET}}(e_L))].$$

#### E.1.2 Welfare of spenders

With this, and the labor-market transitions, the welfare of the spender household is as follows. If employed, without an earnings loss

$$\begin{aligned} W_{0,t}^{L,\text{spend}} = & (1 - \beta^{\text{spend}}) \frac{(c_{0,t}^{L,\text{spend}})^{1-\sigma}}{1-\sigma} \\ & + \beta^{\text{spend}} \pi_{\text{RET}} \mathbb{E}_t \{ W_{R,t+1}^{L,\text{spend}} \} \\ & + \beta^{\text{spend}} (1 - \pi_{\text{RET}}) \mathbb{E}_t \{ (1 - \lambda_x^L - \lambda_e^L (1 - f_{t+1})) W_{0,t+1}^{L,\text{spend}} \} \\ & + \beta^{\text{spend}} (1 - \pi_{\text{RET}}) \mathbb{E}_t \{ (\lambda_x^L + \lambda_e^L (1 - f_{t+1})) W_{U,t+1}^{L,\text{spend}} \}. \end{aligned}$$

If employed, with an earnings loss

$$\begin{aligned}
W_{1,t}^{L,spend} = & (1 - \beta^{spend}) \frac{(c_{1,t}^{L,spend})^{1-\sigma}}{1-\sigma} \\
& + \beta^{spend} \pi_{RET} \mathbb{E}_t \{ W_{R,t+1}^{L,spend} \} \\
& + \beta^{spend} (1 - \pi_{RET}) \pi_L^{emp}(1, 0) \mathbb{E}_t \{ (1 - \lambda_x^L - \lambda_e^L (1 - f_{t+1})) W_{0,t+1}^{L,spend} \} \\
& + \beta^{spend} (1 - \pi_{RET}) \pi_L^{emp}(1, 1) \mathbb{E}_t \{ (1 - \lambda_x^L - \lambda_e^L (1 - f_{t+1})) W_{1,t+1}^{L,spend} \} \\
& + \beta^{spend} (1 - \pi_{RET}) \mathbb{E}_t \{ (\lambda_x^L + \lambda_e^L (1 - f_{t+1})) W_{U,t+1}^{L,spend} \}.
\end{aligned}$$

If unemployed

$$\begin{aligned}
W_{U,t}^{L,spend} = & (1 - \beta^{spend}) \frac{(c_{U,t}^{L,spend})^{1-\sigma}}{1-\sigma} \\
& + \beta^{spend} \pi_{RET} \mathbb{E}_t \{ W_{R,t+1}^{L,spend} \} \\
& + \beta^{spend} (1 - \pi_{RET}) \mathbb{E}_t \{ f_{t+1} \pi_L^{uem}(0) W_{0,t+1}^{L,spend} \} \\
& + \beta^{spend} (1 - \pi_{RET}) \mathbb{E}_t \{ f_{t+1} \pi_L^{uem}(1) W_{1,t+1}^{L,spend} \} \\
& + \beta^{spend} (1 - \pi_{RET}) \mathbb{E}_t \{ (1 - f_{t+1}) W_{U,t+1}^{L,spend} \}.
\end{aligned}$$

If retired

$$\begin{aligned}
W_{R,t}^{L,spend} = & (1 - \beta^{spend}) \frac{(c_{R,t}^{L,spend})^{1-\sigma}}{1-\sigma} \\
& + \beta^{spend} (1 - \pi_{born}) \mathbb{E}_t \{ W_{R,t+1}^{L,spend} \} \\
& + \beta^{spend} \pi_{born} \mathbb{E}_t \{ (\Pr_{0,t}^L (1 - \lambda_x^L - \lambda_e^L (1 - f_{t+1})) + \Pr_{U,t}^L f_{t+1} \pi_L^{uem}(0)) W_{0,t+1}^{L,spend} \} \\
& + \beta^{spend} \pi_{born} \mathbb{E}_t \{ (\Pr_{1,t}^L (1 - \lambda_x^L - \lambda_e^L (1 - f_{t+1})) + \Pr_{U,t}^L f_{t+1} \pi_L^{uem}(1)) W_{1,t+1}^{L,spend} \} \\
& + \beta^{spend} \pi_{born} \mathbb{E}_t \{ [(\Pr_{0,t}^L + \Pr_{1,t}^L) (\lambda_x^L + \lambda_e^L (1 - f_{t+1})) + \Pr_{U,t}^L (1 - f_{t+1})] W_{U,t+1}^{L,spend} \}.
\end{aligned}$$

Where the probabilities  $\Pr^L$  of being reborn into the respective group are defined as

$$\Pr_{0,t}^L = (N_{0,t}^L + N_{1,t}^L \pi^{emp}(1, 0)) / (N_t^L + U_t^L),$$

$$\Pr_{1,t}^L = (N_{1,t}^L \pi^{emp}(1, 1)) / (N_t^L + U_t^L),$$

$$\Pr_{U,t}^L = U_t^L / (N_t^L + U_t^L).$$

The same relations above hold for the highly educated, replacing index  $L$  with  $H$ .

## E.2 Savers

Savers are exposed to the same income risk as spenders. They are not exposed to idiosyncratic consumption risk, however. Rather, savers live in a representative family that encompasses all the different household types (low/high education; employed with/without earnings loss; unemployed; retired). The family pools the incomes of its members.

Saver families maximize expected lifetime utility

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t (c_t^{saver})^{1-\sigma} / (1 - \sigma) \right\}$$

subject to the family's budget constraint

$$\begin{aligned}
(1 + \tau_c)c_t^{\text{saver}} + p_{a,t}a_{t+1} &= (p_{a,t} + d_{a,t})a_t \\
&\quad + \pi^{\text{saver}} U_t^L b_{UI}(e_L) [1 - \tau_t(b_{UI}(e_L))] \\
&\quad + \pi^{\text{saver}} U_t^H b_{UI}(e_H) [1 - \tau_t(b_{UI}(e_H))] \\
&\quad + \pi^{\text{saver}} N_{0,t}^L w_t e_L [1 - \tau_{RET} - \tau_{UI} - \tau_t(w_t e_L)] \\
&\quad + \pi^{\text{saver}} N_{0,t}^H w_t e_H [1 - \tau_{RET} - \tau_{UI} - \tau_t(w_t e_H)] \\
&\quad + \pi^{\text{saver}} N_{1,t}^L w_t e_L (1 - \varrho) [1 - \tau_{RET} - \tau_{UI} - \tau_t(w_t e_L (1 - \varrho))] \\
&\quad + \pi^{\text{saver}} N_{1,t}^H w_t e_H (1 - \varrho) [1 - \tau_{RET} - \tau_{UI} - \tau_t(w_t e_H (1 - \varrho))] \\
&\quad + \pi^{\text{saver}} (1 - \pi^{\text{labforce}}) \pi_E(e_L) b_{RET}(e_L) [1 - \tau_t(b_{RET}(e_L))] \\
&\quad + \pi^{\text{saver}} (1 - \pi^{\text{labforce}}) \pi_E(e_H) b_{RET}(e_H) [1 - \tau_t(b_{RET}(e_H))],
\end{aligned}$$

where  $\pi^{\text{labforce}} (= \pi_S(\mathcal{S}_+))$  is the share of households in the labor force. The exposition above assumes that savers trade shares in the mutual fund. We make this assumption so as to keep the exposition close to the heterogeneous-household's baseline. We could as well have had each saver family decide directly over a portfolio of non-financial firms.

### E.3 Financial firms

The firm side of the TANK variant is identical to the heterogeneous-agent version, with the exception that only saver families own shares in the representative mutual fund. The mutual funds trade the equity of all firms. Being owned by savers only, the mutual funds' discount factor is  $Q_{t,t+1} = \beta^{\text{saver}} (c_{t+1}^{\text{saver}} / c_t^{\text{saver}})^{-\sigma}$ . The central bank steers the inter-fund interest rate,  $R_t$ , resulting in the consumption Euler equation  $1 = \mathbb{E}_t\{Q_{t,t+1}R_t/\Pi_{t+1}\}$ . The mutual fund distributes to shareholders all income that it does not reinvest or use for paying adjustment costs. After-tax dividends are given by

$$d_{a,t} = (1 - \tau_d) \left( y_{f,t} - i_t - N_{0,t}^L w_t e_L - N_{0,t}^H w_t e_H - N_{1,t}^L w_t e_L (1 - \varrho) - N_{1,t}^H w_t e_H (1 - \varrho) \right).$$

### E.4 Non-financial firms

Non-financial firms are identical to the firms in the heterogeneous-agent model.

#### E.4.1 Final goods

There is a representative competitive final-goods firm that transforms differentiated intermediate goods into homogeneous final goods. Final goods can be used for personal consumption expenditures, government consumption, and physical investment. The firm solves

$$\max_{y_{f,t}, (y_{f,j,t})_{j \in [0,1]}} (1 - \tau_d) \left( P_t y_{f,t} - \int_0^1 P_{j,t} y_{f,j,t} dj \right) \text{ s.t. } y_{f,t} = \left( \int_0^1 y_{f,j,t}^{\frac{\vartheta \exp\{\zeta_{P,t}\}}{\vartheta \exp\{\zeta_{P,t}\}-1}} dj \right)^{\frac{\vartheta \exp\{\zeta_{P,t}\}}{\vartheta \exp\{\zeta_{P,t}\}-1}},$$

where  $\vartheta > 1$ .  $y_{f,t}$  marks the output of final goods.  $P_{j,t}$  marks the price of differentiated input  $j$  and  $y_{f,j,t}$  denotes the quantity demanded of that input by final-goods firms.  $P_t$  is the consumer price index.

### E.4.2 Intermediate inputs

Next to final goods, there are intermediate inputs, as in the heterogeneous-household model.

**Differentiated goods producers.** There is a unit mass of producers of differentiated goods. Producer  $j \in [0, 1]$  solves

$$\begin{aligned} & \max_{\{P_{j,t}, \ell_{j,t}, k_{j,t}\}_{t=0}^{\infty}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} Q_{0,t} (1 - \tau_d) \left( -y_{j,t} \left( \frac{P_{j,t}}{P_t} \right) - \Xi - r_t k_{j,t} - h_t \ell_{j,t} - \frac{\psi}{2} \left( \frac{P_{j,t}}{P_{j,t-1}} - \bar{\Pi} \right)^2 y_t \right) \right\} \\ & \text{s.t. } y_{j,t} = \zeta_{TFP,t} k_{j,t}^\theta \ell_{j,t}^{1-\theta}, \\ & \quad y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\vartheta \exp\{\zeta_{P,t}\}} y_t, \\ & \quad P_{j,-1} \text{ given.} \end{aligned}$$

**Labor services.** Labor services are homogeneous. They are produced by employment agencies, under constant returns to scale. Workers come in four types to the agency: low/high education each with/without skill loss. The value to the agency of a household to the employment agency depends on the household's characteristics. The value to the agency of a low-educated worker without skill loss is

$$J_{L,0,t}^L = (1 - \tau_d)(h_t - w_t)e_L + \mathbb{E}_t \{ Q_{t,t+1} (1 - \lambda_x(e_L) + \lambda_n(e_L)) J_{L,0,t+1}^L \}.$$

The value to the agency of a low-educated worker with skill loss is

$$\begin{aligned} J_{L,1,t}^L = & (1 - \tau_d)(h_t - w_t)e_L(1 - \varrho) \\ & + \mathbb{E}_t \{ Q_{t,t+1} (1 - \lambda_x(e_L) + \lambda_n(e_L)) (\pi_L^{emp}(1,0) J_{L,0,t+1}^L + \pi_L^{emp}(1,1) J_{L,1,t+1}^L) \}. \end{aligned}$$

The value to the agency of a high-educated worker without skill loss is

$$J_{L,0,t}^H = (1 - \tau_d)(h_t - w_t)e_H + \mathbb{E}_t \{ Q_{t,t+1} (1 - \lambda_x(e_H) + \lambda_n(e_H)) J_{L,0,t+1}^H \}.$$

The value to the agency of a high-educated worker with skill loss is

$$\begin{aligned} J_{L,1,t}^H = & (1 - \tau_d)(h_t - w_t)e_H(1 - \varrho) \\ & + \mathbb{E}_t \{ Q_{t,t+1} (1 - \lambda_x(e_H) + \lambda_n(e_H)) (\pi_H^{emp}(1,0) J_{L,0,t+1}^H + \pi_L^{emp}(1,1) J_{L,1,t+1}^H) \}. \end{aligned}$$

After separations have occurred, and before production, employment agencies can recruit new households. Let  $V_t$  be the aggregate number of vacancies posted and  $M_t$  the mass of new matches. The job-filling probability is identical for all vacancies, and given by  $q_t = M_t/V_t$ . Letting  $\kappa_t/q_t$  be the average cost per hire, the free-entry condition for recruiting is given by

$$\begin{aligned} & [\tilde{U}_t^L \pi_L^{uem}(0) + \lambda_n(e_L) \tilde{N}_{0,t}^L] J_{L,0,t}^L \\ & + [\tilde{U}_t^L \pi_L^{uem}(1) + \lambda_n(e_L) \tilde{N}_{1,t}^L] J_{L,1,t}^L \\ & + [\tilde{U}_t^H \pi_L^{uem}(0) + \lambda_n(e_H) \tilde{N}_{0,t}^H] J_{L,0,t}^H \\ & + [\tilde{U}_t^H \pi_L^{uem}(1) + \lambda_n(e_H) \tilde{N}_{1,t}^H] J_{L,1,t}^H = (1 - \tau_d)\kappa_t/q_t \cdot \left[ \tilde{U}_t^L + \lambda_n(e_L) \tilde{N}_t^L + \tilde{U}_t^H + \lambda_n(e_H) \tilde{N}_t^H \right]. \end{aligned}$$

Recruiting costs are given by

$$\kappa_t = (\kappa_H q_t + \kappa_v) \left( \frac{M_t / \tilde{N}_t}{\bar{M} / \bar{\tilde{N}}} \right)^2.$$

Matches emerge according to matching function

$$M_t = \left[ \left( \tilde{U}_t + \lambda_n(e_L) \tilde{N}_t^L + \lambda_n(e_H) \tilde{N}_t^H \right) V_t \right] / \left( \tilde{U}_t + \lambda_n(e_L) \tilde{N}_t^L + \lambda_n(e_H) \tilde{N}_t^H \right)^\alpha + V_t^\alpha \right]^{\frac{1}{\alpha}}$$

with  $\alpha > 0$ . The job-finding rate is

$$f_t = \frac{M_t}{\tilde{U}_t + \lambda_n(e_L) \tilde{N}_t^L + \lambda_n(e_H) \tilde{N}_t^H}.$$

The wage rule is

$$\log(w_t/\bar{w}) = \phi_w \log(w_{t-1}/\bar{w}) + \phi_w \log \left( \frac{y_t}{\bar{y}} \right) + \zeta_{w,t}.$$

**Capital services.** The representative producer of capital services faces the following problem

$$\begin{aligned} & \max_{\{v_t, i_t, K_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} (1 - \tau_d)(r_t K_{t-1} v_t - i_t) \right\} \\ \text{s.t. } & K_t = [1 - \delta(v_t)] \cdot K_{t-1} + \zeta_{I,t} \cdot [1 - \Gamma(i_t/i_{t-1})] i_t. \end{aligned}$$

Depreciation of capital evolves as

$$\delta(v_t) = \delta_1 v_t^{\delta_2}, \quad \delta_1 > 0, \delta_2 > 1.$$

The transformation function that governs how investment is transformed into physical capital is given by

$$\Gamma \left( \frac{i_t}{i_{t-1}} \right) = \phi_K / 2 \left( \frac{i_t}{i_{t-1}} - 1 \right)^2, \quad \phi_K \geq 0.$$

**Adjustment services.** The competitive representative adjustment-services firm solves

$$\max_{y_{a,t}, (y_{a,j,t})_{j \in [0,1]}} (1 - \tau_d) \left( P_t y_{a,t} - \int_0^1 P_{j,t} y_{a,j,t} dj \right) \text{ s.t. } y_{a,t} = \left( \int_0^1 y_{a,j,t}^{\frac{\vartheta \exp\{\zeta_{P,t}\} - 1}{\vartheta \exp\{\zeta_{P,t}\} - 1}} dj \right)^{\frac{\vartheta \exp\{\zeta_{P,t}\}}{\vartheta \exp\{\zeta_{P,t}\} - 1}},$$

where  $y_{a,t}$  are total adjustment services produced and  $y_{a,j,t}$  is demand for differentiated good  $j$  by the adjustment-services firm.

## E.5 Central bank and fiscal authority

The central bank sets the gross nominal interest rate according to Taylor rule

$$\begin{aligned}\log\left(\frac{R_t}{\bar{R}}\right) = & \phi_R \log\left(\frac{R_{t-1}}{\bar{R}}\right) \\ & + (1 - \phi_R) \left[ \phi_\Pi \log\left(\frac{\Pi_t}{\bar{\Pi}}\right) - \phi_u \left( \frac{U_t - \bar{U}}{\pi^{\text{labforce}}} \right) \right] + \log \zeta_{R,t}.\end{aligned}$$

The fiscal authority's budget constraint is given by

$$\begin{aligned}g + U_t^L b_{UI}(e_L)[1 - \tau_t(b_{UI}(e_L))] + U_t^H b_{UI}(e_H)[1 - \tau_t(b_{UI}(e_H))] \\ + (1 - \pi^{\text{labforce}})\pi_E(e_L)b_{RET}(e_L)[1 - \tau_t(b_{RET}(e_L))] + (1 - \pi^{\text{labforce}})\pi_E(e_H)b_{RET}(e_H)[1 - \tau_t(b_{RET}(e_H))] \\ = \tau_c c_t + \tau_d \left( y_t - \Xi - \left( \frac{\psi}{2}(\Pi_t - \bar{\Pi})^2 y_t \right) - (\kappa_H q_t + \kappa_v) \left( \frac{M_t/\tilde{N}_t}{\bar{M}/\bar{N}} \right)^2 V_t - i_t \right. \\ \left. - w_t e_L [N_{0,t}^L + N_{1,t}^L(1 - \varrho)] - w_t e_H [N_{0,t}^H + N_{1,t}^H(1 - \varrho)] \right) \\ + N_{0,t}^L w_t e_L [\tau_{RET} + \tau_{UI} + \tau_t(w_t e_L)] + N_{1,t}^L w_t e_L (1 - \varrho) [\tau_{RET} + \tau_{UI} + \tau_t(w_t e_L(1 - \varrho))] \\ + N_{0,t}^H w_t e_H [\tau_{RET} + \tau_{UI} + \tau_t(w_t e_H)] + N_{1,t}^H w_t e_H (1 - \varrho) [\tau_{RET} + \tau_{UI} + \tau_t(w_t e_H(1 - \varrho))].\end{aligned}$$

## E.6 Laws of motion (un)employment

We list (un)employment both at the beginning of the period and at the end of the period.

### E.6.1 (Un)employment at the beginning of the period

Low-education, no skill loss employment at the beginning of the period evolves as

$$\tilde{N}_{0,t}^L = N_{0,t-1}^L + \pi^{\text{emp}}(1, 0)N_{1,t-1}^L.$$

With skill loss, the corresponding law of motion is

$$\tilde{N}_{1,t}^L = \pi^{\text{emp}}(1, 1)N_{1,t-1}^L.$$

Total low-education employment at the beginning of the period is

$$\tilde{N}_t^L = \tilde{N}_{0,t}^L + \tilde{N}_{1,t}^L.$$

High-education, no skill loss employment at the beginning of the period evolves as

$$\tilde{N}_{0,t}^H = N_{0,t-1}^H + \pi^{\text{emp}}(1, 0)N_{1,t-1}^H.$$

With skill loss, the corresponding law of motion is

$$\tilde{N}_{1,t}^H = \pi^{\text{emp}}(1, 1)N_{1,t-1}^H.$$

Total high-education employment at the beginning of the period is

$$\tilde{N}_t^H = \tilde{N}_{0,t}^H + \tilde{N}_{1,t}^H.$$

Total employment at the beginning of the period evolves as

$$\tilde{N}_t = \tilde{N}_t^L + \tilde{N}_t^H.$$

Unemployment at the beginning of the period is defined as

$$\tilde{U}_t^L = \pi_E(e_L)\pi^{\text{labforce}} - \tilde{N}_t^L,$$

$$\tilde{U}_t^H = \pi_E(e_H)\pi^{\text{labforce}} - \tilde{N}_t^H,$$

and

$$\tilde{U}_t = \tilde{U}_t^L + \tilde{U}_t^H.$$

### E.6.2 (Un)employment at the end of the period

Low-education employment at the end of the period evolves as

$$N_{0,t}^L = [1 - \lambda_x(e_L) - \lambda_e(e_L)(1 - ft)]\tilde{N}_{0,t}^L + f_t\pi_L^{uem}(0)U_{t-1}^L.$$

$$N_{1,t}^L = [1 - \lambda_x(e_L) - \lambda_e(e_L)(1 - ft)]\tilde{N}_{1,t}^L + f_t\pi_L^{uem}(1)U_{t-1}^L.$$

$$N_t^L = N_{0,t}^L + N_{1,t}^L.$$

High-education employment at the end of the period evolves as

$$N_{0,t}^H = [1 - \lambda_x(e_H) - \lambda_e(e_H)(1 - ft)]\tilde{N}_{0,t}^H + f_t\pi_H^{uem}(0)U_{t-1}^H.$$

$$N_{1,t}^H = [1 - \lambda_x(e_H) - \lambda_e(e_H)(1 - ft)]\tilde{N}_{1,t}^H + f_t\pi_H^{uem}(1)U_{t-1}^H.$$

$$N_t^H = N_{0,t}^H + N_{1,t}^H.$$

Employment at the end of the period evolves as

$$N_t = N_{0,t}^L + N_{1,t}^L + N_{0,t}^H + N_{1,t}^H.$$

Unemployment at the end of the period evolves as

$$U_t^L = \pi_E(e_L)\pi^{\text{labforce}} - N_t^L,$$

$$U_t^H = \pi_E(e_H)\pi^{\text{labforce}} - N_t^H,$$

and

$$U_t = U_t^L + U_t^H.$$

### E.7 Aggregates

Total per-capita consumption of spenders is

$$\begin{aligned} c_t^{\text{spend}} &= (1 - \pi^{\text{labforce}})\pi_E(e_L)c_{R,t}^{L,\text{spend}} + (1 - \pi^{\text{labforce}})\pi_E(e_H)c_{R,t}^{H,\text{spend}} \\ &\quad + U_t^L c_{U,t}^{L,\text{spend}} + U_t^H c_{U,t}^{H,\text{spend}} \\ &\quad + N_{0,t}^L c_{0,t}^{L,\text{spend}} + N_{1,t}^L c_{1,t}^{L,\text{spend}} \\ &\quad + N_{0,t}^H c_{0,t}^{H,\text{spend}} + N_{1,t}^H c_{1,t}^{H,\text{spend}}. \end{aligned}$$

Total consumption is

$$c_t = \pi^{\text{saver}} c_t^{\text{saver}} + (1 - \pi^{\text{saver}})c_t^{\text{spend}}.$$

## E.8 Market clearing and equilibrium

The market for adjustment services clears if production equals use of adjustment services for price adjustment and labor adjustment,

$$y_{a,t} = \frac{\psi}{2} (\Pi_t - \bar{\Pi})^2 y_t + \kappa_t V_t.$$

The market for capital services clears if (with  $k_{j,t} = k_t$  for all  $j$ )

$$v(X)K_{t-1} = k_t.$$

The market for labor services clears if all labor services supplied are used in the production of differentiated goods (with  $\ell_{j,t} = \ell_t$  for all  $j$ ),

$$N_{0,t}^L e_L + N_{1,t}^L e_L (1 - \varrho) + N_{0,t}^H e_H + N_{1,t}^H e_H (1 - \varrho) = \ell_t.$$

Total demand for differentiated goods is given by

$$y_t = y_{f,t} + y_{a,t}.$$

The market for differentiated goods clears if demand equals production (using symmetry in both price setting and demand for each differentiated good  $j$ ), so

$$y_t = \zeta_{TFP,t} k_t^\theta \ell_t^{1-\theta}.$$

The market for final goods clears if

$$y_{f,t} = c_t + i_t + g.$$

Normalizing the supply of shares to unity, the market for shares in the mutual fund clears if

$$a_t = 1.$$

## F Solution algorithm

This appendix outlines our solution algorithm. We extend the perturbation method developed by Reiter (2009) and Reiter (2010a) to compute a second-order approximation with a parameterized law of motion for the distribution of households.<sup>A10</sup> The parameterized law of motion is obtained from a principal-component decomposition of the first-order dynamics of the distribution of wealth. This step is necessary as, on the one hand, a full second-order solution is infeasible given currently available RAM and the size of the model, and, on the other hand, we need to compute a second-order solution to study welfare along the business-cycle dimension. In earlier versions of the paper we used an approach closer to Krusell and Smith (1998), in which we forecasted the expectation terms in the firms' Euler equations and asset prices, and later a method based on Reiter (2010b). While these methods allow for a global solution of the model, they suffer from a strong curse of dimensionality, limiting the number of aggregate states one can take into account. The algorithm described here overcomes this constraint.

We use splines to approximate households' decision rules along their asset dimension, and approximate the distribution of households as a histogram on the product of skill state, discount factor, education, employment state and a grid on the wealth distribution. All agents use this approximation to construct their forecasts about the evolution of the economy. The solution algorithm takes the following steps.

1. Solve for the model's steady state without aggregates shocks. Collect the values of aggregate variables, the households' decision rules, value functions,<sup>A11</sup> and distributions on their respective grids.
2. Collect all equations characterizing the solution of the model economy. Take first derivatives of these equations with respect to aggregate variables, households' policy functions on the grids, and the mass of agents in each bin of the approximated histogram at the steady state.<sup>A12</sup> Solve for the first-order policy and transition matrices using the algorithm described in Schmitt-Grohé and Uribe (2004).
3. Use the first-order solution to compute the variance-covariance matrix of the deviations of the distribution of assets (the mass points in the respective bins of the histogram) from the steady state. Compute a principal-component decomposition of the matrix. Keep the first  $n$  principal-component vectors so that these  $n$  components explain  $x$  percent of the total variation of the distribution, for example, 99.9 percent, based on the principal-component analysis. Use the principal-component vectors to compute two projection matrices: one,  $D$ , from  $\mathbb{R}^n$  into the linear space spanned by the principal-component vectors, and the other,  $H$ ,

---

<sup>A10</sup>See also Winberry (2018) and Ahn et al. (2017) for closely related solution strategies.

<sup>A11</sup>In practice, we approximate the expected marginal utility of the households and the value function at the beginning of the period before idiosyncratic uncertainty is resolved. This makes it easier to deal with the borrowing constraint when we apply some of the dimension reductions discussed below. The beginning-of-period value function is the object used in the calculation of the welfare effects of policy changes so we approximate it directly.

<sup>A12</sup>In practice, it helps to drop one of the bins from the set of histograms and to use the fact that the total mass has to be 1. In addition, we are keeping track of the aggregate mass of agents in employment with and without skill loss by education, and use this to reduce the number of bins to track further. Here, we are utilizing the fact that the shares of agents in different skill and discount factor states are constant over time and that the job-finding and -loss rate does not depend on these characteristics.

which projects back from this space onto  $\mathbb{R}^n$ , such that  $D \cdot H$  equals the identity matrix in  $\mathbb{R}^n$ .<sup>A13</sup>

4. Collect all equations characterizing the solution of the model economy again. To reduce the number of state variables we now use the sum of the distribution in the steady state,  $\mu^{ss}$ , and  $D \cdot p$  in place of tracking the mass of agents in each bin of the approximated histogram at each point in time separately. Here  $p$  is a vector in  $\mathbb{R}^n$  weighting the different principal components. Its changes over time to allow us to track, approximately, changes in the distribution.<sup>A14</sup> In practice, in our system of equations, the economy starts the period with a vector  $p$  as part of the state. We then assume  $\mu^{ss} + D \cdot p$  as the beginning-of-period distribution, use it in all equations involving the distribution and update it using the law of motion for the distribution. Finally, we subtract  $\mu^{ss}$  from the resulting end-of-period distribution and project the difference on  $H$  to obtain  $p'$ , as part of the new state vector. Add these adjustments to the model equations. Take first and second derivatives of these equations with respect to aggregate variables, households' policy and value functions on the grids, and the new state variables just introduced at the steady state. Solve for the first- and second-order policy and transition matrices.

To reduce the computational complexity of the problem further, we follow Ahn et al. (2017) and Bayer et al. (2019) and approximate the deviations of households' policy and value functions from the steady state using a piece-wise linear spline of a smaller order than the grid used to solve for steady-state policy functions. We verified that increasing the degree of the spline and the number of principal components did not change our conclusions.<sup>A15</sup> We implement the algorithm in MATLAB using a modified version of the codes provided with Schmitt-Grohé and Uribe (2004). We adjust them to allow us to handle the very large derivative matrices resulting from the second-order approximation. We need around 300GB of RAM to perform the calculations; this is so despite making use of the sparsity of the matrices and the reduction in the number of policy functions. We use a spline of order ten, and four principal components for the results in the text and have verified that both the model dynamics and other implications are robust to changes in these numbers.

---

<sup>A13</sup>This procedure follows Reiter (2010a). More details and motivation can be found there. In practice, we found that adding the asset price relative to the steady state as another variable in the decomposition increased both numerical efficiency and stability as it directly relates to the dynamics of the stochastic discount factor in our setting.

<sup>A14</sup>Given the derivation of  $H$  and  $D$  we expect the approximate dynamics to be fairly close to the full ones, and indeed, we verified that the first-order dynamics for aggregate variables of the model with and without this reduction in the state space are extremely close. We can think of  $D \cdot p$  as describing the most likely state of the distribution if a projection of the histogram using  $H$  results in  $p$ .

<sup>A15</sup>We also experimented with Chebyshev polynomials and a smoother spline. In the end, all methods gave similar answers and we used the piece-wise linear spline as it gave us the best trade-off between precision and number of parameters to approximate.

## G Data on wealth and income

Our data source for wealth and income is the U.S. Survey of Consumer Finances. We use the Summary Extract Public Data of the SCF 2004. Acronyms in bold below correspond to the variable names in the dataset. In the construction of different income categories we split business income between labor and financial income under the following assumptions:

- Income from non-actively managed businesses is financial income.
- Income from actively managed businesses is 60 percent labor income and 40 percent financial income, based on the average labor-income share in 2004 according to the BEA.
- As the SCF provides the value of actively and non-actively managed businesses, but does not provide the income from both types of ventures separately, we split total business income into the two categories using the value shares as weights.

We then arrive at the following definitions of income categories:

- “Labor Income” is the sum of wage income plus the labor share of business income constructed as described above. ( $WAGEINC + 0.6 \cdot \frac{ACTBUS}{BUS} \cdot BUSSEFARMINC$ ).
- “Social security” is social security and pension income net of withdrawals from pension accounts ( $SSRETINC - PENACCTWD$ ). We exclude pension account withdrawals, since such pensions will be treated as equity in the model. Withdrawals, therefore, will not be income. Rather, we adjust financial income each period by the putative returns to retirement accounts (see below).
- “Non-SocSec transfer income” is ( $TRANSFOTHINC$ ). This includes among other items unemployment benefits.
- “Financial income,” then is computed as the financial part of business income ( $(0.4 \cdot \frac{ACTBUS}{BUS} + \frac{NONACTBUS}{BUS}) \cdot BUSSEFARMINC$ ), interest and dividend income ( $INTDIVINC$ ), realized capital gains ( $KGINC$ ), and imputed income on other assets (labeled  $IMP\_FININC$  below).

Category  $IMP\_FININC$  we compute ourselves. This is necessary so as to map financial income in the model to the SCF. The SCF does not cover the rents and the service flow from owner-occupied housing. Neither does it capture financial gains on retirement accounts. Rather, both interest and dividend income and realized capital gains in the SCF are taken from IRS Form 1040. We impute the income flow from these categories. Toward this end, we first derive the average rate of return on financial assets for which we have income information, by dividing the sum of the financial part of business income, of interest and dividend income, and of realized capital gains by the stock of wealth generating them. We define this stock as the sum of the value of businesses (BUS) and total financial wealth (FIN) excluding quasi-liquid retirement accounts (RETQLIQ). The resulting real rate of return is  $rret = 4.31$  percent per year (using data for all households aged 25-99).<sup>A16</sup> We use this rate to impute the missing financial income by the return with the value

---

<sup>A16</sup>Here we make the implicit assumption that both income and wealth are measured at the end of the period so that the ratio can be treated as the real rate of return.

of houses (HOUSES, ORESRE, NRESRE), other non-financial assets (OTHNFIN), and quasi-liquid retirement accounts (RETQLIQ). In addition, we also use *rret* to impute negative income from debt secured by a primary residence (MRTHEL), debt secured by other residential property (RESDBT), credit card balances after last payment (CCBAL), other lines of credit (OTHLOC), and other debt (ODEBT). To compute total net worth we sum the value of all asset categories listed above and subtract the value of all debts listed above.

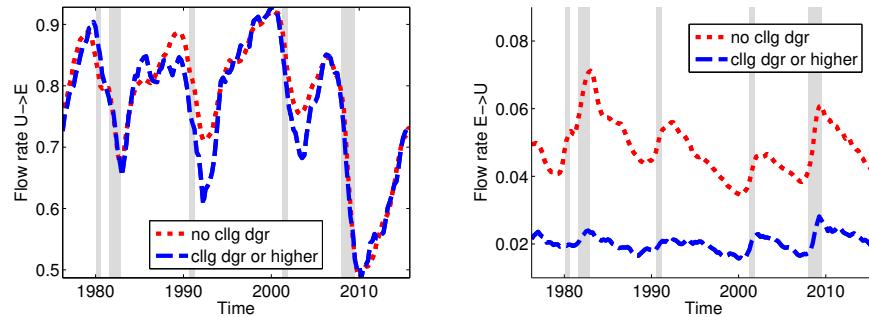
The after-tax real rate of return of 3.2 percent per year, to which we calibrate in Section 3.2.1, is derived as follows. Financial income as reported in the SCF is pre-tax income, where pre-tax refers to taxes paid by households. So as to get the households' after-tax returns, we split the capital tax rate of 36 percent into a part paid by households and a part paid by firms, the latter of which we proxy by the average corporate tax rate over the sample period. Using the same calculations as in Fernández-Villaverde et al. (2015), the average corporate tax rate over the sample is 10 percent. This leaves a 26 percent capital tax rate to be paid by households, meaning that households' after-tax return is  $4.31 \cdot (1 - 0.26) \approx 3.2$ .

## H Labor-market flow rates by education

This appendix provides details on the construction of the labor-market flow rates shown in Section 3.1. The flow rates are based on the Current Population Survey (CPS). We follow the methodology described in Cairó and Cajner (2018). We first compute unemployment rates and monthly flow rates from the survey. From this, we construct quarterly time series. The quarterly flow rate from unemployment to employment (the “job-finding rate”) that we construct is defined as one minus the probability that a worker who was unemployed at the end of a quarter also is unemployed at the end of the next. The flow rate from employment to unemployment is constructed so as to ensure that, combined with the job-finding rate defined above, the flow rate into unemployment replicates the evolution of the unemployment rate in each education group.

Figure A1 shows the resulting quarterly flow rates from unemployment to employment (left panel) and from employment to unemployment (right panel) for working-age individuals. In the low-education group are all those workers with less education than a college degree (red dots), including workers who never went to college as well as college dropouts. The high-education group is composed of workers with a college degree, or more education (blue dashed).

**Figure A1: Data. Flow rates into (un)employment**



*Notes:* Flow rate from unemployment to employment (left panel) and flow rate from employment to unemployment (right panel). Quarterly frequency. Based on the Current Population Survey. Workers ages 25 to 65. Red dotted line: workers without a college degree. Blue dashed: workers with a college or higher degree.

As alluded to in the main text, the level and volatility of flow rates into employment is rather similar for the two education groups. The flow rates into unemployment, instead, differ notably by education. The flow rate into unemployment for the low-educated on average is about twice the level of the flow rate for the high-educated. And it is about twice as volatile. What this means is that the low-educated are exposed to cyclical and average unemployment risk to a larger extent.

## I Calibration details and model fit

This appendix provides further details on the calibration and on how well the model fits the data.

### I.1 List of targeted moments

This appendix lists all data moments that were targeted during the calibration of the model's steady state and the resulting values in the steady state of the model. The list is presented in two tables. The first table lists those targets that we match exactly.<sup>A17</sup> The second table contains moments that were included as targets in the minimum distance criterion described in Footnote 22 of the main text.

**Table A1: Moments matched exactly**

Target Description	Target	Model	Source
Post-Tax Real Rate	3.2%	3.2%	SCF.
Wealth Share Less Educated	30%	30%	SCF.
Wealth Share Poorest 20% Workers	0%	0%	SCF.
Wealth Share Poorest 50% retired	5.25%	5.25%	SCF.
Wealth Share Low Educated	30%	30%	SCF.
Average Working Life	40 years	40 years	Sample choice.
Average Retirement Length	12 years	12 years	SCF.
Share Low Educated Worker	60%	60%	SCF.
Intergen. Elasticity of Income	0.5	0.5	Solon (1992), Mazumder (2005).
Initial earnings loss	25%	25%	Couch/Placzek (2010), Altonji <i>et al.</i> (2013).
Loss six years later	14%	14%	Couch and Placzek (2010).
Stand. Dev. Residual Earnings	0.508	0.508	Floeden and Lindé (2001).
Wealth Gini Working Age	82.4	82.4	SCF.
College Wage Premium	50%	50%	Mukoyama and Sahin (2006).
Capital Depreciation Rate	1.5%	1.5%	NIPA.
Utilization	1	1	Normalization.
Share of exog. separations $e_L$	69.5&	69.5%	Calculations based on Table 2.
Share of exog. separations $e_L$	65.7&	65.7%	Calculations based on Table 2.
Rel. Unempl. Rate by Educ.	44%	44%	CPS.
Unemployment Rate	6%	6%	BLS.
Total Cost per hire Average Wage	50%	50%	Silva and Toledo (2009).
Share Fixed Hiring Cost	94%	94%	Christiano <i>et al.</i> (2016).
Job-Finding Rate	0.82	0.82	CPS.
Job-Filling Rate	0.71	0.71	den Haan <i>et al.</i> (2000).
Labor Share	66%	66%	NIPA.
Investment GDP	18%	18%	NIPA.
Inflation Rate Government Spending GDP	2% p.a.	2% p.a.	Inflation target.
	19%	19%	NIPA.

*Notes:* ‘Target Description’ explains what moment was target. ‘Target’ provides the targeted value. ‘Model’ lists the corresponding value from the calibrated steady state of the HANK model. ‘Source’ adds information on the source of ‘Target Value.’ This table contains moments that were, by design, matched exactly.

<sup>A17</sup>We omit targets in cases where we simply set a parameter to match values found in previous research from this appendix. An example of such a parameter is relative risk aversion, which we target to be 2.5 based on Blundell *et al.* (2016). The same goes for parameters we found through the maximum likelihood estimation of the RANK version of the model.

**Table A2: Fit of percentiles of the wealth distribution**

Target Description	Target	Model
Poorest 5%	-0.1%	0%
Poorest 10%	-0.1%	0%
Poorest 15%	-0.1%	0%
Poorest 20%	-0.1%	$\approx 0\%$
Poorest 25%	$\approx 0\%$	0.01%
Poorest 30%	0.21%	0.07%
Poorest 35%	0.43%	0.15%
Poorest 40%	0.91%	0.30%
Poorest 45%	1.50%	0.55%
Poorest 50%	2.27%	0.96%
Poorest 55%	3.29%	1.72%
Poorest 60%	4.62%	2.90%
Poorest 65%	6.32%	4.75%
Poorest 70%	8.56%	7.40%
Poorest 75%	11.46%	10.92%
Poorest 80%	15.27%	15.49%
Poorest 85%	20.48%	21.62%
Poorest 90%	27.80%	30.53%
Poorest 95%	39.53%	44.25%

*Notes:* In the calibration of  $(\beta_{eL}, \beta_{eH}, \Delta_\beta, \gamma_1, \gamma_2)$ , among other targets, we seek to minimize the distance between percentile of the distribution of net-worth in the data and model; see Footnote 22 of the main text. This table reports the moments used and the fit. All moments in this table are based on the SCF and were rounded to two digits.

## I.2 Calibration of skills

Table A3 provides the targets for calibrating the skills and lists how many restrictions each target delivers. See page 21 of the main text for a detailed discussion of these targets.

**Table A3: Temporary skills. Targets and Parameterization.**

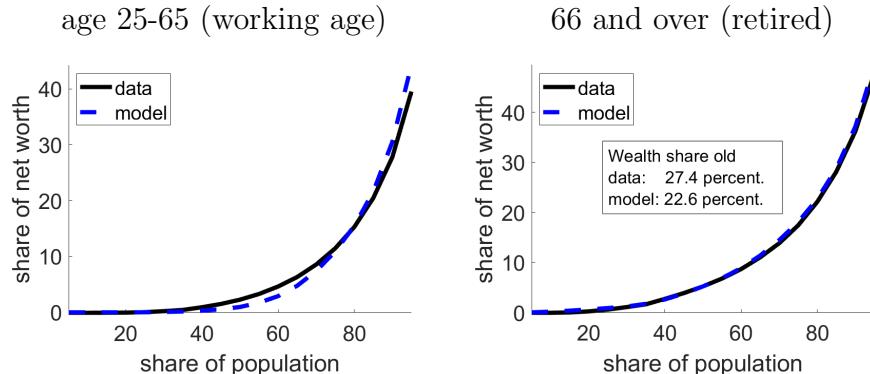
Targets	# restrictions
<u>Assumptions on skill levels</u>	
(i) Average length of a working life: 40 years	1
(ii) Average length of retirement: 12 years	1
(iii) Length of working life independent of skill level	2
(iv) Skills after birth according to ergodic distribution	2
(v) Ergodic mass super-skilled in working-age pop. 1%	1
(vi) Prob. remain super-skilled if not retiring 0.975	1
(vii) Probability of becoming $s_3$ independent of $s_1$ and $s_2$	1
(viii) Ergodic distrib. determines transition from $s_3$ to $s_1, s_2$	1
(ix) Equal ergodic mass of low- and medium-skill agents	1
(x) Persistence of residual earnings of 0.975	1
(xi) Ergodic standard deviation of residual earnings of 0.51	1
(xii) Normalize average skill of workers to 1	1
(xiii) Normalize $s_0 = 0$ (no labor income in retirement)	1
(xiv) Target 0.825 for Gini index of wealth of the working aged	1

*Notes:* Calibration strategy for skills. Section 3.2.1 of the main text provides further details.

## I.3 Wealth distribution in the model

Figure A2 plots the Lorenz curves for net worth and income from the model, for working-age households and households of retirement age.

**Figure A2: Model vs. data. Wealth distribution**



*Notes:* Wealth Lorenz curves within each age group. Comparing model (blue dashed line) and data (solid line). Left: working age (25-65). Right: old age (66 and over).

Note that the wealth Lorenz curves were a target in the calibration. The model matches these well. The share of aggregate net worth held by the old is 27.4 percent in the data. In the model, the retired households hold 22.6 percent of aggregate wealth.

#### I.4 Income shares

Table A4 provides the model-based counterpart to Table 1 in the main text. The calibrated model

**Table A4: Model. Income sources by net worth (percent of total income)**

working age	Percentile of net worth						
	0-20	20-40	40-60	60-80	80-95	Top 5	Top 1
Labor income	96.5	97.0	96.6	87.7	79.4	58.0	32.3
Financial income	0.0	0.3	1.9	10.3	19.6	41.6	67.4
Transfers	3.5	2.7	1.5	2.0	1.0	0.3	0.2

retired							
Financial income	0.6	2.0	7.0	12.1	32.1	73.9	89.3
Social security	99.4	98.0	93.0	87.9	67.9	26.1	10.7

*Notes:* Based on the model calibrated in Section 3. For the respective statistics in the data, see Table 1 in Section 3.

matches the overall pattern of the shape of the distribution of incomes. The bottom part of the wealth distribution in the model has a somewhat lower share of financial income than the data. We wish to note here, however, that our measure of financial income in the data includes imputed income from housing and retirement savings. Also for the retired, the model matches the rough split.

#### I.5 Second moments

For comparing the fit of the model with the business-cycle facts, we rely on the disaggregated data described in Section 3.1 of the main text and further aggregate data. The data are either quarterly to start with or transformed from monthly to quarterly frequency. Unless noted otherwise, this transformation from monthly is done by averaging the monthly data over the quarter. The data are seasonally adjusted where applicable. Unless noted otherwise, the source of the aggregate data is the St. Louis Fed's FRED II database. We start with the series that cover the period 1977Q1 to 2015Q4. After HP-filtering (HP-weight 1600), we drop observations at the beginning and end to arrive at a sample of HP-filtered observations covering the period 1984Q1 to 2008Q3. Nominal variables are deflated by the GDP deflator, which we also use as our measure of inflation. Personal consumption expenditures,  $c$ , include total durable and non-durable consumption expenditures as well as services. Investment,  $i$ , is gross private domestic investment. Government consumption is government consumption and gross investment.

Capacity utilization,  $v$ , is measured by the quarterly average of the Board of Governors' headline index of industrial capacity utilization. We measure vacancies  $V$  using Barnichon's (2010) composite help-wanted index. The wage,  $W(X)$ , is computed as wage and salary accruals from the national accounts divided by the GDP deflator divided by total nonfarm payrolls. The interest rate,  $R$ , is the quarterly average of the effective federal funds rate. The unemployment and separation rates are taken from Table 2.

Table A5 presents the corresponding moments and compares them to the moments in the model.

**Table A5: Model vs. Data – Filtered Second Moments**

	Model						Data		
	heterog. hh.		TANK		RANK		1984Q1-2008Q3		
	Std	Corr	AR1	Std	Corr	AR1	Std	Corr	AR1
<u>Output</u>									
GDP, $y_f$	0.93	1.00	0.70	0.90	1.00	0.70	0.91	1.00	0.70
Consumption, $c$	0.57	0.59	0.67	0.51	0.55	0.66	0.51	0.53	0.66
Investment, $i$	4.27	0.92	0.70	4.27	0.93	0.69	4.34	0.94	0.70
Capacity util., $v$	2.22	0.63	0.24	2.14	0.59	0.22	2.16	0.60	0.22
<u>Labor market</u>									
Unempl. rate ( $e_L$ )	0.81	-0.79	0.77	0.82	-0.77	0.77	0.84	-0.77	0.77
Unempl. rate ( $e_H$ )	0.40	-0.79	0.78	0.40	-0.78	0.78	0.41	-0.77	0.78
Employment	0.69	0.79	0.77	0.70	0.77	0.77	0.71	0.77	0.77
Flow rate $U \rightarrow E$ $f(X)$	4.92	0.79	0.73	5.00	0.77	0.73	5.07	0.77	0.73
Flow rate $E \rightarrow U$ $e_L$	0.47	-0.79	0.72	0.48	-0.77	0.73	0.49	-0.77	0.73
Flow rate $E \rightarrow U$ $e_H$	0.23	-0.79	0.72	0.23	-0.77	0.83	0.23	-0.77	0.73
Vacancies, $V$	9.94	0.73	0.55	10.09	0.71	0.54	10.19	0.71	0.55
<u>Prices</u>									
Wage, $W$	1.02	0.23	0.70	1.01	0.21	0.54	1.02	0.20	0.70
Inflation, $\Pi^{[1]}$	0.76	0.25	0.56	0.77	0.23	0.54	0.77	0.25	0.55
Nominal rate, $R^{[1]}$	0.91	-0.20	0.66	1.10	-0.25	0.63	1.10	-0.24	0.63

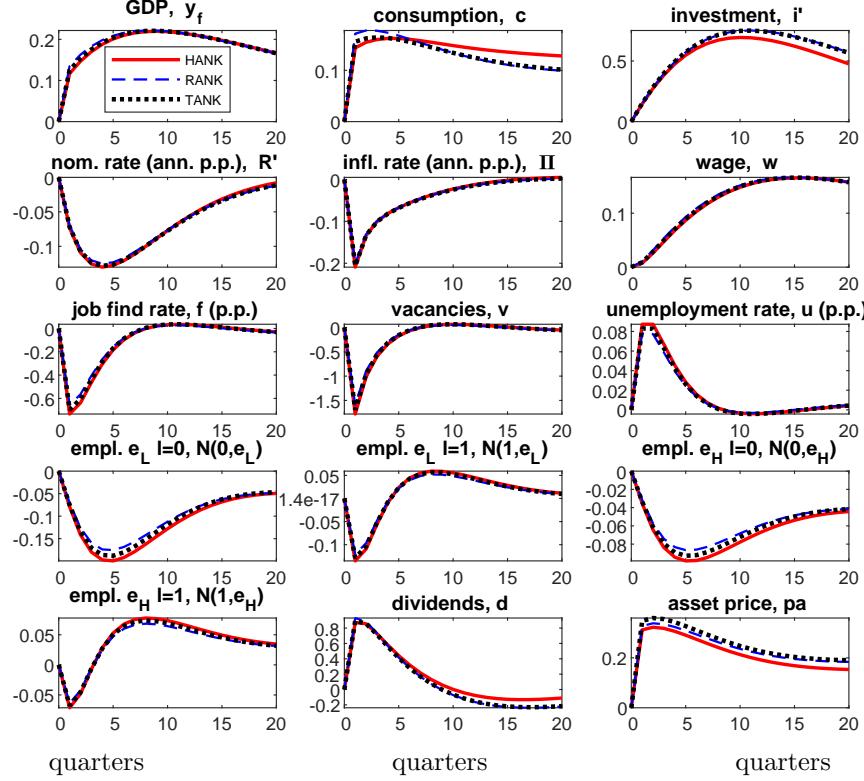
*Notes:* The table compares moments of the data and two variants of the model (heterogeneous households, representative households). Appendix I describes the source of data and construction of data-based moments. The model moments follow the construction of the data. They are based on 100 repeated simulations of the model. Each simulation is initialized with 500 periods of simulations that are dropped for the computation of the moments. The next 156 periods are kept. In each case, we take the natural log of the data and compute the cyclical component of the data multiplied by 100 so as to have percentage deviations from trend. The trend is an H-P-trend with weight 1,600. We then drop the first 28 and last 29 observations and compute moments of interest. Finally, we average across the simulations. The left block shows the model's moments, the block on the right the data's. The first column ("Std.") reports the standard deviation of each series. The second column ("Corr") shows the correlation of the series with GDP. The final column ("AR1") shows the autocorrelation of the series. <sup>[1]</sup>: the nominal interest rate and inflation are reported in annualized percentage points.

A result of economic substance is that allowing for heterogeneity implies notable changes to the business cycle. The heterogeneous-agent model has more procyclical consumption than both the saver-spender variant and the representative-agent model. To some extent, this comes at the expense of making investment less volatile. What is important to note is that employment in the model results from firms making an investment in employment relationships. Just as the investment in physical capital is less volatile in the heterogeneous-agent economy (in spite of output being more volatile), labor-market activity is somewhat *dampened*. Unemployment, vacancies, and the job-finding rate all are somewhat less volatile than in the representative-agent economy. This result holds unconditionally. Section I.6 in this same appendix shows impulse responses to all shocks. For four of them, the heterogeneous-agent economy shows a stronger response of employment than the representative-agent economy. For the wage shock, the response is dampened notably. Before going

there, however, Section I.8 discusses the forecast error decomposition of the three model variants, that is, which of the shocks accounts for how much of the fluctuations in each variable.

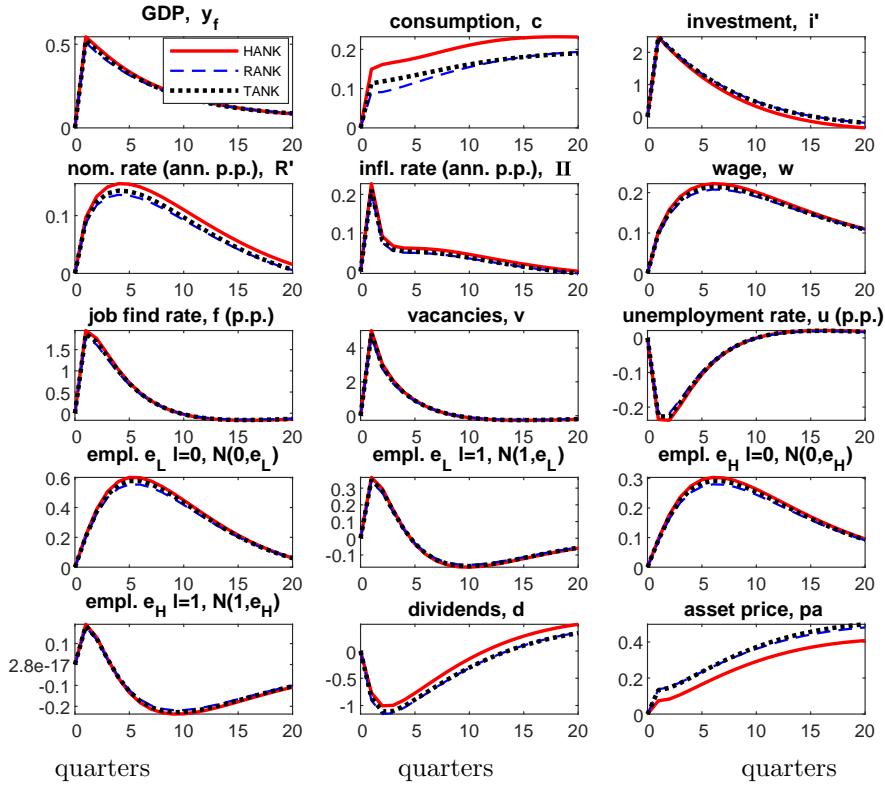
## I.6 Impulse responses, aggregate variables

**Figure A3: Impulse response to TFP shock,  $\zeta_{TFP}$**



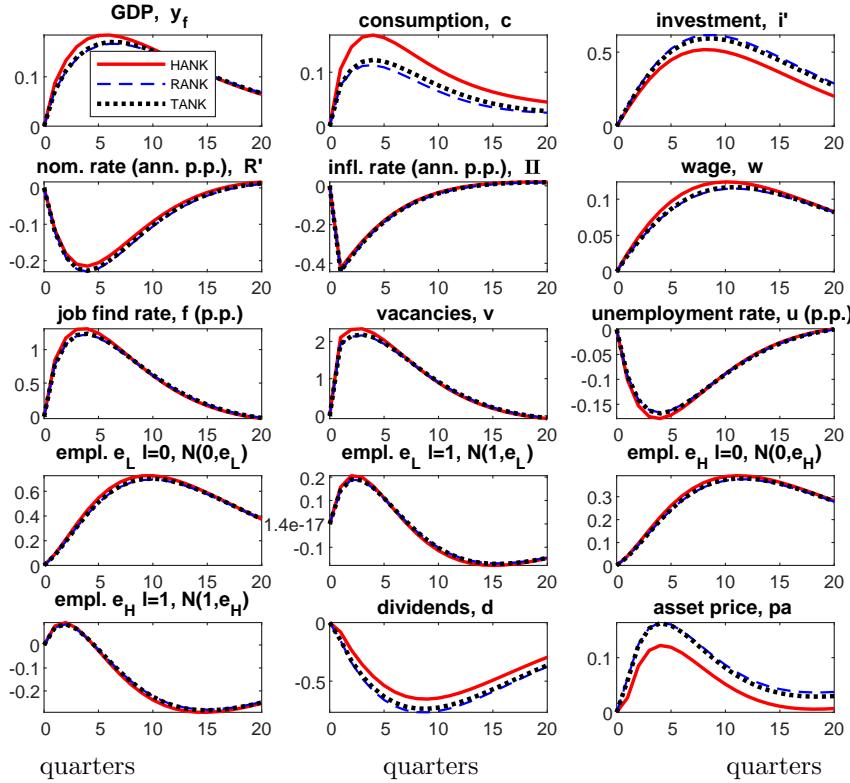
*Notes:* Impulse response to a one-standard-deviation TFP shock, starting at the stochastic steady state. All variables expressed as percent deviations, unless noted otherwise. ann. p.p.: expressed in annualized percentage points. p.p.: expressed in percentage points.

**Figure A4: Impulse response to MEI shock,  $\zeta_I$**



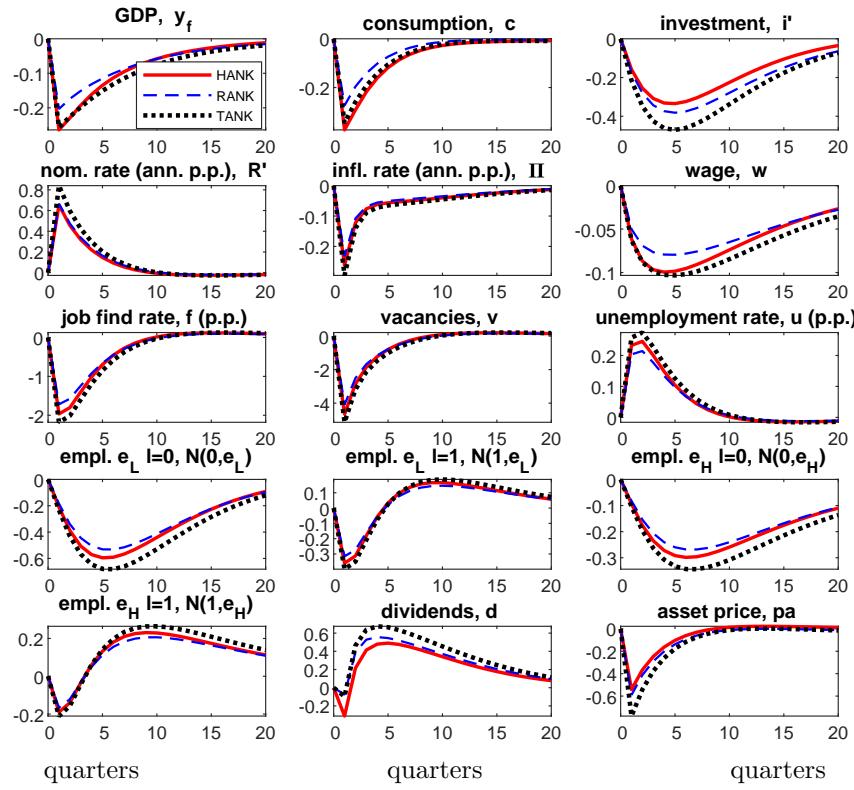
*Notes:* Impulse response to a one-standard-deviation MEI shock, starting at the stochastic steady state. All variables expressed as percent deviations, unless noted otherwise. ann. p.p.: expressed in annualized percentage points. p.p.: expressed in percentage points.

**Figure A5:** Impulse response to demand-elasticity shock (a negative price-markup shock),  $\zeta_P$



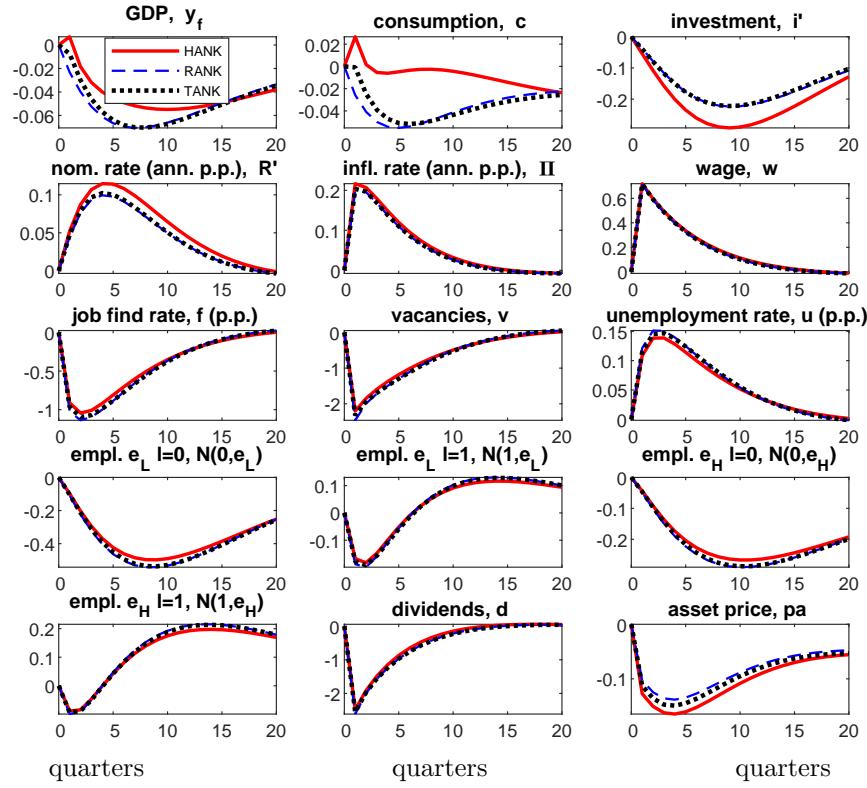
*Notes:* Impulse response to a one-standard-deviation price-markup shock, starting at the stochastic steady state. The shock compresses price markups. All variables expressed as percent deviations, unless noted otherwise. ann. p.p.: expressed in annualized percentage points. p.p.: expressed in percentage points.

**Figure A6: Impulse response to monetary shock,  $\zeta_R$**



*Notes:* Impulse response to a one-standard-deviation monetary shock, starting at the stochastic steady state. All variables expressed as percent deviations, unless noted otherwise. ann. p.p.: expressed in annualized percentage points. p.p.: expressed in percentage points.

**Figure A7: Impulse response to wage shock,  $\zeta_w$**



*Notes:* Impulse response to a one-standard-deviation wage shock, starting at the stochastic steady state. All variables expressed as percent deviations, unless noted otherwise. ann. p.p.: expressed in annualized percentage points. p.p.: expressed in percentage points.

## I.7 Impulse responses cross-sectional inequality

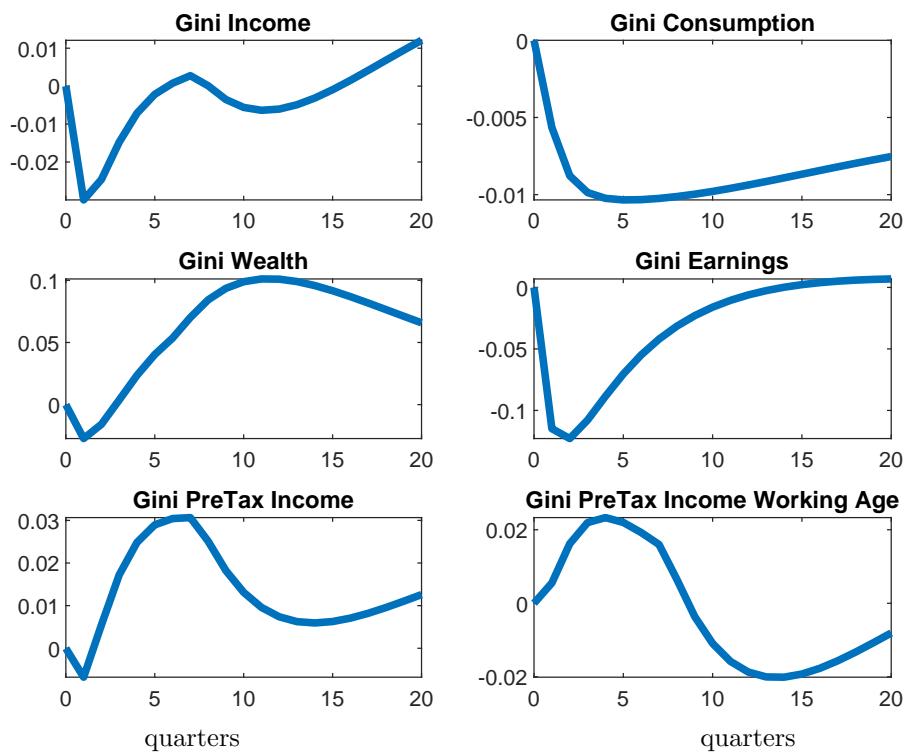
The following figures report the response of inequality in the cross-section of households, as measured by the Gini coefficients. The shocks (and, thus, the aggregate dynamics) are as in the previous graphs that showed the aggregate impulse responses for the HANK model.

**Figure A8: HANK: Impulse response of Ginis to TFP shock,  $\zeta_{TFP}$**



*Notes:* Impulse response of Gini coefficients to a one-standard-deviation TFP shock, starting at the stochastic steady state. The impulse responses are scaled such that a “1” on the y-axis means an increase of the Gini coefficient by one percentage point; say from a Gini coefficient of 0.50 to 0.501.

**Figure A9: HANK: Impulse response of Gini's to MEI shock,  $\zeta_I$**



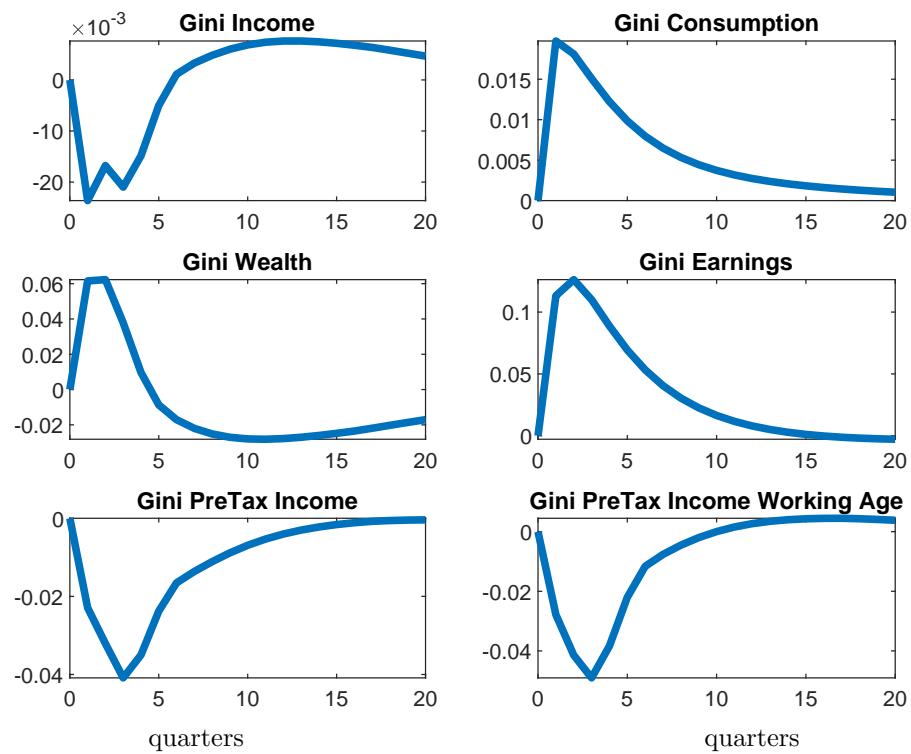
*Notes:* Impulse response of Gini coefficients to a one-standard-deviation MEI shock.

**Figure A10: HANK: Impulse response of Ginis to demand-elasticity shock (a negative price-markup shock),  $\zeta_P$**



*Notes:* Impulse response of Gini coefficients to a one-standard-deviation price-markup shock.

**Figure A11: HANK: Impulse response of Ginis to monetary shock,  $\zeta_R$**



*Notes:* Impulse response of Gini coefficients to a one-standard-deviation monetary shock.

**Figure A12: HANK: Impulse response of Gini's to wage shock,  $\zeta_w$**



*Notes:* Impulse response of Gini coefficients to a one-standard-deviation wage shock.

## I.8 Variance decomposition

Table A6 reports the unconditional variance decomposition for the heterogeneous-agent economy in the main text. To compute these moments we solve for the second-order policy functions with all shocks as described in the calibration and compute the variances of the variables we are interested in, for example, consumption.<sup>A18</sup> We then compute the variances of each variable when we force the realizations of all but one type of shock to zero, while we keep using the policy functions we computed in the model with all the shocks. The ratio of the variances gives us the share of the variance they explain. As the model is non-linear given the second-order solution, these shares will not add up exactly to one as there are potential interaction effects. In practice, we find that for most series listed below most of the variance is explained by the individual shock types.<sup>A19</sup> Therefore, as an approximation, we simply scale the individual shares proportionally to sum up to 1.

For comparison, Tables A7 and A8 report the same results for the representative-agent model and the saver-spender variant. Differences are modest in size. However, focusing for concreteness on consumption, we see that the price-markup shock and the MEI shock play a larger role in driving its dynamics than in the representative-agent model, while the role of TFP shocks is reduced.<sup>A20</sup>

**Table A6: Variance decomposition HANK**

variable	$\zeta_{TFP}$	$\zeta_I$	$\zeta_P$	$\zeta_R$	$\zeta_w$
GDP, $y_f$	28.00	52.47	10.41	6.63	1.50
Consumption, $c$	22.95	62.75	5.64	6.95	1.71
Investment, $i$	14.66	76.04	5.66	1.57	2.07
Unemployment, $U$	2.90	32.06	27.01	24.04	13.98
Flow rate $U \rightarrow E$ , $f$	3.00	33.83	25.29	24.37	13.51
Vacancies, $V$	3.31	36.28	22.11	26.08	12.21
Wage, $W$	19.20	26.98	6.52	2.87	44.42
Inflation, $\Pi$	8.68	16.16	48.08	10.68	16.39
Nominal rate, $R$	7.67	23.65	17.02	46.41	5.25

*Notes:* Forecast error variance decomposition for the heterogeneous-agent model. Contribution of respective shock (TFP, MEI, price-markup, monetary, wage) to the variance. Based on second-order dynamics with pruning. Entries in percent. Rows may not sum to 100 because of rounding error.

## I.9 Marginal propensities to consume

An oft-referenced statistic that helps us to understand the transmission of shocks in many models is household marginal propensity to consume (MPC). To document the MPCs for our model, we perform the following experiment. We are interested in the individual household's consumption response to an exogenous one-time increase in income. A household is characterized by its state  $(n, a, l, e, b, s)$ . At the beginning of the quarter, we give the model-equivalent of \$500 in wealth to a

<sup>A18</sup>We use pruning.

<sup>A19</sup>More than 90 percent.

<sup>A20</sup>The overall volatility of consumption is slightly higher in the heterogeneous-agent model.

**Table A7: Variance decomposition RANK**

variable	$\zeta_{TFP}$	$\zeta_I$	$\zeta_P$	$\zeta_R$	$\zeta_w$
GDP, $y_f$	30.58	53.46	9.53	4.54	1.90
Consumption, $c$	23.71	67.61	3.66	3.44	1.59
Investment, $i$	16.93	72.09	7.79	2.08	1.11
Unemployment, $U$	2.55	30.94	27.33	21.05	18.14
Flow rate $U \rightarrow E$ , $f$	2.63	32.77	25.60	21.33	17.66
Vacancies, $V$	2.96	35.50	22.51	22.83	16.21
Wage, $W$	20.83	28.35	6.08	2.14	42.60
Inflation, $\Pi$	8.92	15.06	53.17	8.52	14.33
Nominal rate, $R$	7.71	20.97	19.25	48.48	3.60

*Notes:* Forecast error variance decomposition for the representative-agent model. Contribution of respective shock (TFP, MEI, price-markup, monetary, wage) to the variance. Based on second-order dynamics with pruning. Entries in percent. Rows may not sum to 100 because of rounding error.

**Table A8: Variance decomposition TANK**

variable	$\zeta_{TFP}$	$\zeta_I$	$\zeta_P$	$\zeta_R$	$\zeta_w$
GDP, $y_f$	28.79	53.07	9.23	7.15	1.76
Consumption, $c$	23.07	65.83	3.84	5.73	1.54
Investment, $i$	17.02	71.51	7.22	3.14	1.11
Unemployment, $U$	2.49	28.81	23.85	29.83	15.03
Flow rate $U \rightarrow E$ , $f$	2.57	30.49	22.35	29.97	14.62
Vacancies, $V$	2.85	32.66	19.45	31.88	13.17
Wage, $W$	20.13	28.51	6.00	3.45	41.91
Inflation, $\Pi$	8.69	15.30	49.08	13.19	13.74
Nominal rate, $R$	6.20	17.44	14.38	59.06	2.92

*Notes:* Forecast error variance decomposition for the saver/spender model. Contribution of respective shock (TFP, MEI, price-markup, monetary, wage) to the variance. Based on second-order dynamics with pruning. Entries in percent. Rows may not sum to 100 because of rounding error.

household (and to that household only).<sup>A21</sup> Then, we record the cumulative increase in consumption expenditure as a share of the initial gift over the next quarter, the next two quarters, three quarters, and four quarters. We focus on the economy's non-stochastic steady state. That is, for the experiment shown here, all aggregate shocks  $\zeta$  are known to be zero in all time periods. Endogenous aggregate variables have settled to their long-run value. While the aggregate state of the economy remains fixed in this experiment, the household still faces idiosyncratic risk (for example, idiosyncratic income and employment risk). We allow the household's individual states to change over time. Table A9 summarizes group averages. It shows the average MPCs for different groups of

<sup>A21</sup>We use year-2000 US\$, in line with the cut-off date for the tax functions employed in the model.

households (first four columns, in percent of the initial increase in income), where households are grouped by their characteristics at the time of the transfer. The last column reports the share of these households in the population.

The first line shows the average MPC for the entire economy, giving equal weight to all households. In the first quarter, the average MPC is 15.4 percent. After a year, on average households have spent – for consumption – a third of the increase in income (an MPC of 33.5 percent). This is in line with the evidence summarized in Carroll et al. (2017), for example. The paper concludes that most empirical estimates in the literature find aggregate annual MPCs of 20 to 60 percent over a yearly horizon.<sup>A22</sup>

The average MPC may be an incomplete guide to the effect of shocks, however. The reason is that shocks impact different sources of income differentially, so that households in different idiosyncratic states might respond differently to a given shock. For example, a rise in wages does not benefit the retired or unemployed directly, while it raises the labor earnings of the employed. Therefore, the table also shows the MPCs for different subgroups of the population. As before, households are assigned to groups on the basis of their characteristics at the time of the gift. But we allow households to transit to other states thereafter. The first set of subgroups splits the population by wealth quartile. Next, we separate households by employment and life-cycle state. The third block shows the MPCs for different levels of idiosyncratic productivity, while the final block looks at MPCs by different time preferences.

The results in the table can be summarized as follows. As is typical in models like ours the MPC decreases in wealth. The wealth-poorest 25 percent of households (“ $\leq$  Wealth25”) spend about 81 percent of the gift within a year. The wealth-richest 25 percent (“Wealth75+”), instead, convert only roughly 9 percent of the gift into consumption within a year. Looking over the employment states we see that employed households without skill loss (“working age,  $n = 1, l = 0$ ”) spend about 1/3 of the gift within a year as they tend to be wealth-richer and tend to save both to insure against employment risk and for retirement. Employed households with a skill loss (“working age,  $n = 1, l = 1$ ”) and unemployed households (“working age,  $n = 0$ ”) have a significantly higher MPC. These households are wealth-poorer on average, as they have earlier used some of their wealth to stabilize consumption. In addition, they hope for a likely rise in income in case their employment or skill-loss state improves. Finally, retired households supplement their low but secure social security income. The same analysis can be applied to the effects of household skills. Finally, the MPC is decreasing in a household’s patience. This is so because of both the direct effect of more forward-looking behavior and the induced stock of savings. Our calibration to the wealth distribution implies that lower-educated households tend to be less patient, and so tend to have higher MPCs.

## I.10 Consumption policies in HANK

Figures A13 through A17 plot the consumption policies as a function of the wealth of a household, where the wealth is expressed as a percent of the average steady-state wage in the economy. For readability, we focus on the lower end of the policy functions. Each panel shows three lines. The blue solid line reports consumption policies under the baseline monetary policy rule, at the aggregate stochastic mean of that economy. A blue dashed line shows the consumption policies under strict inflation targeting, but evaluating the policies at the stochastic mean of the baseline economy. The red dotted line evaluates the consumption policy functions under strict inflation targeting at the

---

<sup>A22</sup>The heterogeneity in discount factors is an important factor for the average MPC, as discussed in Carroll et al. (2017).

**Table A9: Cumulative Marginal Propensity to Consume (in percent)**

	Cumul. MPC after quarter				Fraction of households
	1	2	3	4	
All	15.4	24.0	29.3	33.5	100
<u>By wealth</u>					
$\leq$ Wealth25	46.4	67.3	75.8	80.6	25.0
Wealth25-Wealth50	9.5	17.9	25.3	31.8	25.0
Wealth50-Wealth75	3.3	6.6	9.7	12.7	25.0
Wealth75+	2.2	4.4	6.6	8.7	25.0
<u>By age and employment</u>					
working age, $n = 1, l = 0$	13.7	22.6	28.6	33.1	25.5
working age, $n = 1, l = 1$	21.2	32.3	38.1	42.6	46.8
working age, unempl., $n = 0$	25.8	36.4	42.0	46.0	4.6
retired ( $s_0$ )	3.3	6.5	9.6	12.8	23.1
<u>Working age, by skill</u>					
$s_1$ (low)	28.3	41.3	46.9	50.4	38.1
$s_2$ (medium)	9.9	17.8	24.1	30.0	38.1
$s_3$ (high)	4.2	8.2	11.8	15.3	0.8
<u>By education and patience</u>					
$\beta_{e_L}$ , low edu., patient	4.3	8.1	11.6	14.4	30.0
$\beta_{e_H}$ , high edu., patient	2.0	3.9	5.9	7.8	20.0
$\beta_{e_L} - \Delta\beta$ , low edu., impatient.	28.9	44.4	52.9	58.9	30.0
$\beta_{e_H} - \Delta\beta$ , high edu., impatient.	25.1	37.6	44.1	49.0	20.0

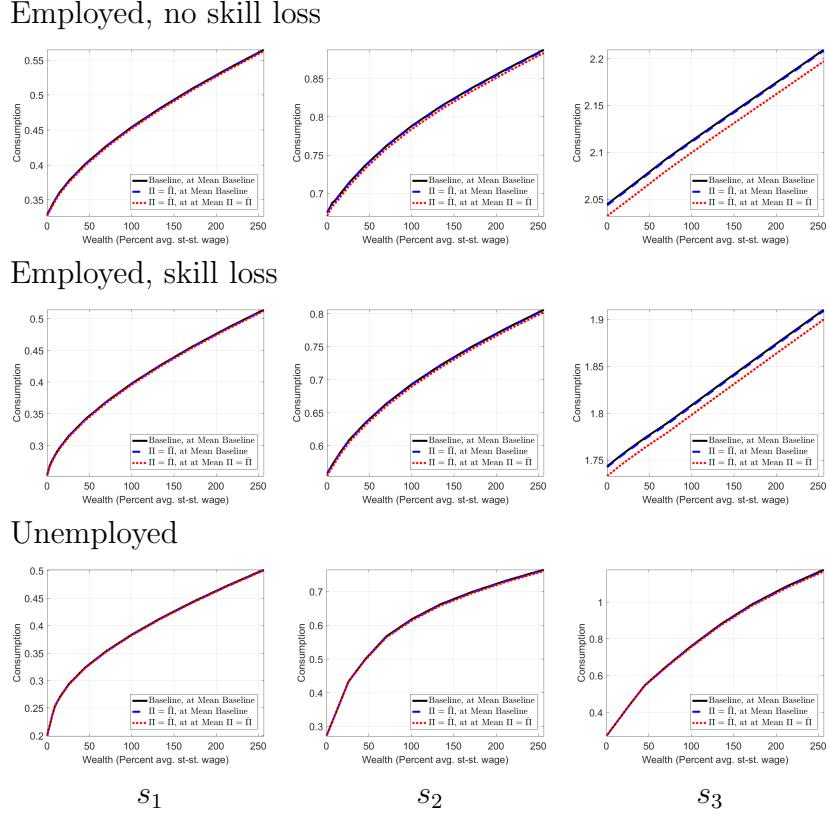
*Notes:* The table shows the share of a \$500 gift to an individual household that is spent after one quarter, two quarters, three quarters, and four quarters in the deterministic steady state. Column ‘Fraction of households’ denotes the percentage of the population in the respective group at the beginning of the first quarter. As per the rows, ‘All’ contains all households and reports the aggregate average MPC. Rows ‘By wealth’ split households into four equally sized groups by wealth and reports their respective MPCs with ‘ $\leq$  Wealth25’ being the lowest wealth quartile and ‘Wealth75+’ being the highest. ‘By age and employment’ reports average MPCs for the retired ( $s_0$ ), and three groups of working-age households. The first are the employed without skill loss ( $n = 1, l = 0, s \in \mathcal{S}_+$ ), the next the employed with skill loss ( $n = 1, l = 1, s \in \mathcal{S}_+$ ). Last, there are the unemployed households ( $n = 0, s \in \mathcal{S}_+$ ). ‘retired’ contains all the retired households ( $s = s_0$ ). ‘Working age, by skill’ groups the working-age households by their skill level (from  $s_1$  (lowest) to  $s_3$  (highest)). ‘By education and patience’ reports results for the different time preferences. ‘ $\beta_{e_L}$ ’ summarizes the low-educated households with high  $\beta$ . ‘ $\beta_{e_H}$ ’ describes the high-educated households with high  $\beta$ . The next two rows contain the same education groups but with lower  $\beta$  (impatient). For each of the groups, membership is determined at the beginning of the first quarter, before the gift is given. Households are allowed to transit to different states thereafter. Shares do not necessarily add up to 100 due to rounding.

stochastic mean state that arises under that policy. That is, differences between the blue solid and the blue dashed lines are due to a change in policy only. Differences between blue dashed lines and

blue dotted lines have the same policy but evaluate it at a different state.

The change toward inflation targeting shifts the policy functions down for lower wealth levels, as households' expected labor income falls. For sufficiently rich individuals the lines cross and consumption increases in response to the rise in dividends and real rates (these wealth levels not shown here). One important point to keep in mind when interpreting the figures is that the wealth of households actually changes with the policy change; everybody with positive wealth becomes richer, as asset prices rise. However, the figure holds wealth constant.

**Figure A13: Consumption policies of the working-aged, by skill: impatient households of low education**

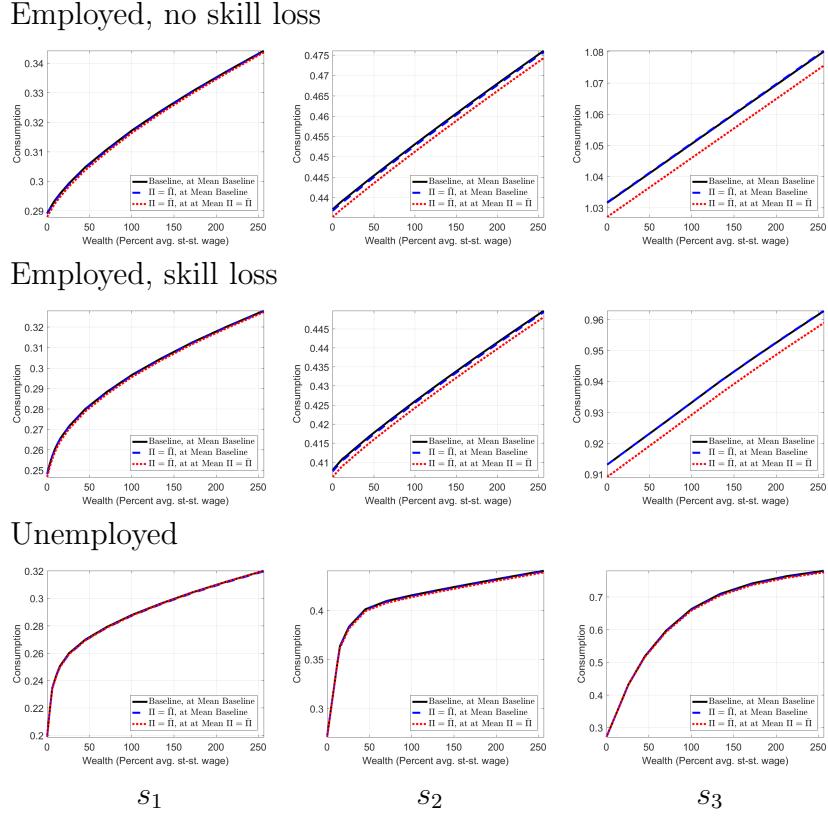


*Notes:* Consumption policies for households of working age that have low education and are impatient.

## I.11 Countercyclical skewness in earnings

Guvenen et al. (2014) document for the United States that cross-sectional logged labor earnings growth becomes more left-skewed during recessions, while the cross-sectional standard deviation of logged labor earnings growth remains fairly stable on average. We would expect this prediction to also hold in our model as the fall in the job-finding rate during a recession implies that more workers fall into or remain in unemployment and a larger share of the population is affected by earnings loss. Conversely, in a cyclical expansion the flows toward employment rise and workers slowly regain their productivity. All these factors should lead to a procyclical skewness in earnings. Furthermore, we would expect a smaller influence of the business cycle on the cross-sectional standard deviation.

**Figure A14: Consumption policies of the working-aged, by skill: patient households of low education**



*Notes:* Consumption policies for households of working age that have low education and are patient.

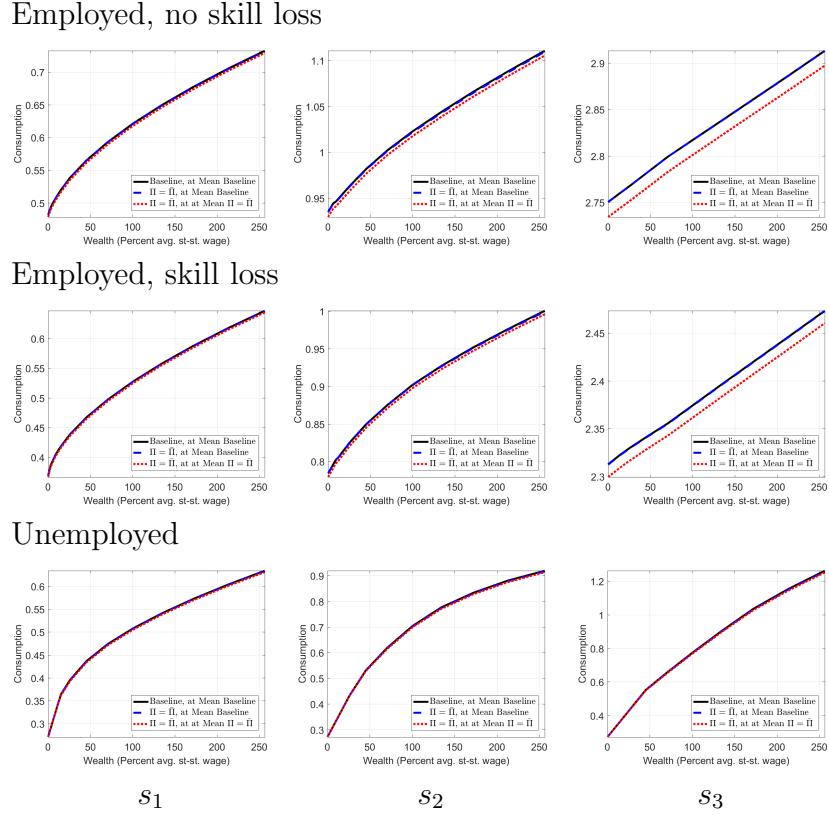
In the following, we document model statistics that are consistent with this idea. To be consistent with their yearly data, we focus on yearly logged labor earnings growth of households that are in the labor force, and that do not retire.<sup>A23</sup> We compute the *cross-sectional* standard deviation and skewness of these earnings growth rates for a 5000-periods-long panel simulation from the HANK model for one million agents.<sup>A24</sup> This gives us 5000 observations of the cross-sectional standard deviation and the cross-sectional skewness of earnings growth. We then compute the correlation of these series with log GDP growth and log employment growth, the latter two being used as cyclical indicators.

The average cross-sectional skewness is -0.24, so the left tail of earnings growth (drops in earnings) is longer than the right tail (rising earnings). The average standard deviation of the cross-sectional skewness is 0.0437, meaning that the cross-sectional skewness fluctuates notably over time. Instead, the cross-sectional standard deviation of earnings growth turns out to be rather stable over time. The average standard deviation of the cross-sectional standard deviation of earnings growth turns out to be one order of magnitude smaller (0.0033 with a mean of 0.22) than the standard

<sup>A23</sup>We include unemployment benefits in our earnings measure as they constitute taxable income in the U.S.

<sup>A24</sup>We allow for 1000 periods of burn-in in each simulation. The simulation uses the version of the model with monetary policy shocks and without adjustment to average inflation - the version we compared the data to.

**Figure A15: Consumption policies of the working-aged, by skill: impatient households of high education**



*Notes:* Consumption policies for households of working age that have high education and are impatient.

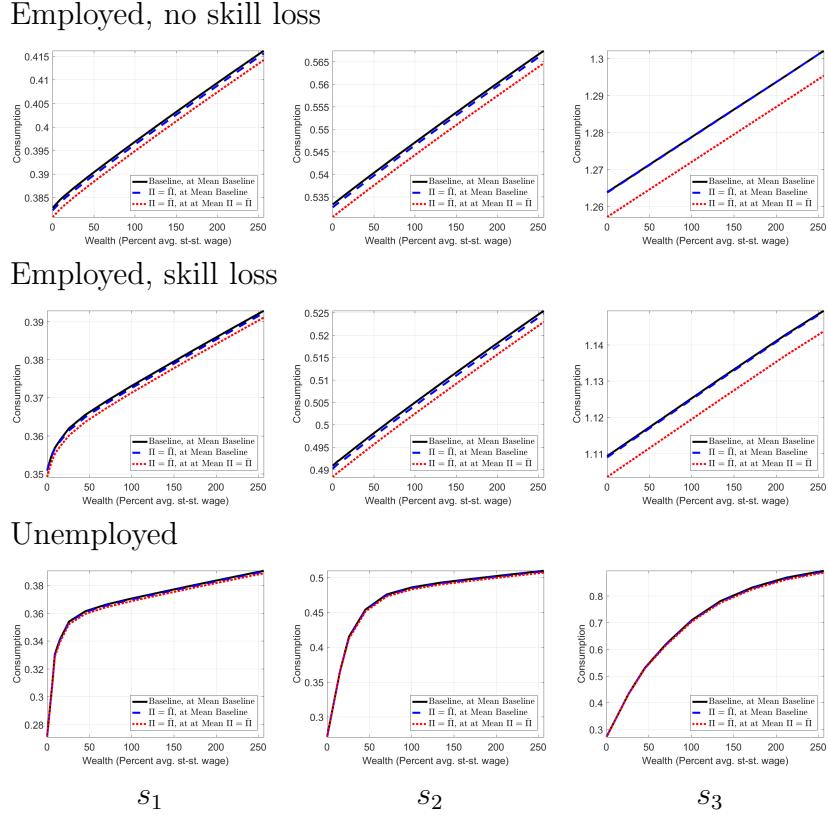
deviation of cross-sectional skewness, consistent with the evidence reported in Guvenen et al. (2014).

The skewness of cross-sectional earnings growth in the model is not only volatile; it is also procyclical. The correlation between GDP growth and the skewness of earnings growth in the model is 0.53 and the correlation of skewness with employment growth is 0.61.<sup>A25</sup> In other words, in booms the left tail of the earnings growth distribution is less pronounced, the opposite in recessions, in line with the above intuition. The smaller correlation with GDP growth than with employment is explained as follows. Unemployment is the main driver of skewness, linking skewness tightly to employment growth. A rise in GDP growth, instead, need not always have a positive effect on employment in a given period, depending on the innovations affecting the economy.<sup>A26</sup>

<sup>A25</sup>When we compute the same correlations for the U.S. using the data in Guvenen et al. (2014) for 1984-2008 we find values of 0.58 and 0.78 for GDP and employment growth, respectively.

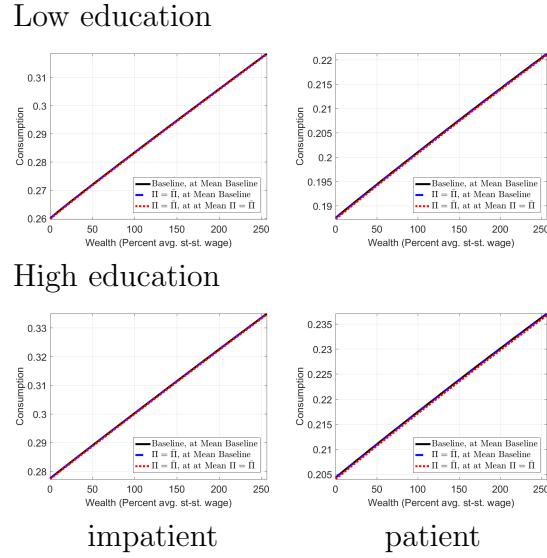
<sup>A26</sup>For example, if innovations to TFP lead to a rise in GDP growth in a period, this would coincide with a temporary fall in the job-finding rate, while a monetary policy shock would imply positive comovement of GDP growth and employment. Compare the impulse responses in Appendix I.6.

**Figure A16: Consumption policies of the working-aged, by skill: patient households of high education**



*Notes:* Consumption policies for households of working age that have high education and are patient.

**Figure A17: Consumption policies of the retired, by education and patience**



*Notes:* Consumption policies for households of retirement age, by education and patience.

## J Adjusting for the effect on average inflation

The effect on average inflation may or may not be a desirable feature of the baseline model. This section reports, for the representative-agent economy, the same results on the inflation unemployment trade-off and on the welfare gains from policy changes that we showed before, with one difference: for each parameter  $\phi_U$ , we now adjust the Taylor rule such that average inflation always is at the steady-state level. That is, it designs policy such that there is no effect on average inflation. Technically, this is done as follows. We adjust the Taylor rule (4) by a term that shifts the nominal rate in the stochastic economy (but leaving the non-stochastic steady state in place). Let  $\epsilon_t^{\text{adjust}}$  be a white noise standard normal shock. The adjusted Taylor rule takes the form:

$$\begin{aligned} \log \left( \frac{R(X)}{\bar{R}} \right) = & \phi_R \log \left( \frac{R_{-1}(X)}{\bar{R}} \right) \\ & + (1 - \phi_R) \left[ \phi_\Pi \log \left( \frac{\Pi(X)}{\bar{\Pi}} \right) + \phi_\epsilon E_t \left\{ \left( \epsilon_{t+1}^{\text{adjust}} \right)^2 \right\} - \phi_u \left( \frac{U(X) - \bar{U}}{\pi_S(\mathcal{S}_+)} \right) \right]. \end{aligned}$$

Note that the term involving the expectation is a constant that appears only in the stochastic version of the model, but not in the non-stochastic steady state. For each value of  $\phi_U$ , we choose a  $\phi_\epsilon$  such that the average inflation rate stays at (very close to) the target level of 2 percent annualized throughout. In other words, whenever we change  $\phi_U$ , we also change  $\phi_\epsilon$ .

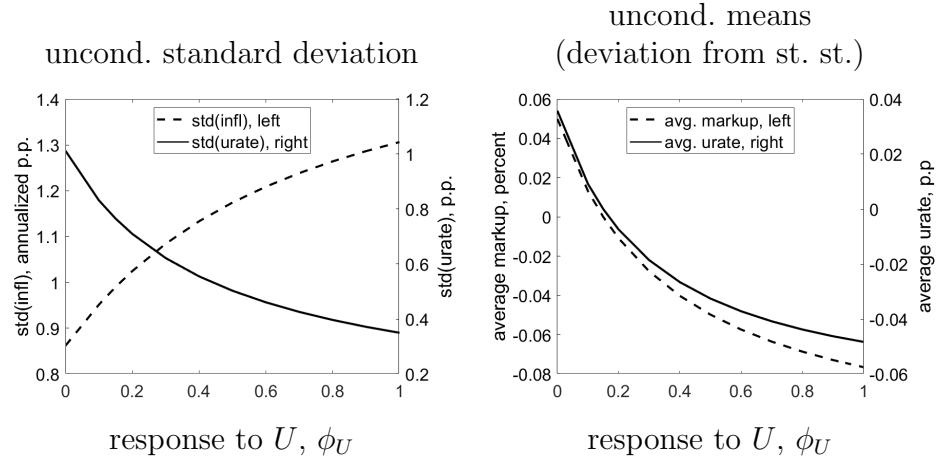
## K Inflation-unemployment trade-off in RANK/TANK

The current section documents the trade-off between inflation and unemployment that is inherent in the RANK/TANK economies. It complements the results for the HANK economy in Section 4.2 of the main text.

### K.1 Inflation-unemployment trade-off and markups — RANK

The current section presents the inflation-unemployment trade-off in the RANK model.

**Figure A18: RANK: Inflation-unemployment trade-off**

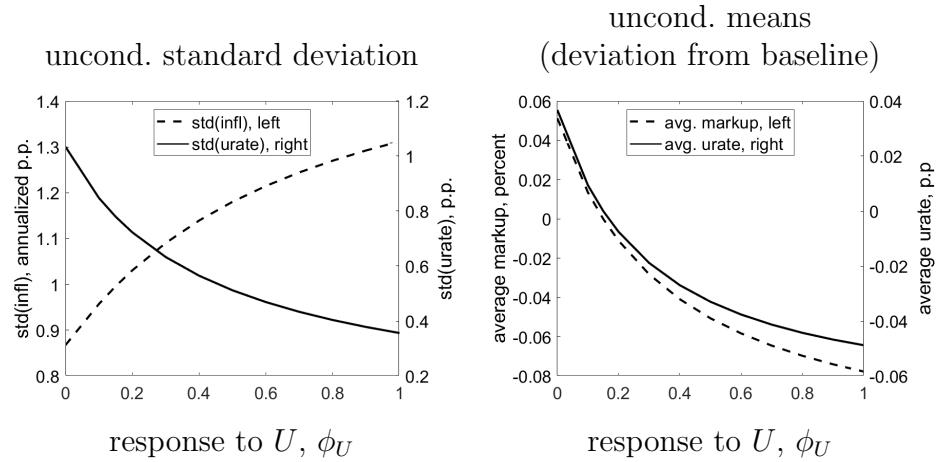


*Notes:* Same as Figure 2 in the main text, but for the RANK economy.

### K.2 Inflation-unemployment trade-off and markups – TANK

The current section presents the inflation-unemployment trade-off in the TANK model.

**Figure A19: TANK: inflation-unemployment trade-off**



*Notes:* Same as Figure 2 in the main text, but for the TANK economy.

## L Welfare gains by shock

This appendix reports welfare results in HANK and RANK/TANK if only one shock is present at a time. Note that this change of scenario changes both the cyclical fluctuations in the economy and the mean of the economy under the baseline policy and, therefore, the starting point for the transition. Table A10 shows the distribution of welfare gains from changing policy if only the TFP shock is present in both the baseline and under the alternative monetary policy. On impact, unemployment rises after a productivity shock, and inflation falls: the Keynesian transmission mechanism. The calibration of the baseline means not only that real wages are rigid, but that they propagate the productivity shock. In spite of the real wage rigidity, though, households' assessment of optimal

**Table A10: Welfare Effects of Policy Change - TFP Shock only**

		Response to unemployment, $\phi_u$							$\Pi = 0$
		0	0.1	0.15	0.2	0.3	0.4	0.5	
Wealth percentile	HANK	Consumption-equivalent welfare gain (in percent)							
	0-20	-0.004	-0.001	—	0.001	0.003	0.004	0.005	<b>0.019</b>
	20-40	-0.004	-0.001	—	0.001	0.003	0.004	0.005	<b>0.018</b>
	40-60	-0.005	-0.001	—	0.002	0.004	0.005	0.006	<b>0.018</b>
	60-80	-0.006	-0.001	—	0.002	0.005	0.006	0.008	<b>0.027</b>
	80-95	-0.008	-0.002	—	0.002	0.005	0.008	0.010	<b>0.049</b>
	95+	-0.007	-0.002	—	0.002	0.005	0.008	0.009	<b>0.051</b>
		Utilitarian sum of welfare (change)							
		-0.004	-0.001	—	0.001	0.003	0.004	0.005	<b>0.021</b>
		Share of households in favor over baseline							
		0.000	0.000	—	1.000	1.000	1.000	1.000	<b>1.000</b>
TANK	RANK	-0.003	-0.001	—	0.001	0.002	0.002	0.003	<b>0.009</b>
	spender	-0.002	0.000	—	0.000	0.001	0.001	0.002	<b>0.005</b>
	saver	-0.003	-0.001	—	0.001	0.002	0.003	0.004	<b>0.011</b>

*Notes:* Welfare effects of a permanent policy change from the baseline policy, if the only shock is the TFP shock. The new policy either has a different response to unemployment,  $\phi_u$  (central panels) or is characterized by strict inflation targeting ( $\Pi = \bar{\Pi}$ , last column). From top to bottom: HANK, lifetime consumption-equivalent welfare gains (in percent of consumption) by wealth, utilitarian consumption-equivalent welfare gains, share of votes in favor of the policy change, all taking the baseline as the alternative. Then, RANK and TANK spender/saver households.

stabilization policy is virtually unanimous if the economy is driven by TFP only. Locally (for a small change in the rule), households prefer a switch toward a stronger unemployment response. Globally, households unanimously favor strict inflation targeting. Virtually all households would be willing to move toward strict inflation targeting if productivity were the only source of fluctuations. We conclude that failure of divine coincidence as in Faia (2009) and Ravenna and Walsh (2011) is not the central driving force of our results. Indeed, for the TFP shock, qualitatively, the HANK models' households' policy assessments are remarkably similar to RANK and to TANK (for both savers and spenders). The stakes are somewhat higher in HANK, however.

**Table A11: Welfare Effects of Policy Change - MEI Shock only**

		Response to unemployment, $\phi_u$							$\Pi = 0$
		0	0.1	0.15	0.2	0.3	0.4	0.5	
Wealth percentile	HANK	Consumption-equivalent welfare gain (in percent)							
	0-20	0.000	-0.002	—	0.001	-0.002	-0.001	<b>0.001</b>	-0.004
	20-40	-0.004	-0.002	—	0.002	0.001	0.002	0.004	<b>0.008</b>
	40-60	-0.008	0.000	—	0.007	0.010	0.014	0.017	<b>0.033</b>
	60-80	-0.013	0.002	—	0.013	0.022	0.028	0.031	<b>0.084</b>
	80-95	-0.024	-0.006	—	0.006	0.015	0.022	0.028	<b>0.130</b>
	95+	-0.021	-0.005	—	0.005	0.013	0.019	0.025	<b>0.128</b>
		Utilitarian sum of welfare (change)							
		-0.008	0.000	—	0.006	0.010	0.013	<b>0.015</b>	<b>0.046</b>
		Share of households in favor over baseline							
		0.102	0.128	—	0.933	0.707	0.724	<b>0.893</b>	<b>0.772</b>
RANK		-0.012	-0.003	—	0.003	0.007	0.011	0.013	<b>0.034</b>
TANK	spender	-0.013	-0.003	—	0.003	0.008	0.011	0.014	<b>0.029</b>
	saver	-0.014	-0.004	—	0.003	0.008	0.012	0.015	<b>0.039</b>

*Notes:* Same as Table A10, but the only shock is the MEI shock.

Table A11 shows the welfare gains of changing the systematic monetary policy rule if only the MEI shock causes business-cycle fluctuations. Under the baseline policy, the MEI shock works like a demand shock, driving inflation and employment in the same direction. Stabilizing unemployment can, therefore, be conducive to stabilizing inflation. This explains why wealth-richer households benefit both from a bigger weight  $\phi_u$  in the Taylor rule, and from strict inflation targeting. For poorer households, the gains from strict inflation targeting are less pronounced. The majority of households favor strict inflation targeting, but roughly 23 percent do not, namely, those in the lower wealth percentiles. For the MEI shock, there is disagreement in HANK, while RANK and TANK do not show any. In TANK, in particular, there is no disagreement between savers and spenders. In this sense, TANK is missing the disagreement present in HANK. Here, too, the stakes of individual groups in HANK can be notably bigger than RANK and TANK would signal.

Next, Table A12 looks at the welfare gains if the price-markup shock is the only shock in the economy. The price-markup shock presents the monetary authority with a trade-off between stabilizing inflation and stabilizing unemployment. Next to this, fluctuations in inflation cause price adjustment costs, which our modeling imparts directly to the owners of capital. The HANK model suggests that households strongly disagree about how the monetary authority should handle cost-push shocks. Roughly 26 percent of households favor strict inflation targeting to the baseline policy. These households are in the top of the wealth distribution. Households at the bottom of the wealth distribution, instead, would be strongly opposed to this. What is important to note is that for the price-markup shock the HANK policy advice notably differs from the advice that the simpler variants give. Both RANK and TANK do not see any support for inflation targeting. And they favor a change to a rule with a larger response to unemployment ( $\phi_u = 0.5$ ). Note that, in HANK, this is a policy change that all but the poorest 20 percent of households would dislike. So,

**Table A12: Welfare Effects of Policy Change - Price-Markup Shock only**

		Response to unemployment, $\phi_u$							$\Pi = 0$
		0	0.1	0.15	0.2	0.3	0.4	0.5	
Wealth percentile	HANK	Consumption-equivalent welfare gain over baseline							
	0-20	-0.012	-0.002	—	0.001	<b>0.002</b>	0.002	0.002	-0.136
	20-40	-0.007	0.000	—	0.000	-0.002	-0.005	-0.007	-0.136
	40-60	<b>0.014</b>	0.006	—	-0.006	-0.016	-0.026	-0.035	-0.060
	60-80	<b>0.028</b>	0.010	—	-0.009	-0.026	-0.041	-0.053	-0.011
	80-95	0.039	0.012	—	-0.011	-0.031	-0.048	-0.063	<b>0.044</b>
	95+	0.038	0.012	—	-0.011	-0.029	-0.045	-0.059	<b>0.061</b>
Utilitarian sum of welfare (change)									
		<b>0.007</b>	0.003	—	-0.003	-0.010	-0.017	-0.023	-0.055
Share of households in favor over baseline									
		0.494	<b>0.554</b>	—	0.313	0.279	0.259	0.245	0.260
RANK		-0.031	-0.007	—	0.005	0.010	0.013	<b>0.014</b>	-0.239
	spender	-0.050	-0.012	—	0.009	0.021	0.028	<b>0.033</b>	-0.289
	saver	-0.028	-0.006	—	0.004	0.008	0.009	<b>0.009</b>	-0.227

*Notes:* Same as Table A10, but the only shock is the price-markup shock.

here too HANK is capturing disagreement that RANK/TANK would miss. And HANK leads to different policy conclusions. In HANK, a majority of households would wish to see slightly *more* inflation-centric policy; RANK/TANK, instead, favor a stronger inflation focus in the rule.

Last, Table A13 looks at the welfare gains from policy changes if the wage-markup shock is the only shock in the economy. The pattern in HANK that emerges from Table A13 is unanimous. The wage-markup shock not only presents the central bank with a trade-off between output and inflation stabilization. It also directly distorts poor households' consumption plans. Even though the wage-markup shock in our model works like a cost-push shock, households unanimously favor a more inflation-centric approach (including strict inflation targeting). This is not the case in RANK or TANK. The RANK households favor a somewhat more inflation-centric approach, but falling short of strict inflation targeting. The preferred policy of TANK spenders is the baseline policy. The preferred policy of TANK savers is the RANK model's optimal policy. In sum, for wage-markup shocks, too, the HANK policy advice differs from the advice that simpler models would give.

**Table A13: Welfare Effects of Policy Change - Wage Shock only**

		Response to unemployment, $\phi_u$							$\Pi = \bar{\Pi}$
		0	0.1	0.2	0.25	0.3	0.4	0.5	
Wealth percentile	HANK	Consumption-equivalent welfare gain over baseline							
	0-20	0.004	0.001	—	-0.001	-0.004	-0.006	-0.007	<b>0.015</b>
	20-40	0.006	0.002	—	-0.002	-0.005	-0.008	-0.011	<b>0.013</b>
	40-60	0.010	0.003	—	-0.003	-0.010	-0.015	-0.020	<b>0.016</b>
	60-80	0.013	0.005	—	-0.005	-0.013	-0.020	-0.026	<b>0.016</b>
	80-95	0.017	0.005	—	-0.005	-0.014	-0.022	-0.029	<b>0.022</b>
	95+	0.016	0.005	—	-0.005	-0.013	-0.020	-0.026	<b>0.022</b>
		Utilitarian sum of welfare (change)							
		0.008	0.003	—	-0.003	-0.007	-0.012	-0.015	<b>0.011</b>
		Share of households in favor over baseline							
		<b>1.000</b>	1.000	—	0.000	0.000	0.000	0.000	<b>1.000</b>
RANK	<b>0.002</b>	0.001	—	-0.001	-0.004	-0.007	-0.009		-0.004
	spender	-0.002	0.000	—	0.000	-0.001	-0.001	-0.002	-0.010
TANK	saver	<b>0.003</b>	0.002	—	-0.002	-0.005	-0.008	-0.011	-0.003

*Notes:* Same as Table A10, but the only shock is the wage shock.

## M Welfare effects of a one-sided monetary shock

As a point of reference, the current section reports the welfare gains or losses from a one-time monetary shock. This serves as a reference both for the magnitude of welfare gains and so as to discuss the magnitude of gains. In the baseline, a contractionary monetary shock reduces lifetime

**Table A14: Welfare gains from a monetary tightening – under baseline policy**

Wealth percentile	Baseline model		Portfolio variant	
	c equiv.	US\$ (2004)	c equiv.	US\$ (2004)
0-20	-0.264	-5,426	-0.264	-5,416
20-40	-0.231	-5,142	-0.225	-5,170
40-60	-0.172	-4,305	-0.269	-6,907
60-80	-0.071	-1,755	-0.103	-2,635
80-95	-0.036	-1,255	-0.021	-826
95-100	-0.019	-1,064	0.008	1,573
top 1% only	0.003	982	0.056	10,148
Utilitarian	-0.067	—	-0.081	—
Vote	0.01	—	0.114	—
All \$	—	-3,567	—	-9,232

*Notes:* Welfare gains from a one-time monetary shock (negative numbers are welfare losses), 100 bps annualized. Shown are two cases: without accounting for the portfolio composition (the baseline) and with accounting for the composition as in Section 5.2 of the main text. And for each of these, the table shows the consumption-equivalent welfare gain (in percent of lifetime consumption) and the dollar-equivalent welfare gain (in 2004 US\$).

welfare for all but the richest 1 percent of households, that is, for the large majority of households. The monetary contraction induces a persistent recession; compare Figure A6. This is particularly costly in terms of consumption-equivalents for low-wealth households for two reasons. On the one hand, these households have few assets to self-insure against the unemployment risk that comes with the recession. On the other, these households tend to be relatively impatient in the first place. For impatient households, the near future gets stronger weight than for patient households; so for the impatient, the welfare losses of a persistent recession in the near term are particularly steep. For the lowest 20 percent of the wealth distribution, the welfare costs of a contractionary one-time monetary shock are twice as large as the welfare costs of systematically more hawkish policy; compare Table 9. For the wealth-richest instead, a change in systematic monetary stabilization policy easily carries the day, relative to the welfare effects of a one-time monetary shock. Indeed, the top 1 percent gain from a contractionary monetary policy shock, but only 0.0033 percent of lifetime consumption, or \$982 in total.

Another important result emerges from Table A14. In particular, compare the columns on the right to column “Leveraged portfolio” of Table 16 in the main text. In both cases, households hold a mix of nominal and real assets, as described in Section 5.2 of the main text. The role that the household portfolio plays in allocating gains and losses is fundamentally different, however. To see this, focus on the middle class in both tables (households in the 40th-60th percentile of net worth). These households tend to hold rather leveraged portfolios, being long in real assets and having

nominal debt. With such portfolios, a surprise monetary tightening is particularly costly for this group of households because their nominal debt (which we assume is short-term debt) exposes them to the ensuing higher real rates and a higher real debt burden; next to the higher risk of losing employment. The role of leverage is considerably different, instead, when considering a move to *systematically* more hawkish policy. Namely, in the latter case leverage exposes the middle class to the windfall gains to financial wealth that, in our model, are associated with a move toward inflation targeting.

The portfolio composition also plays a considerable role for the welfare gains that the richest households have from a one-time monetary tightening. Even then, however, a change toward systematically more hawkish policy brings larger welfare gains than the one-time monetary tightening for the richest households.

## N Welfare gain by dimension of heterogeneity

The current appendix provides the welfare gains of a policy change grouping households by idiosyncratic states. The first block of Table A15 reproduces the entries of Table 10 in Section 4.4 in the main text. The remaining blocks select households based on other idiosyncratic states.

**Table A15: One-time dollar-equivalent gain by dimension of heterogeneity**

		Response to unemployment, $\phi_u$							$\Pi = \bar{\Pi}$
		0	0.1	0.15	0.2	0.3	0.4	0.5	
skills	$s_0$ (retired)	1,672	665	—	-276	-1,087	-1,807	-2,528	<b>7,815</b>
	$s_1$ (low)	-284	-24	—	<b>38</b>	-28	-134	-206	-162
	$s_2$	-337	-12	—	<b>35</b>	-72	-232	-343	<b>166</b>
	$s_3$ (super)	2,048	753	—	-742	-2,279	-3,548	-4,786	<b>19,636</b>
emp., loss	$n = 0$ (unemp.)	-360	-30	—	<b>54</b>	-6	-128	-185	-905
	$n = 1, l = 1$	-318	-20	—	<b>41</b>	-37	-169	-253	-127
	$n = 1, l = 0$	-220	<b>10</b>	—	1	-149	-321	-468	<b>992</b>
	$e_L$	-45	<b>87</b>	—	34	-118	-308	-440	<b>87</b>
pati. educ.	$e_H$	443	221	—	-150	-572	-971	-1,377	<b>4,637</b>
	$b = 1$ (impat.)	-112	<b>23</b>	—	-15	-166	-352	-418	-856
	$b = 0$	421	262	—	-64	-436	-800	-1,223	<b>4,756</b>

*Notes:* Same as first block of Table 9, but sorting the population by dimensions of heterogeneity other than net worth. From top to bottom: residual skill (retired, low skill, medium skill, super-skill), current employment status (unemployed, employed with skill loss, employed without skill loss), education status (low, high), patience (less patient than average comparably educated, more patient than average comparably educated). Average dollar-equivalent gains for each group (2004 US\$).

Table A16 provides the corresponding consumption-equivalent welfare gains in percent.

**Table A16: Consumption-equivalent welfare gain by dimension of heterogeneity**

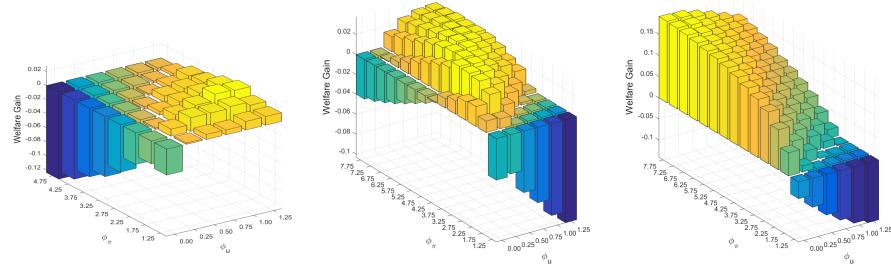
		Response to unemployment, $\phi_u$							
		0	0.1	0.15	0.2	0.3	0.4	0.5	$\Pi = \bar{\Pi}$
skills	$s_0$ (retired)	<b>0.063</b>	0.026	—	-0.009	-0.040	-0.069	-0.096	<b>0.276</b>
	$s_1$	-0.019	-0.003	—	0.004	<b>0.004</b>	0.001	0.000	-0.056
	$s_2$	-0.018	-0.002	—	<b>0.003</b>	0.001	-0.003	-0.005	-0.041
educ.	$s_3$	<b>0.015</b>	0.006	—	-0.007	-0.020	-0.032	-0.043	<b>0.151</b>
	$e_L$	-0.006	<b>0.003</b>	—	0.002	-0.003	-0.011	-0.016	-0.018
	$e_H$	<b>0.008</b>	0.006	—	-0.003	-0.013	-0.024	-0.035	<b>0.092</b>
pati.	$b = 1$	-0.007	<b>0.000</b>	—	-0.000	-0.006	-0.014	-0.016	-0.046
	$b = 0$	0.007	<b>0.008</b>	—	0.001	-0.009	-0.019	-0.032	<b>0.100</b>
empl., loss	$n = 0$	-0.021	-0.003	—	0.004	<b>0.004</b>	0.001	0.001	-0.083
	$n = 1, l = 1$	-0.019	-0.003	—	<b>0.004</b>	0.003	0.000	-0.001	-0.054
	$n = 1, l = 0$	-0.016	-0.002	—	<b>0.002</b>	0.000	-0.004	-0.006	-0.025

*Notes:* Same as Table A15, but reporting average consumption-equivalent welfare gains (in percent).

## O Optimal simple rules

For the three wealth percentiles defined in Section 4.5, Figure A20 reports the consumption-equivalent welfare gains for alternative combinations of systematic monetary policy, ( $\phi_{\Pi}$  and  $\phi_u$ ). The left panel shows the assessment of the bottom 20 percent by wealth, the middle panel presents

**Figure A20: Welfare gains from switching Taylor rule**



Wealth percentile 0-20    Wealth percentile 40-60    Wealth percentile 95-100

*Notes:* Welfare gains from alternative simple policies for different wealth percentiles. Welfare gains from change to new combination of  $\phi_{\Pi}$  and  $\phi_u$ . We used grids  $\phi_{\Pi} \in \{1.25, 1.5, \dots, 7.75\}$  and  $\phi_u \in \{0, 0.25, \dots, 1.25\}$  to search for the preferred policy. For better readability, the left panel shows a smaller range of responses to inflation. Gains are expressed in percent of lifetime consumption. Negative gains are welfare losses.

that of the central wealth percentiles, and on the right is the assessment by the wealthiest 5 percent of households.

## P Long-run policy assessment

This appendix collects information on the long-run welfare counterfactuals, and the effect of policy on average employment, and wages in the long run.

### P.1 Long-run policy assessment — HANK

Table A17 reports the HANK welfare gains in the long run. Here we ask a household: “At your current idiosyncratic state (before the policy change), how much would you pay for an alternative monetary policy if you were to jump (with your shares and other idiosyncratic states) to an economy with that policy, the initial aggregate state of which is that economy’s long-run stochastic mean?”<sup>A27</sup> In the long run, in HANK the vast majority of households would favor notably more accommodative

**Table A17: Long-run Welfare Effects of changing policy – HANK**

		Response to unemployment, $\phi_u$								$\Pi = \bar{\Pi}$
		0	0.1	0.15	0.2	0.3	0.4	0.5	0.6	
Consumption-equivalent welfare gain (in percent)										
Wealth percentile	0-20	-0.141	-0.034	—	0.028	0.068	0.094	0.115	<b>0.129</b>	-0.646
	20-40	-0.131	-0.033	—	0.025	0.060	0.082	0.100	<b>0.114</b>	-0.598
	40-60	-0.110	-0.023	—	0.027	0.059	0.079	0.094	<b>0.105</b>	-0.495
	60-80	-0.085	-0.015	—	0.026	0.054	0.070	0.080	<b>0.087</b>	-0.329
	80-95	-0.066	-0.016	—	0.015	0.036	0.052	0.058	<b>0.065</b>	-0.125
	95+	-0.042	-0.011	—	0.009	0.021	0.033	0.038	<b>0.041</b>	<b>0.002</b>
Utilitarian sum of welfare (change)										
		-0.070	-0.015	—	0.018	0.039	0.053	0.061	<b>0.068</b>	-0.284
Share of households in favor over baseline										
		0.004	0.082	—	0.986	0.995	0.997	0.998	<b>0.995</b>	0.054

*Notes:* Same as Table 8, but looking only at the long run, that is, abstracting from the transition.

monetary policy. The stakes are high. Namely, the poorest 20 percent of households would be willing to give up almost 0.65 percent of their (already lower) lifetime consumption to avoid a move toward strict inflation targeting (top row, right-most column). And even the upper middle class (the 80-95th percentile of wealth) would be willing to pay 0.12 percent of lifetime consumption to avoid that policy change. Of the groups shown here, only the wealth-richest 5 percent of households would marginally favor strict inflation targeting. In contrast to the RANK and TANK models, in HANK putting the focus on the long run only, therefore, does change the policy evaluation notably. In the long run, also in HANK, the utilitarian planner would implement a monetary policy that is focused on unemployment stabilization (see row Utilitarian sum of welfare). Votes would support such a policy by a wide margin (bottom row). The support for inflation targeting, instead, would run at barely 5 percent of the vote.

<sup>A27</sup>In the computation here, households take their share-holding with them, not the market value of net worth. Rather, the long-run share price will differ for different policies, for reasons that we discuss in the text.

## P.2 Long-run policy assessment — RANK and TANK

Table A18 focuses on the long run in the RANK and TANK economies. The table computes welfare gains asking a household: “How much would you be willing to pay to jump to an economy with an alternative monetary policy, and starting in that economy at its ergodic mean?” Compare this

**Table A18: Long-run Consumption-equivalent gain from changing policy – RANK/TANK**

		Response to unemployment, $\phi_u$								$\Pi = \bar{\Pi}$
		0.0	0.1	0.15	0.2	0.4	0.6	0.8	1.0	
RANK		-0.056	-0.013	—	0.009	0.028	<b>0.033</b>	<b>0.033</b>	0.031	-0.279
TANK	saver	-0.055	-0.012	—	0.009	0.024	<b>0.027</b>	0.026	0.023	-0.265
	spender	-0.086	-0.021	—	0.017	0.058	0.078	0.090	<b>0.097</b>	-0.391

*Notes:* Same as Table 14, but looking only at the long run, that is, abstracting from the transition.

with Table 14 in the main text, which accounts for both the long run and the transition path. All households would prefer slightly more accommodative monetary policy than with the transition phase. And welfare gains or losses are somewhat larger than accounting for the transition in Table 14, but qualitatively and in terms of substance, little changes. There is little disagreement across households, and there is no support for inflation targeting.

## Q Transitional dynamics

This appendix details the computation of average transitional dynamics. We build on Andreasen et al. (2018), the notation of which we use below. The first-order dynamics of the state equations are given by

$$x_{t+1}^f = h_x x_t^f + \eta \epsilon_{t+1}.$$

The state equation's second-order dynamics are:

$$x_{t+1}^s = h_x x_t^s + \frac{1}{2} H_{xx} (x_t^f \otimes x_t^f) + \frac{1}{2} h_{\sigma\sigma}.$$

The jump variables' policy function is:

$$y_t^s = g_x(x_t^f + x_t^s) + \frac{1}{2} G_{xx} (x_t^f \otimes x_t^f) + \frac{1}{2} g_{\sigma\sigma}.$$

We want to find the mean change from a point  $(\bar{x}^f, \bar{x}^s)$ .

**Observation 1:** If we can find the component terms for the means of  $x$  we get the mean of  $y$  "for free." That is, we can focus on  $x$ . The mean dynamics for  $E_0(x_h^f)$  are given by

$$E_0(x_h^f) = h_x^h \bar{x}^f.$$

The dynamics for  $\mathbb{E}_0(x_h^f \otimes x_h^f)$  are given by

$$\begin{aligned} \mathbb{E}_0(x_h^f \otimes x_h^f) &= \mathbb{E}_0 \left( \left( h_x^h \bar{x}^f + \sum_{j=1}^h h_x^{h-j} \eta \epsilon_j \right) \otimes \left( h_x^h \bar{x}^f + \sum_{j=1}^h h_x^{h-j} \eta \epsilon_j \right) \right) \\ &= (h_x^h \bar{x}^f) \otimes (h_x^h \bar{x}^f) + \mathbb{E}_0 \left( \sum_{j=1}^h h_x^{h-j} \eta \epsilon_j \otimes \sum_{j=1}^h h_x^{h-j} \eta \epsilon_j \right) \quad (\text{as } \text{corr}(\epsilon_h, \bar{x}^f) = 0) \\ &= (h_x^h \bar{x}^f) \otimes (h_x^h \bar{x}^f) + \sum_{j=1}^h \mathbb{E}_0(h_x^{h-j} \eta \epsilon_j \otimes h_x^{h-j} \eta \epsilon_j) \quad (\text{as } \text{corr}(\epsilon_h, \epsilon_k) = 0, h \neq k) \\ &= (h_x^h \bar{x}^f) \otimes (h_x^h \bar{x}^f) + \sum_{j=1}^h \mathbb{E}_0(h_x^{h-j} \eta \otimes h_x^{h-j} \eta) (\epsilon_j \otimes \epsilon_j) \\ &= (h_x^h \bar{x}^f) \otimes (h_x^h \bar{x}^f) + \sum_{j=1}^h \mathbb{E}_0(h_x^{h-j} \eta \otimes h_x^{h-j} \eta) \text{vec}(I_{nshocks}). \end{aligned}$$

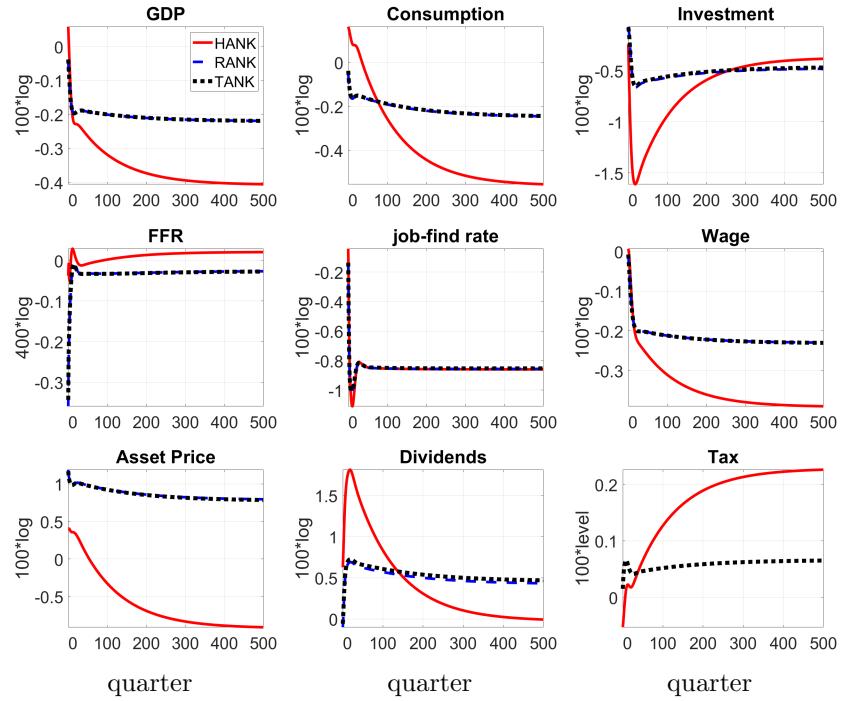
Dynamics for  $\mathbb{E}_0(x_h^s)$ :

$$\mathbb{E}_0(x_h^s) = h_x^h \bar{x}^s + 0.5 \sum_{j=1}^h h_x^{h-j} \left( H_{xx} \mathbb{E}_0(x_{j-1}^f \otimes x_{j-1}^f) + h_{\sigma\sigma} \right).$$

## R Transitions plotted over longer horizon

Where Figure 3 in the main text has reported transition dynamics for 40 quarters, Figure A21 below sketches the entire transition phase by plotting 500 quarters.

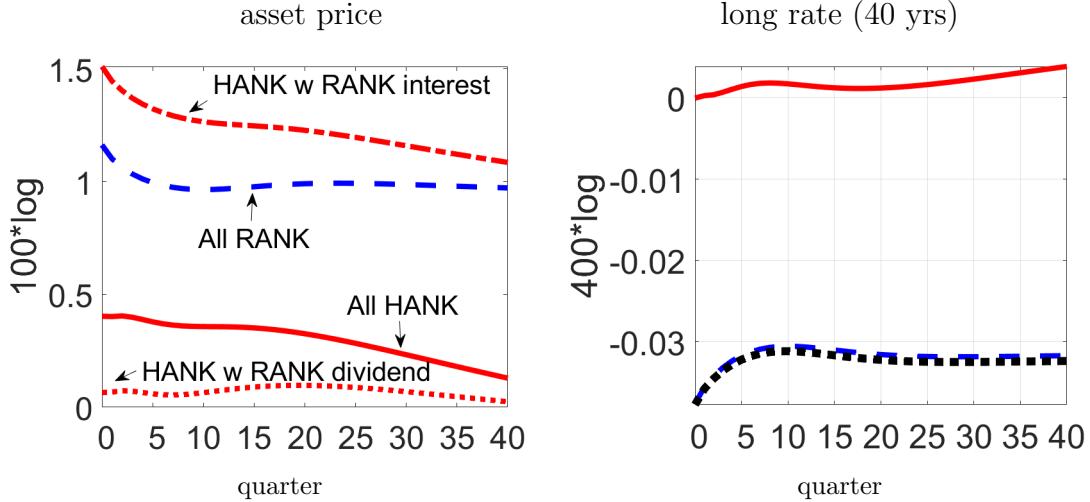
**Figure A21: Transition toward policy of  $\Pi = \bar{\Pi} - 125$  years**



*Notes:* Transition toward strict inflation targeting. Same as Figure 3 in the main text, but plotting 500 quarters of transition.

## S Decomposition of the asset price

**Figure A22: Long rate, decomposition of the asset price**



*Notes:* The right panel shows the long federal funds rate along the transition path; shown here is a 40-year long rate (inflation is constant by assumption, so the nominal rate equals the real rate of interest). The left panel decomposes asset price dynamics for the HANK model, based on present-value calculations. The red solid line gives the asset price transition in HANK. The red dotted line uses the dividend stream for the RANK economy, but the discount rate from HANK. The red dash-dotted line discounts dividends in HANK using the RANK discount rate. The blue dashed line uses both RANK dividends and the RANK discount rate.

The asset price in HANK rises less sharply than in RANK/TANK. The left panel of Figure A22 decomposes the differences in the response of asset prices in HANK and RANK into two sources: discounting and different dividend streams. The panel computes a present-value approximation for the asset price. Namely, we compute the present value of average dividends along the transition path, discounting by the approximate real rate. The red solid line is the approximation for the asset-price in HANK that we obtain; it closely matches the path shown in Figure 3. The same goes for the blue dashed line, which matches RANK asset price dynamics. The asset price in the HANK economy differs from that in the RANK economy for two reasons. First, the discounting differs. Toward this end, the right panel of the figure plots the transition of a 40-year long nominal rate. Since by the nature of the exercise inflation is constant after the policy change (strict inflation targeting), the trajectory of the nominal interest rate also describes the trajectory of the real interest rate. In line with the reduced savings by the wealthier households, documented in Table 13, inflation targeting raises the real rate of interest in the HANK economy. Indeed, in the long run, the effect of a move toward inflation targeting sends the average federal funds rate 4.7 bps (annualized) higher in HANK than in the RANK or TANK counterpart. Second, the dividend stream differs; recall Figure 3. We introduce each of these two elements one by one. The red dashed-dotted line (the one on top) in the left panel of Figure A22 shows the effect of the interest rate alone. It uses the dividend stream from the HANK model alone, but discounts as in the RANK economy. Since dividends are higher in HANK, the dividend effect alone would let the asset price rise by more than in RANK, and much more (about three-fold) than in HANK. The discount-rate effect is quantitatively important. The dotted line at the bottom looks at the asset-price change that would result using the HANK

discounting, but the RANK dividends. Since dividends are lower in HANK, the asset-price effect of the policy change would be lower, too. Combining the dividend and the discounting effect, the RANK price effect results.

## T Further results for the TANK economy

### T.1 TANK welfare when spenders do not pool

This section provides the consumption-equivalent welfare gains for spenders when spender households do not pool incomes across idiosyncratic labor-market state, education, and age. Shown are results for the baseline calibration of the TANK model with 15 percent of spender households.

**Table A19: Spenders – no Pooling – Consumption-equivalent gains in TANK**

Spender type	Response to unemployment, $\phi_u$								
	0.0	0.1	0.15	0.2	0.3	0.4	0.5	0.6	$\Pi = \bar{\Pi}$
	Throughout: Spender welfare only								
	Low education								
retired	-0.024	-0.005	—	0.004	0.009	0.011	0.013	<b>0.014</b>	-0.096
unemployed	-0.088	-0.021	—	0.016	0.040	0.055	0.065	<b>0.073</b>	-0.358
employed, skill loss	-0.076	-0.018	—	0.014	0.034	0.047	0.056	<b>0.062</b>	-0.325
employed, no loss	-0.089	-0.022	—	0.017	0.041	0.057	0.067	<b>0.075</b>	-0.375
	High education								
retired	-0.024	-0.005	—	0.004	0.009	0.011	0.013	<b>0.013</b>	-0.097
unemployed	-0.103	-0.025	—	0.019	0.047	0.064	0.077	<b>0.085</b>	-0.412
employed, skill loss	-0.077	-0.019	—	0.014	0.034	0.047	0.056	<b>0.062</b>	-0.331
employed, no loss	-0.091	-0.022	—	0.017	0.041	0.057	0.068	<b>0.075</b>	-0.389

*Notes:* Same as Table 14, but calculating welfare based on the assumption that spenders do not pool their incomes.

### T.2 Effects of varying the mass of spenders

The baseline TANK model categorizes 15 percent of the population as spenders. This is based on the strict notion of liquidity-constrained households. Table 15 shows the welfare effects for savers in TANK when, instead, wealth is more concentrated, that is, when we assume that a bigger and bigger share of households are spenders. Table A20 reports the corresponding welfare gains for spender households.

**Table A20: Welfare gains for SPENDERS in TANK by share of spenders**

Share of spenders	Response to unemployment, $\phi_u$								
	0.0	0.1	0.15	0.2	0.3	0.4	0.5	0.6	$\Pi = \bar{\Pi}$
50	-0.073	-0.017	—	0.013	0.031	0.042	0.050	<b>0.055</b>	-0.273
70	-0.081	-0.019	—	0.014	0.033	0.045	0.053	<b>0.059</b>	-0.265
75	-0.084	-0.019	—	0.015	0.034	0.047	0.055	<b>0.060</b>	-0.262
80	-0.087	-0.020	—	0.015	0.036	0.049	0.057	<b>0.063</b>	-0.258

*Notes:* Same as Table 15 in the main text, but reporting the welfare gains for spenders.

## U Model with bonds and stocks

This section provides the write-up for the economy with bonds and stocks. We wish to analyze the political economy behind systematic monetary stabilization policy. At the same time, we wish to keep the setup tractable. The household portfolio has two components: the share of the mutual fund and short-term deposits (or loans, which are negative deposits). The latter are issued by competitive banks, which in turn are held by the mutual fund. Households decide how much net worth to accumulate. They do so knowing that the composition of the portfolio into stock-market wealth and nominal savings is determined by their net worth. In particular, let  $nw'$  be the real net worth that the household decides to accumulate for next period. Then, a share  $\phi(nw'; e, s)$  of that wealth will flow into investing in stocks (so  $p_a a' = \phi(nw'; e, s) \cdot nw'$ ); the remaining share will be invested in deposits ( $deposit(nw') = (1 - \phi(nw'; e, s))nw'$ ). The portfolio compositions by net worth depend on education ( $e_L$ , and  $e_H$ ) and retirement status ( $s = s_0, s \neq s_0$ ). The former is meant to capture the effect of permanent income on the portfolio structure. The latter captures age effects on the portfolio structure (in particular, younger households having mortgages, whereas older households tend not to). Mutual funds have to provide those deposits at interest rate  $R(X)$ .

We first walk through the calibration of functions  $\phi(nw'; e, s)$ . Then, we walk through those problems that change relative to Section 2.

### U.1 Definition of share wealth and bond wealth

We build the analysis on the SCF (in 2004). In particular, we define stock-market wealth as containing all non-nominal assets other than the household's vehicles (VEHIC) and SCF category other non-financial assets (OTHNFN, which includes, among other items, furniture). We exclude these two components, since they include consumer durables. We do include housing in our measure of stock-market wealth, though. That is, we define stock-market wealth as

$$swth = EQUITY + BUS + HOUSES + ORESRE + NNRESRE.$$

*EQUITY* is the total value of financial assets invested in stock. *BUS* is the value of businesses in which the household has an active interest. *HOUSES* is the value of the primary residence. *ORESRE* is the value of other residential real estate. *NNRESRE* is the total value of net equity in non-residential real estate held by household.

Turning to savings, we have to make a decision. Namely, the nominal assets that households hold include nominal claims on the government. In keeping with the structure of the model (no government debt, balanced budget), in the modeling we assign all nominal claims by households to private-sector counterparties (the mutual funds). From the nominal claims, we subtract the nominal liabilities. We treat all nominal claims and liabilities as if they were short-term in nature. As in the baseline calibration, we do not include education loans when computing liabilities. With this, the net nominal savings (bond wealth) position of the households is

$$bwth = (FIN - EQUITY) - (MRTHEL + RESDBT + CCBAL + ODEBT + OTHLOC).$$

Nominal assets are financial assets net of equity ( $FIN - EQUITY$ ). Nominal liabilities are mortgages on the primary residence (*MRTHEL*), other residential debt (*RESDBT*), credit card debt (*CCBAL*), other debt (*ODEBT*, for example, loans against pensions or life insurance, margin loans) and other lines of credit not secured by real estate (*OTHLOC*).

A household's net worth then is defined as  $nwth = swth + bwth$ .

### U.1.1 Portfolio composition by net worth

As to determining functions  $\phi(nw'; e, s)$ , we proceed as follows. We split the sample into two education states (high and low) and two age states (working age – age 25-65 ( $s \in S_+$ ), and retired – age 66 and older ( $s = s_0$ ), each by head of household as before).

For each of the four resulting groups, we wish to have a relationship that gives the portfolio composition by net worth. We focus on households with positive net worth. For working age households, we drop households that receive social security income. We are interested in the “typical” evolution of portfolio shares by net worth and so want to guard against outliers. Toward this end, for each group, we split households into bins, by group-specific percentile of net worth. We use the same bins as in the main text (Table 1), but split the lowest net worth bin in two, so as to have finer information. That is, we look at the seven percentile bins 0-10, 10-20, 20-40, 40-60, 60-80, 80-95, 95-100. For each of the these, we compute the interquartile means of  $swth/nwth$  and of  $nwth/annlinc$ . Here  $annlinc$  scales net worth by economy-wide average labor income of households ages 25-65.

Table A21 reports the resulting values. The lowest net worth households in each education-age

**Table A21: Data. Portfolio split by net worth, education, and age**

	Group-specific percentile of net worth						
	0-10	10-20	20-40	40-60	60-80	80-95	Top 5
<b>ages 25-65, low education</b>							
<i>swth/nwth</i> (in percent)	0	20	208	173	133	103	97
<i>nwth/annlinc</i> (in percent)	1	5	31	94	232	644	1880
<b>ages 25-65, high education</b>							
<i>swth/nwth</i> (in percent)	78	234	189	133	108	99	94
<i>nwth/annlinc</i> (in percent)	10	51	144	350	792	2048	10351
<b>ages 66+, low education</b>							
<i>swth/nwth</i> (in percent)	0	79	98	91	80	78	71
<i>nwth/annlinc</i> (in percent)	2	28	103	243	466	1165	2932
<b>ages 66+, high education</b>							
<i>swth/nwth</i> (in percent)	46	104	92	76	79	81	82
<i>nwth/annlinc</i> (in percent)	23	168	342	747	1428	2988	12268

*Notes:* Based on SCF 2004. Households with heads ages 25 to 65 and households with heads ages 66 to 99, low education or high education. For each group, households are placed into the net worth percentile of the group. For each percentile shown: interquartile mean share of stock-market wealth in total net worth (in percent), first row; and interquartile average net worth scaled by economy-wide labor income of households ages 25-65 (in percent), second row.

bin tend to hold none, or only a smaller share, of their net worth in what we measure as stock-market wealth. That is, their savings are largely nominal and the real value of these savings is directly subject to fluctuations in inflation. For all age-education groups, the share of wealth in net worth is hump-shaped. Beyond the lowest net worth groups, as net worth rises, the young households tend to become indebted. For the 20th to 40th percentile, for example, the ratio of stock-market wealth (including housing in the data) to net worth is 2.08. That is, the ratio of bond wealth to net worth is -1.08. Using an average net worth to labor-income ratio of 0.31 for that group, the average

debt in that percentile is worth  $1.08 \cdot 0.31 = 0.33$ , or about a third of the economy's average annual labor income. For higher-educated households the peak in nominal debt occurs earlier. Generally, as households grow wealthier, they grow out of debt. For each age-education group, the richest 5 percent of households in terms of net worth hold, on net, nominal savings. For high-education retirement-age households, for example, stock-market wealth accounts for 82 percent of net worth, so that nominal assets account for the remaining 18 percent.

## U.2 Representing portfolio shares as a function of net worth

From Table A21, for each of the bins of the net worth distribution in the four education-age groups, we have observations on share of stock-market wealth in net worth and of net worth itself, the latter relative to economy-wide average labor income. Since households' savings choice is continuous, we need to have the portfolio composition for all feasible (that is, non-negative) values of net worth. Therefore, we fit a function to the observations in each bin that has the following properties:

- it is continuously differentiable.
- the share of stock-market wealth at zero net worth is zero.
- for low levels of net worth, the share of stock-market wealth can rise rapidly in net worth.
- there is a finite asymptote for the share of stock-market wealth in net worth as net worth rises.

Differentiability is important for the algorithm. The functional form at low net worth is important to fit the data summarized in Table A21. The asymptote is important if we want to be able to extrapolate. In particular, we choose a functional form that folds two differentiable functions  $f_1$  and  $f_2$  as follows

$$\begin{aligned} \text{stockshare} &= f(\log(1 + \text{networth}/\text{annlinc}); \vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4; \vartheta_5, \vartheta_6) \\ &= f_2(\log(1 + \text{networth}/\text{annlinc}); \vartheta_5, \vartheta_6) f_1(\log(1 + \text{networth}/\text{annlinc}); \vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4). \end{aligned}$$

All parameters  $\vartheta_1, \dots, \vartheta_6$  are positive. Here  $f_1$  is chosen to resemble the functional form of the log-logistic probability density function, other than that we add a constant  $\vartheta_1$  that will serve as the asymptote for the stock share, and scale net worth by scaling parameter  $\vartheta_4$ :

$$f_1(x; \vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4) = \vartheta_1 + \frac{\vartheta_3/\vartheta_2 \cdot (\vartheta_4 \cdot x/\vartheta_2)^{\vartheta_3-1}}{(1 + \vartheta_4 \cdot x/\vartheta_2)^{2 \cdot \vartheta_3}}.$$

That part of  $f$  can deliver the skewness in the data and the asymptote.  $f_2$  is responsible for the other two properties, with

$$f_2(x; \vartheta_5, \vartheta_6) = \frac{1}{1 + (x/\vartheta_5)^{-\vartheta_6}}$$

being the cumulative distribution function of the log-logistic distribution.  $\vartheta_5$  determines where the function bends, and  $\vartheta_6$  determines the steepness of  $f_2$  at that point. For  $\vartheta_6 \rightarrow \infty$ ,  $f_2$  would resemble an indicator function with the step at  $\vartheta_5$ . This part of the functional form allows the stock share to rise rapidly in net worth at the low end of the net worth distribution.

**Table A22: Data. Portfolio split by net worth, education, and age**

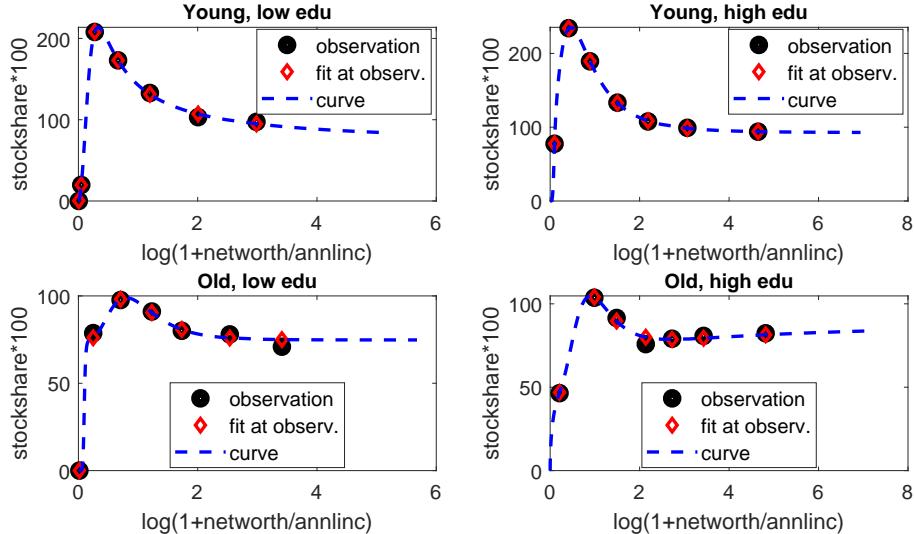
Fitted param value	$\vartheta_1$	$\vartheta_2 \cdot 1000$	$\vartheta_3$	$\vartheta_4 \cdot 1000$	$\vartheta_5$	$\vartheta_6$
low-edu young	0.6865	2240.1273	0.0238	27.9758	0.2319	2.7564
high-edu young	0.9261	19.5075	3.7261	24.4083	0.0872	5.1419
low-edu old	0.7480	0.0856	9.4790	0.0863	0.0990	8.7097
high-edu old	0.9406	0.0264	9.7277	0.0263	0.2197	0.6025

*Notes:* Fitted parameters for stock shares.

For each education/age group, we choose parameters  $\vartheta_1, \dots, \vartheta_6$  to minimize the sum of absolute deviations (summing over the bins) between the data, and the stock wealth implied by  $f$ . Table A22 reports the fitted parameters.

Figure A23 plots the resulting distributions and the fit. Figure A23 zooms in on the implied representation for the lower tail of the distribution.

**Figure A23: Stock share by net worth, education, and age**



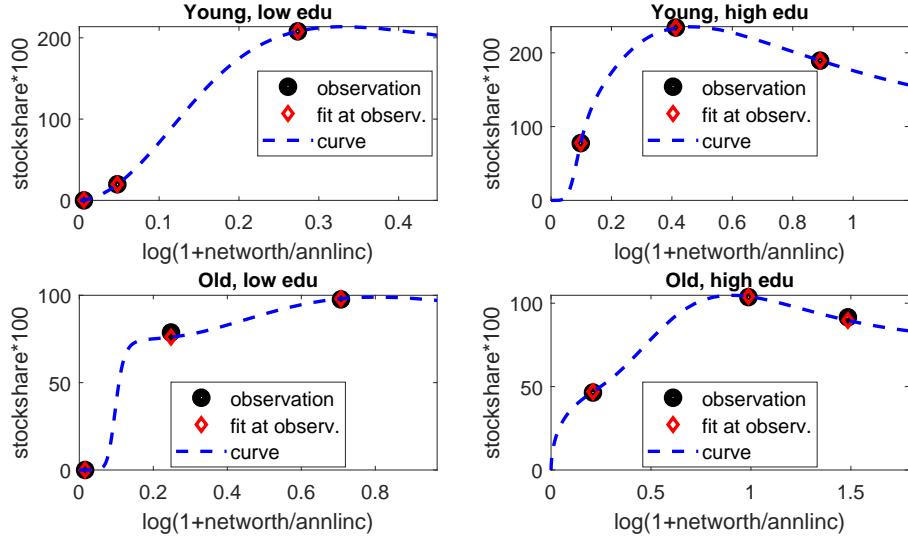
*Notes:* This plots the fitted share of net worth invested in shares for the four education/age groups. In each panel, the y-axis gives the percent of net worth invested in shares. In each panel, the x-axis plots  $\log(1 + \text{networth}/\text{annlinc})$ , where *networth* is the household's net worth and *annlinc* is the average annual labor income of working-age households (where the average is taken over all education groups).

Next, we turn to changes in the modeling implied by introducing the portfolio dimension.

### U.3 States

To be able to track household portfolios without introducing more idiosyncratic states at the household level, we need to keep track of the past price of the stock  $p_{a,-1}$  (so that  $p'_{a,-1} = p_a$ ). Let  $nw'$  denote the net worth that the household carries into the next period. Note that  $nw'$  and  $p_a$  are sufficient to compute the household's asset allocation in stocks  $a$ , and *deposits* (positive or negative) today. At the same time, for allocating returns from the portfolio, we need the same information

**Figure A24: Stock share – focusing on low net worth percentiles**



*Notes:* Same as Figure A23, but plotting only the lower tail of net worth. For each group, the maximum value on the x-axis corresponds to net worth at the 40th percentile of the respective group's distribution of net worth.

to also be available next period. This is why the problem with household portfolios introduces one additional aggregate state.

#### U.4 Households' problems

Relative to the household problem in Section 2.3 of the main text, the household problem changes as it has to account for the borrowing and lending by households. We assume that the household chooses net worth and that household-level net worth chosen last period and the aggregate states are sufficient to uniquely pin down the asset allocation.

##### U.4.1 Employed households

Let  $W(X, n, nw, l, e, b, s)$  be the value of a household at the time of production. Here,  $nw'$  is the net worth that the household chooses to invest in the current period. The employed household's Bellman equation ( $n = 1, s \in \mathcal{S}_+$ ) is given by

$$\begin{aligned} W(X, 1, nw, l, e, b, s) &= \max_{c, nw' \geq 0, a', deposits'} \left\{ u(c) + \pi_{s_0} \mathbb{E}_\zeta \beta(e, b) W(X', 0, nw', 0, e, b, s_0) \right. \\ &+ \sum_{s' \in \mathcal{S}_+} \pi_S(s, s') \beta(e, b) \mathbb{E}_\zeta [1 - \lambda_x(e) - \lambda_n(e)(1 - f(\tilde{X}'))] \sum_{l'} \pi_L^{emp}(l, l') W(X', 1, nw', l', e, b, s') \\ &\left. + \sum_{s' \in \mathcal{S}_+} \pi_S(s, s') \beta(e, b) \mathbb{E}_\zeta [\lambda_x(e) + \lambda_n(e)(1 - f(\tilde{X}'))] W(X', 0, nw', 0, e, b, s') \right\} \end{aligned}$$

$$\begin{aligned} \text{s.t. } (1 + \tau_c)c + p_a(X)a' + deposits' &= [p_a(X) + d_a(X)]a + depositsR_{-1}(X)/\Pi(X) \\ &+ w(X)es(1 - l\varrho) [1 - \tau_{RET} - \tau_{UI} - \tau(X, w(X)es(1 - l\varrho))] \\ p_a a' &= \phi(nw'; e, s) \cdot nw'. \\ nw' &= p_a a' + deposit'. \end{aligned}$$

When choosing to accumulate net worth, the household acquires a certain share of stocks and of deposits, governed by function  $\phi(nw'; e, s)$ . Deposits can be positive or negative. The interest rate applied to deposits is the same as the interest rate for loans. The nominal return on deposits is  $R(X)$  next period. Similarly, with knowledge of the past price of the asset, the net worth carried into the period can be split into stocks and deposits and returns allocated accordingly.

#### U.4.2 Unemployed households

The unemployed household's Bellman equation ( $n = 0, s \in \mathcal{S}_+$ ) is given by

$$\begin{aligned}
W(X, 0, nw, 0, e, b, s) &= \max_{c, nw' \geq 0, a', \text{deposits}'} \left\{ u(c) + \pi_{s_0} \mathbb{E}_\zeta [\beta(e, b) W(X', 0, nw', 0, e, b, s_0)] \right. \\
&+ \sum_{s' \in \mathcal{S}_+} \pi_S(s, s') \beta(e, b) \mathbb{E}_\zeta f(\tilde{X}') [\pi_L^{uem}(1) W(X', 1, nw', 1, e, b, s') + \pi_L^{uem}(0) W(X', 1, nw', 0, e, b, s')] \\
&\left. + \sum_{s' \in \mathcal{S}_+} \pi_S(s, s') \beta(e, b) \mathbb{E}_\zeta (1 - f(\tilde{X}')) W(X', 0, nw', 0, e, b, s') \right\} \\
\text{s.t. } (1 + \tau_c)c + p_a(X)a' + \text{deposits}' &= [p_a(X) + d_a(X)]a + \text{deposits} \cdot R_{-1}(X)/\Pi(X) \\
&+ b_{UI}(es)[1 - \tau(X, b_{UI}(es))], \\
p_a a' &= \phi(nw'; e, s) \cdot nw', \\
nw' &= p_a a' + \text{deposits}'.
\end{aligned}$$

#### U.4.3 Retired households

The retired household's Bellman equation ( $s = s_0$ ) is given by

$$\begin{aligned}
W(X, 0, nw, 0, e, b, s_0) &= \max_{c, nw' \geq 0, a', \text{deposits}'} \left\{ u(c) + \pi_S(s_0, s_0) \beta(e, b) \mathbb{E}_\zeta [W(X', 0, nw', 0, e, b, s_0)] \right. \\
&+ (1 - \pi_S(s_0, s_0)) \mathbb{E}_\zeta [\gamma_1(p_a(X')a + \gamma_2)^{1-\sigma}/(1 - \sigma)] \\
&+ \beta(e, b) \sum_{s' \in \mathcal{S}_+} \sum_{e'} \sum_{b'} \sum_l \pi_S(s_0, s') \pi_E(e, e') \pi_{\Delta_\beta}(b') \Pr(n = 1, l | X, e') \\
&\mathbb{E}_\zeta \left[ [1 - \lambda_x(e') - \lambda_n(e')(1 - f(\tilde{X}'))] \sum_{l'} \pi_L^{emp}(l, l') W(X', 1, nw', l', e', b', s') \right. \\
&\left. + [\lambda_x(e') + \lambda_n(e')(1 - f(\tilde{X}'))] W(X', 0, nw', 0, e', b', s') \right] \\
&+ \beta(e, b) \sum_{s' \in \mathcal{S}_+} \sum_{e'} \sum_{b'} \pi_S(s_0, s') \pi_E(e, e') \pi_{\Delta_\beta}(b') \Pr(n = 0 | X, e') \\
&\mathbb{E}_\zeta \left[ f(\tilde{X}') [\pi_L^{uem}(1) W(X', 1, nw', 1, e', b', s') + \pi_L^{uem}(0) W(X', 1, nw', 0, e', b', s')] \right. \\
&\left. + [1 - f(\tilde{X}')] W(X', 0, nw', 0, e', b', s') \right] \left. \right\} \\
\text{s.t. } (1 + \tau_c)c + p_a(X)a' + \text{deposits}' &= [p_a(X) + d_a(X)]a + \text{deposits} \cdot R_{-1}(X)/\Pi(X) \\
&+ b_{RET}(e)[1 - \tau(X, b_{RET}(e))], \\
p_a a' &= \phi(nw'; e, s) \cdot nw', \\
nw' &= p_a a' + \text{deposits}'.
\end{aligned}$$

## U.5 Non-financial firms

The block of the model that describes the non-financial firms does not change relative to the exposition in the main text. Non-financial firms continue to be owned by competitive mutual funds, the latter to be described next.

## U.6 Financial firms

What needs to be adapted relative to the baseline in Section 2.5 of the main text is the financial sector.

### U.6.1 Banks

Banks are owned by the mutual fund sector. In the current paper, we do not seek to provide a deeper modeling of the banking sector as a source of propagating macroeconomic shocks. Instead, we assume that there are representative, perfectly competitive banks. On the liability side, banks can issue demand deposits ( $hhdeposits' > 0$ ) to households, or attract wholesale funding from mutual funds  $mfdeposits'$ , the former at gross nominal interest rate  $R^D$ , the latter at gross nominal interest rate  $R(X)$ . On the asset side, banks can lend to mutual funds ( $mfloans$ ) at funding rate  $R$ , or they can issue one-period nominal loans to the household sector ( $hhloans' > 0$ ). Lending to households has a one-period rate of return  $R^L$ . We assume that there is a fixed per-unit cost of issuing loans of  $\phi_L > 0$  (say, for monitoring the loan) and that there is a fixed per-unit cost of issuing demand deposits  $\phi_D > 0$ . Let  $mf' := mfdeposits' - mfloans'$  mark the bank's net borrowing from the mutual fund sector. Assuming that the bank does not have net worth to start with, the bank's cash-flow constraint today is

$$hhdeposits' + mf' = hhloans' + (\phi_D hhdeposits' + \phi_L hhloans').$$

The left-hand side is deposits raised from households plus lending by the mutual fund sector. These proceeds are used for (right-hand side) making loans to mutual funds or households, and for covering the costs of issuing deposits or loans. At the beginning of the next period, cash flows in real terms are given by

$$loans' \cdot R^L / \Pi' - (deposits' \cdot R^D / \Pi' + mf' \cdot R / \Pi').$$

Since banks are owned by the mutual funds, the optimality conditions for the banks are as follows. The indifference condition for loans is

$$R^L = (1 + \phi_L)R.$$

The indifference condition for issuing deposits is

$$R^D = (1 - \phi_D)R.$$

With this, it is trivial to show that banks, in equilibrium (due to perfect competition and constant returns to scale), make zero profits. In what follows, we will focus on the case  $\phi_L \rightarrow 0$  and  $\phi_D \rightarrow 0$ , so that  $R^D(X) = R^L(X) = R(X)$ .

## U.6.2 Mutual funds

The mutual funds' problem is the same as in the baseline model, with the one exception that we now have to account for financial investment by the mutual funds  $mf(X)$ . The banks owned by the mutual fund sector make zero profits. To the extent, however, that the banking system's exposure to mutual funds (or vice versa)  $mf(X)$  is not equal to zero, the mutual fund will earn interest income. We continue to apply the cashless limit assumption. The mutual fund distributes to the households all income that is not reinvested in physical or financial capital, after paying taxes to the government. After-tax dividends are given by

$$d_a(X) = (1 - \tau_d) \left( y_f(X) - i(X) - \int_{\mathcal{M}} w(X) se(1 - \varrho l) \mathbb{1}_{n=1} d\mu + mf \cdot R_{-1}/\Pi - mf'(X) \right),$$

where  $\mathbb{1}$  marks the indicator function, meaning  $\mathbb{1}_{n=1}$  marks employment of the household.

## U.7 Central bank and fiscal authority

The descriptions of the central bank and the fiscal authority are exactly as in the baseline.

## U.8 Market clearing and equilibrium

To close the description of the model, in the following we list the market-clearing conditions. We list only those market clearing conditions that differ from the baseline model.

Let  $nw'(X, n, nw, l, e, b, s)$  be the net worth policy function. Total demand for assets is given by  $a'(X, n, nw, l, e, s)$

$$\int_{\mathcal{M}} p_a(X) a' d\mu = \int_{\mathcal{M}} \phi(nw'(X, n, nw, l, e, b, s); e, s) d\mu.$$

With the mass of shares normalized to unity, that is,  $\int_{\mathcal{M}} p_a(X) a' d\mu = 1$ , the market-clearing price for the asset follows

$$p_a(X) = \int_{\mathcal{M}} \phi(nw'(X, n, nw, l, e, b, s); e, s) d\mu.$$

Note that we do not model the frictions or considerations that lead households to opt for a certain split in portfolios. Rather, we assume a fixed relationship between net worth and portfolio shares. This means that the price will have to move to make demand and supply mutually consistent. It also means that *ex-ante* returns on stocks and bonds need not coincide (households cannot arbitrage).

Last, the bond market clears if funding provided by mutual funds makes up for the shortfall of funding provided by deposits, that is, if

$$mf(X) = hhloans(X) - hhdeposits_t(X) := - \int_0^1 deposits' d\mu.$$

## U.9 Calibration

The calibration is the same as in the baseline model, with one difference: we need the functions  $\phi(nw'; e, s)$ . They follow from Appendix U.2.