1 Kolobarji polinomov

Računanje s kompleksnimi števili

$$z = x + iy = re^{i\varphi} = r(\cos\varphi + i\sin\varphi)$$

$$r = |z| = \sqrt{x^2 + y^2} \qquad \varphi = \arg z = \arctan\frac{y}{x}$$

De Moivreova formula

$$z^n = r^n(\cos\varphi n + i\sin\varphi n)$$

Osnovni izrek algebre

Vsak nekonstanten polinom $a_n x^n + \cdots + a_0$ ima natanko n kompleksnih ničel (štetih z večkratnostjo).

Trigonometrične identitete

$$\begin{aligned}
\sin(x \pm y) &= \sin(x)\cos(y) \pm \cos(x)\sin(y)\\
\cos(x \pm y) &= \cos(x)\cos(y) \mp \sin(x)\sin(y)\\
\tan(x \pm y) &= \frac{\tan(x) \pm \tan(y)}{1 \mp \tan(x)\tan(y)}\\
\cot(x \pm y) &= \frac{\cot(x)\cot(y) \mp 1}{\tan(x) \pm \tan(y)}\\
\sin^2(x) + \cos^2(x) &= 1\\
1 + \cot^2(x) &= \frac{1}{\sin^2(x)}\\
1 + \tan^2(x) &= \frac{1}{\cos^2(x)}\\
\sin\frac{x}{2} &= \pm\sqrt{\frac{1-\cos x}{2}}\\
\cos\frac{x}{2} &= \pm\sqrt{\frac{1+\cos x}{2}}
\end{aligned}$$

Mali Fermantov izrek

$$\forall a \in \mathbb{Z}, p \in \mathbb{P}: \; a^p \equiv_p a \tag{1}$$

Polinomi

Polinom je **razcepen**, če ga lahko zapišemo kot produkt dveh nekonstantnih polinomov. Nekonstanten polinom, ki ni razcepen je **nerazcepen**.

Polinom $a_n x^n + \cdots + a_0$ je **primitiven**, če velja $\gcd(a_0, \ldots, a_n) = 0$

Gaussova lema

$$p(x) \in \mathbb{Z}[x] \text{ razcepen nad } \mathbb{Z} \iff p(x) \text{ razcepen nad } \mathbb{Q} \tag{2}$$

Hornerjev algoritem

$$a_n x^n + ... + a_0 = 0$$

- možne cele ničle: $\pm$delitelji a_0
- možne racionalne ničle: $\pm\frac{\text{delitelji } a_0}{\text{delitelji } a_n} = k$

	a_n	a_{n-1}	...	a_0
k		ka_n	...	
	a_n	$ka_n - a_{n-1}$	...	ostanek

Eisensteinov kriterij

Naj bo $a(x) = a_n x^n + \cdots + a_0 \in \mathbb{Z}[x]$ polinom. Če $\exists p \in \mathbb{P} : p | a_0, \ldots, a_{n-1} \land p \nmid a_n \land p^2 \nmid a_0$, potem je $a(x)$ nerazcepen nad $\mathbb{Q}$.

Rodovne funkcije

$$\sum_{n=0}^{\infty} q^n = \frac{1}{1-q} \quad \sum_{n=0}^{b} q^n = \frac{1-q^{b+1}}{1-q}$$

$$\sum_{n=a}^{\infty} q^n = \frac{q^a}{1-q} \quad \sum_{n=a}^{b} q^n = \frac{q^a - q^{b+1}}{1-q}$$

$$a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + ... + ab^{n-2} + b^{n-1})$$

$$\tfrac{a_0 + ... + a_{k-1}x^{k-1}}{1-x^k} = a_0 + ... + a_{k-1}x^{k-1} + a_0^k + ... + a_{k-1}x^{2k-1} + ...$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

$$\frac{1}{(1-x)^n} = \sum_{k=0}^{n} \binom{n+k-1}{k} x^k$$

$$B_\lambda(x) = \sum_n \binom{\lambda}{n} x^n = (1+x)^\lambda; \qquad \binom{\lambda}{n} = \frac{\lambda^{\underline{n}}}{n!}$$