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Accuracy of the line intersect method of post-logging sampling under orientation bias

Gavin Bell [a], Andrew Kerr [a], Donald McNickle [a], Richard Woollons [b,*]

[a] *Department of Management, University of Canterbury, Private Bag, Christchurch, New Zealand*
[b] *School of Forestry, University of Canterbury, Private Bag, Christchurch, New Zealand*

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Abstract

The Line Intersect Method of Post-Logging Residue Sampling is used extensively as a means of estimating waste volume on a clearfelled stand. Whilst the performance of the method under non-random spatial log arrangement conditions has been examined in the literature, similar examinations of method performance when the logs have a non-random orientation distribution have not been rigorously attempted. Thus, a corrected analytic approach to the calculation of expected sampling error under conditions of log orientation angle bias is presented, along with a correction to the Line Intersect Method literature. A simulation model is also used to calculate the standard deviation of the sampling error under orientation bias conditions. The two models are then used to examine the Line Intersect Method in the context of a typical New Zealand stand of *Pinus radiata*, comparing and evaluating the performance of the method under four patterns of line arrangements and varying line lengths. Our principal conclusion is that the fan and L arrangement provide considerable protection against the effects of orientation bias, although the presence of an orientation bias can considerably increase the size of the likely measurement error.

Keywords: Line intersect method; Sampling

1. Introduction

The Line Intersect Method (LIM) of Post-Logging Residue Sampling is an ingenious method of estimating waste volume per hectare in a clearfelled stand, without requiring any measurement of log lengths. The methodology was developed by Van Wagner (1968), based upon the earlier work by Warren and Olsen (1964). In the United States, such assessments have been mainly utilised for fire protection; whereas in New Zealand, the emphasis has been on yield control realisations.

* Corresponding author.

The wood volume per unit area is shown by Van Wagner (1968) to be given by

$$V=\left(\pi^2\sum_i d_i^2\right)/8L \qquad (1)$$

where V is the wood volume per unit area, d_i is the diameter of log i and L is the length of the sample line.

Van Wagner shows that this formula provides an unbiased estimate of the wood volume if the following conditions hold:

1. the pieces are cylindrical;
2. all pieces are horizontal;
3. the pieces are randomly oriented.

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When these assumptions are violated, however, the LIM does not necessarily produce unbiased estimates, and the likely error of these estimates becomes difficult to estimate analytically.

Pickford and Hazard (1978) consider the effects of log taper and population density on LIM by means of simulation. Brown and Roussopoulos (1974) give corrections for bias in populations that are not horizontal. However, there appears to have been little rigorous analysis done on populations where the third assumption is violated. Van Wagner (1968) concludes that the fan (or triangle) arrangement would produce the smallest maximum error of the arrangements considered. However, as we will show, the analysis upon which his conclusion is based is incorrect. One study by Hazard and Pickford (1986) considers the non-random spatial patterns of forest residue produced by cable- or tractor-logging. Their predominant conclusion is to utilise two- or three-line sample designs with random sample orientation.

Kaiser (1983) derives general methods for producing unbiased Line-intersect estimators under the assumption of uniform orientation of logs in the plane. He considers bias in the context of orientation to the plane and suggests sampling at right-angles to this bias. It would appear however, that multiple-line sampling designs are preferable for forestry applications since the bias is usually difficult to measure. General studies on LIM estimators include De Vries (1986) who gives sample formulas for the estimation of the variance of LIM estimators, again under the assumption of uniform log orientation. A good general survey of papers on LIM is given in Warren (1989).

In this contribution we consider the effect of non-random log orientation, and explore the effects of using different arrangements of transect lines and altering the length of sample line. Specifically, we investigate the use of a single transect line, two transect lines at right angles to one another (an 'L' arrangement), three transect lines in a 'fan' or 'triangle' whose internal angles are all 60°, and four transect lines arranged in a square. In all four arrangements, each individual transect line is of the same length. The first, second, and third arrangements were proposed and examined by Van Wagner (1968), whilst the fourth arrangement is commonly used in practice. In New Zealand, a commonly used arrangement is a square of four 100-m transect lines, an arrangement chosen on a subjective and historical basis. Finally, in Appendix A we show that Van Wagner's postulation that the maximum error for a particular orientation angle occurs when all logs in the stand are oriented at that angle is incorrect.

2. Analytic solution

From Van Wagner (1968) the percentage error in the estimate when all the logs are oriented at a constant angle, θ, to the line is

$$\varepsilon = 100(-1 + \pi \sin(\theta)/2) \quad 0 \le \theta \le \pi \tag{2}$$

We extend this to the case in which the orientation angle, u, is not constant, but drawn from a distribution with probability density function $P(u)$. The percentage error is then given by

$$\varepsilon = 100\left(-1 + \frac{\pi}{2} \int |\sin(u)| P(u)\, du\right) \tag{3}$$

If the orientation angles come from a normal distribution, the above formula cannot be evaluated analytically, and hence numerical methods must be used to solve the expression.

It should be noted that when the orientation angles are normally distributed we are in effect truncating both tails of the distribution. However, the error introduced will be minimal: for example, if the standard deviation of the orientation angles is less than $(\pi/2)/3.09$ rad, then fewer than 2 in every 1000 logs will have an angle outside the limits.

3. Simulation analysis

A stand with a particular density, distribution of log lengths and diameters, and orientation bias could be modelled exactly (and reasonably simply) with computer simulation. It is relatively simple to next place lines of different lengths and in any arrangement on each simulated stand to obtain stand wood volume estimates for these lengths and arrangements. The variability of the wood volume estimates for various line transect lengths and arrangements can then be obtained and compared.

The simulation software, LIMBS (Line Intersect Method By Simulation), was written in Turbo Pascal 5.5. This language was selected for program portability and ease of use. Program verification was undertaken following the steps recommended by Law and Kelton (1991), which included interactive debugging, coding and verifying the program in steps of increasing complexity, and, where possible, comparing the results produced with their theoretical counterparts. In addition, an outline version of the simulation was also written in FORTRAN for program verification purposes.

LIMBS simulates the use of the LIM in a harvested stand with a desired level of logging residue, log length and diameter distributions, various line lengths, four common transect line arrangements, and orientation bias. Based on simulations of a large number (typically 10000) of such stands, LIMBS provides estimates of the standard deviation and mean error associated with the volume estimates obtained from the various transect line lengths and arrangement combinations.

The random number generator used in the simulation is that described in Park and Miller (1988).

4. Results

The methods described above were then used to investigate the performance of the LIM. In particular, we wanted to determine whether or not it would be possible to reduce the total length of line used in a thinned stand of *Pinus radiata* from the current 400 m (arranged in a square of four 100-m lines), without a significant decrease in accuracy.

The data used in the investigation were from a typical New Zealand stand of *Pinus radiata*. A Weibull distribution was fitted to both the log lengths and log diameters from this data set, and used as input distributions for the simulation. All errors and their standard deviations are quoted in percentage terms, in order to remove the effects of different stocking levels.

4.1. A note on the display of the results

The actual error in any volume estimate produced by the method will depend upon both the line length and the stand density. By symmetry, under the assumptions of the LIM, the standard deviation of the sampling error obtained using a line length of x m used in a stand of log density y logs ha^{-1} should be the same as those obtained using a line length of x' m in a stand of density y', when $xy = x'y'$. Hence, the standard deviations of the percentage sampling error for each arrangement are quoted, not in terms of line length, but in terms of line length multiplied by the density (number) of logs in the stand.

The standard deviation of the percentage sampling error for a particular arrangement and length × density value presented is simply the standard deviation of the actual errors from the (approximately) 10000 simulation replications of that arrangement and length × density value. It is not a standard error.

Finally, note that the length term in the length × density figures is the length of a single transect line in a particular arrangement. For example, to obtain the standard deviation of the percentage sampling error for a square of four 100-m transect lines arranged in a stand of density 500 logs ha^{-1}, the appropriate length × density value would be 50000. Similarly, for a fan of three 100-m transect lines used on the same stand, the appropriate length × density value would also be 50000. Whilst this method of presentation is biased (the square described above has a total line length of 400 m, whereas the fan has a total line length of 300 m), it is necessary to achieve compact graphical display of the standard deviation of the percentage sampling errors.

4.2. Expected percentage error in the estimate

The analytic solution was evaluated numerically to obtain the expected percentage sampling error in the wood volume estimate under varying log orientation bias conditions. Under no orientation bias conditions (that is, the log orientations are randomly distributed), the analytic solution produced an expected percentage sampling error of 0%.

In the case of orientation bias, it appears reasonable to assume that the orientation angles will be normally distributed about a mean orientation angle. Since log orientation depends on a number of physical effects, it appears reasonable to assume that the orientation of the logs would come from a normal

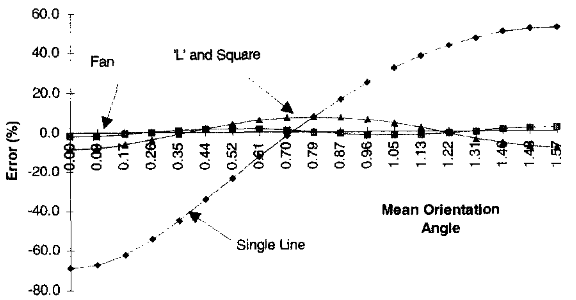

Fig. 1. Expected percentage sampling error in volume estimate.

distribution, with a certain mean (which would probably be close to a logger's intended angle) and standard deviation. Fig. 1 graphs the expected percentage errors obtained when then standard deviation of the orientation angle is 0.25 rad.

Note that the expected percentage sampling error and standard deviation of the percentage sampling error for two transect lines at right angles, and four transect lines arranged in a square are exactly the same. This is because of the symmetry of the percentage sampling error function (Eq. (3)).

4.3. Standard deviation of the percentage sampling error in the estimate

Under conditions of no orientation bias, it was found that the standard deviation of the error was dependent upon the total length of line used, and not on the arrangement used. This was not unexpected, and is due to the lack of orientation bias: hence there is no advantage of one arrangement over another, because any arbitrary line encounters the same distribution of orientation angles as any other arbitrary line. Fig. 2 presents the results in terms of total line length.

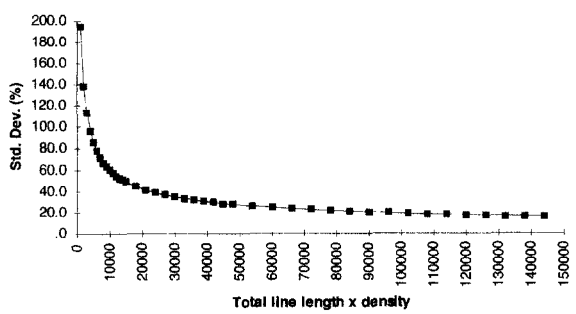

Fig. 2. Standard deviation of the percentage sampling error: no orientation bias.

Table 1
Standard deviation of the percentage sampling error for varying log orientation angle standard deviations

Transect line arrangement	Log orientation angle standard deviation (rad)			
	0.1	0.25	0.5	1.0
Single	58.2	54.0	44.5	35.2
L	26.6	25.7	24.8	24.8
Fan	20.0	19.7	19.6	20.0
Square	19.6	18.1	17.1	17.1

For the orientation bias case, simulations were run with either a 'Random Mean Orientation Bias' or a 'Fixed Mean Orientation Bias' option. The Random option is where the transect lines are placed at a random angle to the mean orientation of the logs. Thus, although the orientation angles of the logs are still normally distributed about the mean orientation angle, the mean orientation angle of the logs to a transect line is essentially random. This replicates the case in practice where little information is available regarding the orientation bias of the logs, and hence no adjustment is made for orientation bias in the placement of any transect line. The Fixed option is where the mean orientation bias to a transect line is known and is the same in each stand of the simulation.

The results from the Random option simulations for a range of orientation angle standard deviations indicated that the standard deviation of the percent-

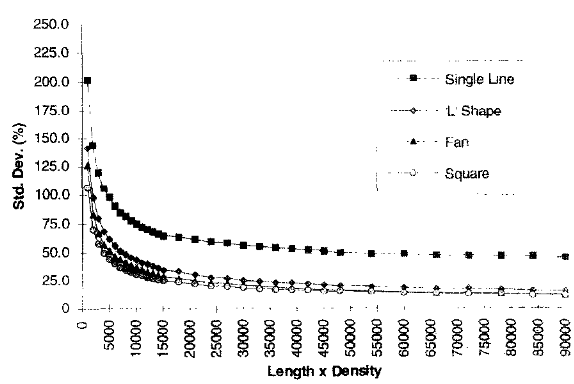

Fig. 3. Standard deviation of the percentage sampling error: random orientation bias.

age sampling error for the L, square, and especially fan, arrangements did not alter greatly with changes in the orientation angle standard deviation. Table 1 illustrates this for a length × density figure of 30000.

Fig. 3 presents the standard deviation of the percentage sampling error results for a single set of simulations where the standard deviation of the orientation angle has been set at 0.25 rad. Examining these standard deviation results in Fig. 3, notice that, for the square arrangement and the L arrangement, the standard deviation of the percentage sampling error was (practically) the same when the same total line length is used in both arrangements. This was expected, since the square arrangement is essentially two L arrangements, where both L arrangements have the same orientation. The standard deviation of the percentage sampling error for the fan arrangement was slightly smaller than that obtained by the square and the L when the same total line length was used for all three arrangements. However, this difference was never large. The standard deviations of the percentage sampling error obtained from all three arrangements were all substantially smaller than the standard deviation obtained when using a single transect line of the same total line length.

For the Fixed option there are a large number of mean orientation angle and standard deviation of the angle combinations that may be simulated. It was found that the standard deviation of the percentage sampling error for the all line lengths in the single line arrangement depended greatly on the mean log orientation angle. However, the standard deviation of the percentage sampling error for all line lengths for the other three arrangements, and the fan of three transect lines in particular, did not appear to be influenced greatly by changes in the mean orientation angle. Further, given a particular orientation angle standard deviation, the standard deviation of the percentage sampling error results obtained from these three arrangements, irrespective of the mean log orientation angle, were similar to those obtained when the mean orientation angle was random. Table 2 presents the standard deviation of the percentage sampling error results for a set of four mean orientation angles under constant angle standard deviation (0.25 rad) and length × density (30000).

Table 2
Standard deviation of the percentage sampling error for a range of mean log orientation angles

Transect line arrangement	Mean log orientation angle (rad)			
	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
Single	29.5	36.0	40.1	41.2
L	25.1	25.7	25.2	24.4
Fan	20.3	20.4	20.3	20.1
Square	17.6	17.9	17.5	16.7

5. Conclusions and recommendations

It is very apparent that the standard deviation of a single line assessment is considerable, a conclusion also reached by Van Wagner (1968). While the method is unbiased, results from a single assessment can be so inaccurate as to be practically worthless.

The question of a suitable line length is not entirely obvious. It is manifestly dependent on stand stocking. In New Zealand, *Pinus radiata* plantations are usually tended at around 300 stems ha^{-1}. Here, the standard deviation of the percentage sampling error, obtained by a square of four 100-m transect lines, has been shown to be approximately 18%. For a fan (or triangle) of three 100-m lines the corresponding percentage sampling error is approximately 20%. Hence, if such a difference is considered trivial, then effectively the standard 400 m configuration can be reduced to 300 m by adopting the fan or triangle structure, without causing a significant decrease in estimate precision.

It is recommended that for any chosen total length of line, a fan (or triangle) of three transect lines be utilised. For a given line length × stand density product, both the expected and standard deviation percentage sampling error are almost always lower than for any other configuration tested. This finding confirms Van Wagner (1968) and Hazard and Pickford (1986), although the former was based on an incorrect assumption as detailed in Appendix A.

Appendix A

Van Wagner (1968) states that, for a set of logs with a mean orientation angle ϕ, the maximum possible error would occur if all pieces had an orientation angle of ϕ. He then uses this result to provide an analytic basis for his conclusion that the fan arrangement is the best of those examined under conditions of orientation bias. Clearly, the maximum possible negative error of -100% (that is, a volume estimate of 0) would occur if all pieces were parallel to the transect line. Similarly, the maximum possible positive error of 57.08% would occur if all logs were perpendicular to the transect line. However, it can be shown by contradiction that Van Wagner's extension of this logic to all orientation angles is incorrect.

Consider the case in which we have three possible orientation angles: $\phi-\gamma$, ϕ, and $\phi+\gamma$. For simplicity, we assume that each of the three angles are greater than 0 and less than π rad. We set the mean orientation angle, again denoted μ, equal to the second possible angle, ϕ. Thus, if we set λ, $0 \leq \lambda \leq 1$, to be the proportion of logs with orientation angle ϕ, the proportion of logs with an orientation angle of $\phi-\lambda$ is therefore $(1-\lambda)/2$, and the proportion of logs with an orientation angle of $\phi+\gamma$ is also $(1-\lambda)/2$.

The expected percentage error is given by

$$\varepsilon = 100(-1+\pi((1-\lambda)\sin(\phi-\gamma)/2+\lambda\sin(\phi) +(1-\lambda)\sin(\phi+\gamma)/2)/2)$$
$$= 100(-1+\pi\sin(\phi)\,\omega(\lambda,\gamma)/2)$$

where $\omega(\lambda,\gamma)=\lambda+(1-\lambda)\cos(\gamma)$. It can be easily verified that

$$\omega(\lambda,\gamma)=\lambda+(1-\lambda)\cos(\gamma)\leq 1$$

Therefore

$$100(-1+\pi\sin(\phi)\,\omega(\lambda,\gamma)/2) \leq 100(-1+\pi\sin(\phi)/2)$$

and we can see that if

$$0<\phi\leq\sin^{-1}(4/\pi(1+\omega(\lambda,\gamma))) \quad \text{or}\ \pi-\sin^{-1}(4/\pi(1+\omega(\lambda,\gamma)))\leq\phi<\pi$$

then

$$|100(-1+\pi\sin(\phi)\,\omega(\lambda,\gamma)/2)| \geq |100(-1+\pi\sin(\phi)/2)| \qquad \text{(A1)}$$

In fact, Eq. A1 will hold regardless of the values of γ and λ if $0<\phi\leq\sin^{-1}(2/\pi)$ or $\pi-\sin^{-1}(2/\pi)\leq\phi\pi$.

In general, the angles for which the expected error is greater in absolute terms than Van Wagner's maximum error term depends on the distribution of the angles. However, this does not weaken the result.

References

Brown, J.K. and Roussopoulos, P.J., 1974. Eliminating biases in the planar intersect method for estimating volumes of small fuels. For. Sci., 20: 350–356.

Hazard, J.W. and Pickford, S.G., 1986. Simulation studies on line intersect sampling of forest residue, part II. For. Sci., 32: 447–470.

Kaiser, L., 1983. Unbiased estimators in line intersect sampling. Biometrics, 39: 965–976.

Law, A.M. and Kelton, W.D., 1991. Simulation Modelling and Analysis, 2nd. edn. McGraw-Hill, New York, 759 pp.

Park, S.K. and Miller, K.W., 1988. Random Number Generators: Good Ones are Hard to Find. Commun. ACM, 31: 1192–1201.

Pickford, S.G. and Hazard, J.W., 1978. Simulation studies on line intersect sampling of forest residue, For. Sci., 24: 469–483.

Van Wagner, C.E., 1968. The line intersect method in forest fuel sampling. For. Sci., 14: 20–26.

De Vries, P.G., 1986. Sampling Theory for Forest Inventory: A Teach-Yourself Course. Springer, Berlin, 399 pp.

Warren, W.G., 1989. Line intersect sampling: a historical perspective. In: State-of-the-art Methodology of Forest Inventory: A Symposium Proceedings. USDA Forest Service, Pacific Northwest Research Station, Portland, OR, pp. 33–38.

Warren, W.G. and Olsen, P.F., 1964. A line intersect technique for assessing logging waste. For. Sci., 10: 267–276.