Exam task 1: Perzyna-type viscoplasticity – linear model with mixed hardening versus closest point projection for non-linear viscoplasticity

Introduction and Theory

A widely used extension to the plasticity models shown in the lectures [1], and especially observed and applied in metal plasticity, is the concept of so-called *viscoplasticity*. In contrast to the *rate-independent* plasticity concepts discussed in the lectures, in the regime of *rate-dependent* plasticity models, a plastically non-admissible state

$$\boldsymbol{\sigma} \notin \bar{\mathbb{E}} = \{\boldsymbol{\sigma} \,|\, \Phi(\boldsymbol{\sigma}) \leq 0\} \tag{1.1}$$

such that the stress $\boldsymbol{\sigma}$ exceeds the elastic region $\bar{\mathbb{E}}$, i.e. $\sigma_\mathrm{e} > \sigma_\mathrm{y}$, is defined to be *temporarily admissible*. In other words, the until now instantaneously applied (radial) return to the yield surface takes place in a viscous manner, i.e. under constant Dirichlet boundary conditions (within the plastic regime), the stress state *relaxes* to the yield surface.

To motivate this material behavior we will at first use a one-dimensional rheological model of the format depicted in fig. 1.1.

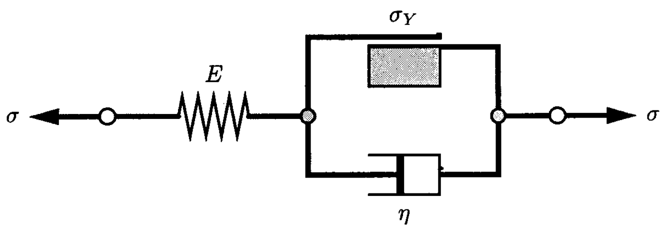


Figure 1.1: Rheological model of a one-dimensional viscoplastic material, from [2].

In the regime of small deformations, one applies an additive split of the strain tensor

$$\varepsilon = \varepsilon^\mathrm{e} + \varepsilon^\mathrm{vp} \tag{1.2}$$

and introduces the viscoplastic stress in the dashpot element

$$\sigma^\mathrm{vp} = [|\sigma| - \sigma_\mathrm{y}]\,\mathrm{sign}(\sigma). \tag{1.3}$$

Assuming the stress in the dashpot element to obey a viscous constitutive relation $\sigma^\mathrm{vp} = \eta\,\dot{\varepsilon}^\mathrm{vp}$, we obtain the evolution equation for the viscoplastic strain as

$$\dot{\varepsilon}^\mathrm{vp} = \frac{1}{\eta}\,\Phi(\sigma)\,\mathrm{sign}(\sigma) \quad \text{if} \quad \Phi(\sigma) \geq 0 \qquad \text{and} \qquad \dot{\varepsilon}^\mathrm{vp} = 0 \quad \text{else.} \tag{1.4}$$

Introducing the ramp function, respectively McCauley brackets $\langle x \rangle = \frac{x+|x|}{2}$, one can rewrite the above evolution equation as the viscoplastic constitutive equation of Perzyna type

$$\dot{\varepsilon}^{\mathrm{vp}} = \frac{\langle \Phi(\sigma) \rangle}{\eta} \frac{\partial \Phi(\sigma)}{\partial \sigma}. \tag{1.5}$$

The above expression can more generally be rewritten with the relaxation time $t^* \equiv \frac{\eta}{E}$ as

$$\dot{\varepsilon}^{\mathrm{vp}} = \frac{E^{-1}}{t^*} \left[\sigma - \mathbf{P}\, \sigma \right] \tag{1.6}$$

where $\mathbf{P} : \mathbb{R} \mapsto \partial \bar{\mathbb{E}}$ is an operator mapping any stress state $\sigma \notin \bar{\mathbb{E}}$ back to the closest point on the yield surface $\partial \bar{\mathbb{E}}$ defined as

$$\mathbf{P}\, \sigma = \sigma_{\mathrm{y}}\, \mathrm{sign}(\sigma) \tag{1.7}$$

and thus is referred to as the so called closest point projection, cf. fig. 1.2.

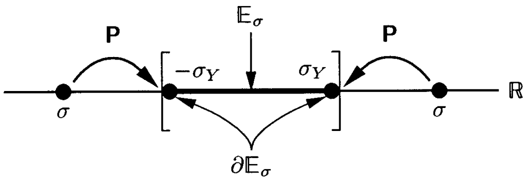


Figure 1.2: Illustration of the closest point mapping operator $\mathbf{P}$ for one-dimensional viscoplasticity, from [2].

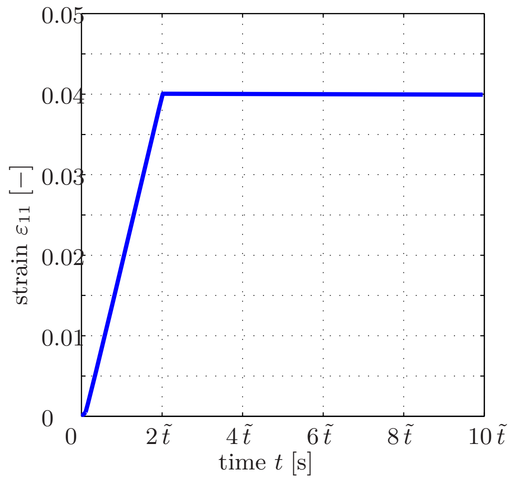
To extend the above model to the three-dimensional case, one can use the algorithm for 3D-plasticity with mixed hardening presented in the lectures, cf. [1], and replace the plastic multiplier λ by the Perzyna type expression

$$\lambda = \frac{\langle \Phi(\boldsymbol{\sigma}, \boldsymbol{\alpha}, \kappa) \rangle}{\eta} \tag{1.8}$$

which renders the algorithmic update for the multiplier as

$$\Delta\lambda := \langle \Phi_{n+1} \rangle \frac{\Delta t}{\eta} = \frac{\langle \Phi^{\mathrm{trial}}_{n+1} \rangle / 2\mu}{\frac{t^*}{\Delta t} + \left[1 + \frac{H}{3\,\mu} \right]} \qquad \text{with} \qquad t^* = \frac{\eta}{2\,\mu}, \tag{1.9}$$

where t^* is the relaxation time, H denotes the hardening modulus and η the viscosity parameter.



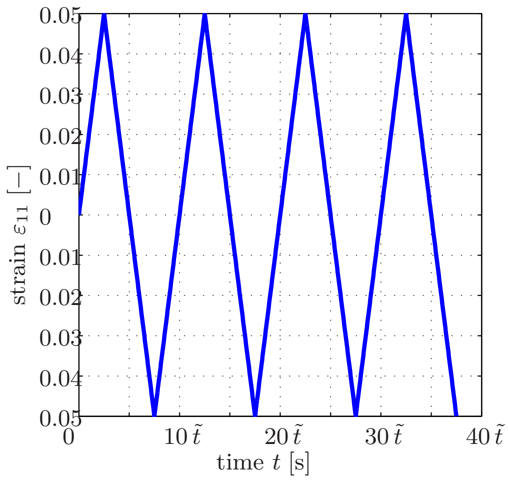


Figure 1.3: Load cases to be investigated: Ramp function with constant section (left) and cyclic load (right).

Task

1. Review the algorithmic correlations for three-dimensional viscoplasticity given in [2], pp. 57 ff. (1D) and pp. 150 ff. and give a short introduction to the topic within your final presentation.
2. Implement the Perzyna type viscoplastic material model in Matlab/Octave for

   a) mixed linear harding of von Mises/J^2 type using the return mapping algorithm (cf. [2], box 3.7, p. 152)

   b) nonlinear viscoplasticity incorporating the closest point projection iteration (cf. [2], box 3.8, p. 153) with a nonlinear yield function incorporating a Norton type power law of the structure $\lambda = [\langle \Phi \rangle / \eta]^{n_c}$

   based on the algorithmic boxes presented in [2], pp. 152 and 153. Please use the framework of the constitutive driver for uniaxial stress states used in the tutorials of this semester. Apply load cases depicted in fig. 3.6, make use of the set of material parameters shown in table 3.4 and vary the deformation speed by setting $\tilde{t} = \{1, 10, 100\}$.
3. After successful implementation, investigate and interprete the results from the above model and plot the results in an illustrative manner, such as the total ε_{11} or viscoplastic strains $\varepsilon_{11}^{\mathrm{vp}}$ over time t, strains ε_{11} or stresses σ_{11}.

Table 1.1: Material parameters to be used.

Parameter	E	ν	σ_{y}	η	n_c	H
Value	200000	0.3	580	$1 \ldots 10$	$1 \ldots 3$	4500

References

[1] A. Menzel. Theory of Materials. Lecture Notes, TU Dortmund, Institute of Mechanics.

[2] J.C. Simo and T.J.R. Hughes. Computational Inelasticity. Volume 7 of Interdisciplinary Applied Mathematics, Springer, 1998.