Soru 1.

$f(x) = x + \frac{2}{x}$ fonksiyonu veriliyor. $X_0 = 1$, $X_1 = 2$ ve $X_2 = 2.5$ alarak $f(x)$ fonksiyonunun $P_2(x)$ Lagrange iç değerlendirme polinomunu bulunuz. $E_3(x)$ hata terimi için hata sınırını da ayrıca bulunuz.

$f(1) = 1 + \frac{2}{1} = 3$; $f(2) = 2 + \frac{2}{2} = 3$; $f(2.5) = 2.5 + \frac{2}{2.5} = 3.3$

$(x_0, y_0) = (1, 3)$; $(x_1, y_1) = (2, 3)$; $(x_2, y_2) = (2.5, 3.3)$

	x_0	x_1	x_2
x	1	2	2.5
y	3	3	3.3

$P_2(x) = y_0 P_{2,0}(x) + y_1 P_{2,1}(x) + y_2 P_{2,2}(x) \Rightarrow 3 \cdot \frac{(x-x_1)}{(x_0-x_1)} \cdot \frac{(x-x_2)}{(x_0-x_2)} + 3 \cdot \frac{(x-x_0)}{(x_1-x_0)} \cdot \frac{(x-x_2)}{(x_1-x_2)} + 3.3 \frac{(x-x_0)}{(x_2-x_0)} \cdot \frac{(x-x_1)}{(x_2-x_1)}$

$= 3 \cdot \frac{(x-2)(x-2.5)}{1.5} + 3 \cdot \frac{(x-1)(x-2.5)}{-0.5} + 3.3 \frac{(x-1)(x-2)}{0.75} \Rightarrow$

$= 2(x-2)(x-2.5) - 6(x-1)(x-2.5) + \frac{3.3}{0.75}(x-1)(x-2)$; $\longleftarrow$

$\boxed{f_n(x) = P(x) = \sum_{i=0}^{n} L_i(x) \cdot f(x_i) \Rightarrow L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x-x_j}{x_i - x_j}}$ $\rightarrow$ bu formülleri biriktirip, yukarıdaki çözümde elde ettik

$f(x) = x + \frac{2}{x}$

$f'(x) = 1 + \left(-\frac{2}{x^2}\right) = 1 - \frac{2}{x^2}$

$f''(x) = 0 - \left(-\frac{4}{x^3}\right) = \frac{4}{x^3}$

$f'''(x) = -\frac{12}{x^4}$

$f''''(x) = \frac{36}{x^5}$

$X_0 = 1$; $X_1 = 2$; $X_2 = 2.5$

$E(x)$ sayısın. 1 ve 2.5 sayılarının arasında olur.

$\boxed{X_{min} = 1 \ ; \ X_{max} = 2.5 \Rightarrow 1 \leq X \leq 2.5}$
bunu elde ettik

$\boxed{|f(x) - P(x)| = \frac{\max |(x-x_1) \cdots (x-x_n)|}{n!} \max |f^n(\xi_n)|}$ Lagrange Polynomial Error

$\Rightarrow \frac{|f''''(E(x))|}{4!} |(x-x_0)(x-x_1)(x-x_2)(x-x_3)| \Rightarrow \frac{\frac{36}{x^5}}{4!} * (x-1)^4$

$\Rightarrow \frac{\frac{36}{(X_{min})^5}}{4!} * (X_{max} - X_{min})^4 \Rightarrow \frac{\frac{36}{1^5}}{24} \cdot (1.5)^4 \Rightarrow \frac{36}{24} \cdot 5.0625 = \boxed{7.59}$ Error field

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Soru 2

$f(x) = 3x \cdot 2^{-x}$ fonksiyonu veriliyor. $X_0 = 0$, $X_1 = 1$, $X_2 = 2$, $X_3 = 3$, $X_4 = 4$

$P_1(x)$, $P_2(x)$, $P_3(x)$, $P_4(x)$, Newton ile değerlendirme?

$f(x_0) = 3 \cdot 0 \cdot 2^0 = 0$

$f(x_1) = 3 \cdot 1 \cdot 2^{-1} = \dfrac{3}{2} = 1.5$

$f(x_2) = 3 \cdot 2 \cdot 2^{-2} = \dfrac{6}{4} = 1.5$

$f(x_3) = 3 \cdot 3 \cdot 2^{-3} = \dfrac{9}{8} = 1.125$

$f(x_4) = 3 \cdot 4 \cdot 2^{-4} = \dfrac{12}{16} = 0.75 \left(\dfrac{3}{4}\right)$

x	x_0	x_1	x_2	x_3	x_4
$f(x)$	0	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{9}{8}$	$\frac{3}{4}$

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$\boxed{\frac{0}{0}1}$

0

$\quad \Rightarrow \boxed{\dfrac{3}{2}}$

$\dfrac{3}{2}$

$\quad \Rightarrow 0$

$\dfrac{3}{2}$

$\quad \Rightarrow \dfrac{9}{8} - \dfrac{3}{2} = \dfrac{9-12}{8} = -\dfrac{3}{8}$

$\dfrac{9}{8}$

$\quad \Rightarrow \dfrac{3}{4} - \dfrac{9}{8} = \dfrac{6-9}{8} = -\dfrac{3}{8}$

$\dfrac{3}{4}$

$\boxed{\frac{0}{0}2}$

$\Rightarrow -\dfrac{3}{2} : 2 = \boxed{\dfrac{-3}{4}}$

$\Rightarrow -\dfrac{3}{8} : 2 = -\dfrac{3}{16}$

$\Rightarrow 0$

$\boxed{\frac{0}{0}3}$

$\Rightarrow \dfrac{-\frac{3}{16} - \left(-\frac{3}{4}\right)}{3} = \dfrac{-\frac{3}{16} + \frac{12}{16}}{3} = \dfrac{\frac{9}{16}}{3} = \boxed{\dfrac{3}{16}}$

$\Rightarrow \dfrac{0 - \left(-\frac{3}{16}\right)}{3} = \dfrac{\frac{3}{16}}{3} = \dfrac{1}{16}$

$\boxed{\frac{0}{0}4}$

$\dfrac{3}{16}$

$\quad \Rightarrow \dfrac{\frac{1}{16} - \frac{3}{16}}{4} = \dfrac{-\frac{2}{16}}{4} = \boxed{\dfrac{-1}{32}}$

$\dfrac{1}{16}$

$a_0 = f(x_0) = 0$

$a_1 = f(x_0 x_1) = \dfrac{3}{2}$

$a_2 = f(x_0, x_1, x_2) = -\dfrac{3}{4}$

$a_3 = f(x_0 x_1 x_2 x_3) = \dfrac{3}{16}$

$a_4 = f(x_0 x_1 x_2 x_3 x_4) = -\dfrac{1}{32}$

$P_1(x) = a_0 + a_1(x - x_0) = 0 + \dfrac{3}{2}(1-0) = \dfrac{3}{2}$

$P_2(x) = P_1(x) + a_2(x - x_0)(x - x_1) = \dfrac{3}{2} + \left(\dfrac{-3}{4}\right) \cdot 2 = \dfrac{3}{2} - \dfrac{6}{4} = 0$

$P_3(x) = P_2(x) + a_3(x - x_0)(x - x_1)(x - x_2) = 0 + \dfrac{3}{16} \cdot 3 \cdot 2 \cdot 1 = \dfrac{9}{8}$

$P_4(x) = P_3(x) + a_4(x - x_0)(x - x_1)(x - x_2)(x - x_3) = \dfrac{9}{8} + \left(-\dfrac{1}{32} \cdot 4 \cdot 3 \cdot 2 \cdot 1\right) = \dfrac{9}{8} - \dfrac{3}{4} = \dfrac{9-6}{8} = \dfrac{3}{8}$

$\boxed{P_1(x) = \dfrac{3}{2}}$; $\boxed{P_2(x) = 0}$; $\boxed{P_3(x) = \dfrac{9}{8}}$; $\boxed{P_4(x) = \dfrac{3}{8}}$;

Soru 3

$f(x) = \ln(x+2)$

i	0	1	2
x	-1	0	1

$[-1, 1]$

$f(-1) = \ln(-1+2) = \ln 1 = 0$

$f(0) = \ln(0+2) = \ln 2 = 0.6931$

$f(1) = \ln(1+2) = \ln 3 = 1.0986$

i	0	1	2
x	-1	0	1
y	0	0.6931	1.0986

$L_0(x) = \dfrac{(x-0)(x-1)}{(-1-0)(-1-1)} = \dfrac{(x-0)(x-1)}{(-1)\cdot(-2)} = \dfrac{(x-0)(x-1)}{2} = \dfrac{x(x-1)}{2}$

$L_1(x) = \dfrac{(x-(-1))(x-1)}{(0-(-1))(0-1)} = \dfrac{(x+1)(x-1)}{1\cdot(-1)} = \dfrac{x^2-1}{-1}$

$L_2(x) = \dfrac{(x-(-1))(x-0)}{(1-(-1))(1-0)} = \dfrac{(x+1)x}{2\cdot 1} = \dfrac{x^2+x}{2}$

$P(x) = \dfrac{x(x-1)}{2}\cdot 0 + \dfrac{x^2-1}{-1}\cdot 0.6931 + \dfrac{x^2+x}{2}\cdot 1.0986$

$\boxed{\pm(x^2-1)(-0.6931) + (x^2+x)(0.5493)}$

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Soru 4.

x	-2	-1	1	2	3
y	0.67	0.98	0.34	0.405	0.43

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$y = x / (A + Bx)$ fonksiyonuna uydurunuz.

① $y = ax + b$

② $\Rightarrow (-2, 0.67) \Rightarrow$ ~~2a+b~~ $-2a + b = 0.67$

$(-1, 0.98) \Rightarrow -a + b = 0.98$

$(1, 0.34) \Rightarrow a + b = 0.34$

$(2, 0.405) \Rightarrow 2a + b = 0.405$

$(3, 0.43) = 3a + b = 0.43$

} Tutarsız denklem

③

$$\begin{bmatrix} -2 & 1 \\ -1 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0.67 \\ 0.98 \\ 0.34 \\ 0.405 \\ 0.43 \end{bmatrix} \quad \Longleftarrow \quad \underline{Ax = b}$$

$A^T \cdot Ax = A^T \cdot b \Rightarrow$

$$\begin{bmatrix} -2 & -1 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 \\ -1 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -2 & -1 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0.67 \\ 0.98 \\ 0.34 \\ 0.405 \\ 0.43 \end{bmatrix} =$$

$4 + 1 + 1 + 4 + 9 = 19$

$-2 - 1 + 1 + 2 + 3 = 3$

$-2 - 1 + 1 + 2 + 3 = 3$

$1 + 1 + 1 + 1 + 1 = 5$

$$= \begin{bmatrix} 19 & 3 \\ 3 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 19 & 3 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \Rightarrow \begin{bmatrix} -1.34 - 0.98 + 0.34 + 0.81 + 1.29 \\ 0.67 + 0.98 + 0.34 + 0.405 + 0.43 \end{bmatrix} = \begin{bmatrix} 0.12 \\ 2.825 \end{bmatrix}$$

$19a + 3b = 0.12 \quad (5)$

$3a + 5b = 2.825 \quad (-3)$

$95a + 15b = 0.6$

$-9a - 15b = -8.475$

$86a = -7.875$

$a = \dfrac{-7.875}{86} \Rightarrow -0.09$

$\boxed{a = -0.09}$

$95a + 15b = 0.6$

$95 \cdot (0.09) + 15b = 0.6$

$-8.55 + 15b = 0.6$

$15b = 0.6 + 8.55$

$b = 9.15 / 15$

$\boxed{b = 0.61}$

④

$y = ax + b \Rightarrow$

$\boxed{y = -0.09x + 0.61}$

$y = x / (A + Bx)$

$$\boxed{y = \frac{x}{-0.09 + 0.61x}}$$