

# Reproducing a Study of Stochastic Volatility + Market Inefficiency

>>> Fun with Multi Linear Regression

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# Original Study:

FEDERAL RESERVE BANK OF SAN FRANCISCO

WORKING PAPER SERIES

## Examining the Sources of Excess Return Predictability: Stochastic Volatility or Market Inefficiency?

Kevin J. Lansing

Federal Reserve Bank of San Francisco

Stephen F. LeRoy

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December 2018

Working Paper 2018-14

<https://www.frbsf.org/economic-research/publications/working-papers/2018/14/>

# Stochastic Volatility

sto·chas·tic

/stə'kastik/

*adjective*

TECHNICAL

1. randomly determined; having a random probability distribution or pattern that may be analyzed statistically but may not be predicted precisely.

[ the volatility of asset prices is not  
constant ]

The efficient-market hypothesis is a theory that asset prices fully reflect all available information.

A direct implication is that it is impossible to "beat the market" consistently on a risk-adjusted basis since market prices should only react to new information.

# Findings:

>>> Not Investment  
Advice

“

We show that the sentiment-momentum variable is positively correlated with fluctuations in Google searches for the term “stock market,” suggesting that the sentiment-momentum variable helps to predict excess returns because it captures shifts in investor attention, particularly during stock market declines.

”

## Data Sources:

Variance Risk Premium: <https://sites.google.com/site/haozhouspersonalhomepage>

EOM Nominal S&P, Nominal Dividends / Nominal Risk Free Rate: <http://www.hec.unil.ch/agoyal/>

University of Michigan Consumer Sentiment: <http://www.sca.isr.umich.edu/>

Google Trends: <https://trends.google.com/trends/?geo=US>

Quandl API: <https://www.quandl.com/>

Yahoo Finance: <https://finance.yahoo.com/>

Study Covers 1990-03 through 2017-12

# Key Terms & Independent Variables:

index: S&P 500

**price-dividend ratio** index closing value / cumulative nominal dividends >> " **pd** "

**fed funds rate delta** 12 month change in federal funds rate >> " **ff12\_D** "

**variance risk premium** 3 month moving average in difference between implied volatility from option on the index and realized volatility of the index >> " **vrp3** "

**fed funds rate delta** 12 month change in federal funds rate >> " **ff12\_D** "

**consumer sentiment delta** 12 month change in UM Consumer Sentiment >> " **sent12\_D** "

**excess stock return delta** 1 month change in excess return (over the risk free rate) - a measure of return momentum >> " **ersf\_D** "

**interaction** consumer sentiment delta X excess stock return delta >> " **sent\_x\_ersf\_D** "

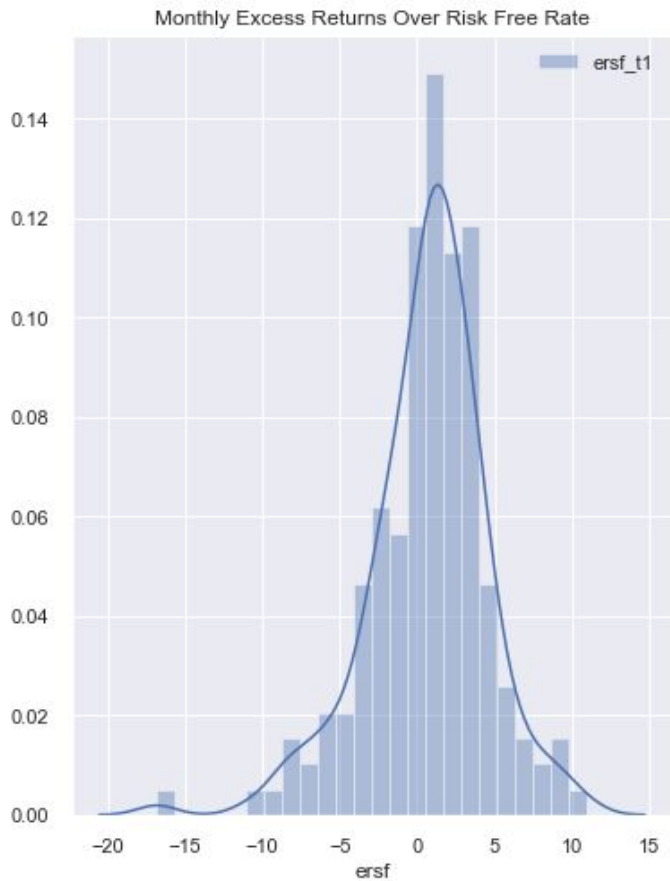
**google search term momentum** 1 month change in volume of google searches for the term "Stock Market" >> " **Google\_D** "

## Target Variable:

**excess return in month t+1** 12 month change in federal funds rate >> " **ersf\_t1** "

# Monthly Excess Returns over Risk Free Rate

## Distribution of Monthly Returns



Mean: 0.65

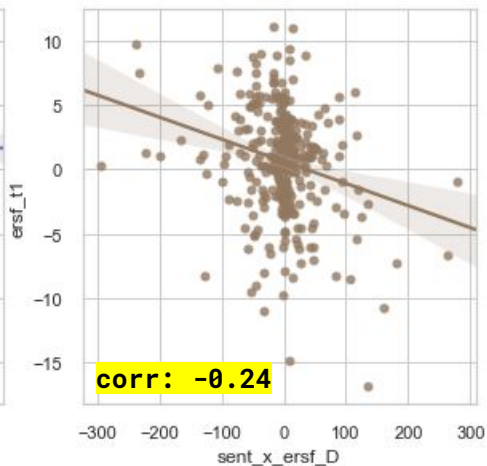
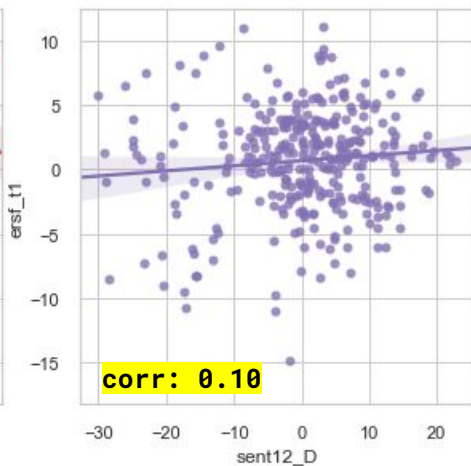
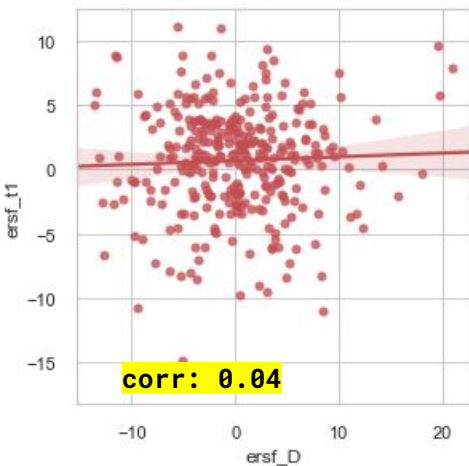
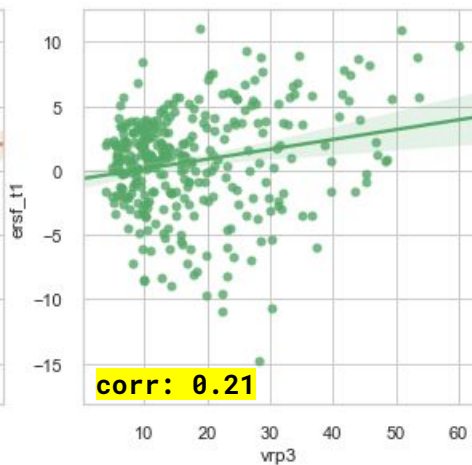
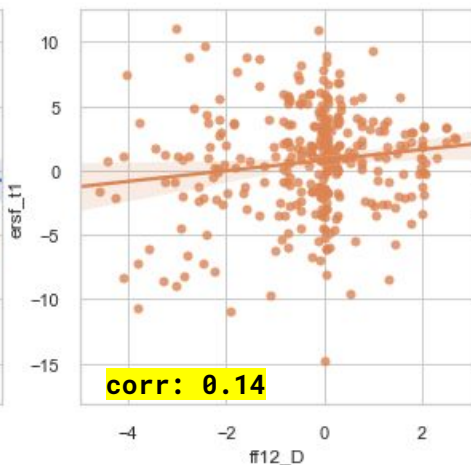
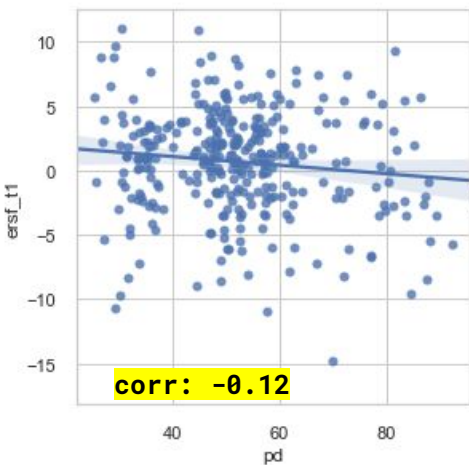
Standard Deviation: 4.1

Minimum: -16.8%

Maximum: 10.9%

## Correlation Plots

target variable vs. independent variables



## Baseline Model

price-dividend / federal funds rate / variance risk premium / sentiment change (12mo) X return momentum

### OLS (Statsmodel)

<b>Dep. Variable:</b>	ersf_t1	<b>R-squared:</b>	0.174
<b>Model:</b>	OLS	<b>Adj. R-squared:</b>	0.164
<b>Method:</b>	Least Squares	<b>F-statistic:</b>	17.29
<b>Date:</b>	Sun, 21 Apr 2019	<b>Prob (F-statistic):</b>	6.93E-13
<b>Time:</b>	23:03:43	<b>Log-Likelihood:</b>	-913.55
<b>No. Observations:</b>	334	<b>AIC:</b>	1837
<b>Df Residuals:</b>	329	<b>BIC:</b>	1856
<b>Df Model:</b>	4		
<b>Covariance Type:</b>	nonrobust		

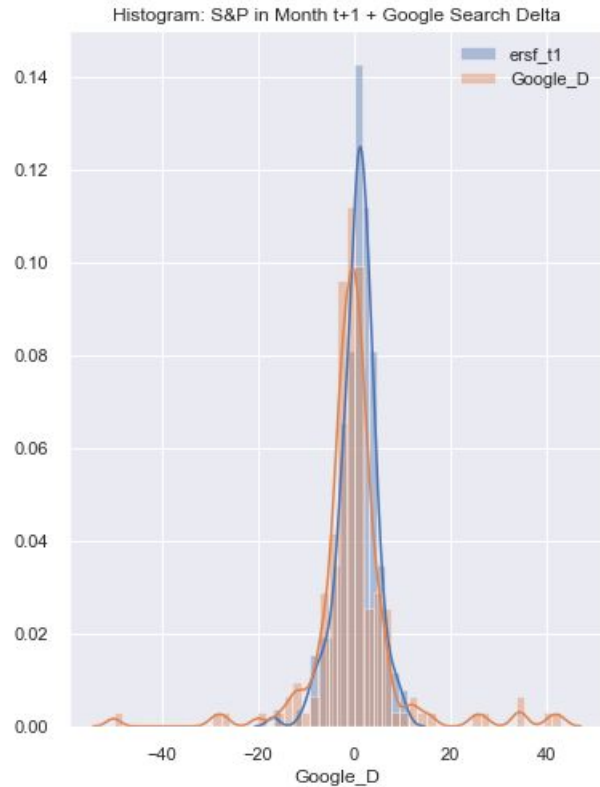
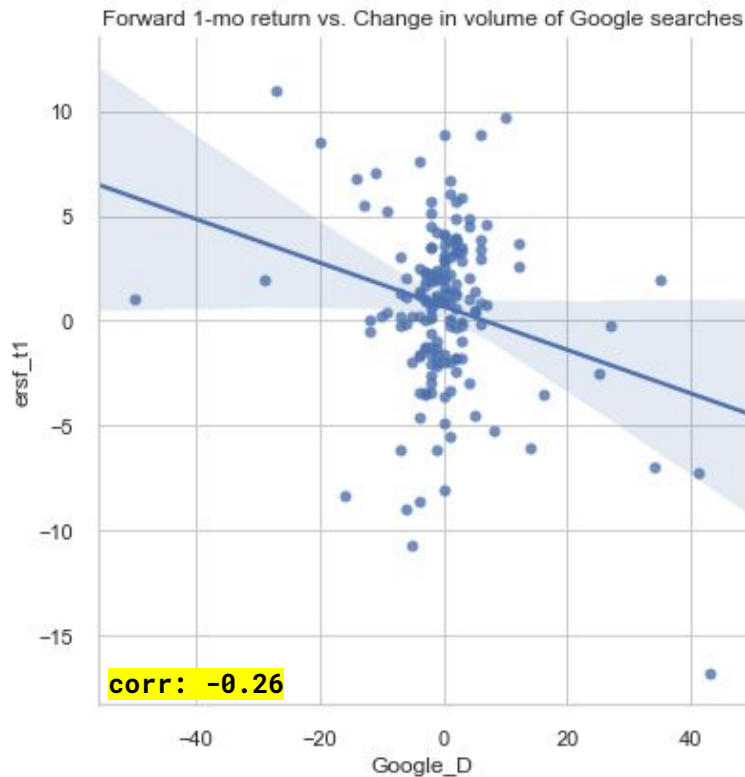
	coef	std err	t	P> t	[0.025	0.975]
<b>Intercept</b>	1.914	0.831	2.305	0.022	0.281	3.548
<b>pd</b>	-0.057	0.015	-3.777	0.000	-0.087	-0.027
<b>ff12_D</b>	0.870	0.162	5.352	0.000	0.550	1.189
<b>vrp3</b>	0.110	0.020	5.391	0.000	0.070	0.150
<b>sent_x_ersf_D</b>	-0.014	0.004	-3.844	0.000	-0.021	-0.007

<b>Omnibus:</b>	29.202	<b>Durbin-Watson:</b>	2.026
<b>Prob(Omnibus):</b>	0	<b>Jarque-Bera (JB):</b>	38.189
<b>Skew:</b>	-0.648	<b>Prob(JB):</b>	5.10E-09
<b>Kurtosis:</b>	4.031	<b>Cond. No.</b>	238

**Original Study R-Squared:**  
**17.3%**

# Adding Change in Volume of Google Searches

**google search term momentum** 1 month change in volume of google searches for the term "Stock Market" >> "Google\_D"





# Final Model

price-dividend / federal funds rate / variance risk premium / sentiment change (12mo) X return momentum / Google Delta

OLS (Statsmodel)

Dep. Variable:	ersf_t1	R-squared:	0.311	<b>Different Timeframe:</b> Google Trends not available until 2004. Covers 2004-01 through 2017-12.
Model:	OLS	Adj. R-squared:	0.29	
Method:	Least Squares	F-statistic:	14.63	
Date:	Sun, 21 Apr 2019	Prob (F-statistic):	7.89E-12	
Time:	23:03:47	Log-Likelihood:	-435.44	
No. Observations:	168	AIC:	882.9	
Df Residuals:	162	BIC:	901.6	
Df Model:	5			

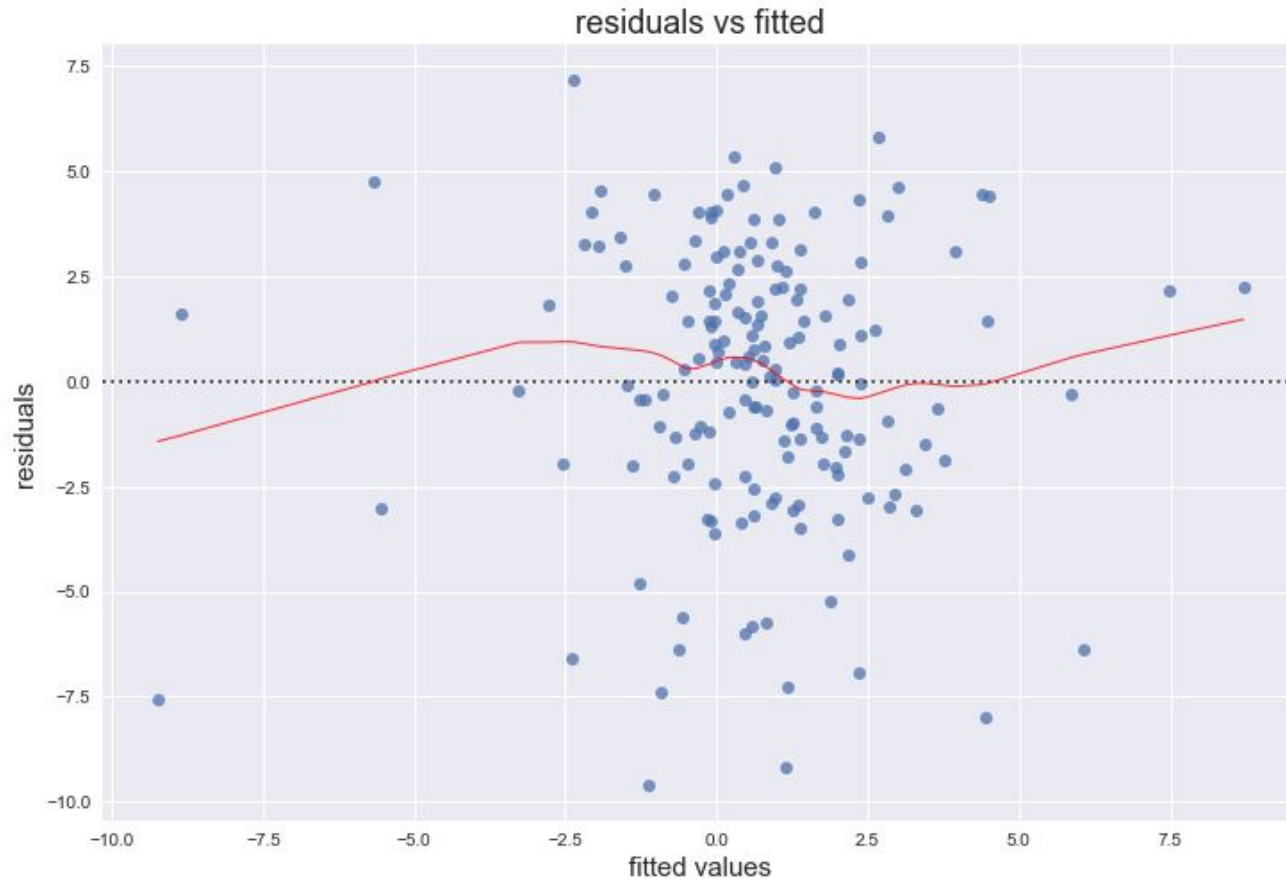
Covariance Type: nonrobust

coef	std err	t	P> t	[0.025	0.975]	
Intercept	6.3421	3.122	2.031	0.0440	0.177	12.508
pd	-0.1484	0.058	-2.557	0.0110	-0.263	-0.034
ff12_D	1.6608	0.287	5.784	0.0000	1.094	2.228
vrp3	0.1386	0.03	4.649	0.0000	0.08	0.197
sent_x_ersf_D	-0.0087	0.005	-1.909	0.0580	-0.018	0
Google_D	-0.0847	0.028	-3.066	0.0030	-0.139	-0.03

Omnibus:	9.6	Durbin-Watson:	1.89
Prob(Omnibus):	0.008	Jarque-Bera	9.844
Skew:	-0.589	Prob(JB):	0.00728
Kurtosis:	3.138	Cond. No.	724

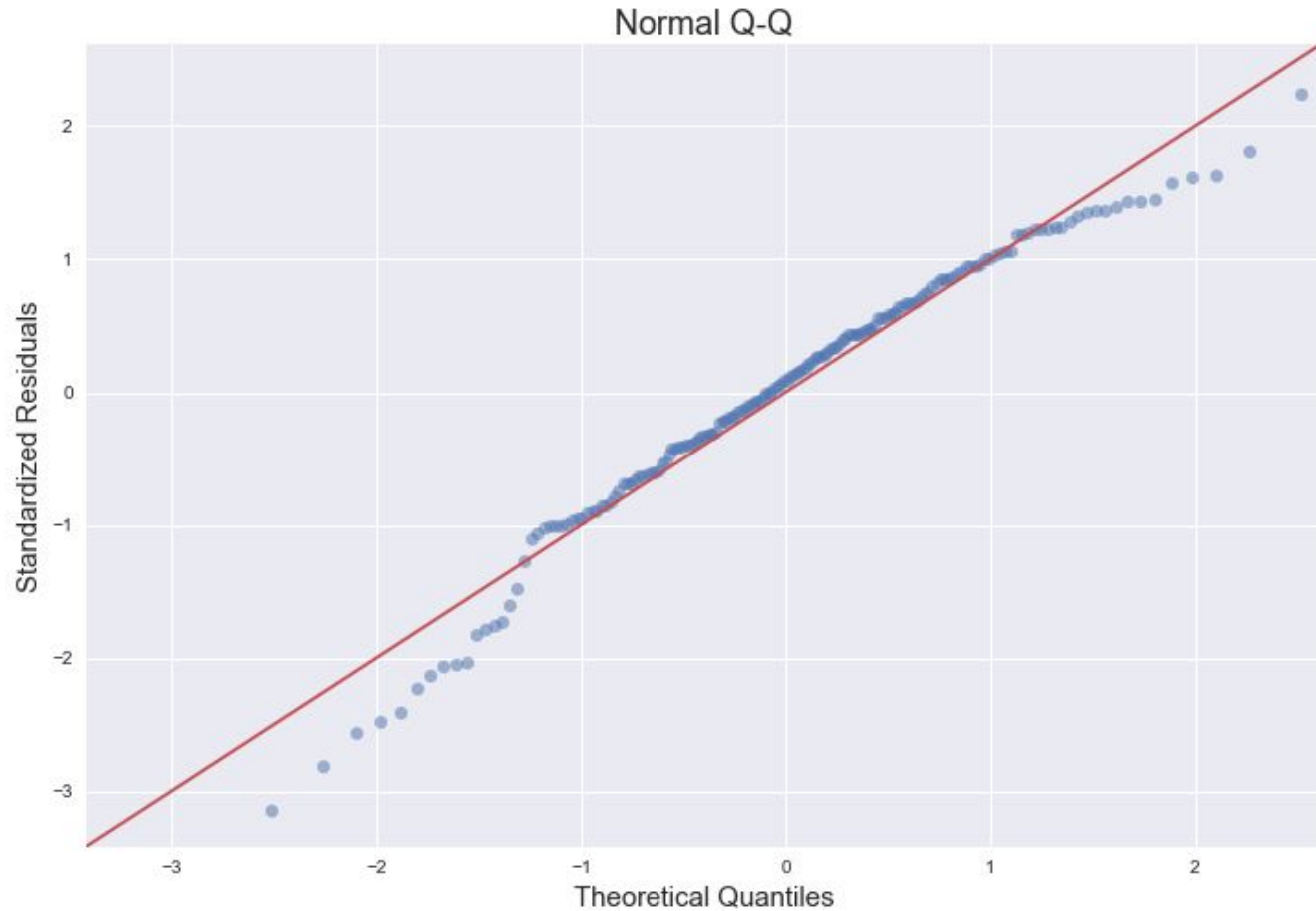
Original Study R-Squared:  
29.1%

## Final Model - Residuals vs. Fitted Values



The residual plots are fairly well distributed around the horizontal line, with no discernible pattern; a linear regression model appears to be appropriate.

## Final Model - QQ Plot



The QQ Plot shows indicates that the residuals are fairly normally distributed, with some significant outliers towards the tails.

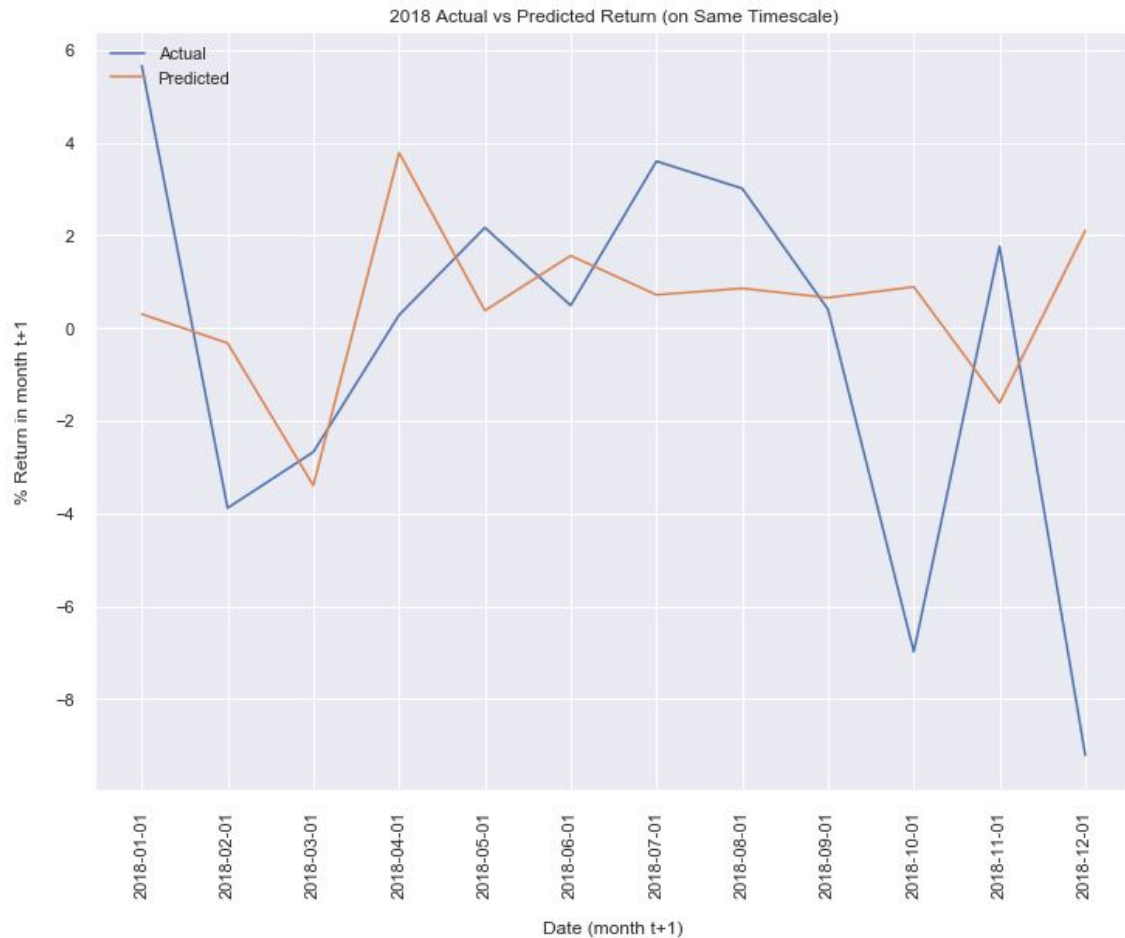
# Predicting 2018 Forward Month Stock Returns

## Running the Final Model on Untrained 2018 Data:

Applying a simple “Buy / Sell” Signal. If the Predicted Return is  $> 0$ , Buy (or Hold), if Predicted Return is  $< 0$ , Sell. Following this signal would have resulted in a -1.6% return on the year versus a -6.3% return with a pure “Buy and Hold” strategy. The amount show the results of investing \$10,000 on 12/31/2017, and following the monthly signal. While this is a useful heuristic for applying the model, the actual model Root Mean Squared Error was fairly high, at 4.76% (compared to 3.23% on the Trained Dataset).

month	pred_ret_next_mo	ersf	invested	correct	model	actual
2018-01	0.30	5.66	TRUE	TRUE	\$10,566	\$10,566
2018-02	-0.32	-3.88	FALSE	TRUE	\$10,566	\$10,155
2018-03	-3.39	-2.67	FALSE	TRUE	\$10,566	\$9,884
2018-04	3.78	0.28	TRUE	TRUE	\$10,595	\$9,912
2018-05	0.38	2.16	TRUE	TRUE	\$10,824	\$10,126
2018-06	1.56	0.49	TRUE	TRUE	\$10,877	\$10,175
2018-07	0.72	3.60	TRUE	TRUE	\$11,268	\$10,541
2018-08	0.85	3.01	TRUE	TRUE	\$11,607	\$10,858
2018-09	0.65	0.40	TRUE	TRUE	\$11,654	\$10,902
2018-10	0.89	-6.98	TRUE	FALSE	\$10,840	\$10,141
2018-11	-1.61	1.76	FALSE	FALSE	\$10,840	\$10,319
2018-12	2.10	-9.21	TRUE	FALSE	\$9,842	\$9,368
					-1.58%	-6.32%

# Predicting 2018 Forward Month Stock Returns



The Model failed to predict some of the extreme market moves in 2018.

The negative 9.6% return in December has a Z-Score of -2.54 compared to the mean return from 2004 - 2017, with only a 1.1% probability of such an extreme move in either direction.

## Next Steps:

Incorporate Additional Variables:

- Volume Weighted Moving Average
- Twitter Sentiment
- Breadth - Advance / Decline
- Sentiment - Put / Call Ratios