Bayesian Data Analysis - Assignment 3

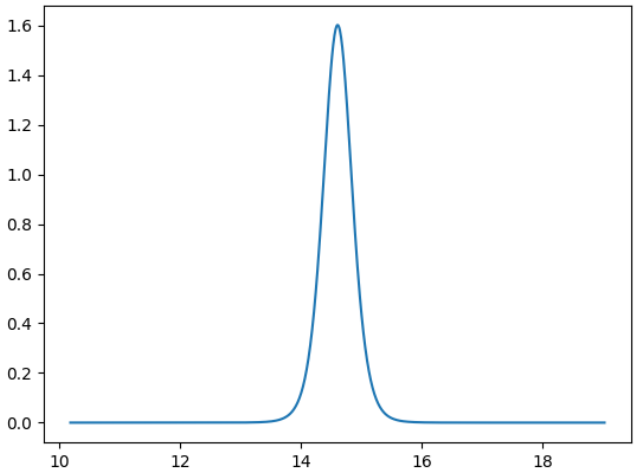
September 30, 2018

1. *Inference for normal mean and deviation.*

   a. *What can you say about the unknown μ?*
   To answer to the questions it would be reasonable to plot the probability distribution function. But before plotting the graph we have to follow the formulas and some conventions. For instance to plot the *pdf* we have to calculate the degree of freedom, which is $n - 1$; the location: $np.mean(data)$ and the estimated variance: $stats.tvar(data)/n$. These are important components for computing μ:

```
y_range = stats.t.pdf(
    x=x_range,
    df=n-1,
    loc=estimated_mean,
    scale=estimated_variance/n
)
```

   It looks like this:



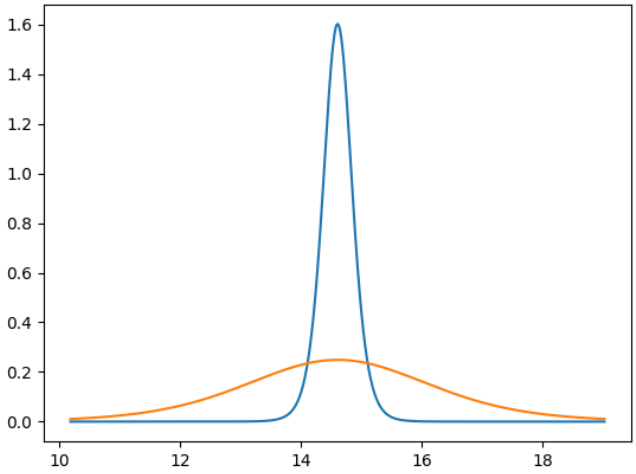
   From that we can say that μ is concentrated between **intervals [14.054, 15.168]** and has an **average hardness of 14.611** with estimated **variance 2.173** and estimated **standard deviation 1.474**.

```
$ intervals: [14.054, 15.168]
$ estimated mean: 14.611
```

```
$ estimated variance: 2.173
$ estimated standard deviation: 1.474
```

Please see *Appendix A Source code for 1 "a" and "b"* for reference.

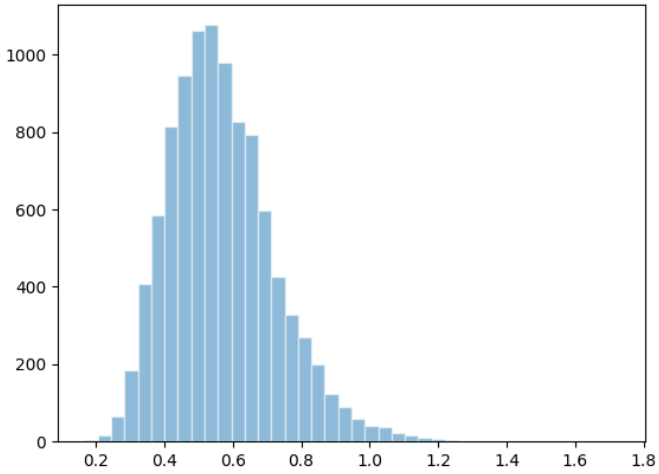
b. *What can you say about the hardness of the next windshield coming from the production line before actually measuring the hardness?*
We can say with the **95% confidence** that the hardness of the next windshield coming from the production line will be between **intervals [14.054, 15.168]**. The same formula as in *a* can be applied to plot it:



2. *Inference for difference between proportions:*

a. *Compute the point and interval estimates and plot the histograms:*
The histogram can be plotted by following the given $(p_1/(1-p_1))/(p_0/(1-p_0))$ and using and python "magic" packages:

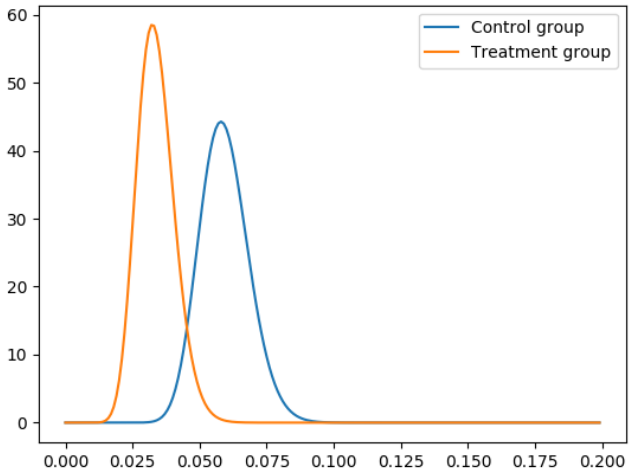
```
p_control = stats.beta.rvs(control_alpha, control_beta, size=10000)
p_treatment = stats.beta.rvs(treatment_alpha, treatment_beta, size=10000)
odd_ratio = (p_treatment/(1-p_treatment))/(p_control/(1-p_control))
plt.hist(odd_ratio, alpha=0.5, bins=40, ec='white')
```




```
$ Control central percentile 95%:  [0.043, 0.0785]
$ Treatment central percentile 95%:  [0.0218, 0.0489]
$ Control posterior mean:  0.0593
$ Treatment posterior mean:  0.0338
```

Please see *Appendix B Source code for 2 "a" and "b"* for reference.

b. *Discuss the sensitivity of your inference to your choice of prior density with a couple of sentences:*
Based on the plotted graph that depicts the control group and treatment group, we can conclude that the treatment does not have a big positive impact.



Since we have chosen an uninformative prior, it does not impact on our inference.

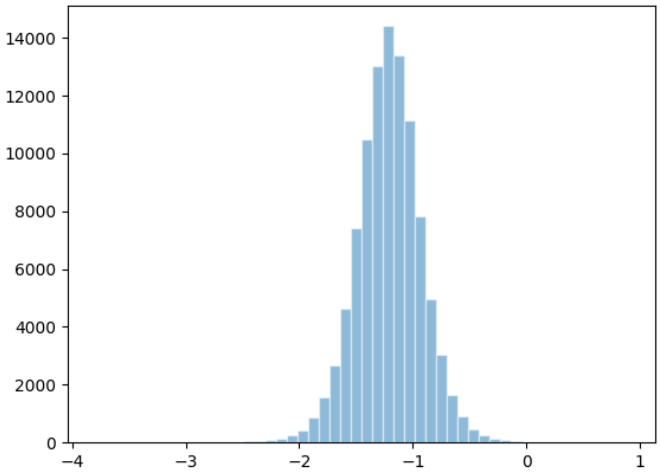
3. *Inference for difference between normal means:*
For this example we can use $uninformativepriors$ and use exactly the same formula as for 3.1 "a", "b". The only issue is that we have to optimize our code and reuse it to

calculate both mu_1 and mu_2. That optimization can then help us to calculate the values quickly.

```
n_1, estimated_mean_1, estimated_variance_1, x_range_1, mu_1 = get_attributes(y1)
n_2, estimated_mean_2, estimated_variance_2, x_range_2, mu_2 = get_attributes(y2)
```

Please see *Appendix C Source code for 3 "a" and "b"* for reference.

a. The histogram of $\mu_d = \mu_1$ - μ_2 looks like this:



b. *Are the means the same? Explain your reasoning with a couple of sentences.*
No they are not the same. Since 99.942% of the points are below 0, then we can conclude that they are not the same.

```
$ Intervals -1.7847 -0.6424
$ Percentile of m < 0: 99.939 %
```

Appendix A Source code for 1 "a" and "b"

```
import matplotlib
matplotlib.use('TkAgg')
from math import sqrt
from scipy import stats
import matplotlib.pyplot as plt
import numpy as np

data = [
    13.357,
    14.928,
    14.896,
    15.297,
    14.82,
    12.067,
```

```
    14.824,
    13.865,
    17.447,
]
n = len(data)
estimated_mean = np.mean(data)
estimated_variance = stats.tvar(data)
x_range = np.arange(
    estimated_mean - 3 * sqrt(estimated_variance),
    estimated_mean + 3 * sqrt(estimated_variance),
    0.01
)

'''
a) What can you say about the unknown $\hat{I}_4^1$?
'''
y_range = stats.t.pdf(
    x=x_range,
    df=n-1,
    loc=estimated_mean,
    scale=estimated_variance/n
)
intervals = stats.t.interval(
    0.95,
    df=n-1,
    loc=estimated_mean,
    scale=estimated_variance/n
)
low, up = intervals
print('a)')
print('-- intervals:', [round(low, 3), round(up, 3)])
print('-- estimated mean:', round(estimated_mean, 3))
print('-- estimated variance:', round(estimated_variance, 3))
print('-- estimated standard deviation:', round(sqrt(estimated_variance), 3))
figure = plt.plot(x_range, y_range)
plt.savefig('./ex3/report/3_1_a_mean_student_distribution.png')

'''
b)
'''
std_y = np.std(data, ddof=1)
scale = sqrt(1 + 1/n) * std_y
y_posterior_range = stats.t.pdf(
    x=x_range,
    df=n-1,
    loc=estimated_mean,
    scale=scale
)
figure = plt.plot(x_range, y_posterior_range)
plt.savefig('./ex3/report/3_1_b_posterior_mean.png')
```

Appendix B Source code for 2 ”a” and ”b”

```
import matplotlib
matplotlib.use('TkAgg')
from scipy import stats
import matplotlib.pyplot as plt
import numpy as np

x_range = np.arange(0, 0.2, 0.001)

'''
a)
'''
control = 674
control_died = 39
control_alpha = control_died + 1
control_beta = control - control_alpha + 1
control_posterior = control_alpha/control
control_pdf = stats.beta.pdf(x_range, control_alpha, control_beta)
control_pdf_line = plt.plot(x_range, control_pdf)

treatment = 680
treatment_died = 22
treatment_alpha = treatment_died + 1
treatment_beta = treatment - treatment_alpha + 1
treatment_posterior = treatment_alpha/treatment
treatment_pdf = stats.beta.pdf(x_range, treatment_alpha, treatment_beta)
treatment_pdf_line = plt.plot(x_range, treatment_pdf)

plt.legend(
    [*control_pdf_line, *treatment_pdf_line],
    ['Control group', 'Treatment group']
)
plt.savefig('./ex3/report/3_2_a_control_beta.png')
plt.figure(0)

'''
b)
'''
p_control = stats.beta.rvs(control_alpha, control_beta, size=10000)
p_treatment = stats.beta.rvs(treatment_alpha, treatment_beta, size=10000)
odd_ratio = (p_treatment/(1-p_treatment))/(p_control/(1-p_control))

plt.hist(odd_ratio, alpha=0.5, bins=40, ec='white')
plt.savefig('./ex3/report/3_2_b_histog.png')
plt.figure(0)

'''
intervals
'''
control_percentile_25 = np.percentile(p_control, q=2.5)
control_percentile_95 = np.percentile(p_control, q=97.5)

treatment_percentile_25 = np.percentile(p_treatment, q=2.5)
```

```
treatment_percentile_95 = np.percentile(p_treatment, q=97.5)
print('-----------')
print(
    'Control central percentile 95%: ',
    [round(control_percentile_25, 4), round(control_percentile_95, 4)]
)
print(
    'Treatment central percentile 95%: ',
    [round(treatment_percentile_25, 4), round(treatment_percentile_95, 4)]
)
print('Control posterior mean: ', round(np.mean(control_posterior), 4))
print('Treatment posterior mean: ', round(np.mean(treatment_posterior), 4))
```

Appendix C Source code for 3 ”a” and ”b”

```
import matplotlib
matplotlib.use('TkAgg')
from math import sqrt
import scipy
from scipy import stats
import matplotlib.pyplot as plt
import numpy as np

def get_attributes(data):
    n = len(data)
    estimated_mean = np.mean(data)
    estimated_variance = stats.tvar(data)
    x_range = np.arange(
        estimated_mean - 3 * sqrt(estimated_variance),
        estimated_mean + 3 * sqrt(estimated_variance),
        0.01
    )
    mu = stats.t.pdf(
        x=x_range,
        df=n-1,
        loc=estimated_mean,
        scale=estimated_variance/n
    )
    return [n, estimated_mean, estimated_variance, x_range, mu]

y1 = [13.357,14.928,14.896,15.297,14.82,12.067,14.824,13.865,17.447]
y2 = [15.98,14.206,16.011,17.25,15.993,15.722,17.143,15.23,15.125,
  16.609,14.735,15.881,15.789]
n_1, estimated_mean_1, estimated_variance_1, x_range_1, mu_1 = get_attributes(y1)
n_2, estimated_mean_2, estimated_variance_2, x_range_2, mu_2 = get_attributes(y2)

mu_1_samples = stats.t.rvs(
  df=n_1-1, loc=estimated_mean_1, scale=estimated_variance_1/n_1, size=100000
)
mu_2_samples = stats.t.rvs(
```

```
  df=n_2-1, loc=estimated_mean_2, scale=estimated_variance_2/n_2, size=100000
)
mu_diff = mu_1_samples - mu_2_samples

plt.hist(mu_diff, bins=50, ec='white',  alpha=0.5)
plt.savefig('./ex3/report/3_3_a_histogram.png')
print(
  'Intervals',
  round(np.percentile(mu_diff, 2.5), 4),
  round(np.percentile(mu_diff, 97.5), 4)
)

'''
b)
'''
print(stats.percentileofscore(mu_diff, 0), '%')
```