Bayesian Data Analysis - Assignment 2

September 23, 2018

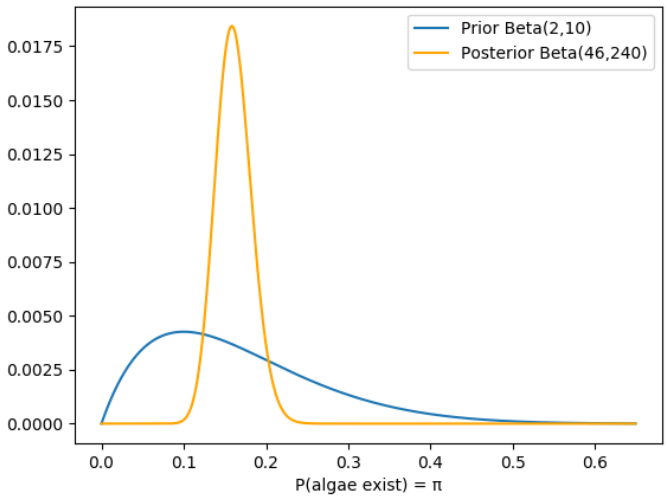
Before responding to questions a, b, c and d; we have to calculate prior and posterior values. From the description and provided data we know that there are 274 lakes, from which 230 did not show any algae sign and 44 of them had algae.

P(algae exist based on data) $= \frac{44}{274} = 0.1605$

prior $= Beta(2, 10) = \frac{2}{2+10} = 0.166\overline{6}$

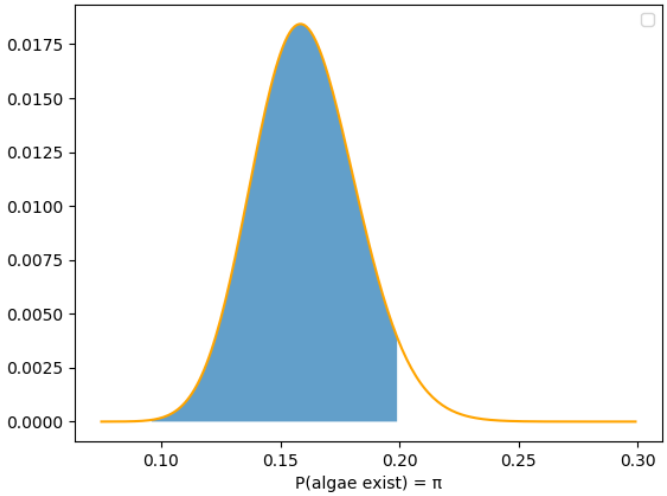
posterior $= Beta(positive(prior + data), negative(prior + data)) = Beta(2 + 44, 10 + 230)$
$= \frac{46}{240} = 0.1608$

a. *Summarize:*
Based on the graph that depicts the probability distribution of prior and posterior we can conclude that the data makes posterior more explicit and concrete. It can be easily observed that the value of π has a higher probability in the intervals: **[0.096, 0.235]**.



b. $P(\pi_0 = 0.2)$:
The probability that the proportion of monitoring sites with detectable algae levels π is smaller than $\pi_0 = 0.2$ is equal to **0.9586**. which can be calculate like

```
stats.beta.cdf(0.2, a=46, b=240)
```



c. *What are the required assumptions:*
All lakes have the same distribution and expected value.

d. *Make prior sensitivity analysis:*

$B(1,5) = \frac{1}{1+5} = 0.16\overline{6}$ $\quad$ $posterior = 0.160$

$B(20,100) = \frac{20}{20+100} = 0.16\overline{6}$ $\quad$ $posterior = 0.162$

$B(40,200) = \frac{40}{40+200} = 0.16\overline{6}$ $\quad$ $posterior = 0.163$

$B(60,300) = \frac{60}{60+300} = 0.16\overline{6}$ $\quad$ $posterior = 0.164$

The above calculation shows the sensitivity of posterior inference about π to the proposed prior distribution. It uses use prior distributions that are increasingly concentrated around 0.162, the proportion of lakes with algae.

Appendix A Source code

```
from scipy import stats
import numpy
import matplotlib
matplotlib.use('TkAgg')
import matplotlib.pyplot as plt

# a) summarize
x_range = numpy.arange(0, 0.65, 0.001)
prior = stats.beta.pdf(x_range, a=2, b=10)/1000
posterior = stats.beta.pdf(x_range, a=46, b=240)/1000

plt.plot(x_range, prior, label='Prior Beta(2,10)')
plt.plot(x_range, posterior, label='Posterior Beta(46,240)', color='orange')

plt.xlabel('P(algae exist) = pi')
```

```
plt.legend()
plt.savefig('./ex2/report.tex/prob_distribution.png')
plt.show()

# a) P(pi0= 0.2)
cumulative = stats.beta.cdf(0.2, a=46, b=240)
print('cumulative at 0.2: ', cumulative)

x_range2_line = numpy.arange(0.075, 0.3, 0.001)
posterior2_line = stats.beta.pdf(x_range2_line, a=46, b=240)/1000

x_range2 = numpy.arange(0.096, 0.2, 0.001)
posterior2 = stats.beta.pdf(x_range2, a=46, b=240)/1000

plt.fill_between(x_range2, posterior2, alpha=0.7)
plt.plot(x_range2_line, posterior2_line, color='orange')

plt.xlabel('P(algae exist) = pi')
plt.legend()
plt.savefig('./ex2/report.tex/cumulative.png')
plt.show()
```