Kernel Methods in Machine Learning

Homework Assignment 1:

student : Donayorov Maksad
id : 571335

Question 1:

a. Prove that $K(x,y) = (\langle x,y \rangle + c)^m$ is a kernel by „Product of kernel function" and „Conic sum of" kernel.

The Binomial theorem says that:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} \cdot b^k$$

And the Conic Sum of Kernels indicates:

$$(k_j)_{j=1}^n \text{ and } (a_j)_{j=1}^n > 0, \sum_{j=1}^{n} d_j k_j \text{ is a kernel.}$$

if we consider „a" to be a kernel and „b" a scaler then following the Binomial formula we can write:

$$(K(x,y)+b)^n = \sum_{k=0}^{n} \binom{n}{k} (\langle x,y \rangle)^{n-k} \cdot b^k =$$

$$= \binom{n}{0} \cdot \langle x,y \rangle^n \cdot b^0 + \binom{n}{1} \langle x,y \rangle^{n-1} \cdot b^1 \ldots \binom{n}{n-1} \langle x,y \rangle^1 \cdot b^{n-1} + \binom{n}{n} \langle x,y \rangle^0 \cdot b^n$$

from the above calculations we know that:

1. both $\binom{n}{k}$ and b^k are scalars
2. kernel multiplied by kernel is a kernel.

$$\underbrace{\binom{n}{k}}_{\text{scalar}} (\langle x,y \rangle)^{n-k} \cdot \underbrace{b^k}_{\text{scalar}}$$

Consequently, we can say that $K(x,y)$ is a kernel based on the combination of „Product of a kernel function is a kernel function" and „Conic sum of kernels is a kernel" using Binomial Theorem.

b. When $K(x,y) = (\langle x, y\rangle)^3$ write down the feature space expansion taking order into account

$x, y \in \mathbb{R}^2 \quad m=3 \quad c=0$

Let's asume that $a = x_1 y_1$ and $b = x_2 y_2$ in that case we will have to compute:

$$(a+b)^3$$

$$(a+b)^3 = (a+b)\cdot(a+b)^2$$

$$(a+b)^2 = (a+b)\cdot(a+b) = aa + ab + ba + bb$$

$$(a+b)\cdot(aa+ab+ba+bb) =$$

$$= aaa + aab + aba + abb + baa + bab + bba + bba$$

Let's now convert a and b to $x_1 y_1$ and $x_2 y_2$

$$(a+b)^3 = aaa + aab + aba + abb + baa + bab + bba + bbb$$

$$(x_1y_1 + x_2y_2)^3 = x_1y_1x_1y_1x_1y_1 + x_1y_1x_1y_1x_2y_2 + x_1y_1x_2y_2x_1y_1$$
$$+ x_1y_1x_2y_2x_2y_2 + x_2y_2x_1y_1x_1y_1 + x_2y_2x_1y_1x_2y_2$$
$$+ x_2y_2x_2y_2x_1y_1 + x_2y_2x_2y_2x_2y_2$$