

Transport and Telecommunication Institute  
Doctoral Faculty  
Telematics and Logistics

**SELF-STUDY ASSIGNMENT**

**Systems Theory**

**Nonlinear Dynamics and Chaos in Urban Logistics Inventory  
Systems**

An Empirical Analysis of FreshRetailNet-50K

Student: Maksim Ilin (83835)

Group: 5402DTL



# Contents

<b>Assignment Alignment and Research Object</b>	<b>3</b>
<b>1 Task 1 — Linear Control System (ACS) Analysis</b>	<b>4</b>
1.1 Structural diagram . . . . .	4
1.2 Element transfer functions . . . . .	4
1.3 Closed-loop transfer function and stability . . . . .	5
<b>2 Task 2 — Nonlinear Dynamic Model (2D ODE) and Local Analysis</b>	<b>6</b>
2.1 Model formulation . . . . .	6
2.2 Fixed point (equilibrium) . . . . .	6
2.3 Local stability (Jacobian and eigenvalues) . . . . .	6
<b>3 Task 3 — Chaos Theory and Time Series Analysis</b>	<b>7</b>
3.1 Data and indicator . . . . .	7
3.2 Methods . . . . .	7
3.3 Visual diagnostics . . . . .	7
3.4 Results and interpretation . . . . .	7
<b>4 Conclusion</b>	<b>12</b>
<b>References</b>	<b>13</b>

## Abstract

This report investigates the dynamic behavior of a perishable inventory system using high-frequency demand data from the FreshRetailNet-50K dataset. The work follows the assignment requirements in three parts: (1) a linear automatic control system (ACS) model of an inventory feedback loop (integrator + proportional controller) with a stability discussion and the effect of transport delay, (2) a nonlinear 2D ODE model with temperature-dependent spoilage and local equilibrium classification, and (3) empirical chaos diagnostics on a time series with more than 1000 samples.

For the chaos task, we analyze the “golden sample” hourly sales series (daytime window 08:00–22:00) and obtain  $H \approx 0.59$  (persistent behavior) and  $D_2 \approx 1.15$  (non-integer correlation dimension), consistent with low-dimensional complex dynamics under feedback, delays, and censoring due to stockouts.

## Assignment Alignment and Research Object

The research object is an urban logistics inventory replenishment system for perishable goods. We use FreshRetailNet-50K, a stockout-annotated censored demand dataset. The empirical indicator for Task 3 is the hourly sales series of the “golden sample” SKU/time window derived in the project pipeline.

This report corresponds to the assignment specification:

- Task 1: Linear ACS model, transfer functions, closed-loop transfer function, stability.
- Task 2: Nonlinear ODE model (order  $\leq 2$ ), fixed point(s), Jacobian stability classification.
- Task 3: Chaos/time-series metrics for  $N \geq 1000$  (Hurst exponent and correlation dimension) and interpretation.

# 1 Task 1 — Linear Control System (ACS) Analysis

## 1.1 Structural diagram

The inventory system is modeled as a unity-feedback loop where the controller generates a replenishment signal based on the inventory error, and the plant integrates the net flow (replenishment minus demand). A transport lead time (delay) is optionally included.

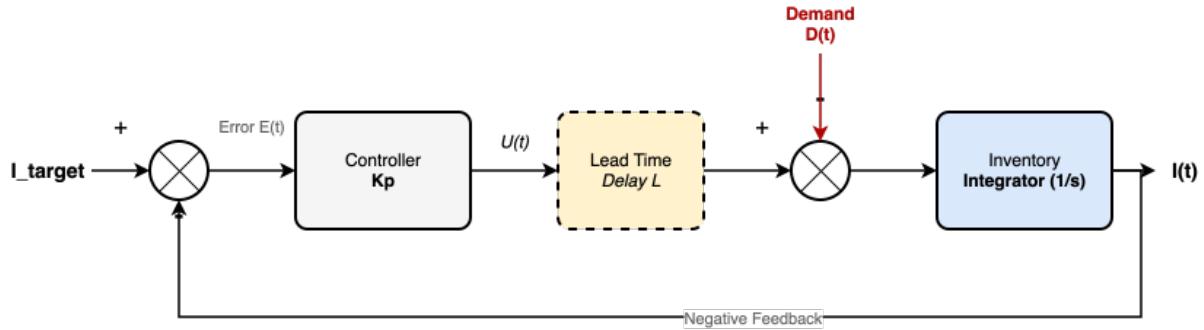


Figure 1: ACS structural diagram (inventory feedback loop)

## Elements and interpretation

- **Reference  $I_{target}$ :** desired inventory level (setpoint).
- **Error  $E(t) = I_{target} - I(t)$ :** deviation from target.
- **Controller output  $U(t)$ :** proportional replenishment policy signal.
- **Lead time / delay  $L$ :** transport delay in the supply chain.
- **Plant / process:** inventory mass balance (integrator in linear approximation).
- **Disturbance  $D(t)$ :** demand (hourly sales), subtracts inventory.

## 1.2 Element transfer functions

We use the standard linear approximation for inventory dynamics:

$$I(s) = \frac{1}{s} (U(s) - D(s)).$$

Controller (proportional control):

$$U(t) = K_p (I_{target} - I(t)).$$

Optional transport delay term is approximated by first-order Padé:  $e^{-Ls}$ .

$$G_d(s) \approx \frac{1 - \frac{Ls}{2}}{1 + \frac{Ls}{2}}.$$

*Ls ~ 5? ✓*

### 1.3 Closed-loop transfer function and stability

With unity feedback and an integrator plant, the reference-to-output closed-loop transfer function is:

$$G_{\text{cl}}(s) = \frac{K_p}{s + K_p}.$$

The characteristic equation is  $s + K_p = 0$  with pole  $s = -K_p$ . Therefore, the baseline system is stable for  $K_p > 0$ .

**Effect of lead time.** Introducing delay increases phase lag and can destabilize the loop for sufficiently large  $K_p$  and  $L$ . In the project implementation, a Padé approximation is used and the resulting poles indicate loss of stability in a scenario with significant delay (e.g.,  $L = 2$  in model time units), consistent with the bullwhip effect under transportation/ordering latency.

$$W(p) = ?$$

Stability analysis?



## 2 Task 2 — Nonlinear Dynamic Model (2D ODE) and Local Analysis

### 2.1 Model formulation

We extend the linear model by adding (i) inventory spoilage that depends on temperature and (ii) inertial dynamics for the replenishment rate.

Temperature-dependent decay (spoilage) is modeled as:

$$\delta(T) = k \cdot (1 + \alpha_T (T - 20)).$$

The 2D nonlinear system is:

$$\frac{dI}{dt} = R - D - \delta(T)I, \quad \frac{dR}{dt} = a(I_{target} - I) - bR.$$

Here  $I$  is inventory,  $R$  is replenishment rate,  $D$  is (approximately) constant demand,  $a$  is the replenishment gain, and  $b$  is the replenishment damping/decay.

### 2.2 Fixed point (equilibrium)

Setting derivatives to zero yields an equilibrium  $(I^*, R^*)$ . For  $I^*$  we use:

$$I^* = \frac{a I_{target} - b D}{a + b \delta(T)}.$$

The steady replenishment is then  $R^* = D + \delta(T)I^*$ .

### 2.3 Local stability (Jacobian and eigenvalues)

The Jacobian matrix is:

$$J = \begin{bmatrix} -\delta(T) & 1 \\ -a & -b \end{bmatrix}.$$

The equilibrium type is determined from the eigenvalues of  $J$ . In the project parameter regime, the eigenvalues are complex with negative real part:

$$\lambda_{1,2} \approx -0.255 \pm 0.970i.$$

which corresponds to a **stable focus** (damped oscillations). This matches the qualitative behavior typically associated with bullwhip-like oscillations: inventory and replenishment oscillate transiently and then settle.

Graphs?



## 3 Task 3 — Chaos Theory and Time Series Analysis

### 3.1 Data and indicator

We analyze the “golden sample” hourly sales time series derived from FreshRetailNet-50K. The full hourly series exceeds 1000 samples ( $90 \text{ days} \times 24 \text{ hours} = 2160$ ). To reduce estimator degeneracy due to long overnight zero runs (stockouts/low demand), we restrict the analysis to daytime hours **08:00–22:00**.

### 3.2 Methods

**Phase space reconstruction (2D embedding):**

$$\mathbf{x}(t) = [x(t), x(t + \tau)].$$

**Hurst exponent (R/S):**

$$\mathbb{E} \left[ \frac{R(n)}{S(n)} \right] = C n^H, \quad \log(R/S) \approx H \log(n) + \text{const.}$$

**Correlation dimension ( $D_2$ ):**

$$C(r) = \frac{2}{N(N-1)} \sum_{i < j} \Theta(r - \|\mathbf{x}_i - \mathbf{x}_j\|), \quad C(r) \sim r^{D_2}, \quad D_2 = \lim_{r \rightarrow 0} \frac{\log C(r)}{\log r}.$$

### 3.3 Visual diagnostics

### 3.4 Results and interpretation

The computed invariants for the daytime hourly series are:

- Hurst exponent:  $H \approx 0.59$ .
- Correlation dimension:  $D_2 \approx 1.15$ .

**Interpretation.** Since  $H > 0.5$ , the process is persistent (long-memory behavior), which is compatible with feedback-driven dynamics rather than a pure random walk. The non-integer value  $D_2 \approx 1.15$  indicates a low-dimensional geometric structure in reconstructed phase space, which is consistent with complex deterministic dynamics (potentially chaotic) rather than a purely stochastic cloud. Given the strong seasonality and the censoring introduced by stockouts, these results should be interpreted cautiously; however, together they support the hypothesis of bounded, non-trivial dynamics under supply-chain feedback.

Interactive report is provided as an attached artifact: task3\_chaos\_report.html.

## Sales with Stockouts

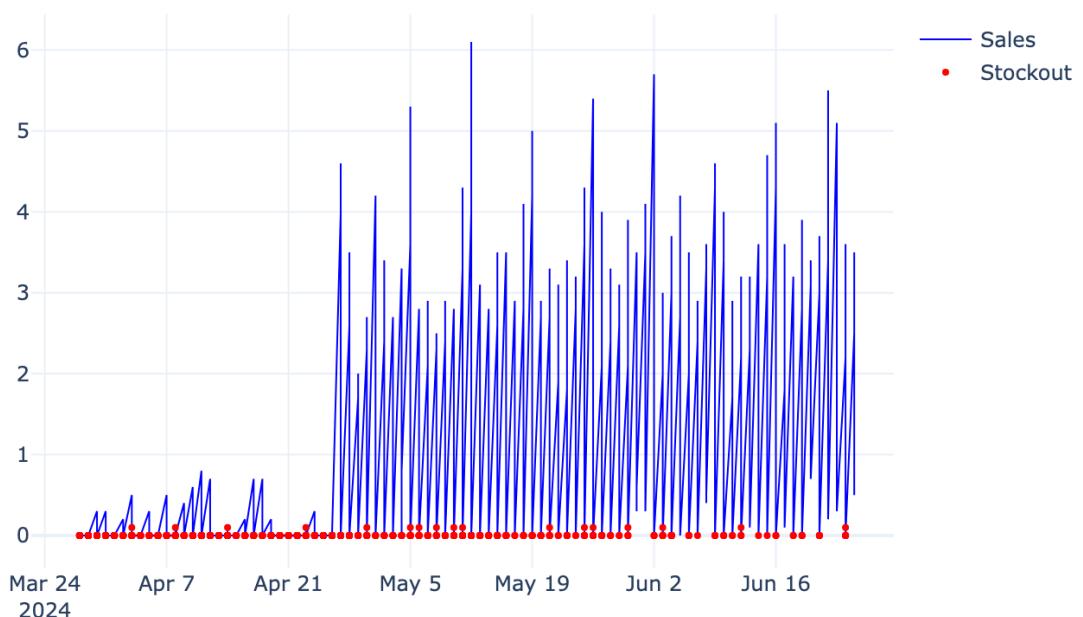


Figure 2: Hourly sales time series with stockout markers

## Phase Space Reconstruction (delay=1)

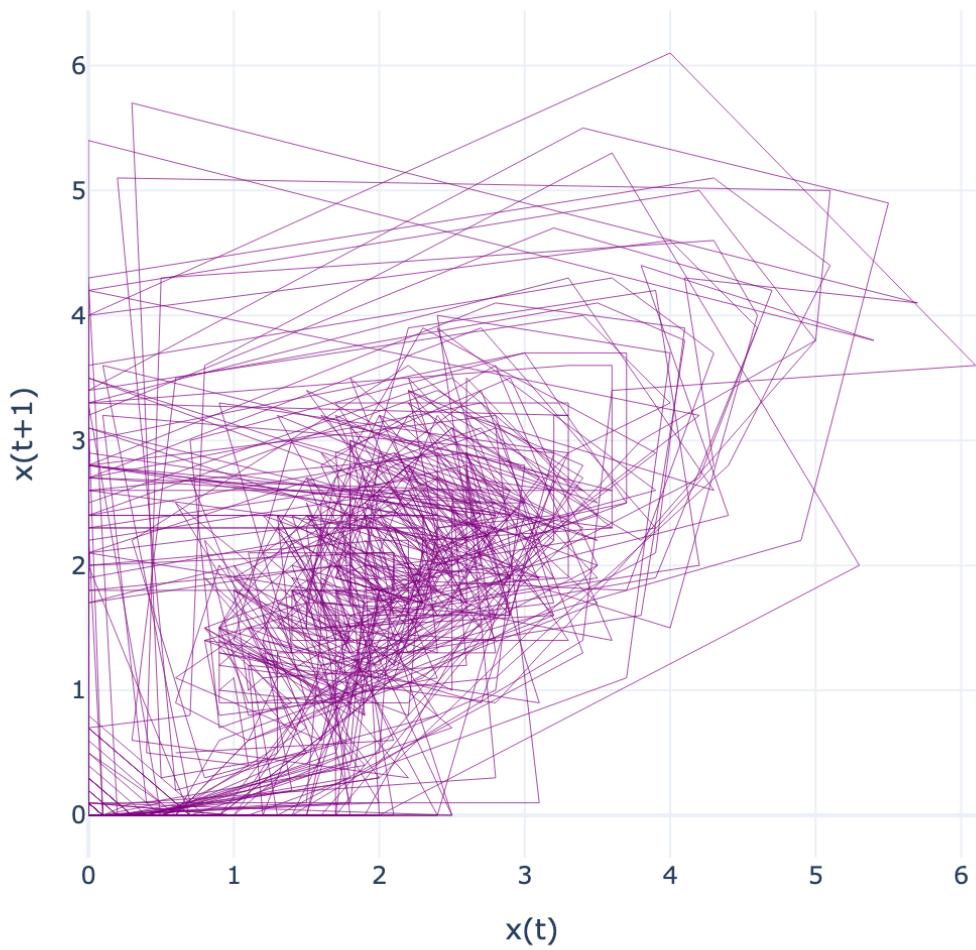


Figure 3: Phase portrait (2D embedding, delay  $\tau = 1$ )

### Hurst Exponent ( $H=0.586$ )

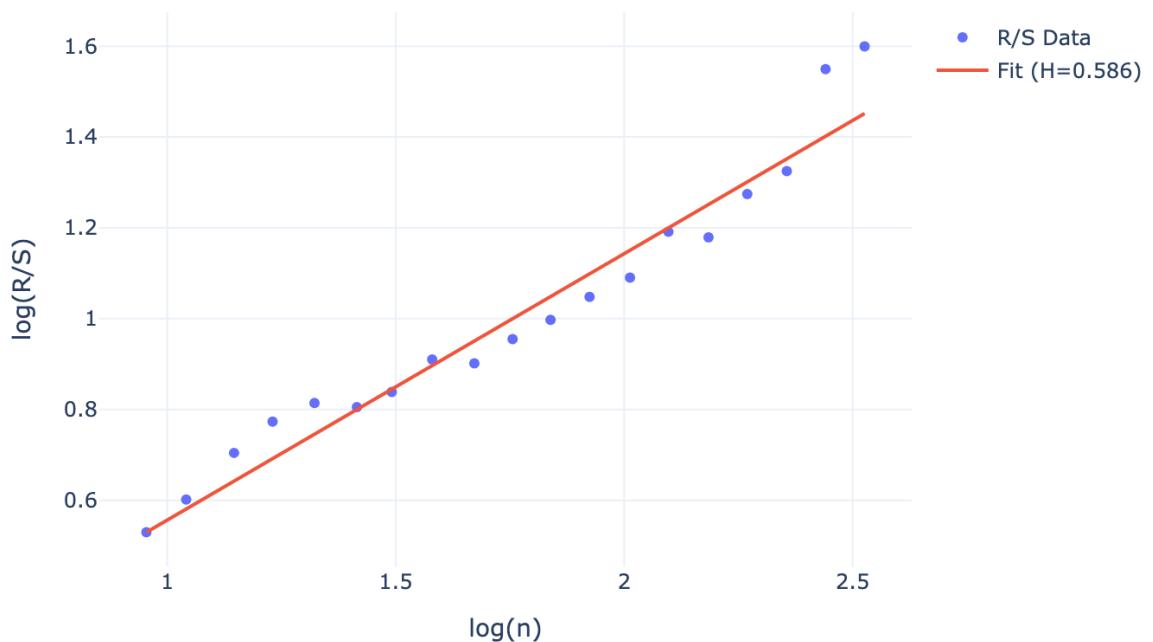


Figure 4: Hurst exponent estimation (R/S log-log fit)

### Correlation Dimension (D2=1.145)

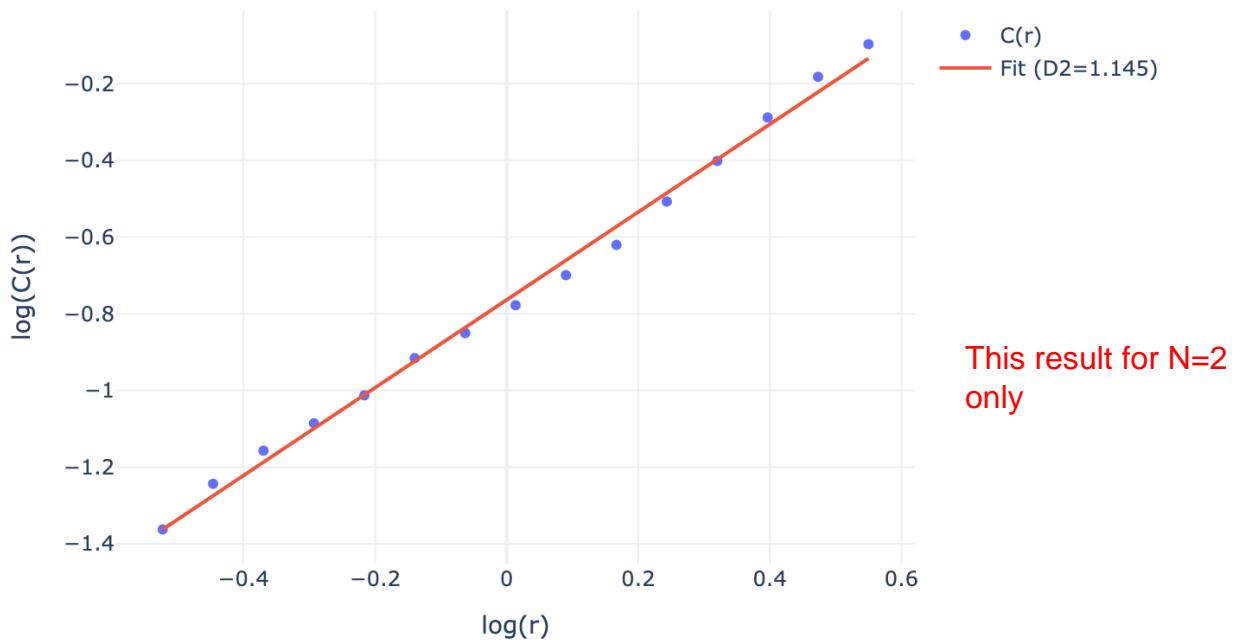


Figure 5: Correlation dimension estimation ( $D_2$  log-log fit)

But what is the dimension if  $N=1, \dots, 20$ ?



## 4 Conclusion

This work demonstrates a coherent systems-theory analysis of an inventory replenishment system using both theory-driven models and empirical diagnostics. The linear ACS formulation provides a baseline stability condition and shows how delays can destabilize the loop. The nonlinear 2D model introduces spoilage and inertia and yields a stable-focus equilibrium consistent with damped oscillations. Finally, the empirical time-series analysis on a long hourly sequence yields a persistent Hurst exponent and a non-integer correlation dimension, suggesting low-dimensional complex behavior influenced by feedback, delays, and stockout censoring.

## References

- Dingdong-Inc (2025). *FreshRetailNet-50K: A Stockout-Annotated Censored Demand Dataset for Latent Demand Recovery and Forecasting in Fresh Retail* [Dataset]. Available at: <https://huggingface.co/datasets/Dingdong-Inc/FreshRetailNet-50K/tree/main/data> (Accessed: 1 February 2026).
- Forrester, J.W. (1961). *Industrial Dynamics*. Waltham, MA: Pegasus Communications.
- Grassberger, P. and Procaccia, I. (1983). ‘Characterization of strange attractors’, *Physical Review Letters*, 50(5), pp. 346–349.
- Hurst, H.E. (1951). ‘Long-term storage capacity of reservoirs’, *Transactions of the American Society of Civil Engineers*, 116, pp. 770–799.
- Ilin, M. (2026). *Systems Theory Task: Analysis of Nonlinear Dynamics and Chaos in Inventory Systems*. GitHub Repository. Available at: <https://github.com/maksim-tsi/systems-theory-task> (Accessed: 1 February 2026).
- Sterman, J.D. (2000). *Business Dynamics: Systems Thinking and Modeling for a Complex World*. Boston: Irwin/McGraw-Hill.
- Strogatz, S.H. (2015). *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering*. 2nd edn. Boulder, CO: Westview Press.