Transport and Telecommunication Institute
Doctoral Faculty
Telematics and Logistics

SELF-STUDY ASSIGNMENT

Systems Theory

Nonlinear Dynamics and Chaos in Urban Logistics Inventory Systems

An Empirical Analysis of FreshRetailNet-50K

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Contents

Abstract

This report investigates the dynamic behavior of a perishable inventory system using high-frequency demand data from the FreshRetailNet-50K dataset. The work follows the assignment requirements in three parts: (1) a linear automatic control system (ACS) model of an inventory feedback loop (integrator + proportional controller) with a stability discussion and the effect of transport delay, (2) a nonlinear 2D ODE model with temperature-dependent spoilage and local equilibrium classification, and (3) empirical chaos diagnostics on a time series with more than 1000 samples.

For the chaos task, we analyze the "golden sample" hourly sales series (daytime window 08:00–22:00) and obtain $H \approx 0.59$ (persistent behavior) and $D_2 \approx 1.15$ (non-integer correlation dimension), consistent with low-dimensional complex dynamics under feedback, delays, and censoring due to stockouts.

Assignment Alignment and Research Object

The research object is an urban logistics inventory replenishment system for perishable goods. We use FreshRetailNet-50K, a stockout-annotated censored demand dataset. The empirical indicator for Task 3 is the hourly sales series of the "golden sample" SKU/time window derived in the project pipeline.

This report corresponds to the assignment specification:

- Task 1: Linear ACS model, transfer functions, closed-loop transfer function, stability.
- Task 2: Nonlinear ODE model (order ≤ 2), fixed point(s), Jacobian stability classification.
- Task 3: Chaos/time-series metrics for $N \geq 1000$ (Hurst exponent and correlation dimension) and interpretation.

1 Task 1 — Linear Control System (ACS) Analysis

1.1 Structural diagram

The inventory system is modeled as a unity-feedback loop where the controller generates a replenishment signal based on the inventory error, and the plant integrates the net flow (replenishment minus demand). A transport lead time (delay) is optionally included.

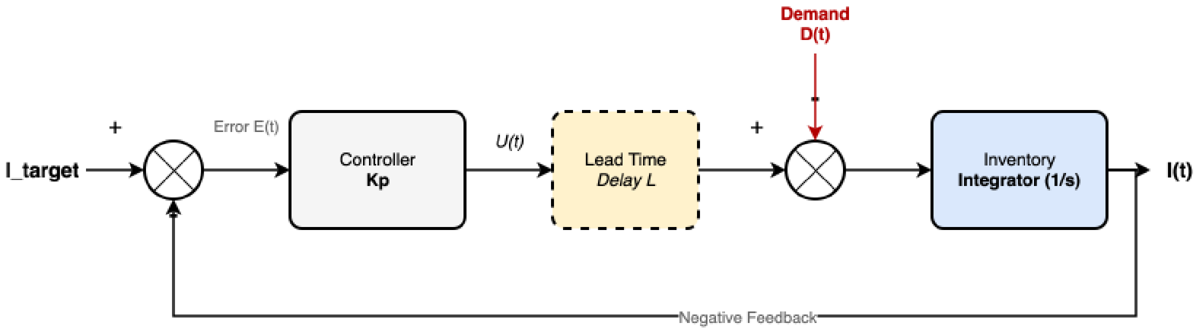

Figure 1: ACS structural diagram (inventory feedback loop)

Elements and interpretation

- **Reference** I_{target}: desired inventory level (setpoint).
- **Error** $E(t) = I_{target} - I(t)$: deviation from target.
- **Controller output** $U(t)$: proportional replenishment policy signal.
- **Lead time / delay** L: transport delay in the supply chain.
- **Plant / process**: inventory mass balance (integrator in linear approximation).
- **Disturbance** $D(t)$: demand (hourly sales), subtracts inventory.

1.2 Element transfer functions

We use the standard linear approximation for inventory dynamics:

$$I(s) = \frac{1}{s}\left(U(s) - D(s)\right).$$

Controller (proportional control):

$$U(t) = K_p\left(I_{target} - I(t)\right).$$

Optional transport delay term is approximated by first-order Padé: e^{-Ls}.

$$G_d(s) \approx \frac{1 - \frac{Ls}{2}}{1 + \frac{Ls}{2}}.$$

1.3 Closed-loop transfer function and stability

With unity feedback and an integrator plant, the reference-to-output closed-loop transfer function is:

$$G_{\mathrm{cl}}(s) = \frac{K_{\mathrm{p}}}{s + K_{\mathrm{p}}}.$$

The characteristic equation is $s + K_{\mathrm{p}} = 0$ with pole $s = -K_{\mathrm{p}}$. Therefore, the baseline system is stable for $K_{\mathrm{p}} > 0$.

Effect of lead time. Introducing delay increases phase lag and can destabilize the loop for sufficiently large K_{p} and L. In the project implementation, a Padé approximation is used and the resulting poles indicate loss of stability in a scenario with significant delay (e.g., $L = 2$ in model time units), consistent with the bullwhip effect under transportation/ordering latency.

2 Task 2 — Nonlinear Dynamic Model (2D ODE) and Local Analysis

2.1 Model formulation

We extend the linear model by adding (i) inventory spoilage that depends on temperature and (ii) inertial dynamics for the replenishment rate.

Temperature-dependent decay (spoilage) is modeled as:

$$\delta(T) = k \cdot (1 + \alpha_T (T - 20)) .$$

The 2D nonlinear system is:

$$\frac{dI}{dt} = R - D - \delta(T)I, \qquad \frac{dR}{dt} = a\,(I_{target} - I) - b\,R.$$

Here I is inventory, R is replenishment rate, D is (approximately) constant demand, a is the replenishment gain, and b is the replenishment damping/decay.

2.2 Fixed point (equilibrium)

Setting derivatives to zero yields an equilibrium (I^*, R^*). For I^* we use:

$$I^* = \frac{a\,I_{target} - b\,D}{a + b\,\delta(T)}.$$

The steady replenishment is then $R^* = D + \delta(T)I^*$.

2.3 Local stability (Jacobian and eigenvalues)

The Jacobian matrix is:

$$J = \begin{bmatrix} -\delta(T) & 1 \\ -a & -b \end{bmatrix}.$$

The equilibrium type is determined from the eigenvalues of J. In the project parameter regime, the eigenvalues are complex with negative real part:

$$\lambda_{1,2} \approx -0.255 \pm 0.970i.$$

which corresponds to a **stable focus** (damped oscillations). This matches the qualitative behavior typically associated with bullwhip-like oscillations: inventory and replenishment oscillate transiently and then settle.

3 Task 3 — Chaos Theory and Time Series Analysis

3.1 Data and indicator

We analyze the "golden sample" hourly sales time series derived from FreshRetailNet-50K. The full hourly series exceeds 1000 samples (90 days × 24 hours = 2160). To reduce estimator degeneracy due to long overnight zero runs (stockouts/low demand), we restrict the analysis to daytime hours **08:00–22:00**.

3.2 Methods

Phase space reconstruction (2D embedding):

$$\mathbf{x}(t) = [x(t),\, x(t+\tau)].$$

Hurst exponent (R/S):

$$\mathbb{E}\left[\frac{R(n)}{S(n)}\right] = C\,n^H, \qquad \log(R/S) \approx H\,\log(n) + \text{const.}$$

Correlation dimension (D_2):

$$C(r) = \frac{2}{N(N-1)} \sum_{i<j} \Theta(r - \|\mathbf{x}_i - \mathbf{x}_j\|), \qquad C(r) \sim r^{D_2}, \qquad D_2 = \lim_{r\to 0} \frac{\log C(r)}{\log r}.$$

3.3 Visual diagnostics

3.4 Results and interpretation

The computed invariants for the daytime hourly series are:

- Hurst exponent: $H \approx 0.59$.
- Correlation dimension: $D_2 \approx 1.15$.

Interpretation. Since $H > 0.5$, the process is persistent (long-memory behavior), which is compatible with feedback-driven dynamics rather than a pure random walk. The non-integer value $D_2 \approx 1.15$ indicates a low-dimensional geometric structure in reconstructed phase space, which is consistent with complex deterministic dynamics (potentially chaotic) rather than a purely stochastic cloud. Given the strong seasonality and the censoring introduced by stockouts, these results should be interpreted cautiously; however, together they support the hypothesis of bounded, non-trivial dynamics under supply-chain feedback.

Interactive report is provided as an attached artifact: task3_chaos_report.html.

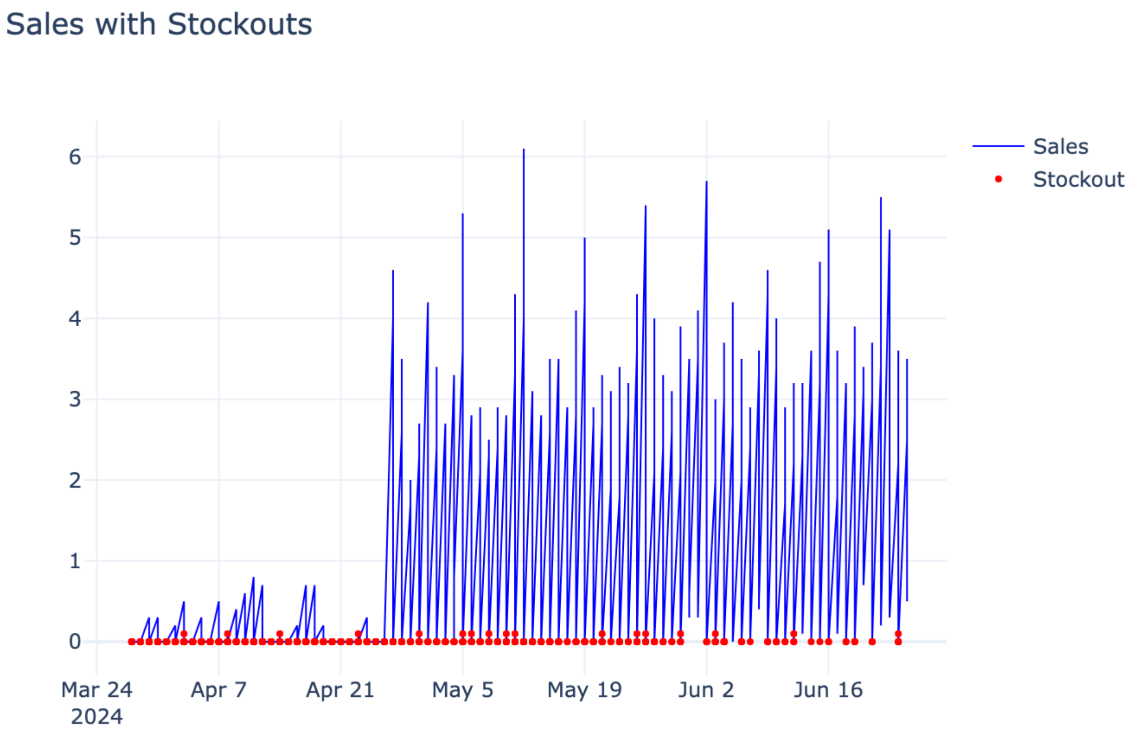

Figure 2: Hourly sales time series with stockout markers

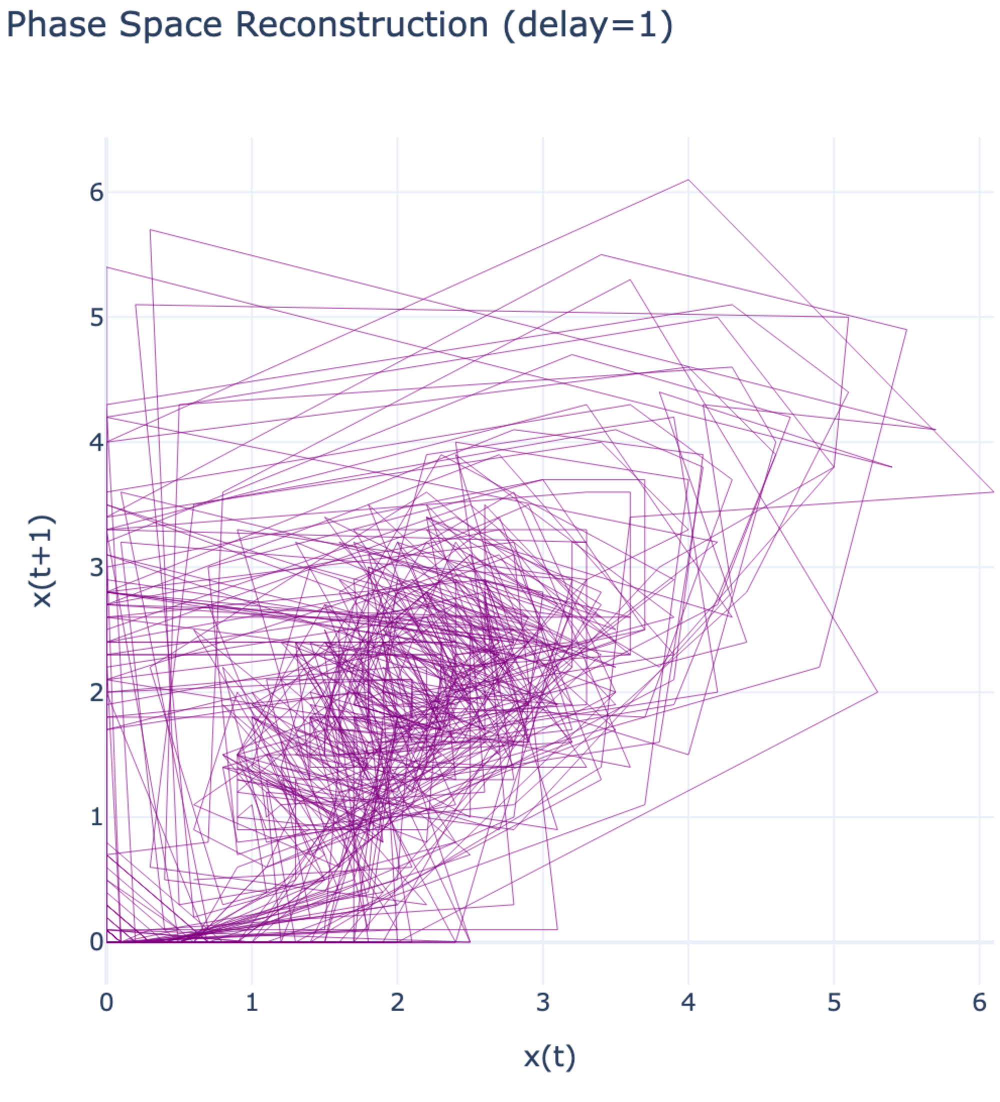

Figure 3: Phase portrait (2D embedding, delay $\tau = 1$)

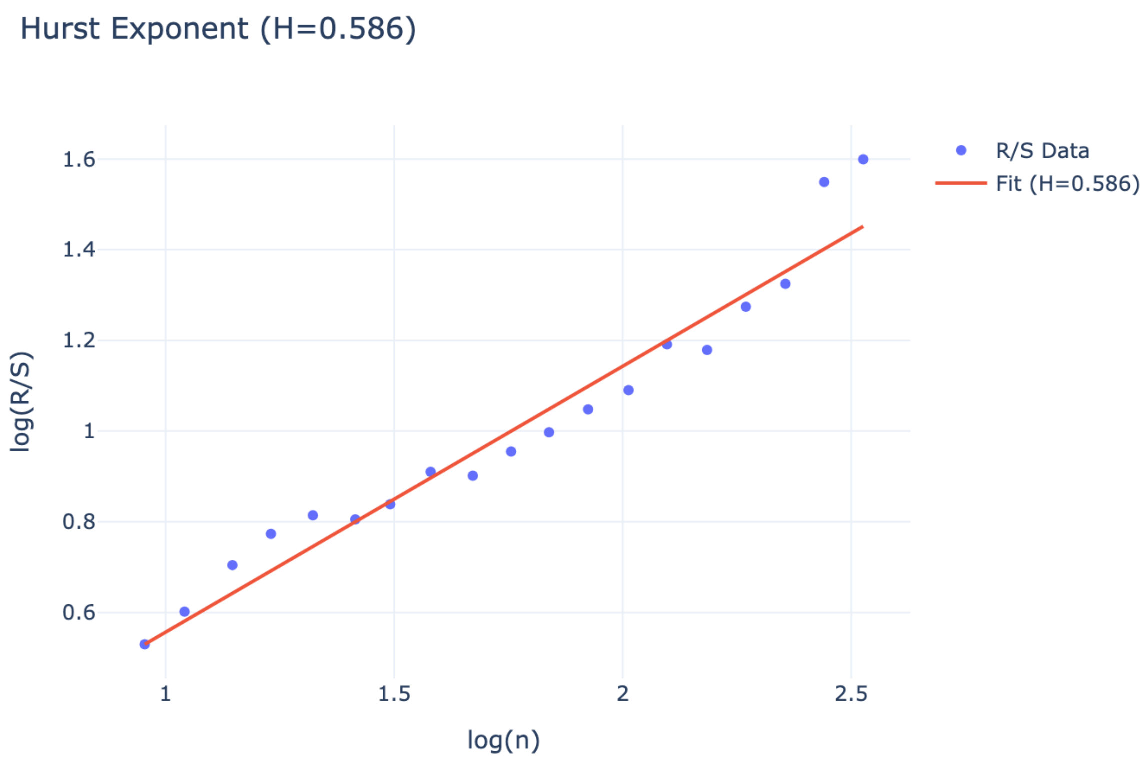

Figure 4: Hurst exponent estimation (R/S log-log fit)

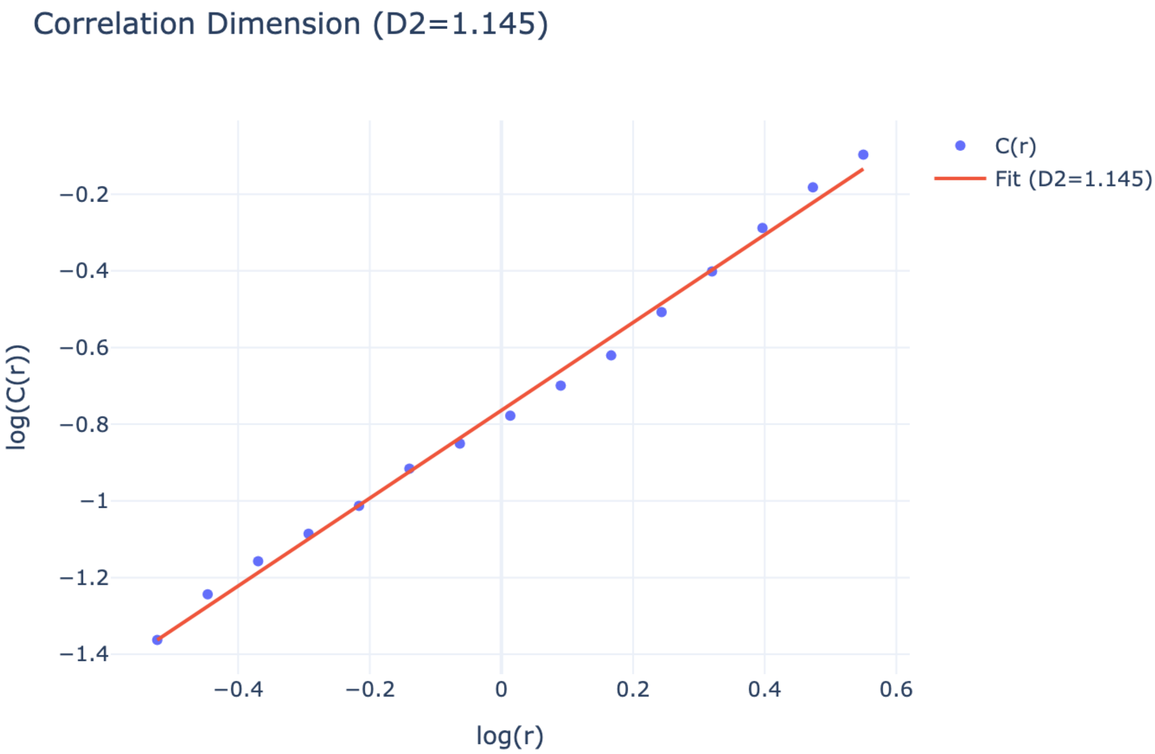

Figure 5: Correlation dimension estimation (D_2 log-log fit)

4 Conclusion

This work demonstrates a coherent systems-theory analysis of an inventory replenishment system using both theory-driven models and empirical diagnostics. The linear ACS formulation provides a baseline stability condition and shows how delays can destabilize the loop. The nonlinear 2D model introduces spoilage and inertia and yields a stable-focus equilibrium consistent with damped oscillations. Finally, the empirical time-series analysis on a long hourly sequence yields a persistent Hurst exponent and a non-integer correlation dimension, suggesting low-dimensional complex behavior influenced by feedback, delays, and stockout censoring.

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