CV

1.1. $P(w_m, w_n) = c(w_m - w_n)^2$ is submodular:

$\alpha < \beta, \quad \gamma < \delta, \quad c > 0$

$P(\beta,\gamma) + P(\alpha,\delta) - P(\beta,\delta) - P(\alpha,\gamma)$

$= c\left((\beta-\gamma)^2 + (\alpha-\delta)^2 - (\beta-\delta)^2 - (\alpha-\gamma)^2\right)$

$= 2c(-\alpha\delta + \beta\delta - \gamma\beta + \alpha\gamma)$

$= 2c(\delta(\beta-\alpha) - \gamma(\beta-\alpha)) \overset{!}{\geq} 0$

$\overset{:2c}{\Leftrightarrow} \quad \delta(\beta-\alpha) - \gamma(\beta-\alpha) \geq 0$

$\overset{:(\beta-\alpha)>0}{\Leftrightarrow} \quad \delta - \gamma \geq 0$ true as $\gamma < \delta$

$\square$

1.2. $P(w_m, w_n) = c(1 - \delta(w_m, w_n))$ is not subm!

assume $c > 0$

$\alpha = 0, \ \beta = \gamma = 1, \ \delta = 2 \ \Rightarrow \ \alpha < \beta, \ \gamma < \delta$

$P(\beta,\gamma) + P(\alpha,\delta) - P(\beta,\delta) - P(\alpha,\gamma)$

$= c(0 + 1 - 1 - 1)$

$= -c < 0 \quad \square$

2. $(a, b, c, d, e, f) \quad \beta\beta\gamma\alpha\alpha\gamma$