

CV

1.1. $P(w_m, w_n) = c(w_m - w_n)^2$ is submodular:

$$\alpha < \beta, \quad r < \delta, \quad c > 0$$

$$\begin{aligned} & P(\beta, r) + P(\alpha, \delta) - P(\beta, \delta) - P(\alpha, r) \\ &= c((\beta - r)^2 + (\alpha - \delta)^2 - (\beta - \delta)^2 - (\alpha - r)^2) \\ &= 2c(-\alpha\delta + \beta\delta - r\beta + \alpha r) \\ &= 2c(\delta(\beta - \alpha) - r(\beta - \alpha)) \geq 0 \\ &\Leftrightarrow \delta(\beta - \alpha) - r(\beta - \alpha) \geq 0 \\ &\Leftrightarrow (\beta - \alpha)(\delta - r) \geq 0 \end{aligned}$$

true as $r < \delta$

□

1.2. $P(w_m, w_n) = c(1 - \delta(w_m, w_n))$ is not submodular:

assume $c > 0$

$$\alpha = 0, \quad \beta = r = 1, \quad \delta = 2 \quad (\Rightarrow \alpha < \beta, \quad r < \delta)$$

$$\begin{aligned} & P(\beta, r) + P(\alpha, \delta) - P(\beta, \delta) - P(\alpha, r) \\ &= c(0 + 1 - 1 - 1) \\ &= -c < 0 \quad \square \end{aligned}$$

Ex 2. (a, b, c, d, e, f) $\beta/\beta \quad r/\alpha \quad r$

