Exercise Sheet 06

Task 1

a) $$r_{ik} = \frac{\frac{1}{K}\frac{1}{\sqrt{2\pi}} e^{-\frac{\|x_i - \mu_k\|^2}{2}}}{\sum_j \frac{1}{K}\frac{1}{\sqrt{2\pi}} e^{-\frac{\|x_i-\mu_j\|^2}{2}}}$$

$$= \frac{e^{\frac{-\|x_i-\mu_k\|^2}{2}}}{\underbrace{\sum_j e^{\frac{-\|x_i-\mu_j\|^2}{2}}}_{=: C_i}}$$

b) $$r_{ik} = \frac{e^{\frac{-\|x_i-\mu_k\|^2}{2}}}{C_i}$$

$$\log r_{ik} = \frac{-\|x_i-\mu_k\|^2}{2} - C_i$$

This is basically just the squared distance between x_i and μ_k

c) $$\mu_k = \frac{\sum_{i=1}^{I} r_{ik} x_i}{\sum_{i=1}^{I} r_{ik}} = \frac{\sum_{i \in \{i \mid r_{ik} = \arg\max_{k'} r_{ik'}\}} x_i}{|\{i \mid r_{ik} = \arg\max_{k'} r_{ik'}\}|}$$

$\Rightarrow \mu_k$ is assigned to the center of all x_i where r_{ik} is maximum

b) $\Rightarrow$ i.e $\max_{k'} r_{ik'} = -\min_{k'} \log r_{ik'} = \min_{k'} \|x_i - \mu_k\|^2$

$\Rightarrow \mu_k$ is assigned to the center of all x_i which are closer to μ_k than to all other $\mu_{k'}$.

$\Rightarrow$ This is exactly <u>k-means</u>