## CV-Sheet 01

**Task 3: Convolutions are associative**

Proof:

$$((f*g)*h)(x) = \int (f*g)(x-u)\, h(u)\, du$$

$$= \int \int f(x-u-u')\, g(u')\, du'\, h(u)\, du$$

h(u) as const. factor

$$\overset{\#}{=} \int \int f(x-u-u')\, g(u')\, h(u)\, du'\, du$$

shift u' by $-u$ (possible as we integrate over $[-\infty, \infty]$)

$$= \int \int f(x-u')\, g(u'-u)\, h(u)\, du'\, du$$

change integration order

$$= \int \int f(x-u')\, g(u'-u)\, h(u)\, du\, du'$$

f(x-u') as constant factor

$$= \int f(x-u') \int g(u'-u)\, h(u)\, du\, du'$$

$$= \int f(x-u')\, (g*h)(u')\, du'$$

$$= (f*(g*h))\, x$$

**Task 6 Multiple Gaussians** // $\varphi_{0,\sigma}$ = pdf of Gaussian with $\mu = 0$, $\sigma = \sigma$

have to show that $\varphi_{0,\sigma} * \varphi_{0,\sigma} = \varphi_{0,\sqrt{2}\sigma}$

$$\varphi_{0,\sigma} * \varphi_{0,\sigma} = \int \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-u)^2}{2\sigma^2}} \cdot \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{\frac{-u^2}{2\sigma^2}}\, du$$

$$= \frac{1}{2\pi\sigma^2} \cdot \int e^{\frac{-x^2+2xu-u^2-u^2}{2\sigma^2}}\, du$$

$$= \frac{1}{2\pi\sigma^2} \cdot \int e^{-\frac{(u^2-xu+\frac{x^2}{2})}{\sigma^2}}\, du$$

$$= \frac{1}{2\pi\sigma^2} \cdot \int e^{\frac{-(u-\frac{1}{2}x)^2 - \frac{1}{4}x^2}{\sigma^2}}\, du$$

$$\cancel{= \frac{1}{2\pi\sigma^2} \cdot \underbrace{\int \varphi_{\frac{1}{2}x, \frac{\sigma}{2}}(u)\, du}_{\frac{1}{\sqrt{2\pi}\frac{\sigma}{2}} = \frac{\sqrt{2\pi}\, 2}{\sigma}}\, e^{\frac{x^2}{4\sigma^2}} \quad \frac{1}{\sqrt{2\pi}\frac{\sigma}{2}}\, e^{-\frac{x^2}{4\sigma^2}}}$$

$$\cancel{= \frac{1}{\sqrt{2\pi}}}$$

$$= \frac{1}{2\pi\sigma^2} \int e^{-\frac{(u-\frac{1}{2}x)^2}{\sigma^2}} du \; e^{-\frac{x^2}{4\sigma^2}}$$

$$= \frac{1}{2\pi\sigma^2} \frac{\sqrt{2\pi}\sigma}{\sqrt{2}} \underbrace{\int \varphi_{\frac{1}{2}x,\frac{\sigma}{\sqrt{2}}} du}_{=1} \; e^{-\frac{x^2}{4\sigma^2}}$$

$$= \frac{1}{2\pi\sigma^2} \frac{\sqrt{2\pi}\sigma}{\sqrt{2}} \; e^{-\frac{x^2}{4\sigma^2}}$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma\sqrt{2}} e^{\frac{-x^2}{4\sigma^2}} = \frac{1}{\sqrt{2\pi}\,\sqrt{2}\sigma} e^{-\frac{x^2}{2(\sqrt{2}\sigma)^2}}$$

$$= \varphi_{0,\sqrt{2}\sigma}(x)$$