```
import pandas as pd
import numpy as np
from IPython.display import display, Latex
import random
from statsmodels.tsa.ar_model import AutoReg
import matplotlib.pyplot as plt
import datetime
from sklearn.metrics import mean_squared_error
from scipy.fft import fft, ifft, fftfreq


import warnings
from statsmodels.tools.sm_exceptions import ConvergenceWarning,
ModelWarning
warnings.simplefilter('ignore', ConvergenceWarning)
warnings.simplefilter('ignore', ModelWarning)
warnings.simplefilter('ignore')

from statsmodels.stats.diagnostic import acorr_ljungbox
from statsmodels.tsa.stattools import adfuller, kpss
from statsmodels.tsa.stattools import acf, pacf
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf

class myAR:
    def __init__(self, c, phis):
        self.c = c
        self.phis = phis
        self.eps = 0.8
        self.burnIn = 0
        self.order = len(phis)
        self.y = []
        self.y_cut = []
        self.autoCor = None
        self.pAutoCor = None
        self.name = self.getEquationStr()

    def predictMany(self, initials, t):
        self.y = np.zeros(t)
        # set initials
        for i in range(len(initials)):
            self.y[i] = initials[i]

        # set the other elements
        for i in np.arange(len(initials), t):
            self.y[i] = self.c
            for idx, phi in enumerate(self.phis):
                self.y[i] += phi * self.y[i-(idx+1)]
            self.y[i] += np.random.normal(0, 1)

        # check autocorrelation
```

```
        _, ax = plt.subplots(4, figsize = (20,8))
        #ax[0].set_xlabel('time')

        timeCorr = 40#(len(self.y) - 1)//10
        self.autoCor = acf(self.y, nlags = timeCorr, fft = True)
        self.pAutoCor = pacf(self.y, nlags= timeCorr)

        # determine burnin
        self.burnin()

        plot_acf(self.y, ax=ax[1], lags = timeCorr, fft = True, zero =
False)
        plot_pacf(self.y, ax=ax[2], lags = timeCorr)

        ax[0].plot(self.y)
        ax[0].set_title(f'Autoregressive model ${self.name}$ with
T={len(self.y)} and init={initials}, burnIn={self.burnIn}')
        ax[3].plot(self.y_cut)
        ax[3].set_title(f'Burnin cut')


        #self.burnin()
    # http://sbfnk.github.io/mfiidd/mcmc_diagnostics.html#33_burn-in
    def tests(self, data):
        # jung - box - whether we are dealing with white noise
        # with hypothesis of stationary ts we look at p_value - close
to one - uncorrelated - up to lags
        jung = acorr_ljungbox(data, lags = [40], return_df=True)
        print("\n\t\t->jung_test (p_value close to 1 ->
uncorrelated):\n\n", jung)

        # augumented Dickey_Fuller test
        # wheather we are dealing with stationary or not
        fuller = adfuller(data) # timeseries in non-stationary
        print("\n\t\t->adfuller (p_value close to 0 -> stationary):\n\
n",fuller)
        # if p value is small - > it is stationary

        #kwiatkowski-phillips-s.. test - > hyphothesis is that it is
stationary -> p_value high -> stationary
        kp = kpss(data)
        print("\n\t\t->kpss (p_value close to 1 -> stationary):\n\
n",kp)

    def burnin(self):

        print("\n\n\t->test before:")
        self.tests(self.y)
        val = np.exp(-1/self.eps)
        for i in np.arange(len(self.autoCor)):
```

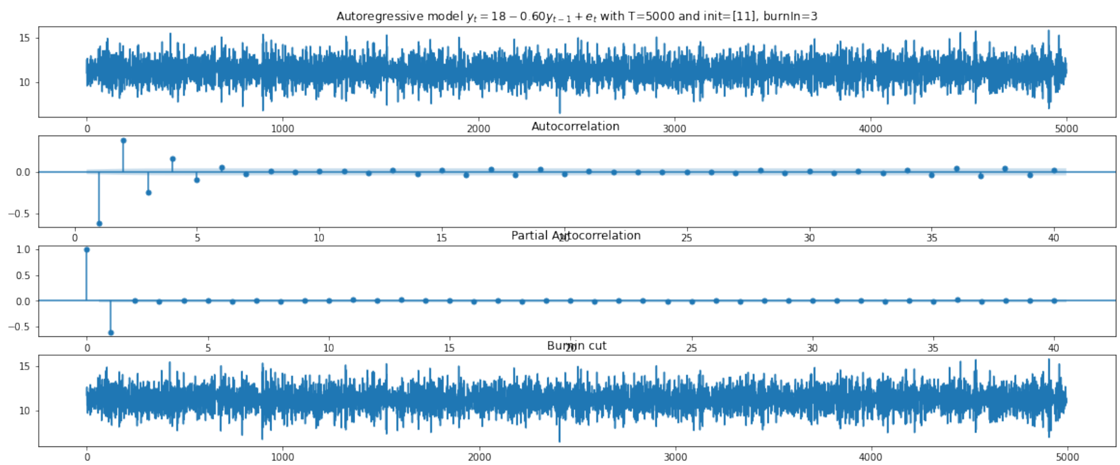
```
            autocor = abs(self.autoCor[i])
            if autocor < val:
                self.burnIn = i
                break
        self.y_cut = self.y[self.burnIn:-1]
        print("\n\n\t->test after:")
        self.tests(self.y_cut)
        print(f'\n\t->Burnin parameter is {self.burnIn}')

    def getEquationStr(self):
        eq = f'y_t={self.c}'
        inside = str([f'{"+" if val > 0 else ""}{val:.2f}'+'y_{t-%s}'%
(str(t+1)) for t, val in enumerate(self.phis)]).replace('\'[]',''')
        for k in ['\'', '[', ']']:
            inside = inside.replace(k,'')
        eq += inside
        eq += '+e_t'
        display(Latex(f" -------------------------------- Doing
AR({len(self.y)}): ${eq}$ -------------------------------- "))
        return  eq
```

first -0.6

```
c = 18
phi = [-0.6]
a = myAR(c, phi)
a.predictMany([random.randrange(-c,c)], 5000)
```

```
<IPython.core.display.Latex object>



     ->test before:

           ->jung_test (p_value close to 1 -> uncorrelated):

         lb_stat  lb_pvalue
40  3211.230687        0.0

           ->adfuller (p_value close to 0 -> stationary):

 (-144.52537996183133, 0.0, 0, 4999, {'1%': -3.431658793968827, '5%':
-2.862118345383404, '10%': -2.567077853953267}, 14069.713687391588)

           ->kpss (p_value close to 1 -> stationary):

 (0.043108636858788914, 0.1, 32, {'10%': 0.347, '5%': 0.463, '2.5%':
0.574, '1%': 0.739})
```

```
      ->test after:

            ->jung_test (p_value close to 1 -> uncorrelated):

        lb_stat  lb_pvalue
40  3211.87973        0.0

            ->adfuller (p_value close to 0 -> stationary):

 (-144.45806143359147, 0.0, 0, 4995, {'1%': -3.43165984259144, '5%': -2.8621188086591505, '10%': -2.5670781005730454}, 14061.586368662223)

            ->kpss (p_value close to 1 -> stationary):

 (0.04221812642800603, 0.1, 32, {'10%': 0.347, '5%': 0.463, '2.5%': 0.574, '1%': 0.739})

      ->Burnin parameter is 3
```

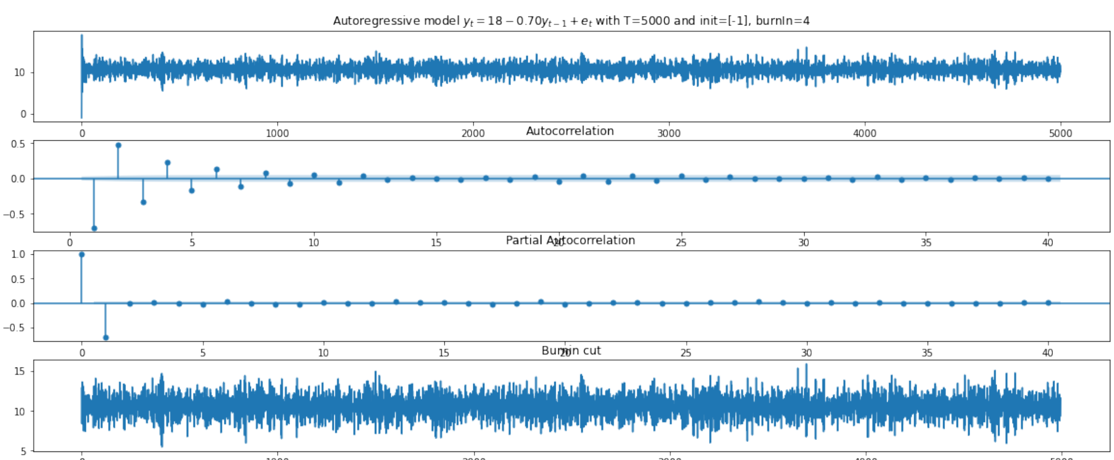


second -0.7

```python
c = 18
phi = [-0.7]

a = myAR(c, phi)
a.predictMany([random.randrange(-c,c)], 5000)
```

```
<IPython.core.display.Latex object>



      ->test before:

            ->jung_test (p_value close to 1 -> uncorrelated):
```

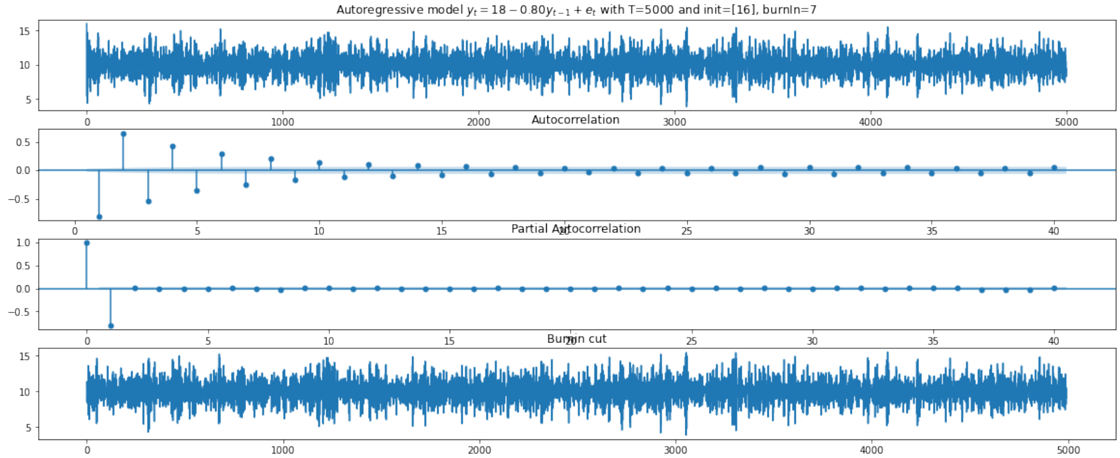
```
         lb_stat  lb_pvalue
40  4824.220559        0.0

           ->adfuller (p_value close to 0 -> stationary):

 (-169.149426439133, 0.0, 0, 4999, {'1%': -3.431658793968827, '5%': -
2.862118345383404, '10%': -2.567077853953267}, 14015.282534113492)

           ->kpss (p_value close to 1 -> stationary):

 (0.10384344131939145, 0.1, 32, {'10%': 0.347, '5%': 0.463, '2.5%':
0.574, '1%': 0.739})



      ->test after:

           ->jung_test (p_value close to 1 -> uncorrelated):

         lb_stat  lb_pvalue
40  4773.241158        0.0

           ->adfuller (p_value close to 0 -> stationary):

 (-166.86528207355198, 0.0, 0, 4994, {'1%': -3.4316601050096995, '5%':
-2.8621189245940792, '10%': -2.5670781622897416}, 13998.629022657227)

           ->kpss (p_value close to 1 -> stationary):

 (0.12432461632629062, 0.1, 32, {'10%': 0.347, '5%': 0.463, '2.5%':
0.574, '1%': 0.739})

      ->Burnin parameter is 4
```



third -0.8

```
c = 18
phi = [-0.8]


a = myAR(c, phi)
a.predictMany([random.randrange(-c,c)], 5000)
```

```
<IPython.core.display.Latex object>



     ->test before:

          ->jung_test (p_value close to 1 -> uncorrelated):

         lb_stat  lb_pvalue
40  10267.983847        0.0

          ->adfuller (p_value close to 0 -> stationary):

 (-218.14508815394817, 0.0, 0, 4999, {'1%': -3.431658793968827, '5%':
-2.862118345383404, '10%': -2.567077853953267}, 14138.656749769565)

          ->kpss (p_value close to 1 -> stationary):

 (0.034474461427187235, 0.1, 32, {'10%': 0.347, '5%': 0.463, '2.5%':
0.574, '1%': 0.739})


     ->test after:

          ->jung_test (p_value close to 1 -> uncorrelated):

         lb_stat  lb_pvalue
40  10321.521065        0.0

          ->adfuller (p_value close to 0 -> stationary):

 (-216.92445738095412, 0.0, 0, 4991, {'1%': -3.4316608928957413, '5%':
-2.862119272677694, '10%': -2.5670783475882715}, 14114.029034710054)

          ->kpss (p_value close to 1 -> stationary):

 (0.0364801198278775, 0.1, 32, {'10%': 0.347, '5%': 0.463, '2.5%':
0.574, '1%': 0.739})

     ->Burnin parameter is 7
```

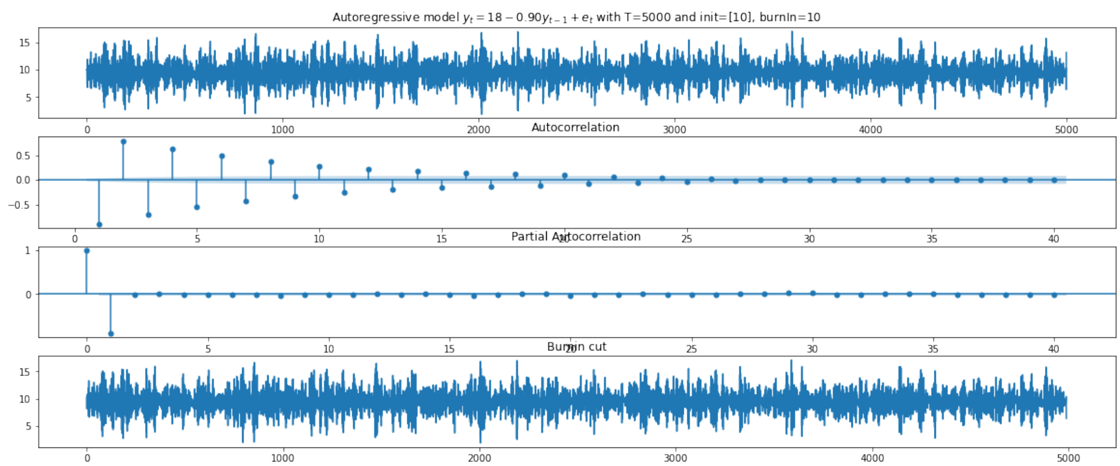


fourth -0.9

```
c = 18
phi = [-0.9]
a = myAR(c, phi)
a.predictMany([random.randrange(-c,c)], 5000)
```

```
<IPython.core.display.Latex object>



     ->test before:

          ->jung_test (p_value close to 1 -> uncorrelated):

         lb_stat  lb_pvalue
40  18334.815674        0.0

          ->adfuller (p_value close to 0 -> stationary):

 (-299.3606105589463, 0.0, 0, 4999, {'1%': -3.431658793968827, '5%': -
2.862118345383404, '10%': -2.567077853953267}, 14002.190328717637)

          ->kpss (p_value close to 1 -> stationary):

 (0.043390436993150 89, 0.1, 32, {'10%': 0.347, '5%': 0.463, '2.5%':
0.574, '1%': 0.739})



     ->test after:

          ->jung_test (p_value close to 1 -> uncorrelated):

         lb_stat  lb_pvalue
```

```
40  18329.607469       0.0

           ->adfuller (p_value close to 0 -> stationary):

 (-299.058368252791, 0.0, 0, 4988, {'1%': -3.43166168173001, '5%': -2.8621196211801374, '10%': -2.5670785331097763}, 13975.562157406333)

           ->kpss (p_value close to 1 -> stationary):

 (0.051361310735682765, 0.1, 32, {'10%': 0.347, '5%': 0.463, '2.5%': 0.574, '1%': 0.739})

      ->Burnin parameter is 10
```

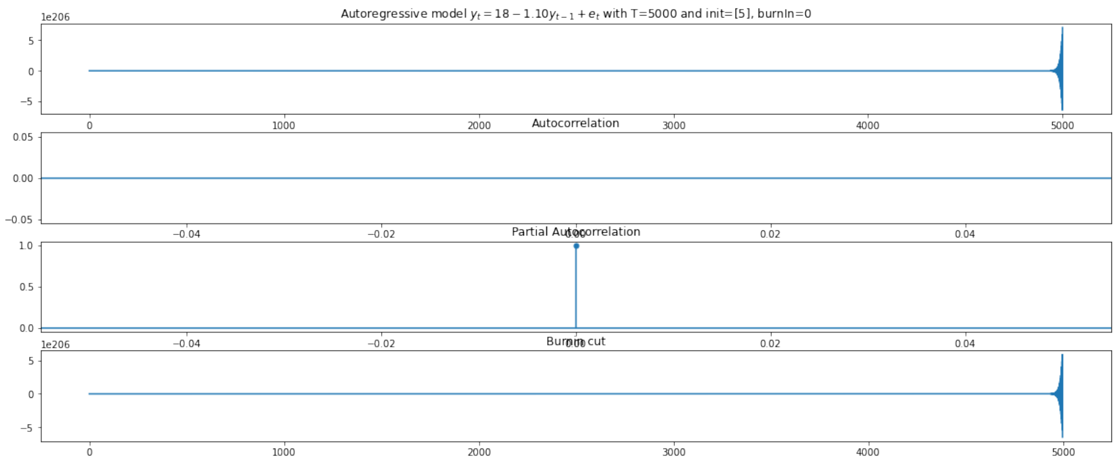


fifth -1.1

```
c = 18
phi = [-1.1]
a = myAR(c, phi)
a.predictMany([random.randrange(-c,c)], 5000)
```

```
<IPython.core.display.Latex object>



      ->test before:

           ->jung_test (p_value close to 1 -> uncorrelated):

     lb_stat  lb_pvalue
40      NaN        NaN

           ->adfuller (p_value close to 0 -> stationary):
```

```
 (nan, nan, 0, 4999, {'1%': -3.431658793968827, '5%': -
2.862118345383404, '10%': -2.567077853953267}, inf)

           ->kpss (p_value close to 1 -> stationary):

 (nan, nan, 32, {'10%': 0.347, '5%': 0.463, '2.5%': 0.574, '1%':
0.739})


     ->test after:

           ->jung_test (p_value close to 1 -> uncorrelated):

     lb_stat  lb_pvalue
40      NaN        NaN

           ->adfuller (p_value close to 0 -> stationary):

 (nan, nan, 0, 4998, {'1%': -3.431659055967043, '5%': -
2.862118461132801, '10%': -2.5670779155711902}, inf)

           ->kpss (p_value close to 1 -> stationary):

 (nan, nan, 32, {'10%': 0.347, '5%': 0.463, '2.5%': 0.574, '1%':
0.739})

     ->Burnin parameter is 0
```

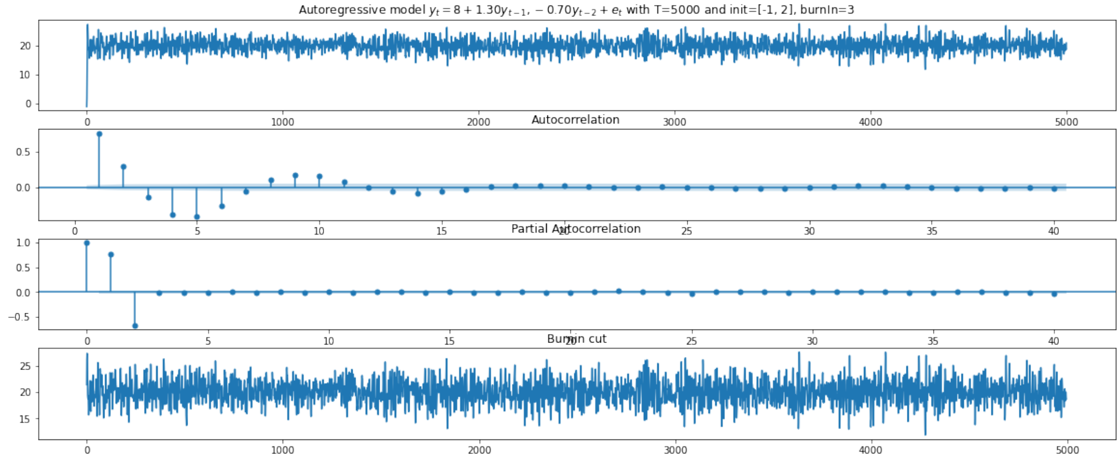


sixth AR(2) 8, 1.3, -0.7

```
c = 8
phi = [1.3, -0.7]
a = myAR(c, phi)
a.predictMany([random.randrange(-c,c), random.randrange(-c,c)], 5000)
```

```
<IPython.core.display.Latex object>
```

```
     ->test before:

          ->jung_test (p_value close to 1 -> uncorrelated):

        lb_stat  lb_pvalue
40  5804.937023        0.0

          ->adfuller (p_value close to 0 -> stationary):

 (-57.42899706589504, 0.0, 1, 4998, {'1%': -3.431659055967043, '5%': -
2.862118461132801, '10%': -2.5670779155711902}, 14174.173917762771)

          ->kpss (p_value close to 1 -> stationary):

 (0.3853920554235468, 0.0834517002484712, 32, {'10%': 0.347, '5%':
0.463, '2.5%': 0.574, '1%': 0.739})


     ->test after:

          ->jung_test (p_value close to 1 -> uncorrelated):

        lb_stat  lb_pvalue
40  5915.465597        0.0

          ->adfuller (p_value close to 0 -> stationary):

 (-56.652696987980036, 0.0, 1, 4994, {'1%': -3.4316601050096995, '5%':
-2.8621189245940792, '10%': -2.5670781622897416}, 14160.980452961647)

          ->kpss (p_value close to 1 -> stationary):

 (0.5743337910545769, 0.024969655358674823, 32, {'10%': 0.347, '5%':
0.463, '2.5%': 0.574, '1%': 0.739})

     ->Burnin parameter is 3
```

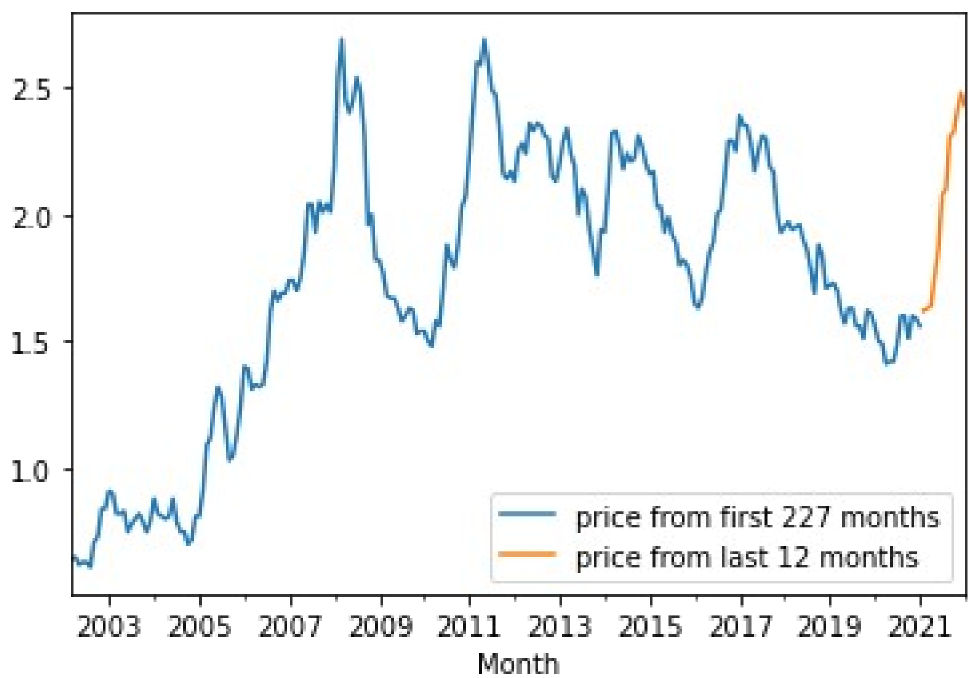


AR FORECASTING

```
from statsmodels.tsa.ar_model import AR, ARResults
def printAR(fit):
    print(f'Lag:\n\t{fit.k_ar}\ncoeff:\n{fit.params}')

def reverseDiff(df, diff, column, shift = 1):
    x, x_diff = df[column].iloc[0], diff.iloc[shift:]
    return pd.Series(np.r_[x, x_diff].cumsum())

shift = 1
maxlag = 20

robusta = pd.read_html('https://www.indexmundi.com/commodities/?
commodity=robusta-coffee&months=240')
robustaDF = robusta[1]
robustaDF['Month'] = pd.to_datetime(robustaDF['Month'])
robustaDF.set_index(['Month'], inplace = True)
robustaDF

            Price  Change
Month
2002-03-01   0.64       -
2002-04-01   0.65   1.56%
2002-05-01   0.62  -4.62%
2002-06-01   0.63   1.61%
2002-07-01   0.63   0.00%
...           ...     ...
2021-09-01   2.31  10.00%
2021-10-01   2.32   0.43%
2021-11-01   2.41   3.88%
2021-12-01   2.48   2.90%
2022-01-01   2.43  -2.02%

[239 rows x 2 columns]
```

Train-Test split

```
forecastLen = 12
trainLen = len(robustaDF)-forecastLen

trainRobusta = robustaDF.iloc[:trainLen]
testRobusta = robustaDF.iloc[trainLen:]
trainRobusta.columns = [f'price from first {len(robustaDF)-
forecastLen} months', 'Change']
testRobusta.columns = [f'price from last {forecastLen} months',
'Change']
ax = trainRobusta.plot()
testRobusta.plot(ax=ax, label = f'last {forecastLen} months', legend =
True)


columnTrain = trainRobusta.columns[0]
columnTest = testRobusta.columns[0]
```

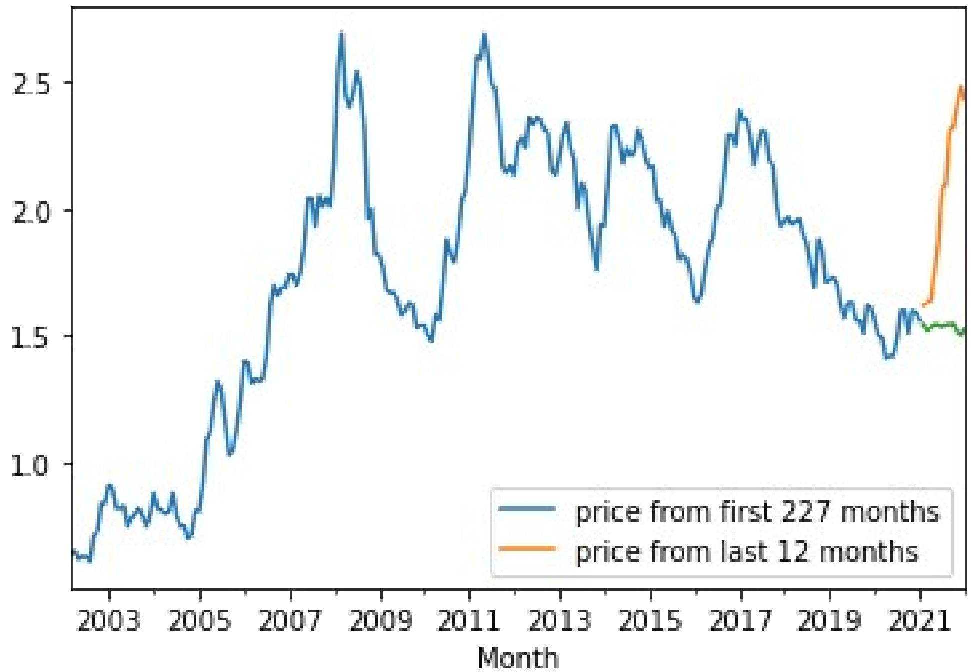


Non-differenced data

```
modelNon = AR(trainRobusta[columnTrain])
# fit
fitNon = modelNon.fit(maxlag = maxlag, method = 'mle')
printAR(fitNon)
# predict
start = len(trainRobusta)
end = len(trainRobusta) + len(testRobusta) - 1
predNon = fitNon.predict(start = start, end = end, dynamic =
False).rename('Non differenced data')
```

```
ax = trainRobusta.plot()
testRobusta.plot(ax=ax)
predNon.plot(ax=ax)
accNon = mean_squared_error(testRobusta[columnTest].to_numpy(),
predNon.to_numpy())
print(f"The accuracy is - mse = {accNon}")
```

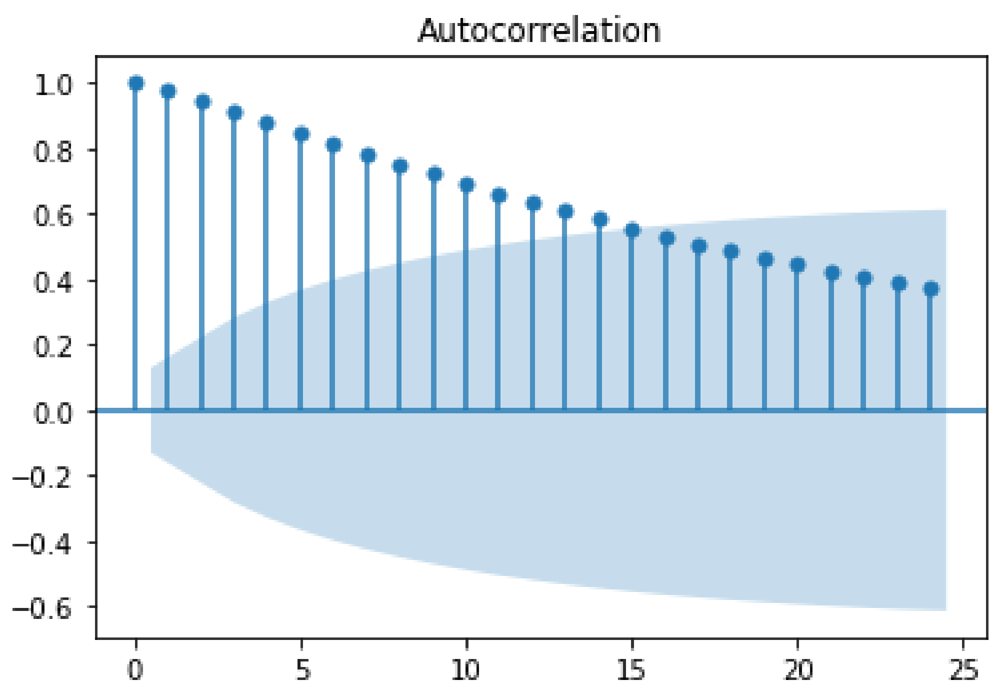
```
Lag:
     20
coeff:
const                              0.024078
L1.price from first 227 months     1.250700
L2.price from first 227 months    -0.263838
L3.price from first 227 months     0.008547
L4.price from first 227 months     0.006578
L5.price from first 227 months     0.012330
L6.price from first 227 months    -0.012362
L7.price from first 227 months    -0.064562
L8.price from first 227 months     0.066119
L9.price from first 227 months     0.042034
L10.price from first 227 months   -0.062886
L11.price from first 227 months   -0.069081
L12.price from first 227 months    0.064803
L13.price from first 227 months    0.014477
L14.price from first 227 months    0.087147
L15.price from first 227 months   -0.233665
L16.price from first 227 months    0.083275
L17.price from first 227 months    0.141037
L18.price from first 227 months   -0.160210
L19.price from first 227 months    0.172394
L20.price from first 227 months   -0.098393
dtype: float64
The accuracy is - mse = 0.3790813009420537
```

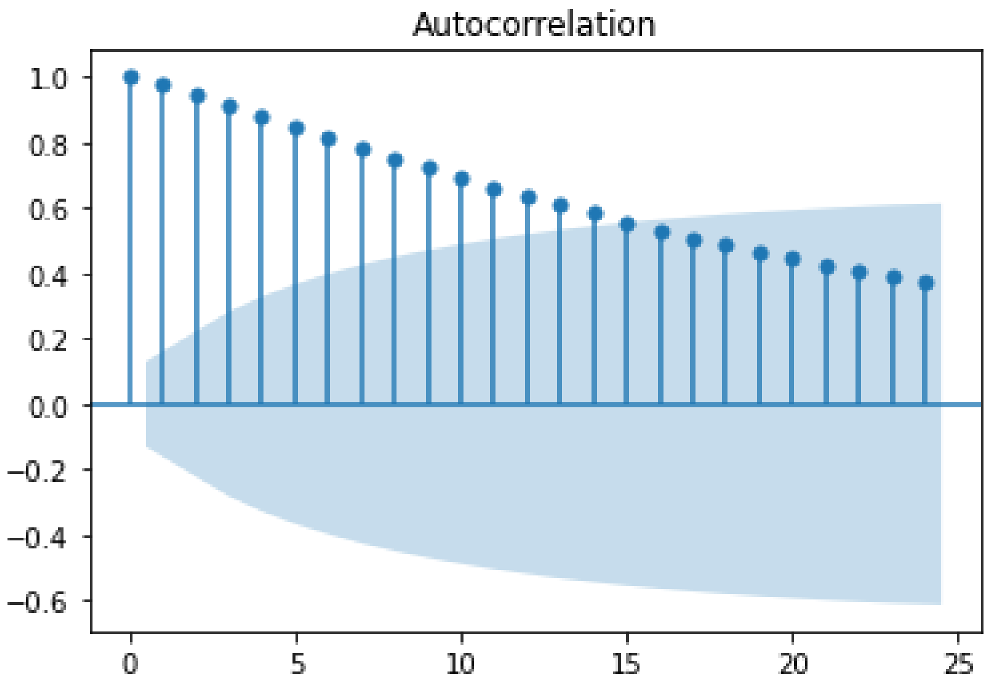


Differenced data

check the autocorrelation

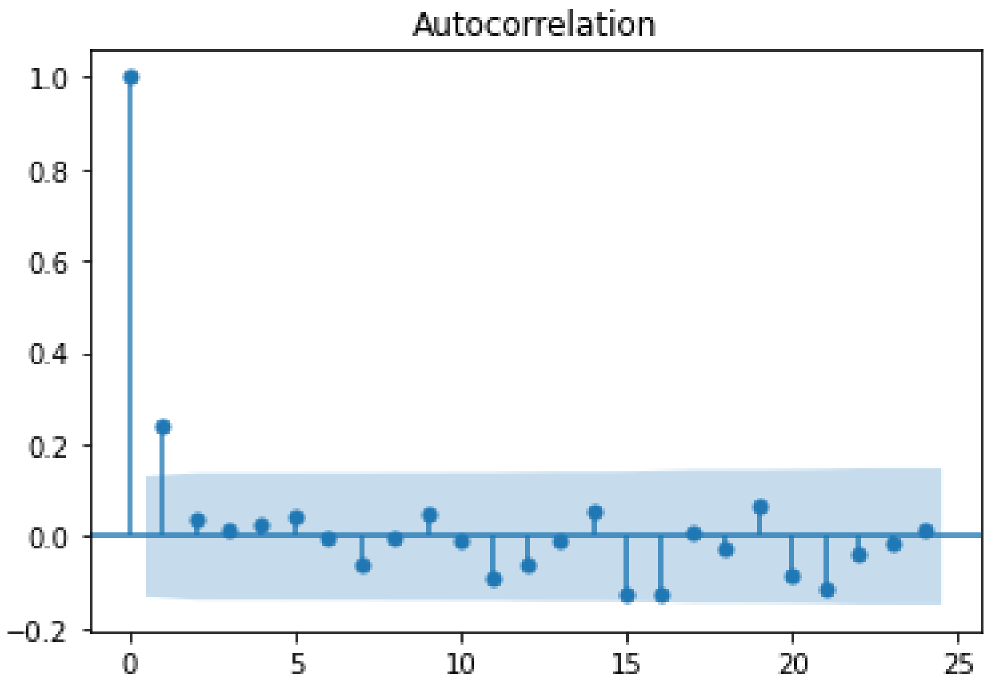
```
plot_acf(trainRobusta[columnTrain])
```





create difference

```
trainRobustaDiff = trainRobusta[columnTrain].diff(shift).dropna()
plot_acf(trainRobustaDiff)
trainRobustaDiff

Month
2002-04-01    0.01
2002-05-01   -0.03
2002-06-01    0.01
2002-07-01    0.00
2002-08-01   -0.02
              ...
2020-09-01    0.00
2020-10-01   -0.09
2020-11-01    0.09
2020-12-01   -0.01
2021-01-01   -0.03
Name: price from first 227 months, Length: 226, dtype: float64
```



fit again

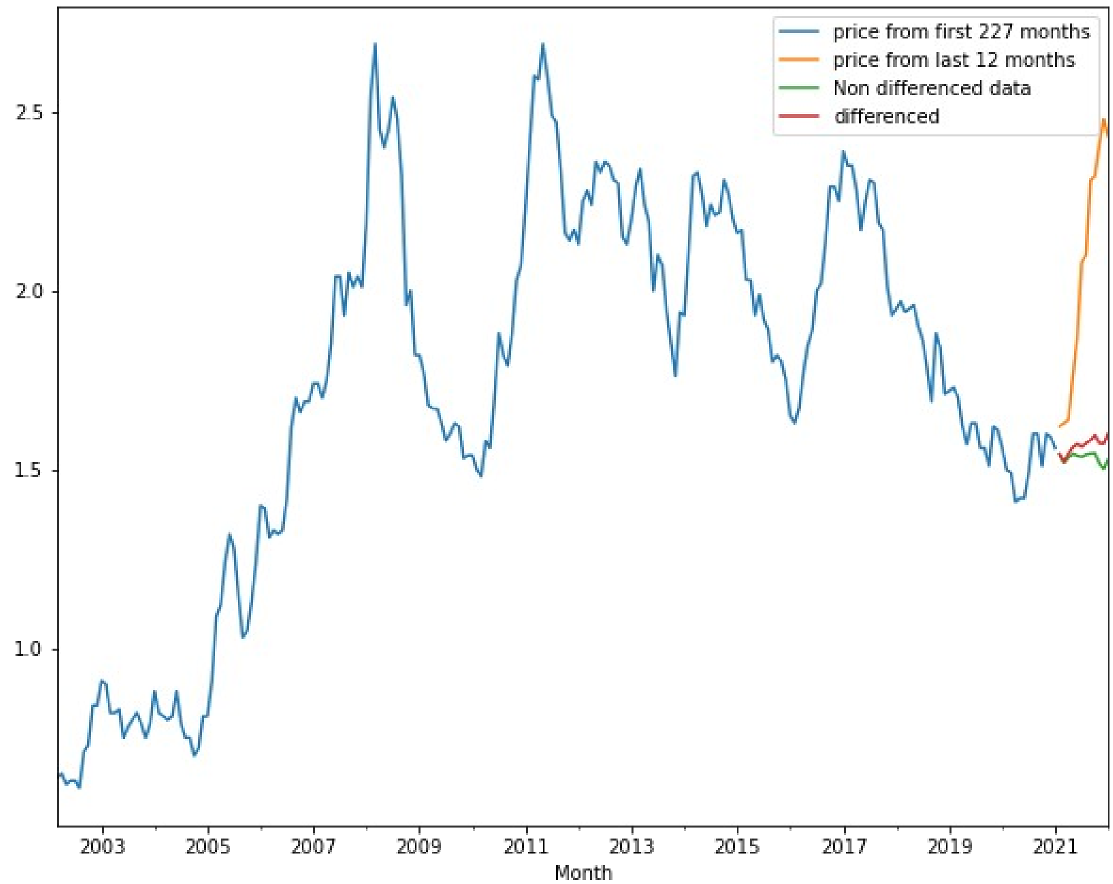
```
model = AR(trainRobustaDiff.to_numpy())
# fit
fit = model.fit(maxlag = maxlag, method = 'mle')
printAR(fit)
# predict
start = len(trainRobustaDiff) #+ diff
end = len(trainRobustaDiff) + len(testRobusta) - 1
pred = fit.predict(start = start, end = end, dynamic = False)
pred = pd.Series(pred, index = testRobusta.index)

fullDifferencedPred =
trainRobusta[columnTrain].diff(shift).append(pred, ignore_index=True)
# return to non-differenced data
diffBack = reverseDiff(trainRobusta,
trainRobusta[columnTrain].diff(shift), columnTrain, shift)
predReal = reverseDiff(trainRobusta, fullDifferencedPred, columnTrain,
shift)

full = pd.DataFrame(columns=['real','diff', 'back'])
full['diff'] = trainRobustaDiff
full['real'] = trainRobusta[columnTrain].iloc[shift:]
full['back'] = diffBack.iloc[shift:].to_numpy()

#"print(predReal)
predReal = pd.Series(predReal[shift:].to_numpy(), index =
robustaDF.index[1:])
predRealTest = pd.DataFrame(columns=['pred','real'])
```

```
predRealTest['differenced'] = predReal.to_numpy()
predRealTest['real'] = robustaDF['Price'].iloc[shift:].to_numpy()
predRealTest.index = robustaDF.index[shift:]
predRealTest
```

```
Lag:
      20
coeff:
[ 0.00373972  0.26837138 -0.01840859  0.00919475 -0.00051124
0.0018554
  0.01896558 -0.05591133  0.00854913  0.04430708 -0.01044202 -
0.07185736
 -0.01494307 -0.00477558  0.08858318 -0.14371465 -0.0616476
0.07925602
 -0.08342745  0.12026067 -0.11652976]

           pred  real  differenced
Month
2002-04-01  NaN  0.65     0.650000
2002-05-01  NaN  0.62     0.620000
2002-06-01  NaN  0.63     0.630000
2002-07-01  NaN  0.63     0.630000
2002-08-01  NaN  0.61     0.610000
...         ...   ...          ...
2021-09-01  NaN  2.31     1.582145
2021-10-01  NaN  2.32     1.596471
2021-11-01  NaN  2.41     1.572697
2021-12-01  NaN  2.48     1.571715
2022-01-01  NaN  2.43     1.600881

[238 rows x 3 columns]
```

```
# compare all predictions
fig, ax = plt.subplots(1, figsize = (10,8))
# plot just the training
trainRobusta[columnTrain].plot(ax = ax)
# plot real data from last part
testRobusta[columnTest].plot(ax = ax)
#ax.plot(testRobusta.index, np.array(testRobusta[columnTest]), label =
'original data')
# plot non-differenced prediction
predNon.plot(ax=ax)
# plot differenced prediction
#ax.plot(testRobusta.index, predNon.diff(), label = 'non-differenced
data predicted')
predRealTest.iloc[trainLen-shift:]['differenced'].plot(ax=ax)
#ax.plot(testRobusta.index, pred, label = 'differenced data
predicted')
ax.legend()
plt.show()
```

```
#accNon =
mean_squared_error(testRobusta[columnTest].diff().to_numpy(),
predNon.to_numpy())
print(f"The accuracy is - mse = {accNon}")
#testRobusta.index, pred
```




```
The accuracy is - mse = 0.3790813009420537
```