In [1]:
```
import pandas as pd
import numpy as np

import warnings
warnings.filterwarnings("ignore")
```

In [2]:
```
from statsmodels.tsa.statespace.sarimax import SARIMAX
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from statsmodels.tsa.seasonal import seasonal_decompose
from pmdarima import auto_arima
```

In [3]:
```
from statsmodels.stats.diagnostic import acorr_ljungbox
from statsmodels.tsa.stattools import adfuller, kpss,grangercausalitytests
```

In [4]:
```
from sklearn.metrics import mean_absolute_percentage_error
```

In [5]:
```
df=pd.read_csv('us_data.csv',sep='\t')
df
```

Out[5]:

	Quarter	Consumption	Income	Production	Savings	Unemployment
1	1970 Q1	0.618566	1.044801	-2.452486	5.299014	0.9
2	1970 Q2	0.451984	1.225647	-0.551459	7.789894	0.5
3	1970 Q3	0.872872	1.585154	-0.358652	7.403984	0.5
4	1970 Q4	-0.271848	-0.239545	-2.185691	1.169898	0.7
5	1971 Q1	1.901345	1.975925	1.909764	3.535667	-0.1
...	...	...	...	...	...	...
194	2018 Q2	0.983112	0.661825	1.117424	-2.723974	0.0
195	2018 Q3	0.853181	0.806271	1.256722	-0.085686	-0.3
196	2018 Q4	0.356512	0.695142	0.948148	5.031337	0.2
197	2019 Q1	0.282885	1.100753	-0.488206	9.760287	-0.1
198	2019 Q2	1.113517	0.593399	-0.539949	-4.264616	-0.1

198 rows × 6 columns

In [6]:
```
df['date']=pd.to_datetime(df['Quarter'].str.replace(' ',''))
df.set_index('date',inplace=True)
df.index.freq='QS-JAN'
df.drop('Quarter',axis=1,inplace=True)
df
```

Out[6]:

date	Consumption	Income	Production	Savings	Unemployment
1970-01-01	0.618566	1.044801	-2.452486	5.299014	0.9
1970-04-01	0.451984	1.225647	-0.551459	7.789894	0.5
1970-07-01	0.872872	1.585154	-0.358652	7.403984	0.5
1970-10-01	-0.271848	-0.239545	-2.185691	1.169898	0.7
1971-01-01	1.901345	1.975925	1.909764	3.535667	-0.1
...	...	...	...	...	...
2018-04-01	0.983112	0.661825	1.117424	-2.723974	0.0
2018-07-01	0.853181	0.806271	1.256722	-0.085686	-0.3
2018-10-01	0.356512	0.695142	0.948148	5.031337	0.2
2019-01-01	0.282885	1.100753	-0.488206	9.760287	-0.1
2019-04-01	1.113517	0.593399	-0.539949	-4.264616	-0.1

198 rows × 5 columns

In [7]:

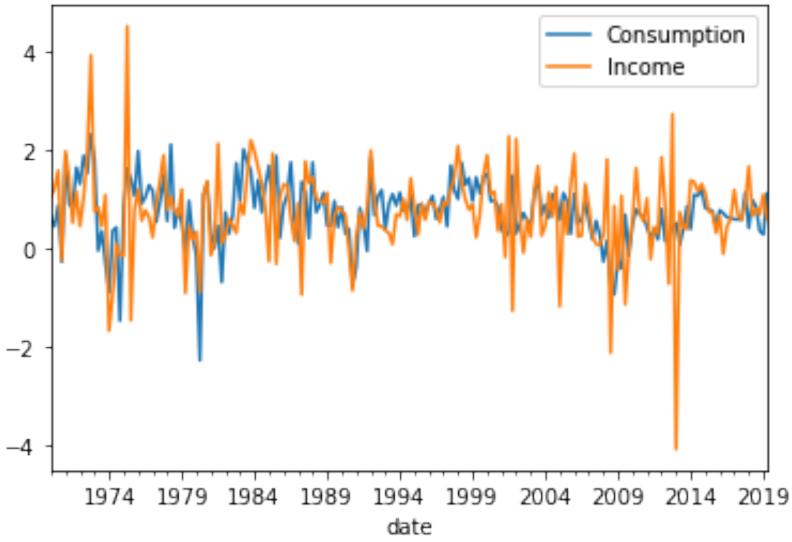
```
df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
DatetimeIndex: 198 entries, 1970-01-01 to 2019-04-01
Freq: QS-JAN
Data columns (total 5 columns):
 #   Column        Non-Null Count  Dtype
---  ------        --------------  -----
 0   Consumption   198 non-null    float64
 1   Income        198 non-null    float64
 2   Production    198 non-null    float64
 3   Savings       198 non-null    float64
 4   Unemployment  198 non-null    float64
dtypes: float64(5)
memory usage: 9.3 KB
```

In [8]:

```
df[['Consumption','Income']].plot(legend=True)
```

Out[8]: <AxesSubplot:xlabel='date'>



In [9]:
```
adfuller(df['Consumption'])
```

Out[9]:
```
(-4.6101231449022055,
 0.00012373240927130436,
 2,
 195,
 {'1%': -3.464337030867007,
  '5%': -2.876478799035722,
  '10%': -2.574733103221565},
 306.84324181648583)
```

In [10]:
```
adfuller(df['Income'])
```

Out[10]:
```
(-6.7656134181274,
 2.722059468837914e-09,
 4,
 193,
 {'1%': -3.4646940755442612,
  '5%': -2.8766348847254934,
  '10%': -2.5748163958763994},
 469.11064516885546)
```

In [12]:
```
grangercausalitytests(df[['Consumption','Income']],maxlag=4)
```

```
Granger Causality
number of lags (no zero) 1
ssr based F test:         F=4.8225  , p=0.0293  , df_denom=194, df_num=1
ssr based chi2 test:   chi2=4.8970  , p=0.0269  , df=1
likelihood ratio test: chi2=4.8372  , p=0.0279  , df=1
parameter F test:         F=4.8225  , p=0.0293  , df_denom=194, df_num=1

Granger Causality
number of lags (no zero) 2
ssr based F test:         F=1.6841  , p=0.1884  , df_denom=191, df_num=2
ssr based chi2 test:   chi2=3.4563  , p=0.1776  , df=2
```

```
likelihood ratio test: chi2=3.4262  , p=0.1803  , df=2
parameter F test:         F=1.6841  , p=0.1884  , df_denom=191, df_num=2

Granger Causality
number of lags (no zero) 3
ssr based F test:         F=1.3557  , p=0.2578  , df_denom=188, df_num=3
ssr based chi2 test:   chi2=4.2186  , p=0.2388  , df=3
likelihood ratio test: chi2=4.1736  , p=0.2433  , df=3
parameter F test:         F=1.3557  , p=0.2578  , df_denom=188, df_num=3

Granger Causality
number of lags (no zero) 4
ssr based F test:         F=1.6619  , p=0.1607  , df_denom=185, df_num=4
ssr based chi2 test:   chi2=6.9711  , p=0.1374  , df=4
likelihood ratio test: chi2=6.8488  , p=0.1441  , df=4
parameter F test:         F=1.6619  , p=0.1607  , df_denom=185, df_num=4
```

Out[12]:
```
{1: ({'ssr_ftest': (4.822459130803696, 0.029276853643505723, 194.0, 1),
   'ssr_chi2test': (4.897033241073856, 0.026902876229972755, 1),
   'lrtest': (4.837158147802484, 0.027852758674570 1, 1),
   'params_ftest': (4.822459130803681, 0.029276853643505945, 194.0, 1.0)},
  [<statsmodels.regression.linear_model.RegressionResultsWrapper at 0x7fac2
0443d30>,
   <statsmodels.regression.linear_model.RegressionResultsWrapper at 0x7fac2
0443b50>,
   array([[0., 1., 0.]])]),
 2: ({'ssr_ftest': (1.6840657336352534, 0.18836203572157817, 191.0, 2),
   'ssr_chi2test': (3.4563024480891067, 0.17761247228772578, 2),
   'lrtest': (3.426181480907303, 0.18030764638208824, 2),
   'params_ftest': (1.6840657336352338, 0.18836203572157892, 191.0, 2.0)},
  [<statsmodels.regression.linear_model.RegressionResultsWrapper at 0x7fac2
0a40ca0>,
   <statsmodels.regression.linear_model.RegressionResultsWrapper at 0x7fac2
0a408b0>,
   array([[0., 0., 1., 0., 0.],
          [0., 0., 0., 1., 0.]])]),
 3: ({'ssr_ftest': (1.3557106457032684, 0.2577711331691011, 188.0, 3),
   'ssr_chi2test': (4.218567700725596, 0.23880946864445363, 3),
   'lrtest': (4.173583750414082, 0.24331999598530307, 3),
   'params_ftest': (1.3557106457032813, 0.2577711331690909, 188.0, 3.0)},
  [<statsmodels.regression.linear_model.RegressionResultsWrapper at 0x7fac2
0a40a60>,
   <statsmodels.regression.linear_model.RegressionResultsWrapper at 0x7fac2
0a40a30>,
   array([[0., 0., 0., 1., 0., 0., 0.],
          [0., 0., 0., 0., 1., 0., 0.],
          [0., 0., 0., 0., 0., 1., 0.]])]),
 4: ({'ssr_ftest': (1.6619230259179782, 0.16067012220182084, 185.0, 4),
   'ssr_chi2test': (6.971093341147844, 0.137423718021706, 4),
   'lrtest': (6.848767339487097, 0.14409929266071153, 4),
   'params_ftest': (1.6619230259179827, 0.16067012220181998, 185.0, 4.0)},
  [<statsmodels.regression.linear_model.RegressionResultsWrapper at 0x7fac2
0a39c10>,
   <statsmodels.regression.linear_model.RegressionResultsWrapper at 0x7fac2
0a39d60>,
   array([[0., 0., 0., 0., 1., 0., 0., 0., 0.],
          [0., 0., 0., 0., 0., 1., 0., 0., 0.],
          [0., 0., 0., 0., 0., 0., 1., 0., 0.],
          [0., 0., 0., 0., 0., 0., 0., 1., 0.]])])}
```

In [13]:
```python
grangercausalitytests(df[['Income','Consumption']],maxlag=4)
```

```
Granger Causality
number of lags (no zero) 1
ssr based F test:         F=22.7569 , p=0.0000  , df_denom=194, df_num=1
ssr based chi2 test:   chi2=23.1088 , p=0.0000  , df=1
likelihood ratio test: chi2=21.8509 , p=0.0000  , df=1
parameter F test:         F=22.7569 , p=0.0000  , df_denom=194, df_num=1

Granger Causality
number of lags (no zero) 2
ssr based F test:         F=9.6515  , p=0.0001  , df_denom=191, df_num=2
ssr based chi2 test:   chi2=19.8083 , p=0.0000  , df=2
```

```
likelihood ratio test: chi2=18.8701 , p=0.0001  , df=2
parameter F test:         F=9.6515  , p=0.0001  , df_denom=191, df_num=2

Granger Causality
number of lags (no zero) 3
ssr based F test:         F=9.8899  , p=0.0000  , df_denom=188, df_num=3
ssr based chi2 test:   chi2=30.7745 , p=0.0000  , df=3
likelihood ratio test: chi2=28.5747 , p=0.0000  , df=3
parameter F test:         F=9.8899  , p=0.0000  , df_denom=188, df_num=3

Granger Causality
number of lags (no zero) 4
ssr based F test:         F=8.9666  , p=0.0000  , df_denom=185, df_num=4
ssr based chi2 test:   chi2=37.6113 , p=0.0000  , df=4
likelihood ratio test: chi2=34.3773 , p=0.0000  , df=4
parameter F test:         F=8.9666  , p=0.0000  , df_denom=185, df_num=4
```

Out[13]:

```
{1: ({'ssr_ftest': (22.756900160084378, 3.606706940851373e-06, 194.0, 1),
   'ssr_chi2test': (23.108810987302178, 1.5308757911111413e-06, 1),
   'lrtest': (21.850903275624944, 2.946778497402558e-06, 1),
   'params_ftest': (22.756900160084353, 3.6067069408514184e-06, 194.0, 1.0)
},
  [<statsmodels.regression.linear_model.RegressionResultsWrapper at 0x7fac2
0443e50>,
   <statsmodels.regression.linear_model.RegressionResultsWrapper at 0x7fac2
0a19df0>,
   array([[0., 1., 0.]])]),
 2: ({'ssr_ftest': (9.6514885484608, 0.00010161291146155666, 191.0, 2),
   'ssr_chi2test': (19.80829063349023, 4.9967122603523374e-05, 2),
   'lrtest': (18.870058954110846, 7.98764511990486e-05, 2),
   'params_ftest': (9.651488548460769, 0.00010161291146156023, 191.0, 2.0)}
,
  [<statsmodels.regression.linear_model.RegressionResultsWrapper at 0x7fac2
0a19b20>,
   <statsmodels.regression.linear_model.RegressionResultsWrapper at 0x7fac2
0a19f10>,
   array([[0., 0., 1., 0., 0.],
          [0., 0., 0., 1., 0.]])]),
 3: ({'ssr_ftest': (9.889918695760983, 4.361586242578561e-06, 188.0, 3),
   'ssr_chi2test': (30.774481047979656, 9.482496572962478e-07, 3),
   'lrtest': (28.574728851674877, 2.7509226577650864e-06, 3),
   'params_ftest': (9.889918695760993, 4.361586242578506e-06, 188.0, 3.0)},
  [<statsmodels.regression.linear_model.RegressionResultsWrapper at 0x7fac2
0a19670>,
   <statsmodels.regression.linear_model.RegressionResultsWrapper at 0x7fac2
0a40910>,
   array([[0., 0., 0., 1., 0., 0., 0.],
          [0., 0., 0., 0., 1., 0., 0.],
          [0., 0., 0., 0., 0., 1., 0.]])]),
 4: ({'ssr_ftest': (8.966609208862248, 1.2192036448451902e-06, 185.0, 4),
   'ssr_chi2test': (37.611290519335704, 1.3477242139193173e-07, 4),
   'lrtest': (34.37725309296604, 6.235542525071482e-07, 4),
   'params_ftest': (8.966609208862259, 1.2192036448451765e-06, 185.0, 4.0)}
,
  [<statsmodels.regression.linear_model.RegressionResultsWrapper at 0x7fac2
0a40820>,
   <statsmodels.regression.linear_model.RegressionResultsWrapper at 0x7fac2
0a401c0>,
   array([[0., 0., 0., 0., 1., 0., 0., 0., 0.],
          [0., 0., 0., 0., 0., 1., 0., 0., 0.],
          [0., 0., 0., 0., 0., 0., 1., 0., 0.],
          [0., 0., 0., 0., 0., 0., 0., 1., 0.]])])}
```

In [31]:

```
stepwise_fit=auto_arima(df['Production'],seasonal=False,d=0,stepwise=True,t
```

```
Performing stepwise search to minimize aic
 ARIMA(2,0,2)(0,0,0)[0]             : AIC=655.018, Time=0.07 sec
 ARIMA(0,0,0)(0,0,0)[0]             : AIC=750.017, Time=0.00 sec
 ARIMA(1,0,0)(0,0,0)[0]             : AIC=653.529, Time=0.01 sec
 ARIMA(0,0,1)(0,0,0)[0]             : AIC=667.267, Time=0.01 sec
 ARIMA(2,0,0)(0,0,0)[0]             : AIC=654.123, Time=0.02 sec
 ARIMA(1,0,1)(0,0,0)[0]             : AIC=653.232, Time=0.01 sec
 ARIMA(2,0,1)(0,0,0)[0]             : AIC=653.562, Time=0.04 sec
 ARIMA(1,0,2)(0,0,0)[0]             : AIC=653.448, Time=0.02 sec
 ARIMA(0,0,2)(0,0,0)[0]             : AIC=659.042, Time=0.01 sec
 ARIMA(1,0,1)(0,0,0)[0] intercept   : AIC=649.179, Time=0.03 sec
 ARIMA(0,0,1)(0,0,0)[0] intercept   : AIC=657.083, Time=0.02 sec
 ARIMA(1,0,0)(0,0,0)[0] intercept   : AIC=650.970, Time=0.02 sec
 ARIMA(2,0,1)(0,0,0)[0] intercept   : AIC=650.095, Time=0.09 sec
 ARIMA(1,0,2)(0,0,0)[0] intercept   : AIC=650.303, Time=0.04 sec
 ARIMA(0,0,0)(0,0,0)[0] intercept   : AIC=731.058, Time=0.01 sec
 ARIMA(0,0,2)(0,0,0)[0] intercept   : AIC=652.145, Time=0.03 sec
 ARIMA(2,0,0)(0,0,0)[0] intercept   : AIC=650.427, Time=0.02 sec
 ARIMA(2,0,2)(0,0,0)[0] intercept   : AIC=651.946, Time=0.10 sec

Best model:  ARIMA(1,0,1)(0,0,0)[0] intercept
Total fit time: 0.551 seconds
```

In [15]:

```
model=SARIMAX(df['Consumption'],order=(3,0,0),seasonal_order=(0,0,0,0))
results=model.fit()
results.summary()
```

```
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.02161D-01    |proj g|=  1.80612D-02

At iterate    5    f=  9.02121D-01    |proj g|=  9.08667D-04

At iterate   10    f=  9.02114D-01    |proj g|=  4.07846D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     10     12      1     0     0   4.078D-06   9.021D-01
```

```
  F =  0.90211402899708149

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
 This problem is unconstrained.
```

Out[15]:

SARIMAX Results

Dep. Variable:	Consumption	**No. Observations:**	198
Model:	SARIMAX(3, 0, 0)	**Log Likelihood**	-178.619
Date:	Wed, 20 Apr 2022	**AIC**	365.237
Time:	09:57:07	**BIC**	378.390
Sample:	01-01-1970	**HQIC**	370.561
	- 04-01-2019		
Covariance Type:	opg		

	coef	std err	z	P>|z|	[0.025	0.975]
ar.L1	0.3011	0.048	6.323	0.000	0.208	0.394
ar.L2	0.2558	0.051	5.057	0.000	0.157	0.355
ar.L3	0.3229	0.056	5.729	0.000	0.212	0.433
sigma2	0.3533	0.025	13.938	0.000	0.304	0.403

Ljung-Box (L1) (Q):	0.16	**Jarque-Bera (JB):**	50.94
Prob(Q):	0.69	**Prob(JB):**	0.00
Heteroskedasticity (H):	0.20	**Skew:**	-0.27
Prob(H) (two-sided):	0.00	**Kurtosis:**	5.43

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).

In [16]:

```
train=df.iloc[:-4]
test=df.iloc[-4:]
```

In [17]:

```
model=SARIMAX(train['Consumption'],order=(3,0,0),seasonal_order=(0,0,0,0))
results=model.fit()
results.summary()
```

```
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10
```

```
At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.07706D-01    |proj g|=  1.84414D-02

At iterate    5    f=  9.07663D-01    |proj g|=  9.62669D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4      9     11      1     0     0   5.100D-06   9.077D-01
  F =  0.90765521326478893

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
 This problem is unconstrained.
```

Out[17]:

SARIMAX Results

Dep. Variable:	Consumption	**No. Observations:**	194
Model:	SARIMAX(3, 0, 0)	**Log Likelihood**	-176.085
Date:	Wed, 20 Apr 2022	**AIC**	360.170
Time:	09:59:12	**BIC**	373.242
Sample:	01-01-1970	**HQIC**	365.463
	- 04-01-2018		
Covariance Type:	opg		

	coef	std err	z	P>|z|	[0.025	0.975]
ar.L1	0.3016	0.048	6.264	0.000	0.207	0.396
ar.L2	0.2604	0.051	5.092	0.000	0.160	0.361
ar.L3	0.3181	0.057	5.552	0.000	0.206	0.430
sigma2	0.3572	0.026	13.759	0.000	0.306	0.408

Ljung-Box (L1) (Q):	0.15	**Jarque-Bera (JB):**	49.65
Prob(Q):	0.70	**Prob(JB):**	0.00
Heteroskedasticity (H):	0.19	**Skew:**	-0.27
Prob(H) (two-sided):	0.00	**Kurtosis:**	5.42

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).

In [18]:

```
start=len(train)
end=start+len(test)-1
```

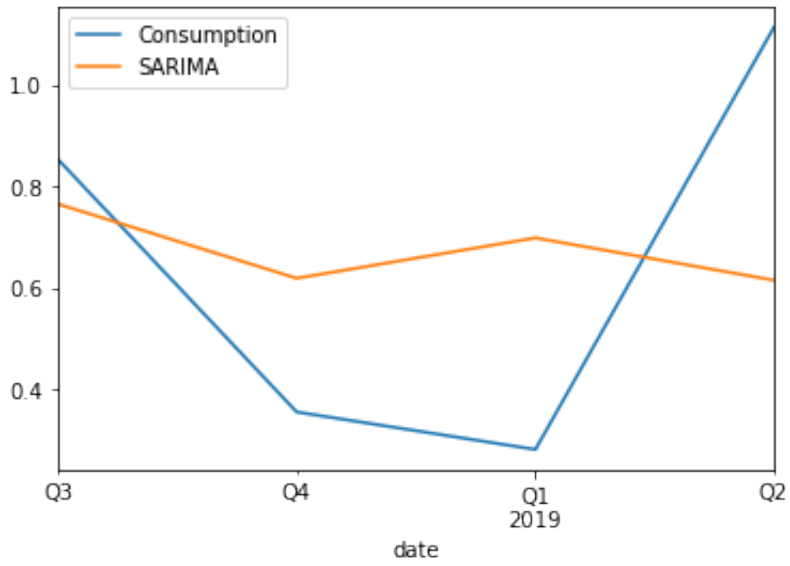
In [19]:

```
predictions=results.predict(start=start,end=end,dynamic=False).rename('SARI
```

In [22]:

```
ax=test['Consumption'].plot(legend=True)
predictions.plot(legend=True)
```

Out[22]: <AxesSubplot:xlabel='date'>



In [23]:
```
mean_absolute_percentage_error(test['Consumption'],predictions)
```

Out[23]: 0.6895530789585476

In [26]:
```
modelExog=SARIMAX(train['Consumption'],exog=train['Income'],order=(3,0,0),s
resultsExog=modelExog.fit()
resultsExog.summary()
```

```
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.00359D+00    |proj g|=  5.34631D-01

At iterate    5    f=  8.73435D-01    |proj g|=  5.91703D-02

At iterate   10    f=  8.70844D-01    |proj g|=  1.62651D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *
```

```
   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     14     16      1     0     0   1.599D-06   8.708D-01
  F =   0.87084105263751344

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
 This problem is unconstrained.
```

Out[26]:

SARIMAX Results

Dep. Variable:	Consumption	**No. Observations:**	194
Model:	SARIMAX(3, 0, 0)	**Log Likelihood**	-168.943
Date:	Wed, 20 Apr 2022	**AIC**	347.886
Time:	10:09:44	**BIC**	364.226
Sample:	01-01-1970	**HQIC**	354.503
	- 04-01-2018		
Covariance Type:	opg		

	coef	std err	z	P>|z|	[0.025	0.975]
Income	0.1782	0.032	5.558	0.000	0.115	0.241
ar.L1	0.2171	0.054	4.003	0.000	0.111	0.323
ar.L2	0.3444	0.057	6.065	0.000	0.233	0.456
ar.L3	0.2838	0.057	4.951	0.000	0.171	0.396
sigma2	0.3321	0.025	13.041	0.000	0.282	0.382

Ljung-Box (L1) (Q):	0.42	**Jarque-Bera (JB):**	36.40
Prob(Q):	0.52	**Prob(JB):**	0.00
Heteroskedasticity (H):	0.22	**Skew:**	-0.19
Prob(H) (two-sided):	0.00	**Kurtosis:**	5.09

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).

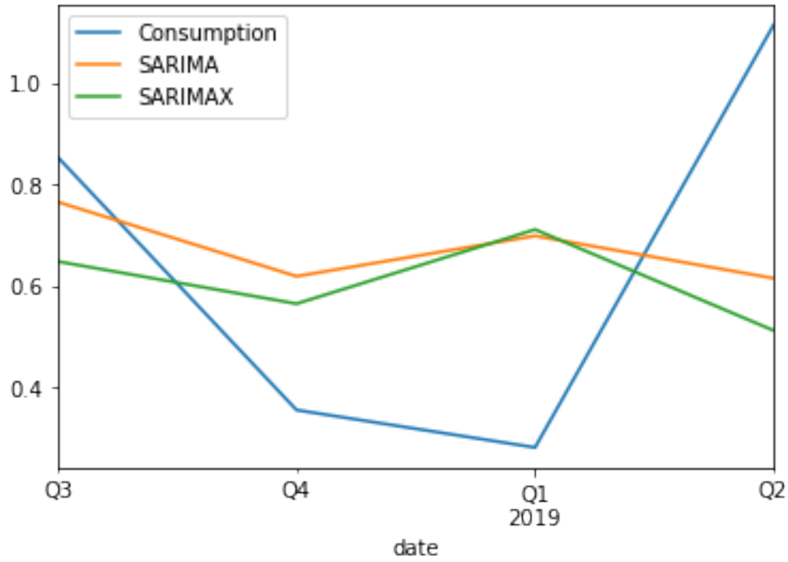
In [27]:

```
predictionsExog=resultsExog.predict(start=start,end=end,exog=test['Income'
```

In [28]:

```
ax=test['Consumption'].plot(legend=True)
predictions.plot(legend=True)
predictionsExog.plot(legend=True)
```

Out[28]: <AxesSubplot:xlabel='date'>



In [29]:

```
mean_absolute_percentage_error(test['Consumption'],predictionsExog)
```

Out[29]: 0.7201050748679051

In [32]:

```
from statsmodels.tsa.api  import VAR
```

In [33]:

```
df1=df[['Consumption','Income']]
df1
```

Out[33]:

date	Consumption	Income
1970-01-01	0.618566	1.044801
1970-04-01	0.451984	1.225647
1970-07-01	0.872872	1.585154
1970-10-01	-0.271848	-0.239545
1971-01-01	1.901345	1.975925
...	...	...
2018-04-01	0.983112	0.661825
2018-07-01	0.853181	0.806271
2018-10-01	0.356512	0.695142
2019-01-01	0.282885	1.100753
2019-04-01	1.113517	0.593399

198 rows × 2 columns

In [34]:

```
trainVAR=df1.iloc[:-4]
testVAR=df1.iloc[-4:]
```

In [37]:

```
for i in range(1,10):
    modelVAR=VAR(trainVAR)
    resultsVAR=modelVAR.fit(i)
    print('order= ',i)
    print('AIC= ', resultsVAR.aic)
    print('')
```

```
order=  1
AIC=  -1.44355774811431

order=  2
AIC=  -1.4427021710347265

order=  3
AIC=  -1.47905868013255

order=  4
AIC=  -1.4929139211167433

order=  5
AIC=  -1.555900188730903

order=  6
AIC=  -1.5281062869097655

order=  7
AIC=  -1.5045248025067974

order=  8
AIC=  -1.4866123239137767

order=  9
AIC=  -1.438980776981068
```

In [38]:

```
modelVAR=VAR(trainVAR)
resultsVAR=modelVAR.fit(5)
```

In [39]:

```
resultsVAR.summary()
```

Out[39]:

```
  Summary of Regression Results
==================================
Model:                         VAR
Method:                        OLS
Date:           Wed, 20, Apr, 2022
Time:                     10:30:11
--------------------------------------------------------------------
No. of Equations:         2.00000    BIC:                  -1.17855
Nobs:                     189.000    HQIC:                 -1.40303
Log likelihood:          -367.326    FPE:                  0.211055
AIC:                     -1.55590    Det(Omega_mle):       0.188477
--------------------------------------------------------------------
Results for equation Consumption
===========================================================================
======
                    coefficient       std. error           t-stat
prob
---------------------------------------------------------------------------
------
const                  0.349013         0.087649            3.982
0.000
L1.Consumption         0.189323         0.077975            2.428
```

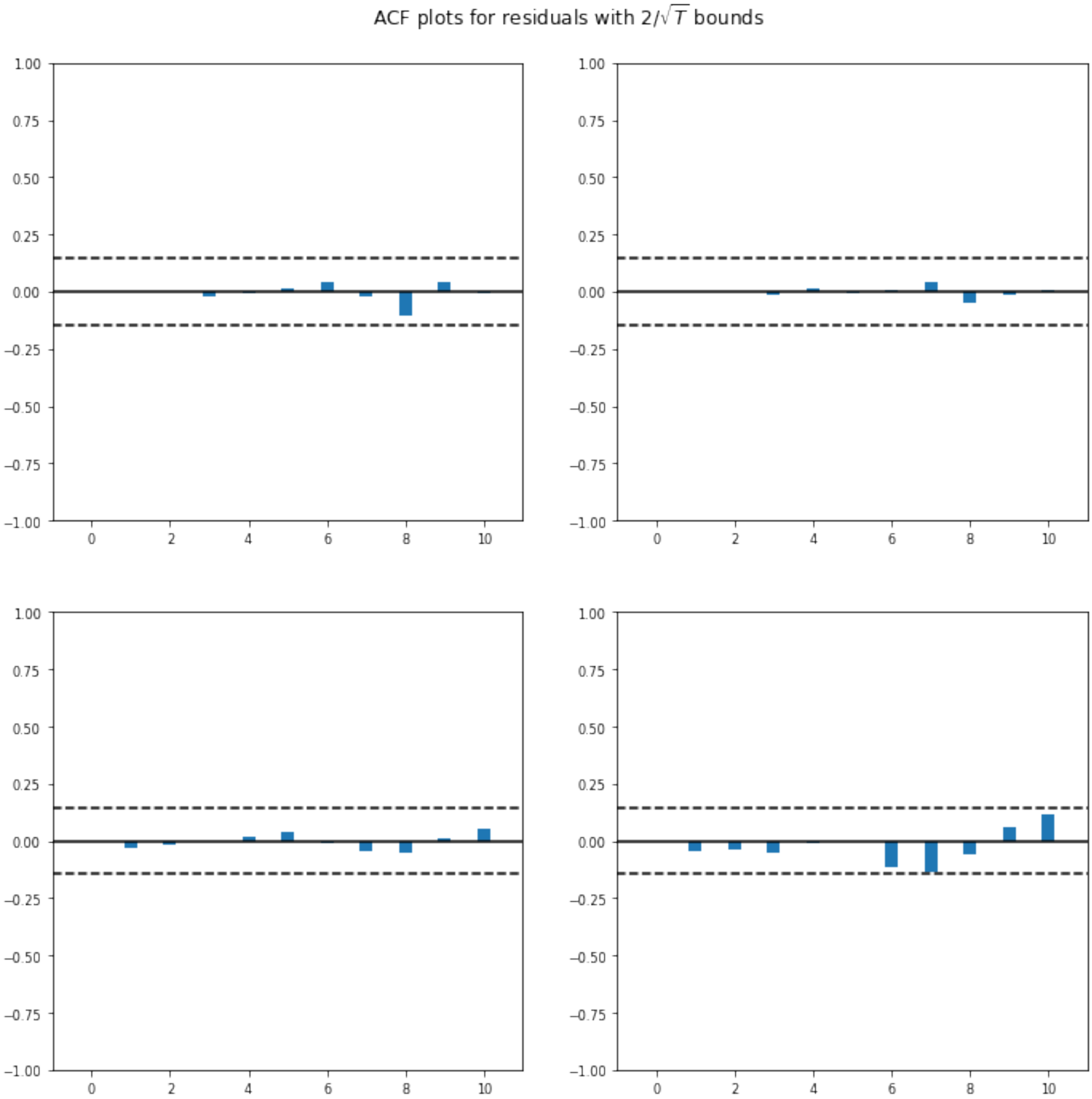
```
0.015
L1.Income              0.108857         0.054103            2.012
0.044
L2.Consumption         0.170980         0.080227            2.131
0.033
L2.Income             -0.013760         0.057062           -0.241
0.809
L3.Consumption         0.296786         0.079875            3.716
0.000
L3.Income             -0.028260         0.055760           -0.507
0.612
L4.Consumption        -0.033752         0.082758           -0.408
0.683
L4.Income             -0.071837         0.055894           -1.285
0.199
L5.Consumption        -0.043607         0.079010           -0.552
0.581
L5.Income             -0.046964         0.053585           -0.876
0.381
===========================================================================
======

Results for equation Income
===========================================================================
======
                    coefficient       std. error           t-stat
prob
---------------------------------------------------------------------------
------
const                  0.572394         0.123709            4.627
0.000
L1.Consumption         0.410378         0.110055            3.729
0.000
L1.Income             -0.303253         0.076362           -3.971
0.000
L2.Consumption         0.101852         0.113234            0.899
0.368
L2.Income             -0.045916         0.080538           -0.570
0.569
L3.Consumption         0.468458         0.112737            4.155
0.000
L3.Income             -0.150672         0.078701           -1.914
0.056
L4.Consumption         0.266858         0.116805            2.285
0.022
L4.Income             -0.260842         0.078890           -3.306
0.001
L5.Consumption        -0.125637         0.111515           -1.127
0.260
L5.Income             -0.202112         0.075631           -2.672
0.008
===========================================================================
======

Correlation matrix of residuals
               Consumption    Income
Consumption       1.000000  0.314089
```

```
Income            0.314089  1.000000
```

In [40]:
```
resultsVAR.plot_acorr();
```



In [42]:
```
lag_order=resultsVAR.k_ar
lag_order
```

Out[42]: 5

In [44]:
```
predictedValues=resultsVAR.forecast(df1.values[-lag_order:],4)
predictedValues
```

Out[44]:
```
array([[0.58313943, 0.65003969],
       [0.62821021, 0.31107678],
       [0.76946343, 0.80071605],
       [0.69544088, 0.75463026]])
```

In [45]:
```
testValuesConsumption=testVAR['Consumption'].values
testValuesConsumption
```

Out[45]:
```
array([0.85318135, 0.35651203, 0.28288547, 1.1135167 ])
```

In [46]:
```
testValuesIncome=testVAR['Income'].values
testValuesIncome
```

Out[46]:
```
array([0.80627128, 0.69514222, 1.10075295, 0.5933995 ])
```

In [47]:
```
mean_absolute_percentage_error(testValuesConsumption,predictedValues[:,0])
```

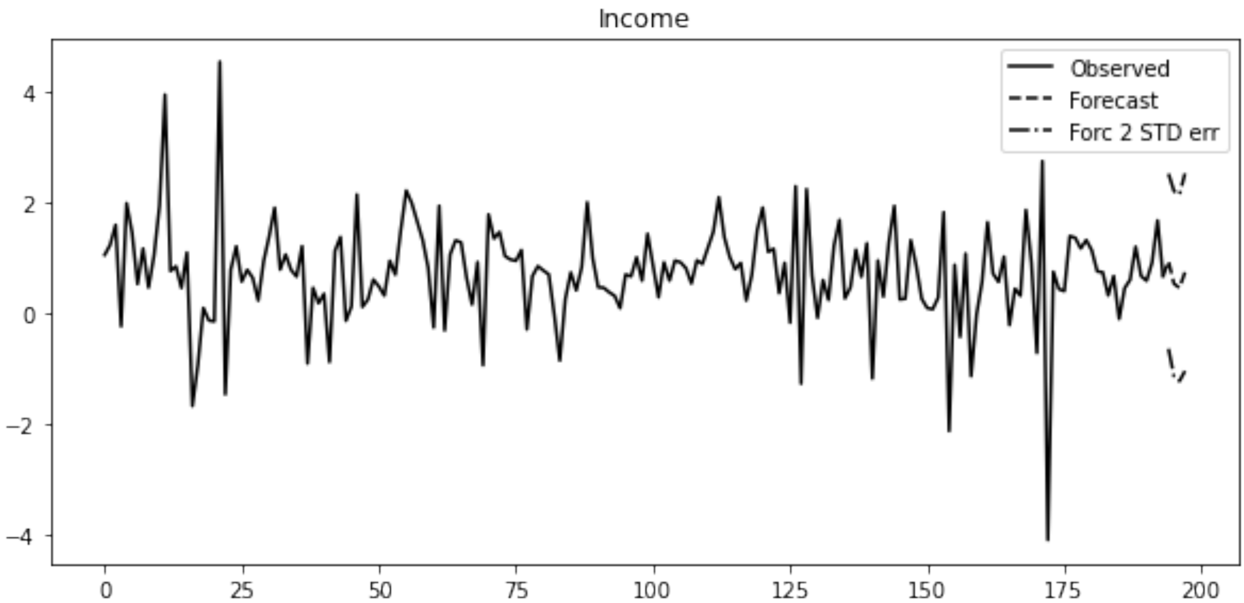
Out[47]:
```
0.7935302535890587
```

In [48]:
```
mean_absolute_percentage_error(testValuesIncome,predictedValues[:,1])
```

Out[48]:
```
0.32263769174292994
```

In [50]:
```
resultsVAR.plot_forecast(4);
```


Consumption
Observed
Forecast
Forc 2 STD err
2
1
0
−1
−2
0
25
50
75
100
125
150
175
200


Income
Observed
Forecast
Forc 2 STD err
4
2
0
−2
−4
0
25
50
75
100
125
150
175
200

In [ ]: