In [2]:
```
import pandas as pd
import numpy as np
```

In [3]:
```
from statsmodels.tsa.statespace.sarimax import SARIMAX
from statsmodels.tsa.seasonal import seasonal_decompose
from pmdarima import auto_arima
```

```
/Users/mirek/opt/anaconda3/lib/python3.8/site-packages/statsmodels/compat/p
andas.py:61: FutureWarning: pandas.Int64Index is deprecated and will be rem
oved from pandas in a future version. Use pandas.Index with the appropriate
dtype instead.
  from pandas import Int64Index as NumericIndex
```

In [4]:
```
from sklearn.metrics import mean_absolute_error,mean_absolute_percentage_e
```

In [5]:
```
df=pd.read_csv('airline_passengers.csv',index_col='Month',parse_dates=True
df
```

Out[5]:

Month	Thousands of Passengers
1949-01-01	112
1949-02-01	118
1949-03-01	132
1949-04-01	129
1949-05-01	121
...	...
1960-08-01	606
1960-09-01	508
1960-10-01	461
1960-11-01	390
1960-12-01	432

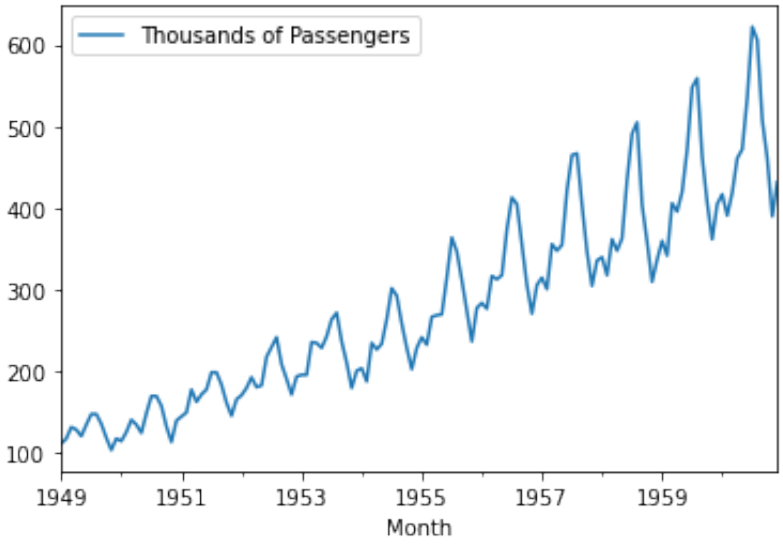
144 rows × 1 columns

In [6]:
```
df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
DatetimeIndex: 144 entries, 1949-01-01 to 1960-12-01
Data columns (total 1 columns):
 #   Column                  Non-Null Count  Dtype
---  ------                  --------------  -----
 0   Thousands of Passengers  144 non-null    int64
dtypes: int64(1)
memory usage: 2.2 KB
```

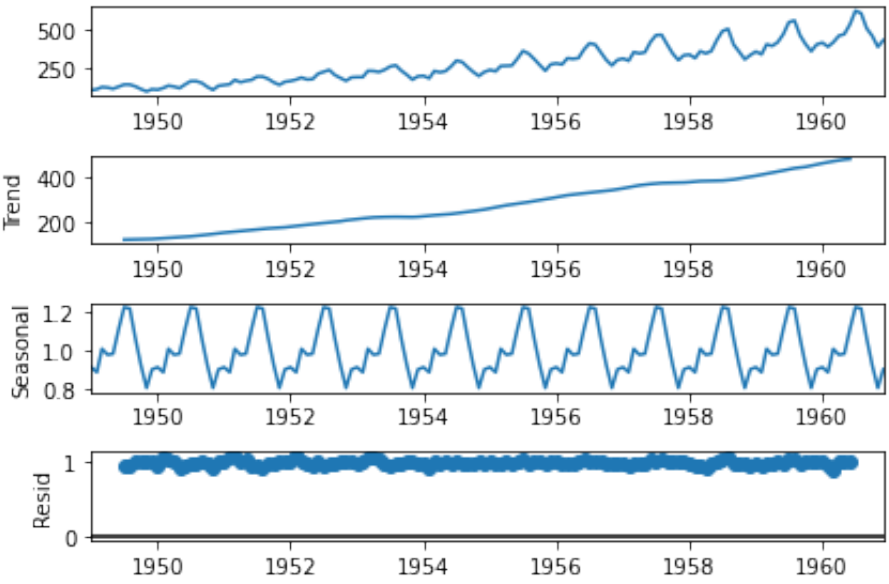
In [7]:
```
df.index.freq='MS'
```

In [8]:
```
df.plot();
```



In [9]:
```
seasonalDecomposition=seasonal_decompose(df,model='mul')
```

In [10]:
```
seasonalDecomposition.plot();
```



In [13]:
```
auto_arima(df,seasonal=True,m=12,trace=True).summary()
```

```
Performing stepwise search to minimize aic
 ARIMA(2,1,2)(1,1,1)[12]             : AIC=1020.048, Time=0.48 sec
 ARIMA(0,1,0)(0,1,0)[12]             : AIC=1031.508, Time=0.01 sec
 ARIMA(1,1,0)(1,1,0)[12]             : AIC=1020.393, Time=0.05 sec
 ARIMA(0,1,1)(0,1,1)[12]             : AIC=1021.003, Time=0.07 sec
 ARIMA(2,1,2)(0,1,1)[12]             : AIC=1019.935, Time=0.28 sec
 ARIMA(2,1,2)(0,1,0)[12]             : AIC=1019.290, Time=0.11 sec
 ARIMA(2,1,2)(1,1,0)[12]             : AIC=1019.546, Time=0.28 sec
 ARIMA(1,1,2)(0,1,0)[12]             : AIC=1024.160, Time=0.06 sec
 ARIMA(2,1,1)(0,1,0)[12]             : AIC=1017.847, Time=0.11 sec
 ARIMA(2,1,1)(1,1,0)[12]             : AIC=1017.914, Time=0.25 sec
 ARIMA(2,1,1)(0,1,1)[12]             : AIC=1018.359, Time=0.24 sec
 ARIMA(2,1,1)(1,1,1)[12]             : AIC=1018.248, Time=0.47 sec
 ARIMA(1,1,1)(0,1,0)[12]             : AIC=1022.393, Time=0.03 sec
 ARIMA(2,1,0)(0,1,0)[12]             : AIC=1022.393, Time=0.03 sec
 ARIMA(3,1,1)(0,1,0)[12]             : AIC=1019.084, Time=0.11 sec
 ARIMA(1,1,0)(0,1,0)[12]             : AIC=1020.393, Time=0.02 sec
 ARIMA(3,1,0)(0,1,0)[12]             : AIC=1023.666, Time=0.05 sec
 ARIMA(3,1,2)(0,1,0)[12]             : AIC=1021.083, Time=0.20 sec
 ARIMA(2,1,1)(0,1,0)[12] intercept   : AIC=inf, Time=0.26 sec

Best model:  ARIMA(2,1,1)(0,1,0)[12]
Total fit time: 3.138 seconds
```

Out[13]:

SARIMAX Results

Dep. Variable:	y	**No. Observations:**	144
Model:	SARIMAX(2, 1, 1)x(0, 1, [], 12)	**Log Likelihood**	-504.923
Date:	Tue, 26 Apr 2022	**AIC**	1017.847
Time:	06:45:11	**BIC**	1029.348
Sample:	0	**HQIC**	1022.520
	- 144		
Covariance Type:	opg		

	coef	std err	z	P>|z|	[0.025	0.975]
ar.L1	0.5960	0.085	6.987	0.000	0.429	0.763
ar.L2	0.2143	0.091	2.343	0.019	0.035	0.394
ma.L1	-0.9819	0.038	-25.602	0.000	-1.057	-0.907
sigma2	129.3150	14.557	8.883	0.000	100.784	157.846

Ljung-Box (L1) (Q):	0.00	**Jarque-Bera (JB):**	7.68
Prob(Q):	0.98	**Prob(JB):**	0.02
Heteroskedasticity (H):	2.33	**Skew:**	-0.01
Prob(H) (two-sided):	0.01	**Kurtosis:**	4.19

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).

In [14]:

```
train=df.iloc[:-12]
test=df.iloc[-12:]
len(test)
```

Out[14]:

```
12
```

In [15]:

```
model=SARIMAX(train,order=(2,1,1),seasonal_order=(0,1,0,12))
results=model.fit()
```

```
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  3.42129D+00    |proj g|=  2.27842D-01

At iterate    5    f=  3.39217D+00    |proj g|=  1.21741D-02

At iterate   10    f=  3.39006D+00    |proj g|=  2.06863D-03

At iterate   15    f=  3.39003D+00    |proj g|=  1.15951D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     18     32      1     0     0   1.039D-05   3.390D+00
  F =   3.3900246510399326

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
/Users/mirek/opt/anaconda3/lib/python3.8/site-packages/statsmodels/tsa/stat
espace/sarimax.py:966: UserWarning: Non-stationary starting autoregressive
parameters found. Using zeros as starting parameters.
  warn('Non-stationary starting autoregressive parameters'
/Users/mirek/opt/anaconda3/lib/python3.8/site-packages/statsmodels/tsa/stat
espace/sarimax.py:978: UserWarning: Non-invertible starting MA parameters f
ound. Using zeros as starting parameters.
  warn('Non-invertible starting MA parameters found.'
 This problem is unconstrained.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
```

In [16]:

```
results.summary()
```

Out[16]:

SARIMAX Results

Dep. Variable:	Thousands of Passengers	**No. Observations:**	132
Model:	SARIMAX(2, 1, 1)x(0, 1, [], 12)	**Log Likelihood**	-447.483
Date:	Tue, 26 Apr 2022	**AIC**	902.967
Time:	06:45:41	**BIC**	914.083
Sample:	01-01-1949	**HQIC**	907.481
	- 12-01-1959		
Covariance Type:	opg		

	coef	std err	z	P>|z|	[0.025	0.975]
ar.L1	-0.5253	0.929	-0.566	0.572	-2.345	1.295
ar.L2	0.0110	0.275	0.040	0.968	-0.529	0.551
ma.L1	0.3064	0.935	0.328	0.743	-1.527	2.140
sigma2	108.0093	13.498	8.002	0.000	81.553	134.466

Ljung-Box (L1) (Q):	0.00	**Jarque-Bera (JB):**	1.39
Prob(Q):	0.97	**Prob(JB):**	0.50
Heteroskedasticity (H):	1.47	**Skew:**	-0.09
Prob(H) (two-sided):	0.23	**Kurtosis:**	3.50

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).

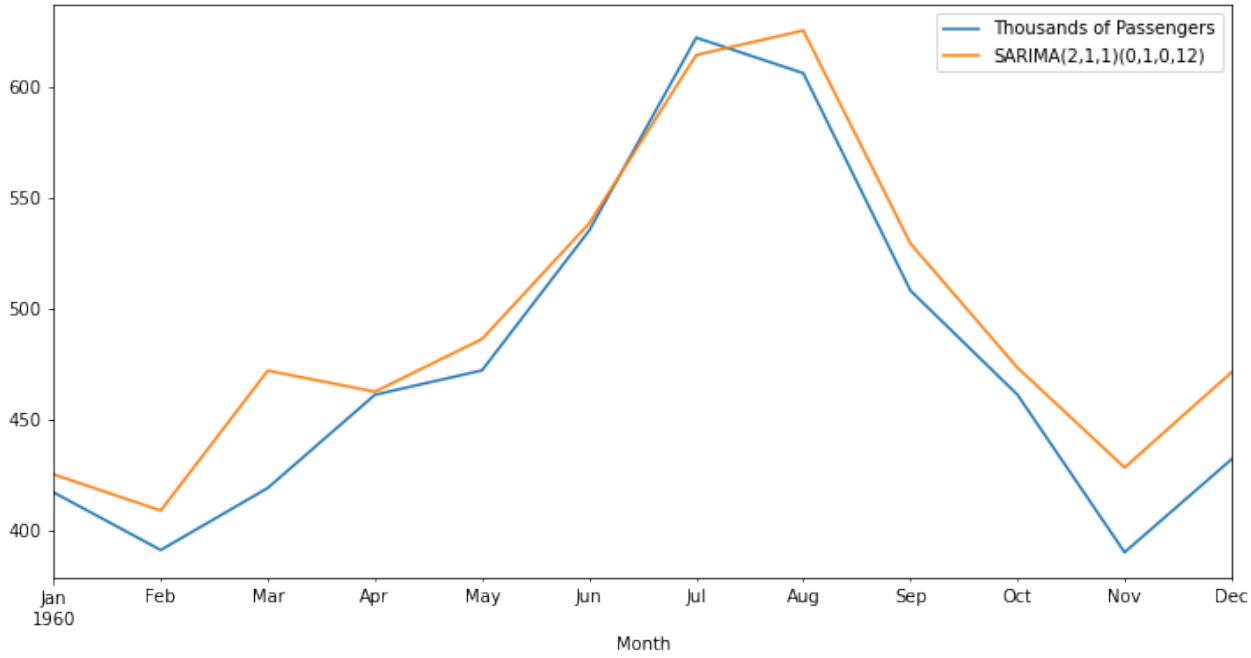
In [17]:

```
start=len(train)
end=start+len(test)-1
predictions=results.predict(start=start,end=end,dynamic=False).rename('SARI
```

In [18]:

```
ax=test.plot(legend=True,figsize=(12,6))
predictions.plot(legend=True)
```

Out[18]: <AxesSubplot:xlabel='Month'>



In [19]:
```
mean_absolute_percentage_error(test,predictions)
```

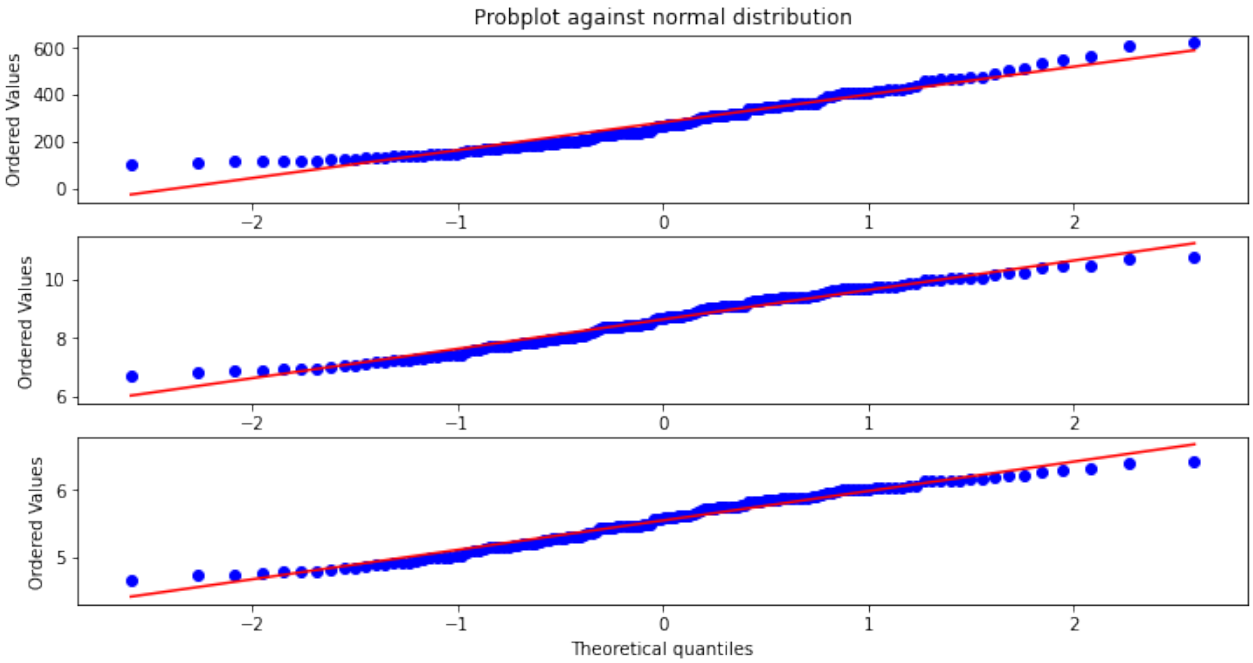
Out[19]: 0.044317548784737405

In [20]:
```
import scipy.stats as stats
import matplotlib.pyplot as plt
```

In [21]:
```
x=df['Thousands of Passengers']
xt, lambda_mle=stats.boxcox(x)
x_log=np.log(x)

fig, (ax1,ax2,ax3)=plt.subplots(3,figsize=(12,6))
stats.probplot(x,dist=stats.norm,plot=ax1);
stats.probplot(xt,dist=stats.norm,plot=ax2);
stats.probplot(x_log,dist=stats.norm,plot=ax3);
ax1.set_title('Probplot against normal distribution')
ax2.set_title('')
ax3.set_title('')
ax1.set_xlabel('')
```

Out[21]:
```
Text(0.5, 0, '')
```



In [22]:
```
lambda_mle
```

Out[22]:
```
0.14802265137037945
```

In [23]:
```
df['Log']=np.log(df["Thousands of Passengers"])
df
```

Out[23]:

Month	Thousands of Passengers	Log
1949-01-01	112	4.718499
1949-02-01	118	4.770685
1949-03-01	132	4.882802
1949-04-01	129	4.859812
1949-05-01	121	4.795791
...	...	...
1960-08-01	606	6.406880
1960-09-01	508	6.230481
1960-10-01	461	6.133398
1960-11-01	390	5.966147
1960-12-01	432	6.068426

144 rows × 2 columns

In [24]:
```
auto_arima(df['Log'],seasonal=True,m=12,trace=True).summary()
```

```
Performing stepwise search to minimize aic
 ARIMA(2,0,2)(1,1,1)[12] intercept   : AIC=-480.365, Time=0.81 sec
 ARIMA(0,0,0)(0,1,0)[12] intercept   : AIC=-358.005, Time=0.03 sec
 ARIMA(1,0,0)(1,1,0)[12] intercept   : AIC=-472.836, Time=0.28 sec
 ARIMA(0,0,1)(0,1,1)[12] intercept   : AIC=-423.080, Time=0.30 sec
 ARIMA(0,0,0)(0,1,0)[12]             : AIC=-152.756, Time=0.01 sec
 ARIMA(2,0,2)(0,1,1)[12] intercept   : AIC=-485.489, Time=0.64 sec
 ARIMA(2,0,2)(0,1,0)[12] intercept   : AIC=-455.362, Time=0.29 sec
 ARIMA(2,0,2)(0,1,2)[12] intercept   : AIC=-483.632, Time=1.63 sec
 ARIMA(2,0,2)(1,1,0)[12] intercept   : AIC=-479.075, Time=0.74 sec
 ARIMA(2,0,2)(1,1,2)[12] intercept   : AIC=-482.918, Time=1.32 sec
 ARIMA(1,0,2)(0,1,1)[12] intercept   : AIC=-486.812, Time=0.54 sec
 ARIMA(1,0,2)(0,1,0)[12] intercept   : AIC=-455.568, Time=0.29 sec
 ARIMA(1,0,2)(1,1,1)[12] intercept   : AIC=-477.167, Time=0.45 sec
 ARIMA(1,0,2)(0,1,2)[12] intercept   : AIC=-484.812, Time=1.31 sec
 ARIMA(1,0,2)(1,1,0)[12] intercept   : AIC=-480.056, Time=0.58 sec
 ARIMA(1,0,2)(1,1,2)[12] intercept   : AIC=-483.074, Time=1.67 sec
 ARIMA(0,0,2)(0,1,1)[12] intercept   : AIC=-455.593, Time=0.41 sec
 ARIMA(1,0,1)(0,1,1)[12] intercept   : AIC=-489.188, Time=0.50 sec
 ARIMA(1,0,1)(0,1,0)[12] intercept   : AIC=-457.177, Time=0.12 sec
 ARIMA(1,0,1)(1,1,1)[12] intercept   : AIC=-487.318, Time=0.61 sec
 ARIMA(1,0,1)(0,1,2)[12] intercept   : AIC=-487.300, Time=1.13 sec
 ARIMA(1,0,1)(1,1,0)[12] intercept   : AIC=-482.595, Time=0.47 sec
 ARIMA(1,0,1)(1,1,2)[12] intercept   : AIC=-485.198, Time=1.18 sec
 ARIMA(1,0,0)(0,1,1)[12] intercept   : AIC=-482.026, Time=0.27 sec
 ARIMA(2,0,1)(0,1,1)[12] intercept   : AIC=-487.314, Time=0.55 sec
 ARIMA(0,0,0)(0,1,1)[12] intercept   : AIC=-367.061, Time=0.10 sec
 ARIMA(2,0,0)(0,1,1)[12] intercept   : AIC=-489.295, Time=0.37 sec
 ARIMA(2,0,0)(0,1,0)[12] intercept   : AIC=-458.262, Time=0.11 sec
 ARIMA(2,0,0)(1,1,1)[12] intercept   : AIC=-487.388, Time=0.66 sec
 ARIMA(2,0,0)(0,1,2)[12] intercept   : AIC=-487.321, Time=0.68 sec
 ARIMA(2,0,0)(1,1,0)[12] intercept   : AIC=-482.539, Time=0.90 sec
 ARIMA(2,0,0)(1,1,2)[12] intercept   : AIC=-486.907, Time=1.21 sec
 ARIMA(3,0,0)(0,1,1)[12] intercept   : AIC=-487.303, Time=0.52 sec
 ARIMA(3,0,1)(0,1,1)[12] intercept   : AIC=-485.297, Time=0.61 sec
 ARIMA(2,0,0)(0,1,1)[12]             : AIC=-481.419, Time=0.33 sec

Best model:  ARIMA(2,0,0)(0,1,1)[12] intercept
Total fit time: 21.662 seconds
```

Out[24]:

SARIMAX Results

Dep. Variable:	y	**No. Observations:**	144
Model:	SARIMAX(2, 0, 0)x(0, 1, [1], 12)	**Log Likelihood**	249.648
Date:	Tue, 26 Apr 2022	**AIC**	-489.295
Time:	06:46:57	**BIC**	-474.881
Sample:	0	**HQIC**	-483.438
	- 144		
Covariance Type:	opg		

	coef	std err	z	P>|z|	[0.025	0.975]
intercept	0.0194	0.008	2.340	0.019	0.003	0.036
ar.L1	0.5750	0.075	7.632	0.000	0.427	0.723
ar.L2	0.2616	0.084	3.122	0.002	0.097	0.426
ma.S.L12	-0.5549	0.106	-5.225	0.000	-0.763	-0.347
sigma2	0.0013	0.000	8.856	0.000	0.001	0.002

Ljung-Box (L1) (Q):	0.02	**Jarque-Bera (JB):**	6.47
Prob(Q):	0.90	**Prob(JB):**	0.04
Heteroskedasticity (H):	0.58	**Skew:**	0.07
Prob(H) (two-sided):	0.07	**Kurtosis:**	4.08

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).

In [25]:

```
train=df.iloc[:-12]
test=df.iloc[-12:]
len(test)
```

Out[25]:

```
12
```

In [27]:

```
model2=SARIMAX(train['Log'],order=(2,0,0),seasonal_order=(0,1,1,12))
results2=model2.fit()
results2.summary()
```

```
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10
```

```
At X0         0 variables are exactly at the bounds

At iterate    0    f= -1.60346D+00    |proj g|=  1.42556D+00

At iterate    5    f= -1.67277D+00    |proj g|=  4.61090D+00

At iterate   10    f= -1.68643D+00    |proj g|=  7.10030D-02

At iterate   15    f= -1.68739D+00    |proj g|=  5.02994D-02

At iterate   20    f= -1.68740D+00    |proj g|=  1.44260D-01

At iterate   25    f= -1.68879D+00    |proj g|=  4.28816D-01
 This problem is unconstrained.
At iterate   30    f= -1.69873D+00    |proj g|=  2.38197D-01

At iterate   35    f= -1.69904D+00    |proj g|=  1.53912D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     38     79      2     0     0   1.568D-03  -1.699D+00
  F =  -1.6990374795045518

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 Bad direction in the line search;
   refresh the lbfgs memory and restart the iteration.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
```

Out[27]:

SARIMAX Results

Dep. Variable:	Log	**No. Observations:**	132
Model:	SARIMAX(2, 0, 0)x(0, 1, [1], 12)	**Log Likelihood**	224.273
Date:	Tue, 26 Apr 2022	**AIC**	-440.546
Time:	06:47:26	**BIC**	-429.396
Sample:	01-01-1949	**HQIC**	-436.018
	- 12-01-1959		
Covariance Type:	opg		

	coef	std err	z	P>|z|	[0.025	0.975]
ar.L1	0.6787	0.074	9.183	0.000	0.534	0.824
ar.L2	0.3143	0.076	4.161	0.000	0.166	0.462
ma.S.L12	-0.5626	0.094	-5.992	0.000	-0.747	-0.379
sigma2	0.0013	0.000	8.170	0.000	0.001	0.002

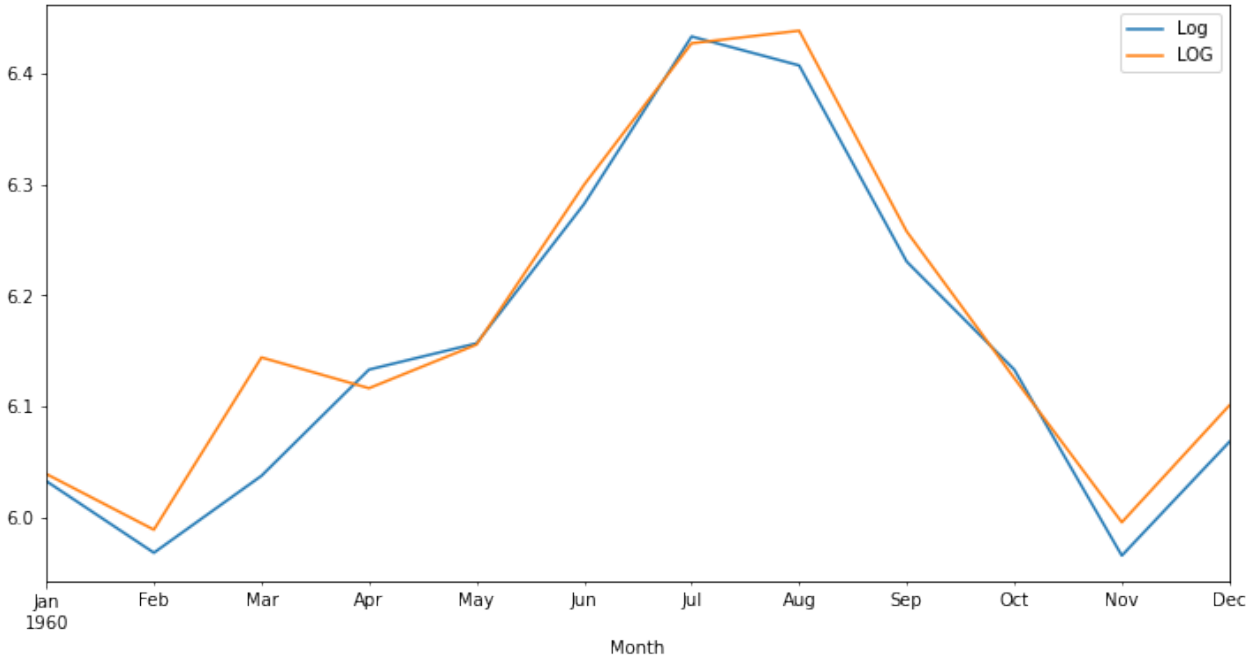
Ljung-Box (L1) (Q):	0.00	**Jarque-Bera (JB):**	0.98
Prob(Q):	0.97	**Prob(JB):**	0.61
Heteroskedasticity (H):	0.35	**Skew:**	0.02
Prob(H) (two-sided):	0.00	**Kurtosis:**	3.44

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).

In [28]:
```
predictions2=results2.predict(start=start,end=end,dynamic=False).rename('L(
```

In [29]:
```
ax=test['Log'].plot(legend=True,figsize=(12,6))
predictions2.plot(legend=True)
```

Out[29]: <AxesSubplot:xlabel='Month'>



In [30]:
```
mean_absolute_percentage_error(test['Log'],predictions2)
```

Out[30]: 0.004140185361718522

In [31]:
```
mean_absolute_percentage_error(test['Thousands of Passengers'],np.exp(predi
```

Out[31]: 0.02596284258776781

In [ ]:
```

```