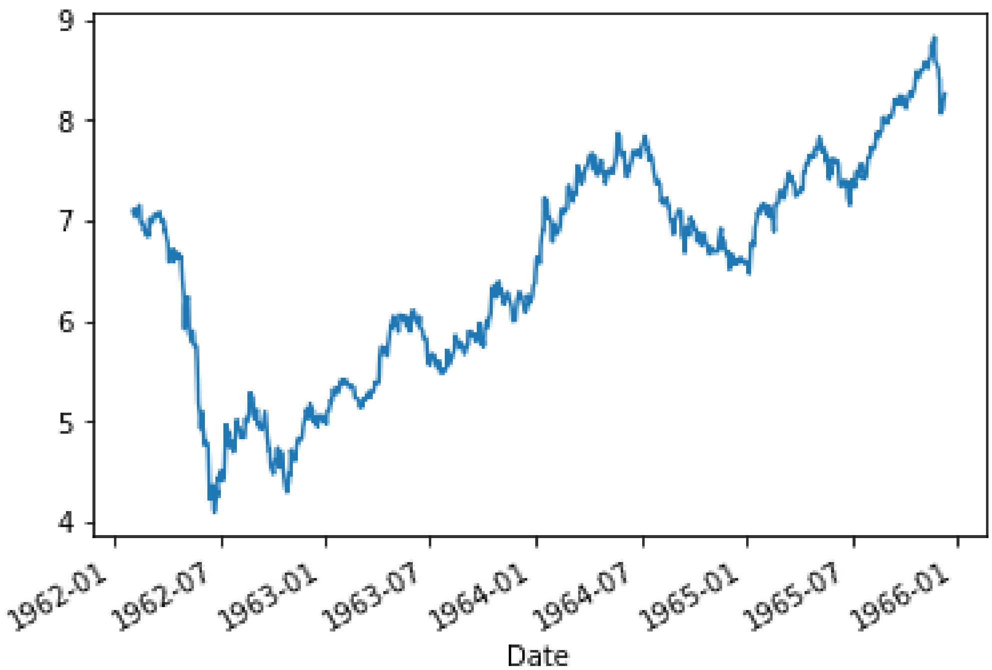
```
import pandas as pd
import numpy as np
from  statsmodels.tsa.api import SimpleExpSmoothing, Holt
import matplotlib.pyplot as plt
import datetime
from sklearn.metrics import mean_squared_error
from scipy.fft import fft, ifft, fftfreq
import warnings
from statsmodels.tsa.api import ExponentialSmoothing,
SimpleExpSmoothing, Holt
from statsmodels.tools.sm_exceptions import ConvergenceWarning,
ModelWarning
warnings.simplefilter('ignore', ConvergenceWarning)
warnings.simplefilter('ignore', ModelWarning)

class myHolt:
    def __init__(self, y, alpha, beta):
        self.alpha = alpha
        self.beta = beta
        self.y0 = y[0]
        self.data = np.zeros(len(y))
        self.data[0] = self.y0
        self.l = np.zeros(len(y))
        self.l[0] = y[0]
        self.b = np.zeros(len(y))
        self.b[0] = 1 if y[1] > y[0] else -1

        for i in np.arange(1,len(y)):
            self.calcNextL(i, y[i-1])
            self.calcNextB(i)
            self.data[i] = self.l[i] + self.b[i]

    def calcNextL(self, t, val_real):
        l_before = self.l[t-1]
        b_before = self.b[t-1]
        self.l[t] = self.alpha * val_real + (1-self.alpha) * (l_before
+ b_before)

    def calcNextB(self, t):
        self.b[t] = self.beta * (self.l[t] - self.l[t-1]) + (1-
self.beta)*self.b[t-1]
```

IBM HOLT TEST

```
ibm = pd.read_csv('../Lab2/IBM.csv', index_col=['Date'])
ibm.index=pd.to_datetime(ibm.index, format='%d/%m/%Y')
freq = pd.infer_freq(ibm.index)
ibm['High'].plot()
#
ibm
```

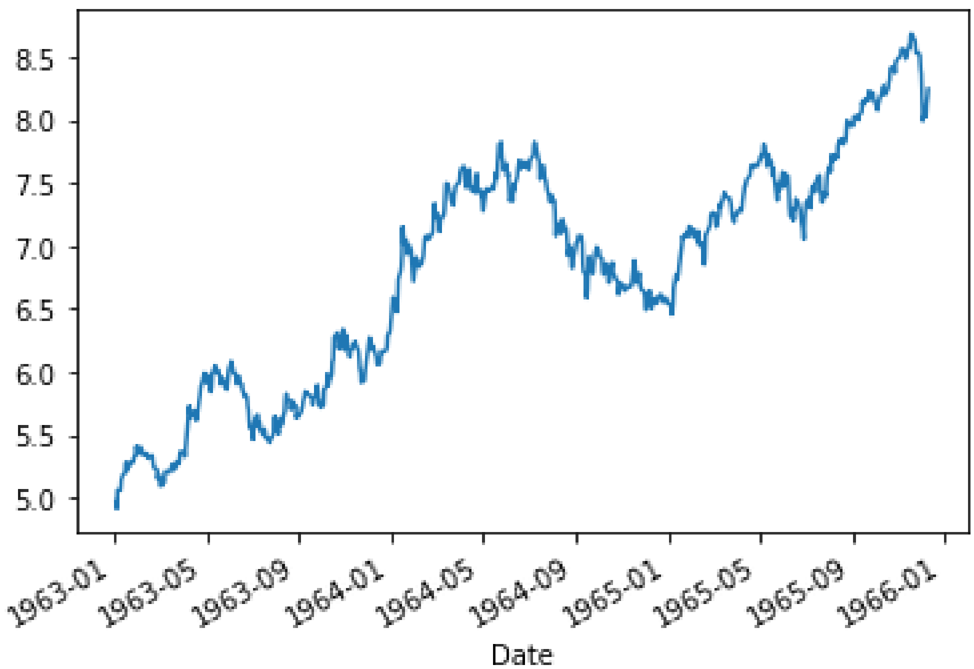
```
                 Open      High       Low     Close  Adj Close   Volume
Date
1962-02-01  6.978967  7.087317  6.978967  7.068196   1.577106   674670
1962-02-02  7.068196  7.112811  7.036329  7.112811   1.587062   533460
1962-02-05  7.112811  7.112811  6.985341  7.023582   1.567152   329490
1962-02-06  7.023582  7.036329  6.998088  7.029955   1.568787   274575
1962-02-07  7.036329  7.074570  7.036329  7.036329   1.570211   266730
...              ...       ...       ...       ...        ...      ...
1965-12-06  8.070427  8.102294  7.934990  7.998725   1.791781  1041816
1965-12-07  8.026609  8.142129  8.026609  8.102294   1.814982   407940
1965-12-08  8.134162  8.205864  8.134162  8.150096   1.825689   395388
1965-12-09  8.173996  8.221797  8.173996  8.217814   1.840859   332628
1965-12-10  8.245698  8.261632  8.245698  8.253665   1.848891   282420

[972 rows x 6 columns]
```




```
# take the linear part
timestart = datetime.datetime(1963,1,1)
timeend = datetime.datetime(1966,1,1)
print((timestart))
ibm_linear = ibm.loc['1963-01-02':'1966-01-01']
ibm_linear['Open'].plot()

params = [
    "smoothing_level",
    "smoothing_trend",
    "damping_trend",
    "initial_level",
```

```
    "initial_trend",
]
```

```
1963-01-01 00:00:00
```




```
fig, ax = plt.subplots(2, figsize = (10,16))
plt.suptitle("Optimization of exponential smoothing Holt's forecasting
via mean squared error")
col = 'Open'
numpy_df = ibm_linear[col].to_numpy()
#print(numpy_df)
best_mse = 1e18
best_mse_mine = [1e18, -1, -1]

bestFit = None
bestFitMine = None

# create grid for mse phase plot
start = 0.05
end = 1.0
step = 0.06
param_num =int((end - start)/step) + 1
alphas = np.arange(start, end, step)
betas = np.arange(start, end, step)
y, x = np.meshgrid(alphas, betas)

# take mse's
mses = []
mses_mine = []
```

```python
l0 = None
b0 = None
# sweep through all alphas
for alpha in alphas:
    for beta in betas:
        # implemented one
        fit = Holt(numpy_df, exponential=True,

initialization_method="estimated").fit(smoothing_level=alpha,
smoothing_trend=beta, optimized = False)
        fitted = fit.fittedvalues

        # mine
        mHolt = myHolt(numpy_df, alpha, beta)
        l0 = mHolt.l[0]
        b0 = mHolt.b[0]
        fitMine = mHolt.data

        # calculate mse's
        mse = mean_squared_error(fitted, numpy_df)
        mseMine = mean_squared_error(fitMine, numpy_df)

        mses.append(mse)
        mses_mine.append(mseMine)
        #print(r"For $\alpha =$"+ f"{alpha:.3f}" "and $\beta =$"+
f"{beta:.3f} the mse is {mse:.3f}")
        if mse < best_mse:
            best_mse = mse
            bestFit = fit
        if mseMine < best_mse_mine[0]:
            best_mse_mine = [mseMine, alpha, beta]
            bestFitMine = fitMine

# make optimized prediction from the library
fit = Holt(ibm_linear[col], exponential=True,
initialization_method="estimated",).fit()
m=mean_squared_error(np.array(fit.fittedvalues), numpy_df)


# plot the results
results = pd.DataFrame(
    index=[r"$\alpha$", r"$\beta$", r"$\phi$", r"$l_0$", "$b_0$"],
    columns=["Holt's try to find manually", "Holt's model estimated"],
)

results["Holt's try to find manually"] = [bestFit.params[p] for p in
params]
results["Holt's model estimated"] = [fit.params[p] for p in params]
results["My Holt's model estimation"] = [best_mse_mine[1],
```

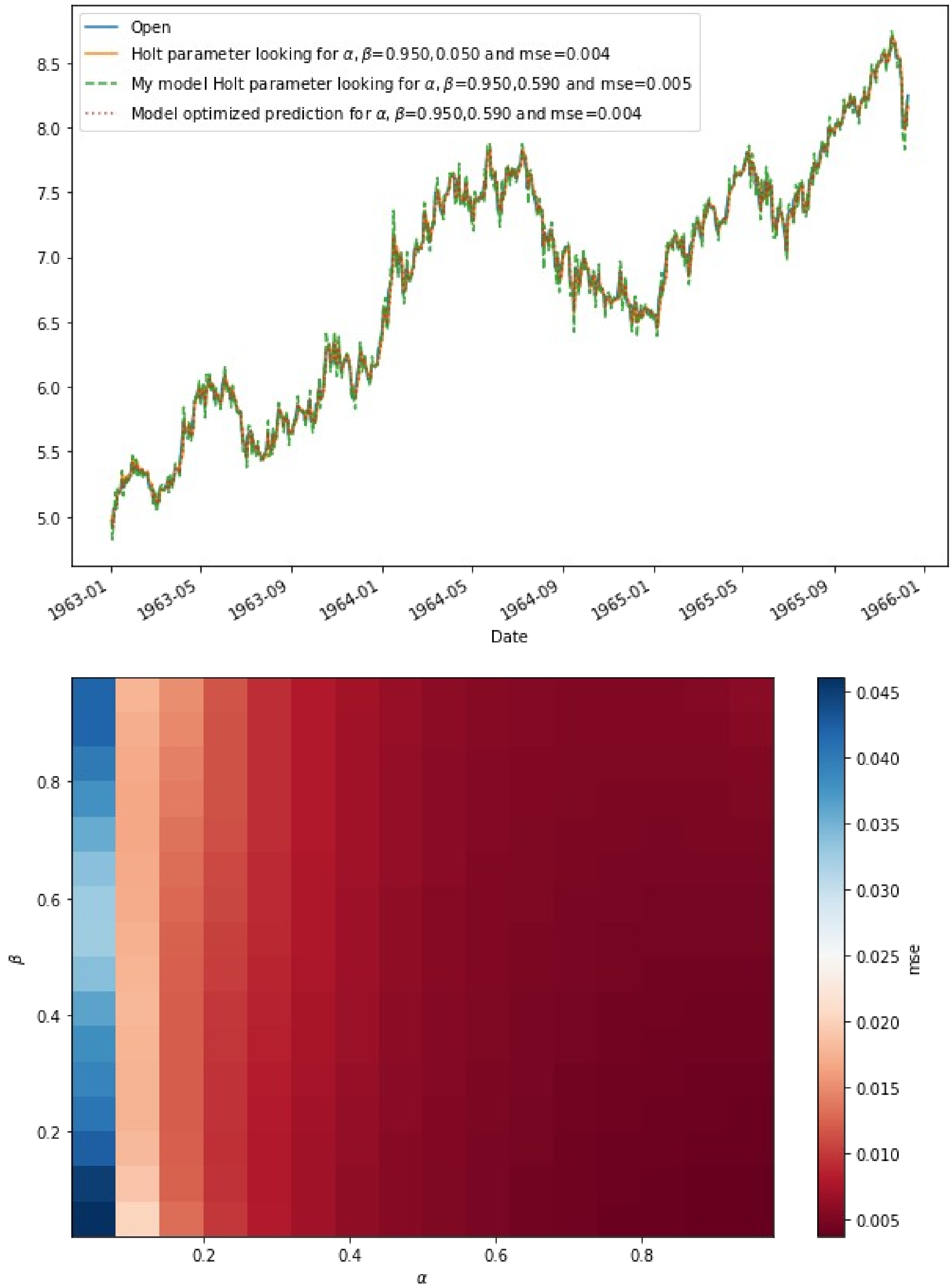
```
best_mse_mine[2], np.nan, l0, b0]
alphas = results.iloc[0]
betas = results.iloc[1]
print(alphas, betas, best_mse_mine)
# plot
#ax[0].plot(ibm_linear.index, numpy_df, label = 'original data')
ibm_linear[col].plot(ax=ax[0])
ax[0].plot(ibm_linear[col].index, bestFit.fittedvalues, '-',
alpha=0.9,
           label = r'Holt parameter looking for $\alpha,\beta$=' +
f'{alphas[0]:.3f},{betas[0]:.3f} and mse={best_mse:.3f}')
ax[0].plot(ibm_linear[col].index, fitMine, '--', alpha=0.9,
           label = r'My model Holt parameter looking for $\alpha,\
beta$=' + f'{best_mse_mine[1]:.3f},{best_mse_mine[2]:.3f} and
mse={best_mse_mine[0]:.3f}')
ax[0].plot(ibm_linear[col].index, fit.fittedvalues, ':', alpha=0.8,
label = r'Model optimized prediction for $\alpha,\beta$=' +
f'{alphas[-1]:.3f},{betas[-1]:.3f} and mse={m:.3f}')
ax[0].legend()

c = ax[1].pcolormesh(x, y, np.array(mses).reshape((param_num,
param_num)), cmap='RdBu', vmin=np.min(mses), vmax=np.max(mses),
shading='auto')
cbar = fig.colorbar(c, ax = ax[1])
cbar.ax.set_ylabel('mse')

#ax[1].plot(alphas, mses_stat, label = 'from statsmodel')
#ax[1].plot(alphas, mses_mine, label = 'from mymodel')
ax[1].set_xlabel(r"$\alpha$")
ax[1].set_ylabel(r"$\beta$")
#ax[1].set_zlabel(r"Holt mse")
#ax[1].set_title(r"Model parameter $\alpha$ optimization")
results
```

```
Holt's try to find manually    0.95
Holt's model estimated         1.00
My Holt's model estimation     0.95
Name: $\alpha$, dtype: float64 Holt's try to find manually
5.000000e-02
Holt's model estimated         1.175507e-13
My Holt's model estimation     5.900000e-01
Name: $\beta$, dtype: float64 [0.005080765714657853, 0.95,
0.5900000000000001]
```

```
          Holt's try to find manually  Holt's model estimated  \
$\alpha$                     0.950000            1.000000e+00
$\beta$                      0.050000            1.175507e-13
$\phi$                            NaN                     NaN
$l_0$                        4.911196            4.968435e+00
$b_0$                        1.007098            1.000580e+00
```

	My Holt's model estimation
α	0.950000
β	0.590000
ϕ	NaN
l_0	4.971319
b_0	-1.000000

Optimization of exponential smoothing Holt's forecasting via mean squared error



Tripple exponential smoothing (Holt-Winter's method)

The forecasts generated by Holt's linear method display a constant trend (increasing or decreasing) indefinitely into the future. Empirical evidence indicates that these methods tend to over-forecast, especially for longer forecast horizons. Motivated by this observation, Gardner & McKenzie (1985) introduced a parameter that "dampens" the trend to a flat line some time in the future.

Holt (1957) and Winters (1960) extended Holt's method to capture seasonality. The Holt-Winters seasonal method comprises the forecast equation and three smoothing equations

There are two variations to this method that differ in the nature of the seasonal component. The additive method is preferred when the seasonal variations are roughly constant through the series, while the multiplicative method is preferred when the seasonal variations are changing proportional to the level of the series.

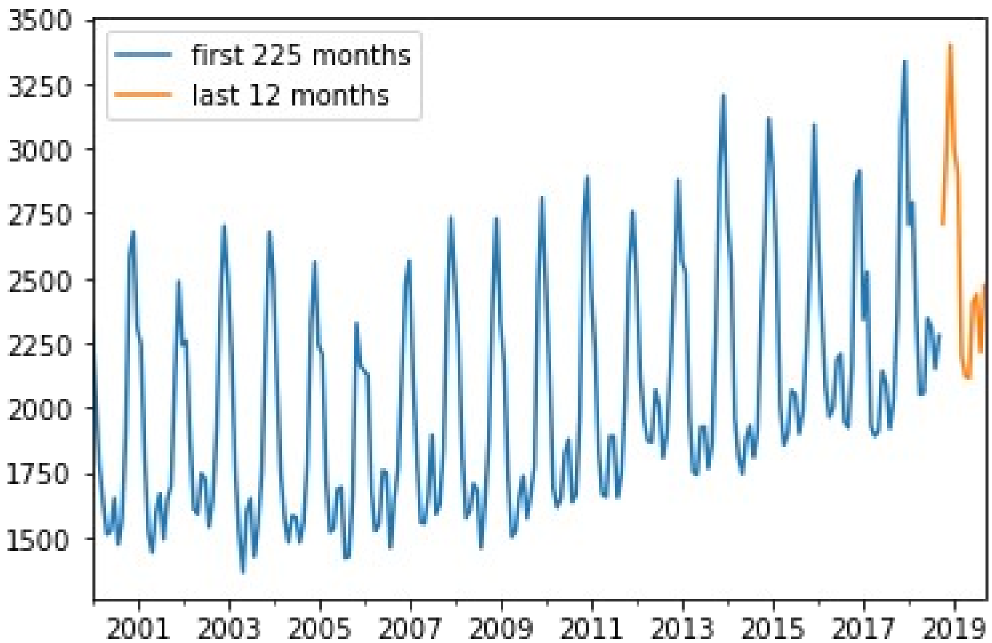
With the additive method, the seasonal component is expressed in absolute terms in the scale of the observed series, and in the level equation the series is seasonally adjusted by subtracting the seasonal component. Within each year, the seasonal component will add up to approximately zero

With the multiplicative method, the seasonal component is expressed in relative terms (percentages), and the series is seasonally adjusted by dividing through by the seasonal component. Within each year, the seasonal component will sum up to approximately m.

```
time = 12
usgas = pd.read_csv('USGas.csv')
datelist = pd.date_range('2000-01-01','2019-10-01', freq='M')
datelist
usgas.index = datelist
usgas.columns = ['price']
# take last time elements
usgas_last = usgas.tail(time)
usgas_last.columns = [f'last {time} months']
usgas_first = usgas.head(len(usgas) - time)
usgas_first.columns = [f'first {len(usgas)-time} months']

ax0=usgas_first.plot(label=f'first {len(usgas)-time} months')
usgas_last.plot(ax = ax0, label=f'last {time} months')
ax0.legend()
usgas
```

```
             price
2000-01-31  2330.7
2000-02-29  2050.6
2000-03-31  1783.3
2000-04-30  1632.9
2000-05-31  1513.1
...            ...
2019-05-31  2115.2
```

```
2019-06-30  2407.5
2019-07-31  2437.2
2019-08-31  2215.6
2019-09-30  2472.3

[237 rows x 1 columns]
```




```
# simple smoothing
fit1 = SimpleExpSmoothing(usgas_first,
initialization_method="estimated").fit()
# simple Holt's linear trend
fit2 = Holt(usgas_first, initialization_method="estimated").fit()
# Holt's exponential
fit3 = Holt(usgas_first, exponential=True,
initialization_method="estimated").fit()
# Holt's damped
fit4 = Holt(usgas_first, damped_trend=True,
initialization_method="estimated").fit(
    damping_trend=0.98
)
# Holt's damped exponential
fit5 = Holt(
    usgas_first, exponential=True, damped_trend=True,
initialization_method="estimated"
).fit()


fit_add_add= ExponentialSmoothing(
    usgas_first,
    seasonal_periods=2,
```

```
    trend="add",
    seasonal="add",
    damped_trend=True,
    use_boxcox=True,
    initialization_method="estimated",
).fit()

fit_add_mult= ExponentialSmoothing(
    usgas_first,
    seasonal_periods=2,
    trend="add",
    seasonal="mul",
    damped_trend=True,
    use_boxcox=True,
    initialization_method="estimated",
).fit()

fit_mult_add= ExponentialSmoothing(
    usgas_first,
    seasonal_periods=2,
    trend="mul",
    seasonal="add",
    damped_trend=True,
    use_boxcox=True,
    initialization_method="estimated",
).fit()

fit_mult_mult= ExponentialSmoothing(
    usgas_first,
    seasonal_periods=2,
    trend="mul",
    seasonal="mul",
    use_boxcox=True,
    damped_trend=True,
    initialization_method="estimated",
).fit()


params = [
    "smoothing_level",
    "smoothing_trend",
    "damping_trend",
    "smoothing_seasonal",
    "initial_level",
    "initial_trend",
]
```

```
C:\Users\maxgr\anaconda3\lib\site-packages\statsmodels\tsa\
holtwinters\model.py:80: RuntimeWarning: overflow encountered in
```

```
matmul
  return err.T @ err


results = pd.DataFrame(
    index=[r"$\alpha$", r"$\beta$", r"$\phi$",r"$\gamma$", r"$l_0$",
"$b_0$", "SSE"],
    columns=["SES", "Holt's linear", "Holt's exponential", "Damped
additive", "Damped multiplicative",
             "Holt's Winters trend:add-seas:add", "Holt's Winters
trend:mult-seas:add",
             "Holt's Winters trend:add-seas:mult", "Holt's Winters
trend:mult-seas:mult"]
)
results
```

```
          SES Holt's linear Holt's exponential Damped additive  \
$\alpha$  NaN           NaN                NaN             NaN
$\beta$   NaN           NaN                NaN             NaN
$\phi$    NaN           NaN                NaN             NaN
$\gamma$  NaN           NaN                NaN             NaN
$l_0$     NaN           NaN                NaN             NaN
$b_0$     NaN           NaN                NaN             NaN
SSE       NaN           NaN                NaN             NaN

         Damped multiplicative Holt's Winters trend:add-seas:add  \
$\alpha$                   NaN                               NaN
$\beta$                    NaN                               NaN
$\phi$                     NaN                               NaN
$\gamma$                   NaN                               NaN
$l_0$                      NaN                               NaN
$b_0$                      NaN                               NaN
SSE                        NaN                               NaN

         Holt's Winters trend:mult-seas:add  \
$\alpha$                                NaN
$\beta$                                 NaN
$\phi$                                  NaN
$\gamma$                                NaN
$l_0$                                   NaN
$b_0$                                   NaN
SSE                                     NaN

         Holt's Winters trend:add-seas:mult  \
$\alpha$                                NaN
$\beta$                                 NaN
$\phi$                                  NaN
$\gamma$                                NaN
$l_0$                                   NaN
$b_0$                                   NaN
```

```
SSE                                     NaN

         Holt's Winters trend:mult-seas:mult
$\alpha$                                 NaN
$\beta$                                  NaN
$\phi$                                   NaN
$\gamma$                                 NaN
$l_0$                                    NaN
$b_0$                                    NaN
SSE                                      NaN
```

```
# put to results non-seasonal
results["SES"] = [fit1.params[p] for p in params] + [fit1.sse]
results["Holt's linear"] = [fit2.params[p] for p in params] +
[fit2.sse]
results["Holt's exponential"] = [fit3.params[p] for p in params] +
[fit3.sse]
results["Damped additive"] = [fit4.params[p] for p in params] +
[fit4.sse]
results["Damped multiplicative"] = [fit5.params[p] for p in params] +
[fit5.sse]
results
```

```
                   SES  Holt's linear  Holt's exponential  Damped
additive  \
$\alpha$  1.000000e+00   9.951292e-01        9.950000e-01
9.963088e-01
$\beta$            NaN   2.308149e-02        1.000000e-04
7.392281e-05
$\phi$             NaN            NaN                 NaN
9.800000e-01
$\gamma$           NaN            NaN                 NaN
NaN
$l_0$     2.626320e+03   2.048957e+03        2.032320e+03
2.163984e+03
$b_0$              NaN   3.514904e-01        9.742113e-01    -
3.819292e+01
SSE       1.841552e+07   1.889183e+07        1.866914e+07
1.842298e+07

          Damped multiplicative Holt's Winters trend:add-seas:add  \
$\alpha$           9.950074e-01                               NaN
$\beta$            9.990454e-05                               NaN
$\phi$             9.900074e-01                               NaN
$\gamma$                    NaN                               NaN
$l_0$              2.424100e+03                               NaN
$b_0$              9.898477e-01                               NaN
SSE                1.831165e+07                               NaN

         Holt's Winters trend:mult-seas:add  \
```

```
$\alpha$                                NaN
$\beta$                                 NaN
$\phi$                                  NaN
$\gamma$                                NaN
$l_0$                                   NaN
$b_0$                                   NaN
SSE                                     NaN

         Holt's Winters trend:add-seas:mult  \
$\alpha$                                NaN
$\beta$                                 NaN
$\phi$                                  NaN
$\gamma$                                NaN
$l_0$                                   NaN
$b_0$                                   NaN
SSE                                     NaN

         Holt's Winters trend:mult-seas:mult
$\alpha$                                 NaN
$\beta$                                  NaN
$\phi$                                   NaN
$\gamma$                                 NaN
$l_0$                                    NaN
$b_0$                                    NaN
SSE                                      NaN
```

```
# put to results seasonal
results["Holt's Winters trend:add-seas:add"] = [fit_add_add.params[p]
for p in params] + [fit1.sse]
results["Holt's Winters trend:mult-seas:add"] =
[fit_mult_add.params[p] for p in params] + [fit2.sse]
results["Holt's Winters trend:add-seas:mult"] =
[fit_add_mult.params[p] for p in params] + [fit3.sse]
results["Holt's Winters trend:mult-seas:mult"] =
[fit_mult_mult.params[p] for p in params] + [fit4.sse]
results
```

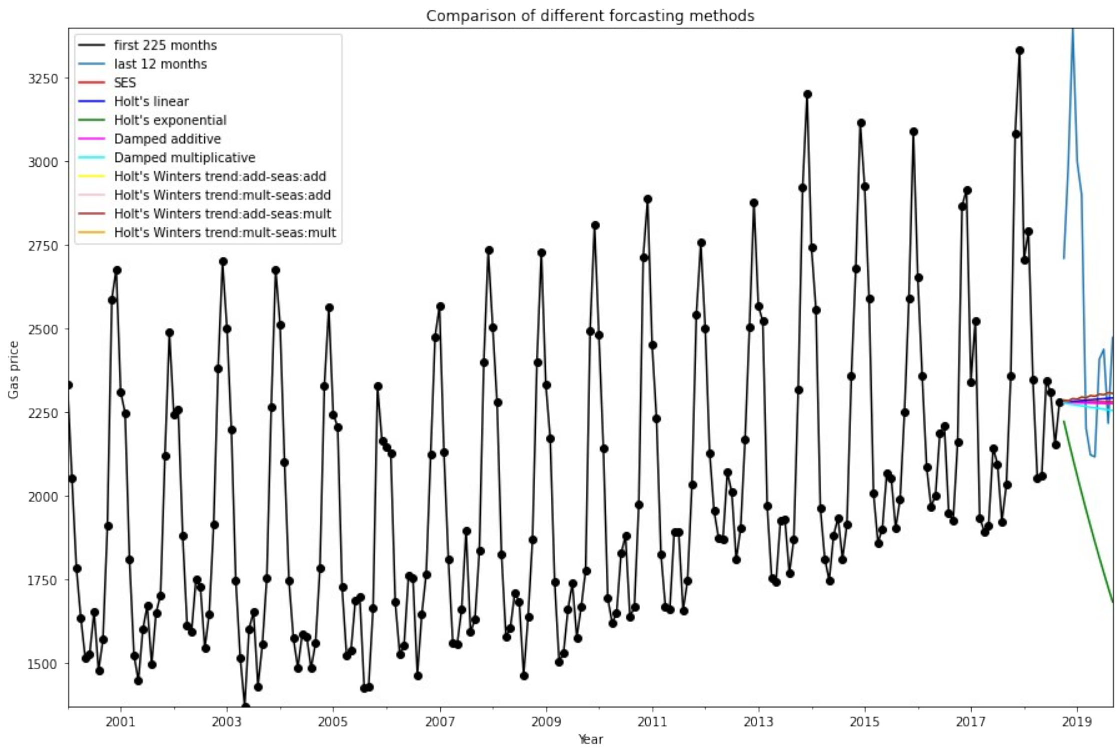
```
                   SES  Holt's linear  Holt's exponential  Damped
additive  \
$\alpha$  1.000000e+00   9.951292e-01        9.950000e-01
9.963088e-01
$\beta$            NaN   2.308149e-02        1.000000e-04
7.392281e-05
$\phi$             NaN            NaN                 NaN
9.800000e-01
$\gamma$           NaN            NaN                 NaN
NaN
$l_0$     2.626320e+03   2.048957e+03        2.032320e+03
2.163984e+03
$b_0$              NaN   3.514904e-01        9.742113e-01    -
```

```
3.819292e+01
SSE        1.841552e+07   1.889183e+07        1.866914e+07
1.842298e+07

           Damped multiplicative  Holt's Winters trend:add-seas:add  \
$\alpha$           9.950074e-01                        9.950000e-01
$\beta$            9.990454e-05                        9.999681e-05
$\phi$             9.900074e-01                        9.900000e-01
$\gamma$                    NaN                        5.000007e-03
$l_0$              2.424100e+03                        1.013720e+00
$b_0$              9.898477e-01                        4.860738e-06
SSE                1.831165e+07                        1.841552e+07

           Holt's Winters trend:mult-seas:add  \
$\alpha$                         1.490116e-08
$\beta$                          3.106468e-10
$\phi$                           9.950000e-01
$\gamma$                         4.983185e-17
$l_0$                            1.016091e+00
$b_0$                            1.005026e+00
SSE                              1.889183e+07

           Holt's Winters trend:add-seas:mult  \
$\alpha$                         9.950000e-01
$\beta$                          9.999690e-05
$\phi$                           9.900000e-01
$\gamma$                         5.000007e-03
$l_0$                            1.013720e+00
$b_0$                            4.892010e-06
SSE                              1.866914e+07

           Holt's Winters trend:mult-seas:mult
$\alpha$                          9.950000e-01
$\beta$                           9.999690e-05
$\phi$                            9.900000e-01
$\gamma$                          5.000007e-03
$l_0$                             1.013720e+00
$b_0$                             1.010106e+00
SSE                               1.842298e+07
```

```
ax = usgas_first.plot(
    figsize=(15, 10),
    marker="o",
    color="black",
    title="Comparison of different forcasting methods",
)
ax.set_ylabel("Gas price")
ax.set_xlabel("Year")
```

```
usgas_last.plot(ax=ax, legend=True)
# forecast
time = 12
fcast1 = fit1.forecast(time).rename("SES")
fcast2 = fit2.forecast(time).rename("Holt's linear")
fcast3 = fit3.forecast(time).rename("Holt's exponential")
fcast4 = fit4.forecast(time).rename("Damped additive")
fcast5 = fit5.forecast(time).rename("Damped multiplicative")

# seasonal
fcast6 = fit_add_add.forecast(time).rename("Holt's Winters trend:add-
seas:add")
fcast7 = fit_mult_add.forecast(time).rename("Holt's Winters
trend:mult-seas:add")
fcast8 = fit_add_mult.forecast(time).rename("Holt's Winters trend:add-
seas:mult")
fcast9 = fit_mult_mult.forecast(time).rename("Holt's Winters
trend:mult-seas:mult")
fcast1 = pd.Series(fcast1)
fcast2 = pd.Series(fcast2)
fcast3 = pd.Series(fcast3)
fcast4 = pd.Series(fcast4)
fcast5 = pd.Series(fcast5)
fcast6 = pd.Series(fcast6)
fcast7 = pd.Series(fcast7)
fcast8 = pd.Series(fcast8)
fcast9 = pd.Series(fcast9)

colors = ['red', 'blue', 'green', 'magenta', 'cyan', 'yellow', 'pink',
'brown', 'orange']
fcast1.plot( ax=ax, color=colors[0], legend = True)
fcast2.plot( ax=ax, color=colors[1], legend = True)
fcast3.plot( ax=ax, color=colors[2], legend = True)
fcast4.plot( ax=ax, color=colors[3], legend = True)
fcast5.plot( ax=ax, color=colors[4], legend = True)
fcast6.plot( ax=ax, color=colors[5], legend = True)
fcast7.plot( ax=ax, color=colors[6], legend = True)
fcast8.plot( ax=ax, color=colors[7], legend = True)
fcast9.plot( ax=ax, color=colors[8], legend = True)

ax.set_ylim(np.min(usgas.to_numpy()), np.max(usgas.to_numpy()))

plt.show()
#usgas_last.plot(ax=ax, legend=True)
```

```
C:\Users\maxgr\anaconda3\lib\site-packages\statsmodels\tsa\base\
tsa_model.py:132: FutureWarning: The 'freq' argument in Timestamp is
deprecated and will be removed in a future version.
  date_key = Timestamp(key, freq=base_index.freq)
```



Comparison of different forcasting methods
first 225 months
last 12 months
SES
Holt's linear
Holt's exponential
Damped additive
Damped multiplicative
Holt's Winters trend:add-seas:add
Holt's Winters trend:mult-seas:add
Holt's Winters trend:add-seas:mult
Holt's Winters trend:mult-seas:mult
Gas price
3250
3000
2750
2500
2250
2000
1750
1500
2001
2003
2005
2007
2009
2011
2013
2015
2017
2019
Year