Super Resolution for Automated Target Recognition

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***Abstract*—Super resolution is the process of producing high-resolution images from low-resolution images while preserving ground truth about the subject matter of the images and potentially inferring more such truth. Algorithms that successfully carry out such a process are broadly useful in all circumstances where high-resolution imagery is either difficult or impossible to obtain. In particular we look towards super resolving images collected using longwave infrared cameras since high resolution sensors for such cameras do not currently exist. We present an exposition of motivations and concepts of super resolution in general, and current techniques, with a qualitative comparison of such techniques. Finally we suggest directions for future research.**

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1.1 Differential Geometry

1.1.1 Directional Derivative

Elements of the *tangent space* $T_p(\mathbb{R}^n)$ anchored at a point $p = (p^1, \ldots, p^n) \in \mathbb{R}^n$ are called *tangent vectors*

$$\boldsymbol{v} = \begin{bmatrix} v^1 \\ \vdots \\ v^n \end{bmatrix} = \left[v^1, \ldots, v^n\right] \tag{1}$$

The line through a point p with direction $\boldsymbol{v}$ has parameterization

$$c(t) = \left(p^1 + tv^1, \ldots, p^n + tv^n\right) \tag{2}$$

If $f \in C^\infty$ in a neighborhood of p and $\boldsymbol{v}$ is a tangent vector at p, the *directional derivative* of f in the direction of $\boldsymbol{v}$ at p is defined

$$D_{\boldsymbol{v}} f = \lim_{t \to 0} \frac{f(c(t)) - f(p)}{t} = \frac{\mathrm{d}}{\mathrm{d}t} f(c(t))\bigg|_{t=0} \tag{3}$$

By the chain rule

$$D_{\boldsymbol{v}} f = \sum_{i=1}^{n} \frac{\partial f}{\partial x^i}\bigg|_p \frac{\mathrm{d}c^i}{\mathrm{d}t}\bigg|_{t=0} \tag{4}$$

$$= \sum_{i=1}^{n} \frac{\mathrm{d}c^i}{\mathrm{d}t}\bigg|_{t=0} \frac{\partial f}{\partial x^i}\bigg|_p \tag{5}$$

$$= \sum_{i=1}^{n} v^i \frac{\partial f}{\partial x^i}\bigg|_p \tag{6}$$

$$\tag{7}$$

The directional derivative operator at p is defined

$$D_{\boldsymbol{v}} = \sum_{i=1}^{n} v^i \frac{\partial}{\partial x^i}\bigg|_p \tag{8}$$

The association $\boldsymbol{v} \mapsto D_{\boldsymbol{v}}$ offers a way to characterize tangent vectors as operators on functions.

1.1.2 Germs

Consider the set of all pairs (f, U), where U is a neighborhood of p and $f\colon U \to \mathbb{R}$ is a C^∞ function. We say that $(f, U) \sim (g, U')$ if there is an open W such that $p \in W \subset U \cap U'$ and $f = g$ when restricted to W. The equivalence class of (f, U) is the *germ* of f at p. We write $C_p^\infty(\mathbb{R}^n)$ for the set all germs of C^∞ functions on $\mathbb{R}^n$ at p.

An *algebra* over *field* K is a vector space A over K with a multiplication map

$$\mu\colon A \times A \to A \tag{9}$$

usually written $\mu(a, b) = a \cdot b$, such that μ is associative, distributive, homogeneous

$$r(a \cdot b) = (ra \cdot b) = (a \cdot rb) \tag{10}$$

Let L be a $K-$*linear operator* $L\colon V \to W$. If A, A' are algebras then an *algebra homomorphism* is a linear operator L that respects algebra multiplication $L(ab) = L(a)L(b)$. It's the case that addition and multiplication of functions induce addition and multiplication on the set of germs C_p^∞, making it into an $\mathbb{R}-$algebra of germs.

1.1.3 Derivations

For each tangent vector $\boldsymbol{v}$ at a point $p \in \mathbb{R}^n$, the directional derivative at p gives a map of vector spaces

$$D_{\boldsymbol{v}}\colon C_p^\infty \to \mathbb{R}$$

By eqn. (6), $D_{\boldsymbol{v}}$ is a linear map that satisfies the *Leibniz rule*

$$D_{\boldsymbol{v}}(fg) = (D_{\boldsymbol{v}} f)g(p) + f(p)(D_{\boldsymbol{v}} g) \tag{11}$$

In general, any linear map $L\colon C_p^\infty \to \mathbb{R}$ that satisfies the Leibniz rule is called a *derivation* at p or a *point derivation* of C_p^∞. Denote the set of all derivations at p by $\mathcal{D}_p(\mathbb{R}^n)$ This set is also a real vector space. Thus, there is a map

$$\begin{aligned} \phi\colon T_p(\mathbb{R}^n) &\to \mathcal{D}_p(\mathbb{R}^n) \\ \boldsymbol{v} &\mapsto D_{\boldsymbol{v}} \end{aligned}$$

Theorem 1: The linear map ϕ is an *isomorphism* of vector spaces

What this means is we may identify tangent vectors at p with derivations at p (by way of directional derivatives against germs). Under this isomorphism $T_p(\mathbb{R}^n) \simeq \mathcal{D}_p(\mathbb{R}^n)$, the standard basis $\{e_1, \ldots, e_n\}$ for $T_p(\mathbb{R}^n)$ maps to

$$\left\{ \left.\frac{\partial}{\partial x^1}\right|_p, \ldots, \left.\frac{\partial}{\partial x^n}\right|_p \right\}$$

hence from now on we write

$$\boldsymbol{v} = \sum_{i=1}^n v^i \left.\frac{\partial}{\partial x^i}\right|_p \tag{12}$$

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