Super Resolution for Automated Target Recognition

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Abstract

Super resolution is the process of producing high-resolution images from low-resolution images while preserving ground truth about the subject matter of the images and potentially inferring more such truth. Algorithms that successfully carry out such a process are broadly useful in all circumstances where high-resolution imagery is either difficult or impossible to obtain. In particular we look towards super resolving images collected using longwave infrared cameras since high resolution sensors for such cameras do not currently exist. We present an exposition of motivations and concepts of super resolution in general, and current techniques, with a qualitative comparison of such techniques. Finally we suggest directions for future research.

1 Appendix

1.1 Automatic Differentiation

1.2 Splines

A *spline* is a piecewise defined polynomial function. For example a simple *order* $d+1$ spline s could be defined

$$s(x) \coloneqq \sum_{i=0}^{d} \mathbb{1}_{[k_j, k_{j+1})} P_{ij} \cdot (x - k_j)^i$$

where the $n+1$ increasing *knots* $k_0 < \cdots < k_n$ bracket n intervals over which the component polynomials are defined, and the $(d+1)$ polynomial coefficients P_{ij} define the spline (see figure **??**) for $x \in [k_j, k_{j+1})$ ($\mathbb{1}$ enforces this[1]). Splines need not be differentiable (nor even continuous) at the knots but can be specified with such continuity constraints (at the knots).

The number of degrees of freedom (the dimension of the vector space[2] of such splines) of a spline over $n+1$ knots and of order $d+1$ is the number of polynomial coefficients minus the number of continuity conditions. For example if the spline is constrained to be maximally continuous ($d-1$ times[3]) then the spline has $d+1$ polynomial coefficients for every one of the n knot intervals and d continuity constraints at every one of the $n-1$ interior knots (continuity constraints cannot be specified at the boundary knots), which implies

$$n(d+1) - (n-1)d = n + d$$

degrees of freedom.

[1] The indicator (or characteristic) $\mathbb{1}_{[a,b)}$ function is 1 on the interval $[a, b)$ and 0 otherwise.

[2] A set whose members (*vectors*) can be decomposed as linear combinations of elementary elements (a *basis*).

[3] Note that even though a d-degree polynomial has d derivatives we only require agreement at the first $d-1$ derivatives (and continuity itself). This is owing to the fact that if we required all d derivatives to agree the number of degrees of freedom would be $(n(d+1) - (n-1)(d+1) = d+1$ and therefore the spline would no longer be a piecewise polynomial but simply a polynomial (of degree d).

A Basis-spline (B-spline) is a spline defined as a linear combination of a particular set of basis functions: the B-spline basis element $B_{i,d+1}$ of order $d+1$ (degree d) over knots $k_i < \cdots < k_{i+d+1}$ is defined recursively according to the Cox-de Boor recursion formula **de1971subroutine**:

$$B_{j,1}(x) \coloneqq \mathbb{1}_{[k_j, k_{j+1})} \tag{1}$$

$$B_{j,d+1}(x) \coloneqq \frac{x - k_j}{k_{j+d} - k_j} B_{j,d}(x) + \frac{k_{j+d+1} - x}{k_{j+d+1} - k_{i+1}} B_{j+1,d}(x) \tag{2}$$

where $\frac{x-k_j}{k_{j+d}-k_j}$ increases smoothly from zero to one as x goes from k_j to k_{j+d} and $\frac{k_{j+d+1}-x}{k_{j+d+1}-k_{i+1}}$ decreases smoothly from one to zero as x goes from k_{i+1} to k_{j+d+1}.

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