Evidence of accurate logical reasoning in online sentence comprehension

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slides available at `https://ling.auf.net/lingbuzz/005989`

Introduction

- what is the status of logic in thought?

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- logic studies relations among propositions

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└─Introduction

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└─logic in thought

Logic, to characterize it broadly, is the formal study of relations which obtain among propositions.

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- what is the status of logic in thought?
- logic studies relations among propositions

Dictum de omni

All rats love to eat.

∴ All spotted rats love to eat.

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Logical schemata capture rules which govern correct inference. The example
here is the *dictum de omni*, which is the principle that whatever is affirmed of
a kind can be affirmed of its subkind.

- what is the status of logic in thought?
- logic studies relations among propositions

Dictum de omni

All rats love to eat.
∴ All spotted rats love to eat.

- do such schemata capture the nature of thought?

While such schemata capture normatively accurate inferences, there has been a long standing debate over what their status is in human cognition. This is to say, are such logical schemata the rules of thought?

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└─Introduction

└─logic in psychology

The status of logic in thought has been viewed very differently by the disciplines of psychology and linguistics.

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- **psychology** has focused on difficulties in logical reasoning
  - Wason's (1968) selection tasks easier when ecologically valid (Cheng and Holyoak, 1985, 1989; Cheng, Holyoak, et al., 1986)
  - dual-process theories (Evans and Stanovich, 2013; Kahneman, 2011)

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Research in psychology has tended to focus on documenting which logical tasks are more difficult than others and understanding why.

logic in linguistics

- **formal semantics** presupposes logical ability
  the logical notions are embedded in our deepest nature, in the very form of our language and thought

  Chomsky (1988, p. 99)

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On on the hand, certain branches of linguistics posit that language has a kind of logical form. Thus, implicitly, linguists credit language-users with a rather high degree of logical sophistication.

- **formal semantics** presupposes logical ability
  *the logical notions are embedded in our deepest nature, in
  the very form of our language and thought*

  Chomsky (1988, p. 99)

- linguists predict some logical thought as effortless as language

This view of language predicts that at least some inferences should be as intuitive, automatic, and effortless as thought and language themselves.

- **formal semantics** presupposes logical ability
  the logical notions are embedded in our deepest nature, in the very form of our language and thought

  Chomsky (1988, p. 99)

- linguists predict some logical thought as effortless as language

- can we find evidence for **spontaneous logical computation**?

Thus, the following question arises: Can we find some evidence for fast and automatic reasoning?

- **formal semantics** presupposes logical ability
  the logical notions are embedded in our deepest nature, in the very form of our language and thought

  Chomsky (1988, p. 99)

- linguists predict some logical thought as effortless as language
- can we find evidence for **spontaneous logical computation**?
- **entailment**: if *p* is true, then *q* is also true

Dictum de omni

All rats love to eat.
∴ All spotted rats love to eat.

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└─Introduction

└─logic in linguistics

To answer this question, in our study, we focused on the notion of entailment. Entailment is a relation between propositions such that when one proposition is true, another proposition is always also true. Entailment has been argued to be based in the structure of language. Thus, it is a good candidate for a logical notion that is processed fast and automatically.

Methods

- two novel self-paced reading experiments

We designed two novel self-paced reading experiments.

- two novel self-paced reading experiments
- tested for signatures of accurate inferences between quantified sentences

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We tested for signatures of normatively accurate logical inferences between quantified sentences during sentence comprehension.

- two novel self-paced reading experiments
- tested for signatures of accurate inferences between quantified sentences
- experiment 1 involved detecting logical contradictions

Experiment 1 tested whether speakers detect logical contradictions.

- two novel self-paced reading experiments
- tested for signatures of accurate inferences between quantified sentences
- experiment 1 involved detecting logical contradictions
- experiment 2 leveraged variable entailments of the first and second arguments of quantifiers to detect incorrect inferences

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And Experiment 2 tested detecting something subtler... which is incorrect inferences in the absence of downright contradictions.

- two novel self-paced reading experiments
- tested for signatures of accurate inferences between quantified sentences
- experiment 1 involved detecting logical contradictions
- experiment 2 leveraged variable entailments of the first and second arguments of quantifiers to detect incorrect inferences
- preregistered design and analyses on OSF

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Experiment 1

- tested whether speakers detect logical contradictions

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└─Experiment 1
  └─Methods
    └─experiment 1

But first, experiment 1.

experiment 1

- tested whether speakers detect logical contradictions
- 400 participants on Amazon Mechanical Turk

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└─Experiment 1
  └─Methods
    └─experiment 1

We ran 400 participants on Amazon Mechanical Turk.

experiment 1

- tested whether speakers detect logical contradictions
- 400 participants on Amazon Mechanical Turk
- 12 target items displayed line by line

Participants read 12 target items displayed line-by-line, with line breaks at clausal boundaries.

experiment 1

- tested whether speakers detect logical contradictions
- 400 participants on Amazon Mechanical Turk
- 12 target items displayed line by line
- 6 conditions differing in quantifiers

Across participants, each item appeared in six conditions which differed in what quantifiers were used.

- tested whether speakers detect logical contradictions
- 400 participants on Amazon Mechanical Turk
- 12 target items displayed line by line
- 6 conditions differing in quantifiers

Test item

(1) *A group of scientists wanted to know whether spotted rats,*

(2) *who are pickier eaters than other rats, liked a new kind of food.*

(3) *They tested white, black, and spotted rats of both sexes.*

(4) *The scientists discovered that QUANT1 of the rats loved the food.*

(5) *Now that they knew that QUANT2 of the rats loved the food,*

(6) *they decided to issue a recommendation based on their findings.*

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Each item contained a "premise" in line 4 and a "conclusion" in line 5, which began with *now that they knew that ...,* presupposing that what comes next appeared earlier in the discourse. Otherwise, the two lines differed only in the quantifiers they used.

- tested whether speakers detect logical contradictions
- 400 participants on Amazon Mechanical Turk
- 12 target items displayed line by line
- 6 conditions differing in quantifiers

Test item

(1) *A group of scientists wanted to know whether spotted rats,*

(2) *who are pickier eaters than other rats, liked a new kind of food.*

(3) *They tested white, black, and spotted rats of both sexes.*

(4) *The scientists discovered that QUANT1 of the rats loved the food.*

(5) *Now that they knew that QUANT2 of the rats loved the food,*

(6) *they decided to issue a recommendation based on their findings.*

- measured variable: RT of the conclusion line (5)
- participants were asked unrelated comprehension questions
  - *The researchers studied rodents.* [TRUE] [FALSE]

We measured the reading times of the boxed conclusion line as a proxy for processing costs.

	QUANT1	*QUANT2*
IDENTITY	*some* of the rats loved	they knew that *some* of the rats . . .
IDENTITY	*not all* of the rats loved . . .	they knew that *not all* of the rats . . .
ENTAILMENT	*all* of the rats loved	they knew that *some* of the rats . . .
ENTAILMENT	*none* of the rats loved	they knew that *not all* of the rats . . .
CONTRADICTION	*none* of the rats loved	they knew that *some* of the rats . . .
CONTRADICTION	*all* of the rats loved	they knew that *not all* of the rats . . .

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└─Experiment 1
  └─Methods
    └─experiment 1 conditions

There were two conditions where the premise was identical to the conclusion, two conditions where it differed from but entailed the conclusion, and two conditions where it contradicted it.
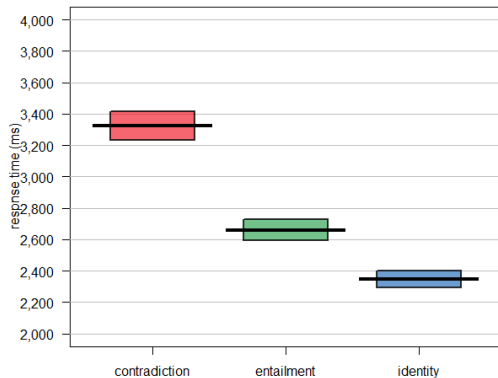
identity

entailment

contradiction

LMER effect between contradiction and entailment: $\chi^2 = 161.31$

$p < 0.001$

Evidence of accurate logical reasoning in online sentence comprehension
└─Experiment 1
  └─Results
    └─experiment 1 results

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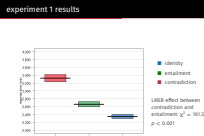
All of the conditions were significantly different from each other. Participants took significantly longer to advance to the conclusion line when it contradicted the premise than when it was entailed by the premise. This is consistent with rapid, normatively accurate sensitivity to the logical relations between these clauses.

Experiment 2

- same paradigm to detect subtler unlicensed inferences (n = 400)

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└─Experiment 2

  └─Methods

    └─experiment 2

Experiment 2 used the same paradigm to test for the capacity to detect subtler unlicensed inferences, even in the absence of strict contradictions.

- same paradigm to detect subtler unlicensed inferences (n = 400)
- manipulated quantifiers and premise quantifier's 1st arg

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experiment 2

- same paradigm to detect subtler unlicensed inferences (n = 400)
- manipulated quantifiers and premise quantifier's 1st arg

Evidence of accurate logical reasoning in online sentence comprehension
└─Experiment 2
  └─Methods
    └─experiment 2

We manipulated the quantifiers in both the premise and the conclusion ... ... as well as the noun phrase in first argument of the quantifier in the premise.

- same paradigm to detect subtler unlicensed inferences (n = 400)
- manipulated quantifiers and premise quantifier's 1st arg

Test item

(1) *A group of scientists wanted to know whether spotted rats,*
(2) *who are pickier eaters than other rats, liked a new kind of food.*
(3) *They tested white, black, and spotted rats of both sexes.*
(4) *The scientists discovered that QUANT of the ((male) spotted) rats loved the food.*
(5) *Now that they knew that QUANT of the spotted rats loved the food,*
(6) *they decided to issue a recommendation based on their findings.*

experiment 2

- same paradigm to detect subtler unlicensed inferences (n = 400)
- manipulated quantifiers and premise quantifier's 1st arg

Test item

(1) *A group of scientists wanted to know whether spotted rats,*
(2) *who are pickier eaters than other rats, liked a new kind of food.*
(3) *They tested white, black, and spotted rats of both sexes.*
(4) *The scientists discovered that QUANT of the ((male) spotted) rats loved the food.*
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Evidence of accurate logical reasoning in online sentence comprehension
└─ Experiment 2
  └─ Methods
    └─ experiment 2

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In the test items, we changed lines four and five to manipulate whether the conclusion was entailed by the premise.

Unlike experiment 1, here, the quantifier was kept constant between the premise and the conclusion. The premise noun phrase appeared with two, one, or zero modifiers. The conclusion noun phrase always appeared with one modifier.

- same paradigm to detect subtler unlicensed inferences (n = 400)
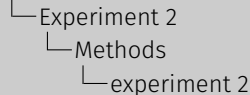- manipulated quantifiers and premise quantifier's 1st arg

Test item

(1) *A group of scientists wanted to know whether spotted rats,*
(2) *who are pickier eaters than other rats, liked a new kind of food.*
(3) *They tested white, black, and spotted rats of both sexes.*
(4) *The scientists discovered that QUANT of the ((male) spotted) rats loved the food.*
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- 4 quantifiers × 3 containment relations = 12 conditions

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└─Experiment 2
  └─Methods
    └─experiment 2

Thus, the premise noun phrase was a subset of, identical to, or a superset of the conclusion noun phrase. **Four** quantifiers and **three** containment relations yielded twelve experimental conditions in total, which will ultimately reduce to a two-by-two.

Depending on the combination of the quantifier and containment,

- same paradigm to detect subtler unlicensed inferences (n = 400)
- manipulated quantifiers and premise quantifier's 1st arg

Test item

(1) *A group of scientists wanted to know whether spotted rats,*
(2) *who are pickier eaters than other rats, liked a new kind of food.*
(3) *They tested white, black, and spotted rats of both sexes.*
(4) *The scientists discovered that QUANT of the ((male) spotted) rats loved the food.*
(5) *Now that they knew that QUANT of the spotted rats loved the food,*
(6) *they decided to issue a recommendation based on their findings.*

- 4 quantifiers × 3 containment relations = 12 conditions
  - 4 conditions: premise identical to (trivially entails) conclusion

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  └─Methods
    └─experiment 2

there were four conditions where the premise was identical to the conclusion, and so it trivially entailed it,

- same paradigm to detect subtler unlicensed inferences (n = 400)
- manipulated quantifiers and premise quantifier's 1st arg

Test item

(1) *A group of scientists wanted to know whether spotted rats,*
(2) *who are pickier eaters than other rats, liked a new kind of food.*
(3) *They tested white, black, and spotted rats of both sexes.*
(4) *The scientists discovered that QUANT of the ((male) spotted) rats loved the food.*
(5) *Now that they knew that QUANT of the spotted rats loved the food,*
(6) *they decided to issue a recommendation based on their findings.*

- 4 quantifiers × 3 containment relations = 12 conditions
  - 4 conditions: premise identical to (trivally entails) conclusion
  - 4 conditions: premise entails conclusion

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four conditions where the premise differed from but entailed the conclusion,

- same paradigm to detect subtler unlicensed inferences (n = 400)
- manipulated quantifiers and premise quantifier's 1st arg

Test item

(1) *A group of scientists wanted to know whether spotted rats,*
(2) *who are pickier eaters than other rats, liked a new kind of food.*
(3) *They tested white, black, and spotted rats of both sexes.*
(4) *The scientists discovered that QUANT of the ((male) spotted) rats loved the food.*
(5) *Now that they knew that QUANT of the spotted rats loved the food,*
(6) *they decided to issue a recommendation based on their findings.*

- 4 quantifiers × 3 containment relations = 12 conditions
  - 4 conditions: premise identical to (trivally entails) conclusion
  - 4 conditions: premise entails conclusion
  - 4 conditions: premise does not entail conclusion

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and four conditions where the premise did not entail the conclusion.

- same paradigm to detect subtler unlicensed inferences (n = 400)
- manipulated quantifiers and premise quantifier's 1st arg

Test item

(1) *A group of scientists wanted to know whether spotted rats,*
(2) *who are pickier eaters than other rats, liked a new kind of food.*
(3) *They tested white, black, and spotted rats of both sexes.*
(4) *The scientists discovered that QUANT of the ((male) spotted) rats loved the food.*
(5) *Now that they knew that QUANT of the spotted rats loved the food,*
(6) *they decided to issue a recommendation based on their findings.*

- 4 quantifiers × 3 containment relations = 12 conditions
  - 4 conditions: premise identical to (trivially entails) conclusion
  - 4 conditions: premise entails conclusion
  - 4 conditions: premise does not entail conclusion
- within quantifier, critical lines have identical lexical content

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    └─experiment 2

An elegant feature of our design is that within quantifier, we're looking at differences in reading times among lines that have identical lexical content, so whatever the difference is on the critical line, that difference *must be* due to preceding context.

	SOME	*NOT ALL*	*ALL*	*NONE*
SUBSET → of spotted rats →	*... **some** of the* **male spotted rats** *loved the food. Now that they knew that **some** of the* **spotted rats** *...*	*... **not all** of the* **male spotted rats** *loved the food. Now that they knew that **not all** of the* **spotted rats** *...*	*... **all** of the* **male spotted rats** *loved the food. Now that they knew that **all** of the* **spotted rats** *...*	*... **none** of the* **male spotted rats** *loved the food. Now that they knew that **none** of the* **spotted rats** *...*
IDENTICAL → to spotted rats →	*... **some** of the* **spotted rats** *loved the food. Now that they knew that **some** of the* **spotted rats** *...*	*... **not all** of the* **spotted rats** *loved the food. Now that they knew that **not all** of the* **spotted rats** *...*	*... **all** of the* **spotted rats** *loved the food. Now that they knew that **all** of the* **spotted rats** *...*	*... **none** of the* **spotted rats** *loved the food. Now that they knew that **none** of the* **spotted rats** *...*
SUPERSET → of spotted rats →	*... **some** of the* **rats** *loved the food. Now that they knew that **some** of the* **spotted rats** *...*	*... **not all** of the* **rats** *loved the food. Now that they knew that **not all** of the* **spotted rats** *...*	*... **all** of the* **rats** *loved the food. Now that they knew that **all** of the* **spotted rats** *...*	*... **none** of the* **rats** *loved the food. Now that they knew that **none** of the* **spotted rats** *...*

- 🟦 trivially entailed
- 🟩 entailed
- 🟧 not entailed

experiment 2 conditions, full

	SOME	NOT ALL	ALL	NONE
SUBSET → of spotted rats →	… **some** _of the_ **male spotted rats** _loved the food. Now that they knew that_ **some** _of the_ **spotted rats** _…_	… **not all** _of the_ **male spotted rats** _loved the food. Now that they knew that_ **not all** _of the_ **spotted rats** _…_	… **all** _of the_ **male spotted rats** _loved the food. Now that they knew that_ **all** _of the_ **spotted rats** _…_	… **none** _of the_ **male spotted rats** _loved the food. Now that they knew that_ **none** _of the_ **spotted rats** _…_
IDENTICAL → to spotted rats →	… **some** _of the_ **spotted rats** _loved the food. Now that they knew that_ **some** _of the_ **spotted rats** _…_	… **not all** _of the_ **spotted rats** _loved the food. Now that they knew that_ **not all** _of the_ **spotted rats** _…_	… **all** _of the_ **spotted rats** _loved the food. Now that they knew that_ **all** _of the_ **spotted rats** _…_	… **none** _of the_ **spotted rats** _loved the food. Now that they knew that_ **none** _of the_ **spotted rats** _…_
SUPERSET → of spotted rats →	… **some** _of the_ **rats** _loved the food. Now that they knew that_ **some** _of the_ **spotted rats** _…_	… **not all** _of the_ **rats** _loved the food. Now that they knew that_ **not all** _of the_ **spotted rats** _…_	… **all** _of the_ **rats** _loved the food. Now that they knew that_ **all** _of the_ **spotted rats** _…_	… **none** _of the_ **rats** _loved the food. Now that they knew that_ **none** _of the_ **spotted rats** _…_

- ■ trivially entailed
- ■ entailed
- ■ not entailed

experiment 2 conditions, full

|	SOME	NOT ALL	ALL	NONE
SUBSET → of spotted rats →	… **some** of the **male spotted rats** loved the food. Now that they knew that **some** of the **spotted rats** …	… **not all** of the **male spotted rats** loved the food. Now that they knew that **not all** of the **spotted rats** …	… **all** of the **male spotted rats** loved the food. Now that they knew that **all** of the **spotted rats** …	… **none** of the **male spotted rats** loved the food. Now that they knew that **none** of the **spotted rats** …
IDENTICAL → to spotted rats →	… **some** of the **spotted rats** loved the food. Now that they knew that **some** of the **spotted rats** …	… **not all** of the **spotted rats** loved the food. Now that they knew that **not all** of the **spotted rats** …	… **all** of the **spotted rats** loved the food. Now that they knew that **all** of the **spotted rats** …	… **none** of the **spotted rats** loved the food. Now that they knew that **none** of the **spotted rats** …
SUPERSET → of spotted rats →	… **some** of the **rats** loved the food. Now that they knew that **some** of the **spotted rats** …	… **not all** of the **rats** loved the food. Now that they knew that **not all** of the **spotted rats** …	… **all** of the **rats** loved the food. Now that they knew that **all** of the **spotted rats** …	… **none** of the **rats** loved the food. Now that they knew that **none** of the **spotted rats** …

- ■ trivially entailed
- ■ entailed
- ■ not entailed

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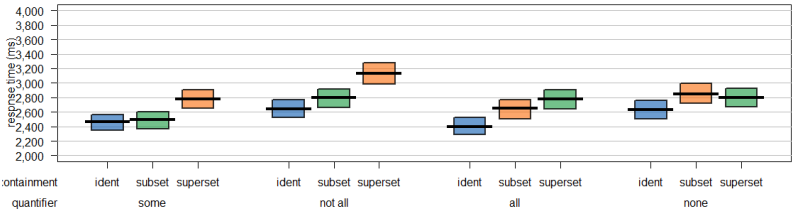└─Experiment 2
 └─Methods
  └─experiment 2 conditions, full

When the premise noun phrase is a superset of the conclusion noun phrase we get an unlicensed inference.

	SOME	NOT ALL	ALL	NONE
SUBSET → of spotted rats →	… **some** of the **male spotted rats** loved the food. Now that they knew that **some** of the **spotted rats** …	… **not all** of the **male spotted rats** loved the food. Now that they knew that **not all** of the **spotted rats** …	… **all** of the **male spotted rats** loved the food. Now that they knew that **all** of the **spotted rats** …	… **none** of the **male spotted rats** loved the food. Now that they knew that **none** of the **spotted rats** …
IDENTICAL → to spotted rats →	… **some** of the **spotted rats** loved the food. Now that they knew that **some** of the **spotted rats** …	… **not all** of the **spotted rats** loved the food. Now that they knew that **not all** of the **spotted rats** …	… **all** of the **spotted rats** loved the food. Now that they knew that **all** of the **spotted rats** …	… **none** of the **spotted rats** loved the food. Now that they knew that **none** of the **spotted rats** …
SUPERSET → of spotted rats →	… **some** of the **rats** loved the food. Now that they knew that **some** of the **spotted rats** …	… **not all** of the **rats** loved the food. Now that they knew that **not all** of the **spotted rats** …	… **all** of the **rats** loved the food. Now that they knew that **all** of the **spotted rats** …	… **none** of the **rats** loved the food. Now that they knew that **none** of the **spotted rats** …

- ■ trivially entailed
- ■ entailed
- ■ not entailed

And when we look at a quantifier that is downward-entailing, such as "none," the pattern reverses.
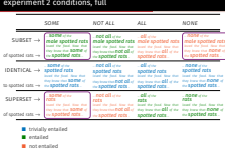
- ■ trivial
- ■ entailed
- ■ not entailed

subset: *male spotted rats ≺ spotted rats*
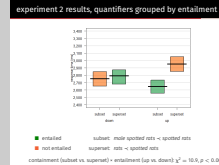ident: *spotted rats ≺ spotted rats*
superset: *rats ≺ spotted rats*

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Evidence of accurate logical reasoning in online sentence comprehension
└─Experiment 2
  └─Results
    └─experiment 2 results

- ■ trivial
- ■ entailed
- ■ not entailed

subset: *male spotted rats ≺ spotted rats*
ident: *spotted rats ≺ spotted rats*
superset: *rats ≺ spotted rats*

In the results of Experiment 2, the twelve conditions are grouped by quantifiers.

We find a main effect of containment, with superset conditions generally slower than subset. What matters, however, is the interaction of entailment and containment, which you can see as the difference between the green bars and the orange bars within each quantifier.

experiment 2 results, quantifiers grouped by entailment
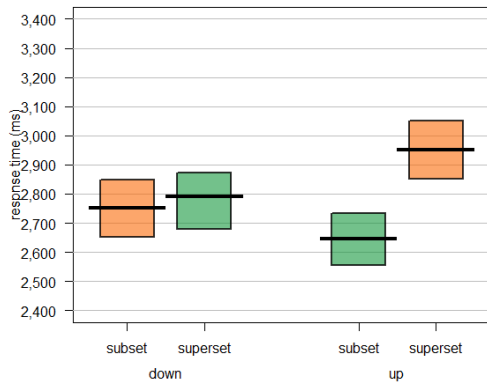
entailed — subset: *male spotted rats ≺ spotted rats*

not entailed — superset: *rats ≺ spotted rats*

containment (subset vs. superset) × entailment (up vs. down): $\chi^2 = 10.9$, $p < 0.001$

Discussion

- language involves accurate and spontaneous logical computations

In summary, our findings suggest that language processing involves automatic, accurate, and spontaneous logical computations, even in the absence of a task that requires making these inferences to verify text comprehension.

- language involves accurate and spontaneous logical
  computations

- differs from dual-process theories of cognition
    *it is assumed that people's intuitive logical knowledge
    emerges from a learning process in which key principles have
    been practiced to automaticity*

    De Neys and Pennycook (2019)

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This view differs from what is assumed by dual-process theories of cognition.

- language involves accurate and spontaneous logical computations
- differs from dual-process theories of cognition
    it is assumed that people's intuitive logical knowledge emerges from a learning process in which key principles have been practiced to automaticity

    De Neys and Pennycook (2019)
- consistent with some logic being naturally intuitive
    - natural logic in reasoning (e. g. Braine and O'Brien, 1998)
    - logic (L-analyticity) in grammar (e. g. Gajewski, 2002)

On the other hand, our findings are anticipated by frameworks which take some logical inferences as naturally intuitive.

- language involves accurate and spontaneous logical computations
- differs from dual-process theories of cognition
  it is assumed that people's intuitive logical knowledge emerges from a learning process in which key principles have been practiced to automaticity

  De Neys and Pennycook (2019)

- consistent with some logic being naturally intuitive
  - natural logic in reasoning (e. g. Braine and O'Brien, 1998)
  - logic (L-analyticity) in grammar (e. g. Gajewski, 2002)
- inference derives from compositionality?

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Thus, we conclude that the compositionality of language and at least *some* inferential thought may both derive from the same cognitive mechanisms.

thank you!

slides available at

`https://ling.auf.net/lingbuzz/005989`

Thank you very much!

📄 Braine, Martin D. S. and David P. O'Brien (1998). *Mental logic*. Psychology Press.

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