Lecture Note 9: Causality

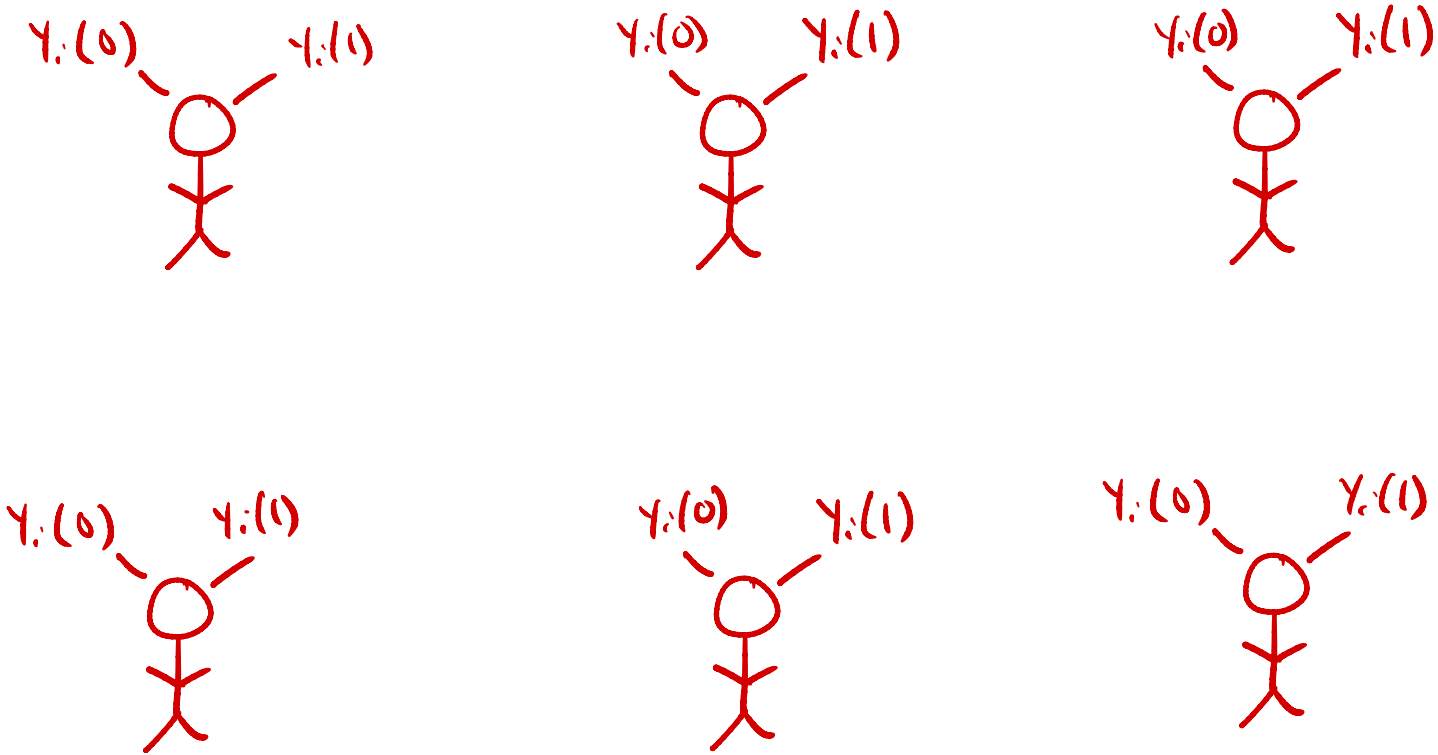
- Potential outcomes framework (Rubin Causal Model)
- $Y_i(t)$ - potential outcome at treatment level t

  → we are interested in $t = 0, 1$ → $Y_i(0)$ $Y_i(1)$

- heterogeneous treatment effect: $\alpha_i = Y_i(1) - Y_i(0)$
- only observe: Y_i, T_i

  treatment status

$$Y_i = Y_i(T_i) = T_i Y_i(1) + (1 - T_i) Y_i(0)$$


$Y_i(0)$
$Y_i(1)$
$Y_i(0)$
$Y_i(1)$
$Y_i(0)$
$Y_i(1)$
$Y_i(0)$
$Y_i(1)$
$Y_i(0)$
$Y_i(1)$
$Y_i(0)$
$Y_i(1)$

Estimands of interest

① Average treatment effect: $ATE = E[Y_i(1) - Y_i(0)] = E[\alpha_i]$

② Treatment on treated: $TOT = ATT = E[Y_i(1) - Y_i(0) \mid T_i = 1]$

$$= E[\alpha_i \mid T_i = 1]$$

not bias! heterogeneity.

Selection bias

$$E[Y_i \mid T_i = 1] - E[Y_i \mid T_i = 0]$$

$$= E[Y_i(1) \mid T_i = 1] - E[Y_i(0) \mid T_i = 0]$$

$$+ E[Y_i(0) \mid T_i = 1] - E[Y_i(0) \mid T_i = 1]$$

$$= \underbrace{E[Y_i(1) \mid T_i = 1] - E[Y_i(0) \mid T_i = 1]} + E[Y_i(0) \mid T_i = 1] - E[Y_i(0) \mid T_i = 0]$$

$$= E[Y_i(1) \mid T_i = 1] - E[Y_i(0) \mid T_i = 1] + \cdots$$

$$= \underbrace{E[\overbrace{Y_i(1) - Y_i(0)}^{\alpha_i} \mid T_i = 1]}_{\text{TOT}} + \underbrace{E[Y_i(0) \mid T_i = 1] - E[Y_i(0) \mid T_i = 0]}_{\text{selection bias}}$$

if T_i is randomized, then

1. no selection bias
2. TOT = ATE

- For causal interpretation, need $E[Y_i(0) \mid T_i = 1] = E[Y_i(0) \mid T_i = 0]$

- Unconfoundedness:

$$\{Y_i(0), Y_i(1)\} \perp T_i$$

- Selection on observables (conditional independence):

$$\{Y_i(0), Y_i(1)\} \perp T_i \mid X_i$$

Randomized experiments

① Direct randomization: researcher directly randomizes T_i

② Eligibility randomization: randomizes elig. Z_i, not treatment T_i

$$ITT = E[Y_i \mid Z_i = 1] - E[Y_i \mid Z_i = 0]$$

intent to treat

can we still learn about effect of T_i?

To learn about effect of T_i, 3 assumptions:

① Z_i is randomly assigned

② Z_i affects Y_i <u>only</u> through T_i

↳ potential outcomes $Y_i(z,t) = Y_i(t)$

③ Ineligibles can't participate in treatment: $\Pr[T_i = 1 \mid Z_i = 0] = 0$

$$Z_i \to T_i \to Y_i$$

$Z_i = 1$

	$T_i = 0$	$T_i = 1$
$T_i = 0$	never takers	compliers
$T_i = 1$		

$Z_i = 0$

by ass. 3

← compliance rate $= \Pr[T_i = 1 \mid Z_i = 1]$

↳ by ass. 1, share of eligibles receiving treatment is an unbiased & consistent estimator for compliance rate

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$$\text{ITT} = (\text{avg. effect of eligibility among compliers})(\text{compliance rate}) + \cancel{(\text{avg. effect of eligibility among never})(1 - \text{compliance})}$$

← ass 2

$$\text{ITT} = \text{TOT} \times \text{compliance rate}$$

$$\text{TOT} = \frac{\text{ITT}}{\text{compliance rate}}$$

← share $T_i = 1$ among $Z_i = 1$