LECTURE NOTE 11: REGRESSION DISCONTINUITY DESIGNS

1 Introduction

Regression discontinuity (RD) designs take advantage of precise rules governing treatment assignment. The setup involves a *running variable* X_i and a *cutoff* or *threshold* c, such that the probability of receiving some treatment T_i changes discontinuously as X_i crosses c. *Sharp* RDs deal with the case in which an individual is treated if and only if $X_i \geq c$. *Fuzzy* RDs deal with the case in which individuals below the cutoff can access the treatment, but individuals above the cutoff have greater access. We will discover that fuzzy RDs can be seen as an application of instrumental variables methods.

2 Sharp Regression Discontinuity Designs

To develop intuition for sharp RDs, we return to the potential outcomes setup of Lecture Note 9. Let $Y_i(t)$ be the potential outcome for treatment level t. Here, we consider a binary treatment $t \in \{0, 1\}$, such that the realized outcome Y_i can be expressed as follows:

$$Y_i = T_i Y_i(1) + (1 - T_i) Y_i(0) = \begin{cases} Y_i(0) & \text{if } T_i = 0 \\ Y_i(1) & \text{if } T_i = 1 \end{cases}$$

where T_i is a dummy for treatment status. The following threshold rule determines T_i:

$$T_i = \begin{cases} 1 & \text{if } X_i \geq c \\ 0 & \text{if } X_i < c \end{cases}$$

This rule expresses the idea that an individual i is treated if and only if X_i exceeds the cutoff.

To use the threshold rule to identify the effect of T_i on Y_i we make the following continuity assumption:

$$E[Y_i(0)|X_i = x] \quad \text{and} \quad E[Y_i(1)|X_i = x] \quad \text{are continuous in } x.$$

The conditional expectations of the potential outcomes are continuous in X_i. As a result, were it not for the change in treatment status at c, average outcomes would change continuously at c. Then the discontinuity in the conditional expectation of Y_i given X_i at c is can be interpreted as an average treatment effect:

$$\begin{aligned}\alpha_{SRD} &= \lim_{x\downarrow c} E[Y_i|X_i = x] - \lim_{x\uparrow c} E[Y_i|X_i = x] \\ &= \lim_{x\downarrow c} E[Y_i(1)|X_i = x] - \lim_{x\uparrow c} E[Y_i(0)|X_i = x] \\ &= E[Y_i(1) - Y_i(0)|X_i = c]\end{aligned}$$

That is to say, the discontinuity in the conditional expectation of Y_i given X_i is equal to the average treatment effect at the cutoff. This result is conceptually similar to the TOT and the $LATE$; we are identifying an average treatment effect among compliers.

For illustration, we list a few applications of this research design. First, many university admissions or academic honors criteria use a cutoff score or grade point average. Suppose we are studying such a case, and we want to know the effect of being admitted on later earnings. The discontinuity in the conditional expectation of earnings at the cutoff score is the average effect for the marginal admitted person. Second, elections use a cutoff rule. Suppose we want to know the effect of a Democratic mayor on a city's fiscal outcomes. Assuming that each election has a single Democratic candidate and a single Republican candidate (an assumption that is easily relaxed), we can look at the discontinuity in the conditional expectation of fiscal outcomes at the Democratic vote share of 50%. That discontinuity is the effect of electing a Democratic mayor in a close election. Third, many social programs use cutoff rules based on age. For instance, in the U.S., the legal drinking age is 21. Suppose we want to know the effect of being of legal drinking age on motor vehicle fatalities. If we assume that no other eligibility criteria change discontinuously at 21, then the discontinuity in motor vehicle fatality rates at age 21 has exactly that interpretation.

In all these cases, a key assumption is that outcomes would not change discontinuously in the absence of the threshold rule. As in past lecture notes, we may be a bit uncomfortable with this arcane statistical assumption that bears no obvious relation to reality. Lee (2008, also reviewed in Lee and Lemieux 2010) has suggested a more intuitive framework that motivates the continuity assumption. The details of his framework are beyond the scope of this course, but the basic idea is that if individuals have imperfect control over X_i, then we can think of RD as being based on local random assignment around $X_i = c$. Lee's framework applies well to the first two examples above. A student can influence her test score by studying, but some component of her final score will be random. Similarly, a politician can influence the distribution of votes by campaigning, but in a democratic system with a sufficiently large electorate, some component of the final vote distribution will be random. In both cases, assignment to either side of c can be treated as random, so that individuals with X_i just below c are similar to individuals with X_i just above c. However, the third example above does not fit as cleanly into Lee's framework because age is deterministic (i.e., it has no random component). For the age discontinuity, the original continuity assumption is more natural than the local random assignment framework. We will see that Lee's framework suggests ways to check the validity of an RD design.

To implement the sharp RD design, we will use the following regression specification:

$$
\begin{aligned}
Y_i &= \alpha T_i + f(X_i) + U_i \\
&= \alpha 1[X_i \geq c] + f(X_i) + U_i
\end{aligned}
$$

where $1[X_i \geq c]$ is a dummy variable that equals 1 if and only if $X_i \geq c$, and $f(X_i)$ is a continuous function of X_i. A major issue concerns how we approximate the function $f(X_i)$. Two approaches are common: (1) fit a global polynomial using all the data, and (2) fit a local linear regression only using data near the cutoff. The global polynomial approach involves running the regression:

$$
Y_i = \alpha T_i + \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \cdots + \beta_K X_i^K + \beta_{K+1} T_i \cdot X_i + \beta_{K+2} T_i \cdot X_i^2 + \cdots + \beta_{K+K} T_i \cdot X_i^K + U_i
$$

where K is the order of the polynomial. Researchers often use a cubic ($K = 3$) or a quartic ($K = 4$) polynomial. By interacting each of the polynomial terms with T_i, we allow the shape of the polynomial to differ above and below the cutoff. The local linear approach involves running the regression:

$$
Y_i = \alpha T_i + \beta_0 + \beta_1 X_i + \beta_2 T_i \cdot X_i + U_i \quad \text{for} \quad X_i \in [c-h, c+h]
$$

where h is the *bandwidth*. Basically, we estimate the regression only using data within a window of the cutoff. By interacting X_i with T_i, we allow the slope of the regression line to be different to the right and left of the cutoff.

3 Fuzzy Regression Discontinuity Designs

Sometimes a threshold rule is not binding, so that some individuals with $X_i < c$ might have $T_i = 1$ and some individuals with $X_i \geq c$ might have $T_i = 0$. In this case, we can think of the threshold as affecting the *probability* of treatment:

$$
\lim_{x \downarrow c} Pr[T_i = 1 | X_i = x] > \lim_{x \uparrow c} Pr[T_i = 1 | X_i = x]
$$

In the university admissions example, the sharp RD identifies the effect of being *admitted*. But we might want to know the effect of *attending* the university, and some admitted students might not attend. This situation is similar to an eligibility experiment with non-compliance. Because the threshold rule does not change the probability of attending from 0 to 1, we call the estimation strategy a *fuzzy* RD.

To estimate a meaningful average causal effect using fuzzy RD, we make the following monotonicity assumption for potential treatment status, $T_i(x)$:

$$T_i(x) \text{ is non-decreasing in } x \text{ at } x = c.$$

This assumption is similar to the monotonicity condition in Lecture Note 9. It says that if crossing from below to above the cutoff increases T_i for any individual, then for no individual does it decrease T_i. We refer to individuals induced into treatment when they cross c as *compliers*. Compliers are characterized by:

$$\lim_{x\downarrow c} T_i(x) = 1 \quad \text{and} \quad \lim_{x\uparrow c} T_i(x) = 0$$

We can then obtain a local average treatment effect by dividing the discontinuity in Y_i by the discontinuity in X_i. This procedure is intuitively similar to dividing the *ITT* by the compliance rate in an eligibility experiment and to dividing the reduced form coefficient by the first stage coefficient in a more general IV setting. Specifically, we have:

$$\begin{aligned}
\alpha_{FRD} &= \frac{\lim_{x\downarrow c} E[Y_i|X_i = x] - \lim_{x\uparrow c} E[Y_i|X_i = x]}{\lim_{x\downarrow c} E[T_i|X_i = x] - \lim_{x\uparrow c} E[T_i|X_i = x]} \\
&= E[Y_i(1) - Y_i(0)|X_i = c \text{ and } i \text{ is a complier}]
\end{aligned}$$

As with IV, we can take two approaches: (1) estimate the numerator and the denominator separately and then divide, and (2) run two-stage least squares (TSLS). For the TSLS option, the first stage would be:

$$T_i = \pi 1[X_i \geq c] + g(X_i) + V_i$$

where $g(X_i)$ is a continuous function of X_i. We would then predict $\hat{T}_i$ and run the second stage:

$$Y_i = \alpha \hat{T}_i + f(X_i) + U_i$$

where $f(X_i)$ is a continuous function of X_i. We *always* use the same technique to estimate $g(X_i)$ and $f(X_i)$.

4 Assessing the Validity of the RD Design

Lee's local random assignment framework suggests some useful ways to check the validity of an RD design:

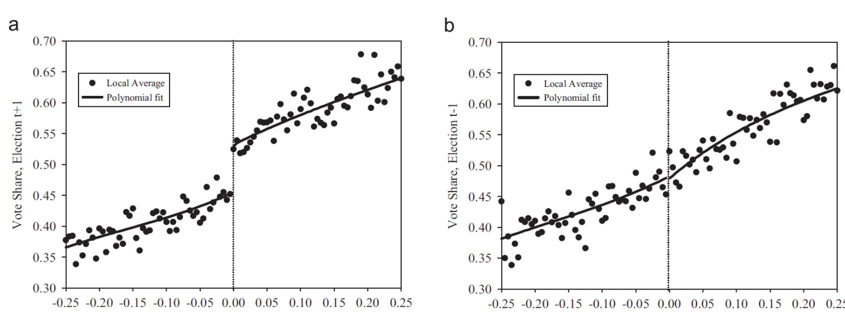
1. If local random assignment holds, then the density of X_i should in most cases be continuous at c. For example, if we observe a discontinuity in the test score distribution at an admissions cutoff, then we

might conclude that at least some individuals have *perfect* control over their test scores, which would violate local random assignment. Similarly, if Democrats are disproportionately likely to win close elections, we might be concerned about post-election manipulation of the vote returns, which would again violate local random assignment. McCrary (2008) provides a technical way to test whether the density of the running variable is discontinuous at c, but that method is beyond the scope of this course. Often, a simple histogram of the running variable is instructive enough. When plotting the histogram, one should always make sure to have separate bins above and below the cutoff. If the bars just above and below the cutoff look very different, the RD may not be valid.

2. Local random assignment also implied that the conditional expectations of predetermined variables will be continuous at c. We can test this implication by running an RD with a predetermined variable as the dependent variable. If predetermined variables change discontinuously at the cutoff, we might be concerned about a violation of local random assignment.
3. Similar to the logic in (2), our RD estimates should be robust to the inclusion of the predetermined variables as controls.

All three approaches are useful checks regardless of whether the local random assignment framework is natural for the RD design under consideration.

RD designs are quite powerful because they can by illustrated graphically. Usually, researchers will plot estimates of the conditional expectation function, along with a series of local means for bins of X_i. For example, Lee (2008) examines the political party incumbency advantage in U.S. congressional elections by estimating the effect of a Democratic victory on the Democratic vote share in the next election. His running variable is the Democratic margin of victory, or the Democratic vote share minus the vote share of the best-performing non-Democrat. A Democrat wins the election if and only if the Democratic margin of victory is greater than zero. In graph (a) below, Lee plots the conditional expectation for the Democratic vote share in the next election. The observed discontinuity represents the effect of a Democratic victory in a close election. In graph (b), Lee plots the conditional expectation for the Democratic vote share in the *last* election. Since the last election outcome is predetermined, we expect to see no discontinuity, and indeed, we do not.



5 External Validity and the RD Design

Both sharp and fuzzy RD designs are only able to estimate average treatment effects among compliers who are exactly at the cutoff. This subpopulation is very specific. In fact, if X_i is continuous, then technically, no individual in our sample will have X_i exactly equal to c. So in a sense, RD is estimating an average treatment effect for a subpopulation that does not exist. In David Lee's local random assignment setup, it is possible to reframe the RD estimator. Rather than viewing it as the average effect *at* $X_i = c$, we can instead interpret it as a weighted average treatment effect for the population, where observations are weighted by their probabilities of being close to the cutoff. However, while this reframing does allow us to estimate an average treatment effect for a real population, the weighting remains very specific.

The specificity of the RD estimator is important for interpretation. In the test score example, RD measures the average treatment effect among individuals who marginally passed the test. In the election example, RD measure the average treatment effect in close elections. These subpopulations may have different treatment effects from the general population.