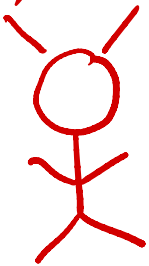
Lecture Note 9: Causality

→ Potential outcomes framework (Rubin Causal Model)

→ $Y_i(t)$ = potential outcome at treatment level t

↳ today, focus on $t = 0, 1$ → $Y_i(0)$ $Y_i(1)$



→ heterogeneous causal effect:

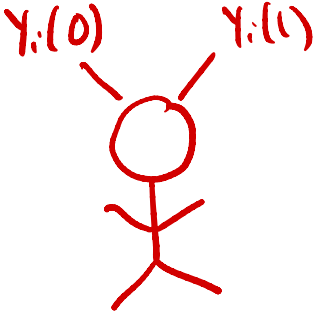
$$\alpha_i = Y_i(1) - Y_i(0)$$

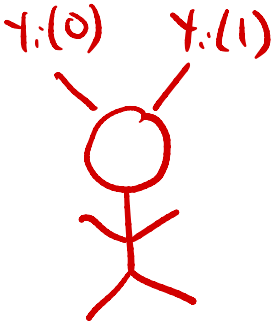
→ T_i = treatment assignment

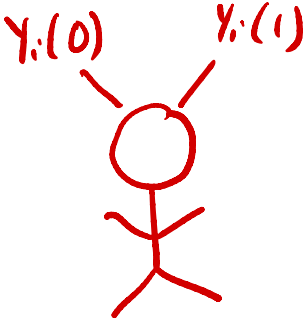
→ observe: $Y_i = Y_i(T_i) = T_i Y_i(1) + (1 - T_i) Y_i(0)$

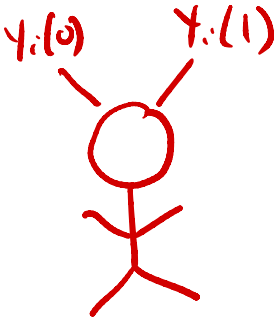
→ want to know $\alpha_i = Y_i(1) - Y_i(0)$

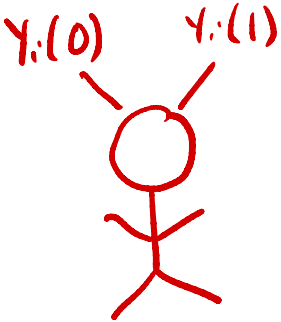
→ only observe (Y_i, T_i)

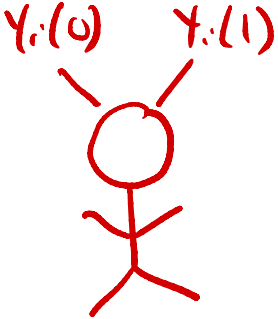

$Y_i(0)$
$Y_i(1)$


$Y_i(0)$
$Y_i(1)$


$Y_i(0)$
$Y_i(1)$


$Y_i(0)$
$Y_i(1)$


$Y_i(0)$
$Y_i(1)$


$Y_i(0)$
$Y_i(1)$

Estimands of interest

- Average treatment effect: $ATE = E[Y_i(1) - Y_i(0)] = E[\alpha_i]$
- Treatment on treated = $TOT = ATT = E[Y_i(1) - Y_i(0) \mid T_i = 1]$
  $= E[\alpha_i \mid T_i = 1]$

- heterogeneity, <u>not</u> bias

Selection bias

$$E[Y_i \mid T_i = 1] - E[Y_i \mid T_i = 0]$$

$$= E[Y_i(1) \mid T_i = 1] - E[Y_i(0) \mid T_i = 0]$$

$$+ E[Y_i(0) \mid T_i = 1] - E[Y_i(0) \mid T_i = 1]$$

$$= \underbrace{E[Y_i(1) \mid T_i = 1] - E[Y_i(0) \mid T_i = 1]} + \underbrace{E[Y_i(0) \mid T_i = 1] - E[Y_i(0) \mid T_i = 0]}$$

$$= \underbrace{E[\overbrace{Y_i(1) - Y_i(0)}^{\alpha_i} \mid T_i = 1]}_{\text{TOT}} + \underbrace{E[Y_i(0) \mid T_i = 1] - E[Y_i(0) \mid T_i = 0]}_{\text{selection bias}}$$

if T_i randomized,
then selection bias $= 0$
<u>and</u> ATE = TOT

For causal effect, need "no selection bias:"

$$E[Y_i(0) \mid T_i = 1] = E[Y_i(0) \mid T_i = 0]$$

Unconfoundedness:

$$\{Y_i(1), Y_i(0)\} \perp T_i$$

$Y_i = T_i Y_i(1) + (1 - T_i) Y_i(0)$

Selection on observables (conditional independence):

$$\{Y_i(1), Y_i(0)\} \perp T_i \mid X_i$$

Randomized experiments

① Direct randomization: experimenter has direct control over T_i

② Eligibility randomization: randomize elig. Z_i, not treatment T_i

$$ITT = E[Y_i \mid Z_i = 1] - E[Y_i \mid Z_i = 0]$$

intent to treat

how to get TOT?

3 assumptions to get TOT:

1. Eligibility is randomly assigned.
2. Effect of Z_i operates only through T_i.
   ↳ potential outcome $Y_i(z,t) = Y_i(t)$
3. Ineligibles can't participate: $Pr[T_i=1 \mid Z_i=0] = 0$

$$\begin{tikzcd} Z_i \arrow[r] \arrow[rr, dashed, bend right, "\times\times\times\times"'] & T_i \arrow[r] & Y_i \end{tikzcd}$$

		$Z_i=1$	
		$T_i=0$	$T_i=1$
$Z_i=0$	$T_i=0$	never takers	compliers
	$T_i=1$		ass. 3

← $Pr[T_i=1 \mid Z_i=1]$ = compliance rate
= share of compliers

ITT = (avg. effect of Z_i on compliers)(compliance rate)
+ ~~(avg. effect of Z_i on never)(1 − compliance)~~

ass. 2

ITT = TOT × compliance rate

$$\frac{\text{ITT}}{\text{compliance rate}} = \text{TOT}$$

treated share among $Z_i=1$ by ass 1