

Lecture 11: RD designs

Sharp RD design

Fuzzy RD design

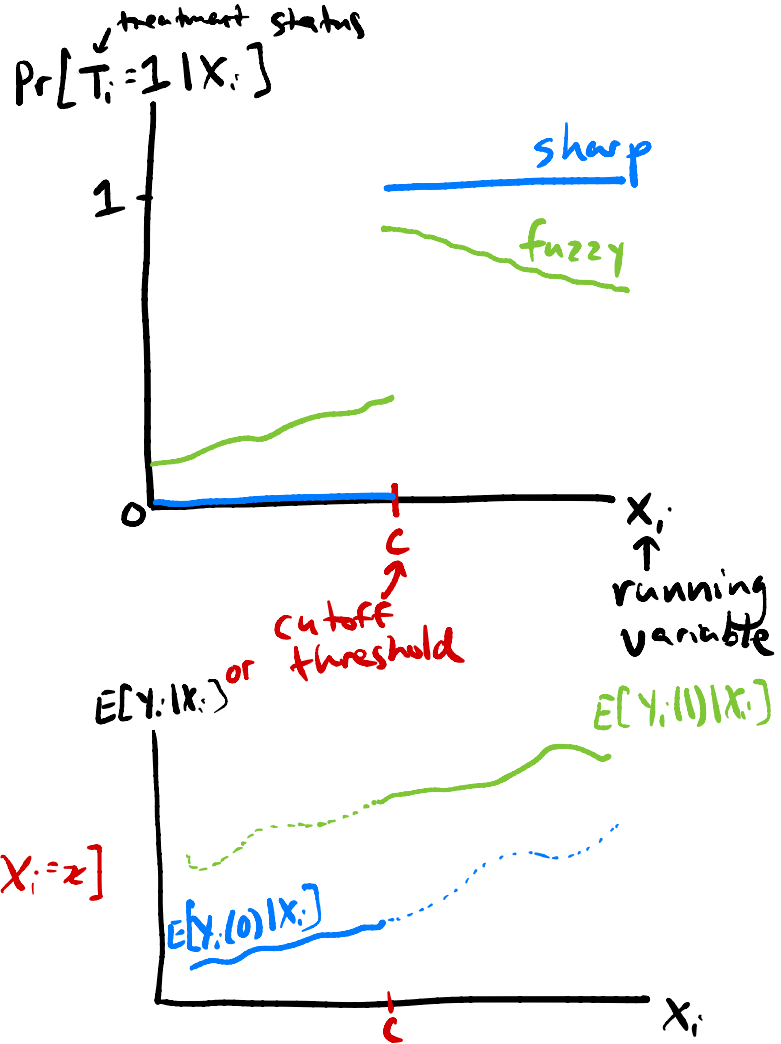
$$\begin{aligned} Y_i &= Y_i(T_i) = T_i Y_i(1) + (1 - T_i) Y_i(0) \\ &= \begin{cases} Y_i(0) & \text{if } T_i = 0 \\ Y_i(1) & \text{if } T_i = 1 \end{cases} \end{aligned}$$

In a sharp RD:

$$T_i = \begin{cases} 1 & \text{if } X_i \geq c \\ 0 & \text{if } X_i < c \end{cases}$$

Assumption:

$E[Y_i(0) | X_i = x]$ and $E[Y_i(1) | X_i = x]$
are continuous in x



Under Assumption:

$$\begin{aligned}\alpha_{SRD} &= \lim_{x \downarrow c} E[Y_i | X_i = x] - \lim_{x \uparrow c} E[Y_i | X_i = x] \\&= \lim_{x \downarrow c} E[Y_i(1) | X_i = x] - \lim_{x \uparrow c} E[Y_i(0) | X_i = x] \quad \text{continuity} \\&= E[Y_i(1) | X_i = c] - E[Y_i(0) | X_i = c] \\&= E[\underbrace{Y_i(1) - Y_i(0)}_{\alpha_i} | X_i = c]\end{aligned}$$

average effect
of T_i for i with $X_i = c$

Alternative assumption: local random assignment
imperfect control over X_i .

Checks on RD designs:

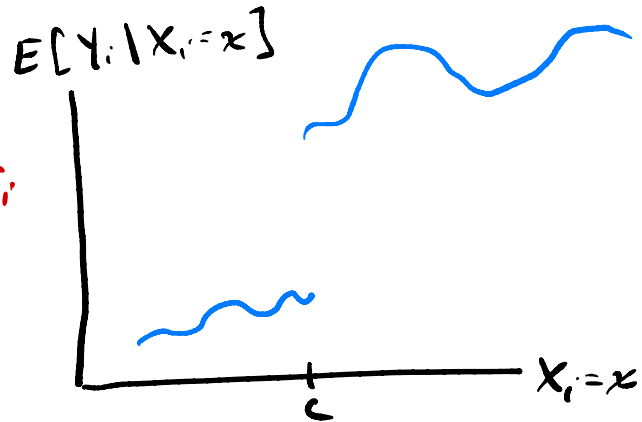
- balance checks: are predetermined variables diff above and below c ?
- density continuity: discontinuity in histogram of X_i ?

Two approaches to estimation:

$$Y_i = \alpha T_i + f(X_i) + U_i \\ = \alpha 1(X_i \geq c) + f(X_i) + U_i$$

- Global polynomial

$$Y_i = \alpha T_i + \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \beta_3 X_i^3 \\ + \beta_4 X_i T_i + \beta_5 X_i^2 T_i + \beta_6 X_i^3 T_i \\ + U_i$$



- Local linear

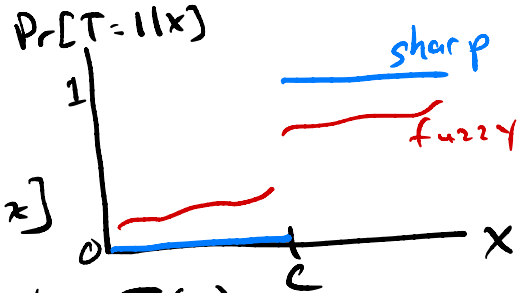
$$Y_i = \alpha T_i + \beta_0 + \beta_1 X_i + \beta_2 X_i T_i + U_i \quad \text{for } X_i \in [c-h, c+h]$$

↑
bandwidth

Fuzzy designs

→ Discontinuity in probability of treatment:

$$\lim_{x \downarrow c} \Pr[T_i=1|X_i=x] > \lim_{x \uparrow c} \Pr[T_i=1|X_i=x]$$



→ Monotonicity: for potential treatment status $T_i(x)$, $T_i(x)$ is non-decreasing in x at $x=c$

→ Under monotonicity, can estimate LATE at cutoff

→ compliers: $\lim_{x \downarrow c} T_i(x) = 1$ and $\lim_{x \uparrow c} T_i(x) = 0$

→ Can estimate:

$$\alpha_{FRD} = \frac{\lim_{x \downarrow c} E[Y_i|X_i=x] - \lim_{x \uparrow c} E[Y_i|X_i=x]}{\lim_{x \downarrow c} E[T_i|X_i=x] - \lim_{x \uparrow c} E[T_i|X_i=x]} \leftarrow \text{RF}$$

$\leftarrow 1^{st} \text{ stage}$

→ Or TSLS: 1st: $T_i = \pi 1[X_i \geq c] + \underline{g(X_i)} + V_i$
 2nd: $Y = \alpha \hat{T}_i + \underline{f(X_i)} + U_i$

flexible functions in X_i : use same method!

Assessing internal validity

- ① Density discontinuity?
- ② Discontinuities in predetermined variables? (balance)
- ③ Include predetermined variables as covariates