Lecture 11: RD designs

Sharp RD design

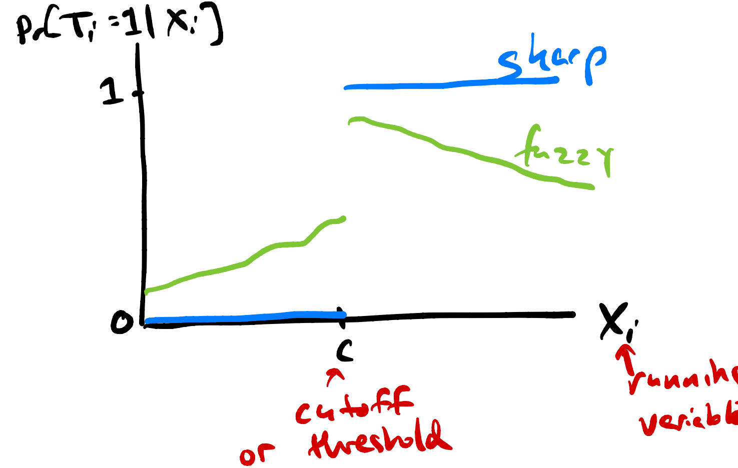
Fuzzy RD design

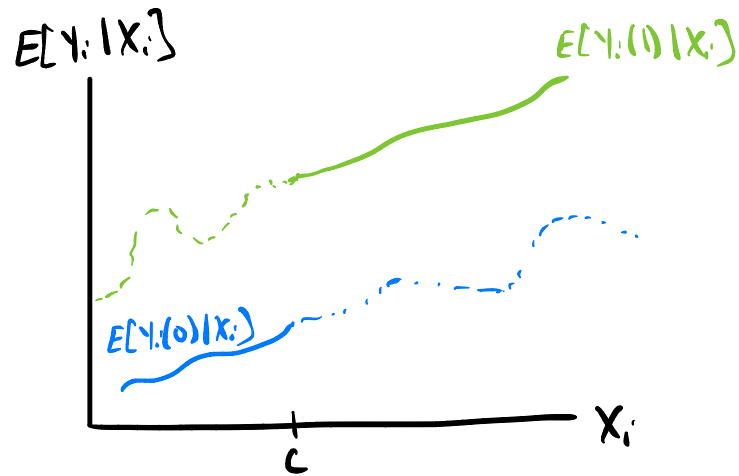
$$Y_i = Y_i(T_i) = T_i Y_i(1) + (1-T_i) Y_i(0)$$
$$= \begin{cases} Y_i(0) & \text{if } T_i = 0 \\ Y_i(1) & \text{if } T_i = 1 \end{cases}$$

$$T_i = \begin{cases} 1 & \text{if } X_i \geq c \\ 0 & \text{if } X_i < c \end{cases} \quad \text{sharp}$$

Assumption:

$E[Y_i(0) \mid X = x]$ and $E[Y_i(1) \mid X_i = x]$ are continuous





Under Assumption,

$$\alpha_{SRD} = \lim_{x \downarrow c} E[Y_i \mid X_i = x] - \lim_{x \uparrow c} E[Y_i \mid X_i = x]$$

$$= \lim_{x \downarrow c} E[Y_i(1) \mid X_i = x] - \lim_{x \uparrow c} E[Y_i(0) \mid X_i = x]$$

assumption

$$= E[Y_i(1) \mid X_i = c] - E[Y_i(0) \mid X_i = c]$$

$$= \underbrace{E[\underbrace{Y_i(1) - Y_i(0)}_{\alpha_i} \mid X_i = c]}_{\text{average effect of } T_i \text{ among } i \text{ with } X_i = c}$$

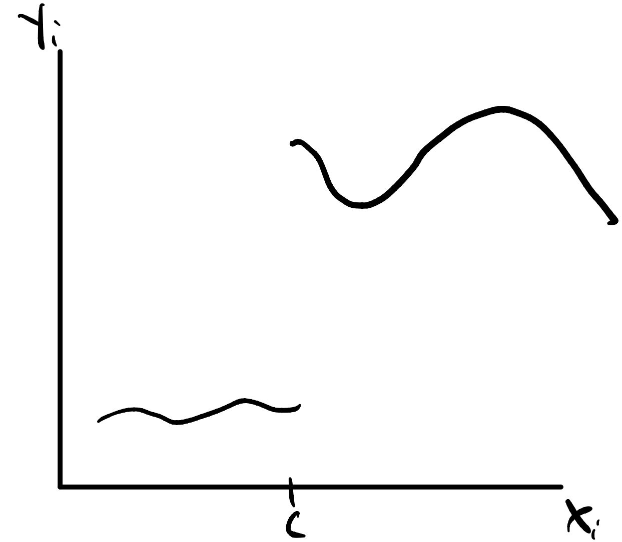
Alternative assumption: local random assignment
imperfect control

Checks on RD designs:

- balance check: predetermined variables similar above/below c.
- density continuity: histogram of X_i is continuous

Two methods:

$$Y_i = \alpha T_i + f(X_i) + U_i$$
$$= \alpha \mathbb{1}[X_i \geq c] + f(X_i) + U_i$$



① Global polynomial

$$Y_i = \alpha T_i + \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \beta_3 X_i^3$$
$$+ \beta_4 X_i T_i + \beta_5 X_i^2 T_i + \beta_6 X_i^3 T_i$$

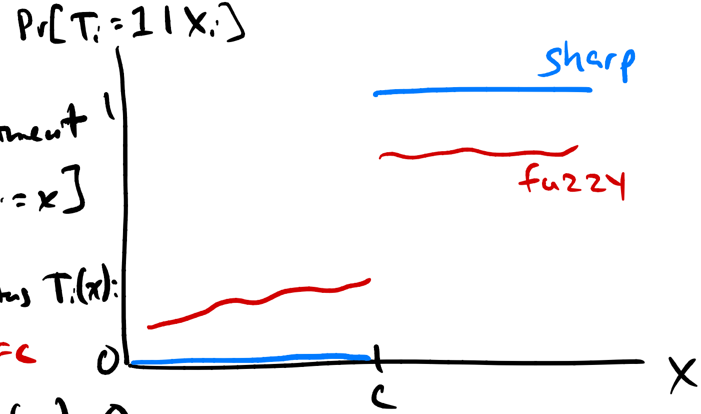
② Local linear

$$Y_i = \alpha T_i + \beta_0 + \beta_1 X_i + \beta_2 X_i T_i$$

for $X_i \in [c-h, c+h]$

bandwidth

Fuzzy RD design



- → discontinuity in probability of treatment

$$\lim_{x \downarrow c} Pr[T_i = 1 | X_i = x] > \lim_{x \uparrow c} Pr[T_i = 1 | X_i = x]$$

- → monotonicity: for potential treatment status $T_i(x)$:

  $T_i(x)$ is non-decreasing in x at $x = c$

- → compliers: $\lim_{x \downarrow c} T_i(x) = 1$ and $\lim_{x \uparrow c} T_i(x) = 0$
- → Use IV to estimate LATE at $x = c$
- → RF / 1st stage:

$$\alpha_{FRD} = \frac{\lim_{x \downarrow c} E[Y_i | X_i = x] - \lim_{x \uparrow c} E[Y_i | X_i = x]}{\lim_{x \downarrow c} E[T_i | X_i = x] - \lim_{x \uparrow c} E[T_i | X_i = x]} \quad \begin{matrix} \leftarrow \text{RF} \\ \leftarrow \text{1st stage} \end{matrix}$$

- → TSLS:
  1st: $T_i = \pi 1[X_i \geq c] + g(X_i) + V_i$

  2nd: $Y_i = \alpha \hat{T}_i + f(X_i) + U_i$

$g(X_i)$ and $f(X_i)$ are flexible functions in X_i. use same method!