Algorithmic and Theoretical Foundations of RL

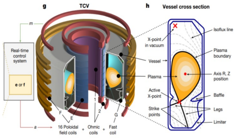
Introduction

Ke Wei
School of Data Science
Fudan University

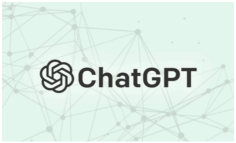
Success of RL

(Nature, 2016)

(Nature, 2019)



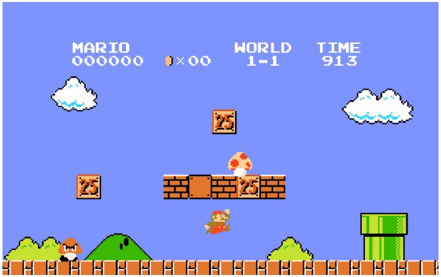
(Nature, 2022)



(ChatGPT, 2023)

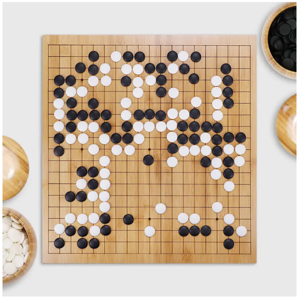
- RL has also been used to computationally difficult problems like traveling salesman problem and plays an important role in “AI for Science”.

Illustration: Super Mario



Super Mario makes a decision, then receives a reward and transfers to the next state; Goal: high long term cumulative reward by making right decisions.

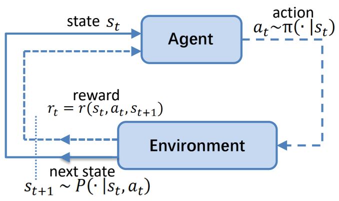
Challenges in RL



RL is a sequential decision problem and is essentially about efficient search (dynamic programming, control, game theory).

- High dimension (large state/action spaces)
- Highly nonconvex (distribution optimization, parameterization)
- Computational efficiency vs Reliability
- Plenty of scenarios $\mathcal{M} = \langle \mathcal{S}, \mathcal{A}, P, r, \gamma \rangle$
- ......

Formal Model: Markov Decision Process (MDP)

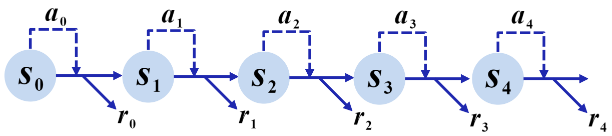


state s_t
Agent
action
$a_t \sim \pi(\cdot|s_t)$
reward
$r_t = r(s_t, a_t, s_{t+1})$
Environment
next state
$s_{t+1} \sim P(\cdot|s_t, a_t)$

$$\mathcal{M} = \langle \mathcal{S}, \mathcal{A}, P, r, \gamma \rangle$$

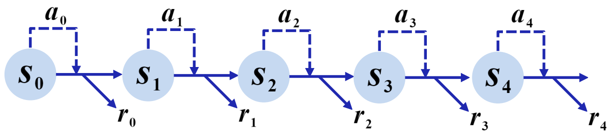
- $\mathcal{S}$: State space
- $\mathcal{A}$: Action space
- $P(\cdot|s, a)$: Transition probability
- $r(s, a, s')$: Immediate reward
- $\pi(\cdot|s)$: Policy (probability distribution on action space)

Agent selects action based on policy $a_t \sim \pi(\cdot|s_t)$ at state s_t, receives reward $r(s_t, a_t, s_{t+1})$, and transits to new state following $s_{t+1} \sim P(\cdot|s_t, a_t)$.



Trajectory: $\tau = (s_0, a_0, r_0, \cdots, s_t, a_t, r_t, \cdots)$

Formal Model: Markov Decision Process (MDP)



Trajectory: $\tau = (s_0, a_0, r_0, \cdots, s_t, a_t, r_t, \cdots)$

State value function at s and state-action value function at (s, a):

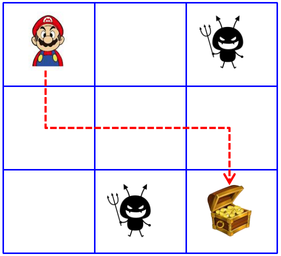
$$V^\pi(s) := \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t r(s_t, a_t, s_{t+1}) | s_0 = s, \pi\right],$$

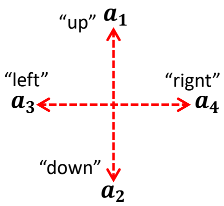
$$Q^\pi(s, a) := \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t r_t | s_0 = s, a_0 = a\right].$$

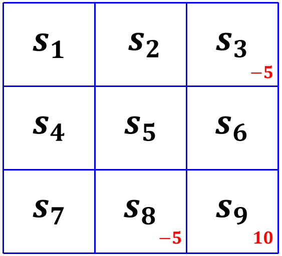
Goal of RL is to a find a policy that maximizes weighted state values:

$$\max_\pi V^\pi(\mu), \quad \text{where } V^\pi(\mu) := \mathbb{E}_{s\sim\mu}\{V^\pi(s)\}.$$

Simple RL Example: GridWorld

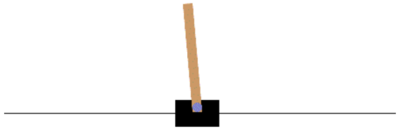






- State space: $\mathcal{S} = \{s_i\}_{i=1}^{9}$
- Action space: $\mathcal{A} = \{a_i\}_{i=1}^{4}$
- Reward: $r = -5$ if hitting "obstacle" grid; $r = 10$ if arriving at "goal" grid
- Goal: Arriving "goal" grid while avoiding "obstacle" grid

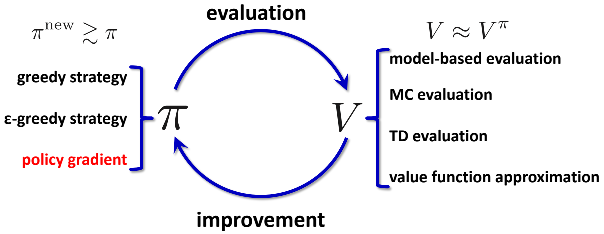
Simple RL Example: CartPole



- State: $s = [x, y, \theta, \omega] \in \mathbb{R}^4$
  - $x \in [-4.8, 4.8]$: cart position
  - $y \in \mathbb{R}$: cart velocity
  - $\theta \in [-24^\circ, 24^\circ]$: pole angle
  - $\omega \in \mathbb{R}$: pole velocity at tip
- Action space: $\mathcal{A} = \{$left, right $\}$
- Reward: $r = 1$ if the pole remains upright, $r = 0$ otherwise
- Goal: prevent pole from falling over

More typical examples, such as Mountain Car and Cliff Walking, can be found in OpenAI Gym (https://github.com/openai/gym).

Basic RL Methods



- Value-based Methods: Learn optimal values and policy is implicitly inferred;
- Policy Optimization: Parametrize policy and conduct search in policy space.

Basic RL Methods

- Valued-based methods: not directly optimize policy but seek optimal state or action values based on fixed point iteration or dynamic programming:

$$\begin{cases}\text{Value Iteration}\\ \text{Policy Iteration}\end{cases} \xrightarrow{\textit{sampling}} \begin{cases}\text{MC Learning}\\ \text{SARSA}\\ \text{Q-Learning}\end{cases} \xrightarrow[\textit{approximation}]{\textit{function}} \text{Deep Q-Learning}$$

Basic RL Methods

- Policy optimization: directly optimize policy via parameterization $\pi_\theta(\cdot|s)$:

$$V^{\pi_\theta}(\mu) = \mathbb{E}_{\tau \sim P_\mu^{\pi_\theta}} \left[\sum_{t=0}^{\infty} \gamma^t r(s_t, a_t, s_{t+1}) \right],$$

  where $P_\mu^{\pi_\theta}(\tau) = \mu(s_0) \prod_{t=0}^{\infty} \pi_\theta(a_t|s_t) p(s_{t+1}|s_t, a_t)$. Then maximize $V^{\pi_\theta}(\mu)$ is finite dimensional optimization problem about θ. Value exists in expression of policy gradient and policy optimization+value update= Actor-Critic.

Policy optimization is also known as policy search, i.e., search over policy space directly. In contrast, value-based methods update state/action values and retrieve (optimal) policy from (optimal) state/action values.

Exploration & Exploitation

Recall the RL is a sequential decision problem that pursues a long term return, but one can only make somehow the best decision at each time step based on the current/local information. If the local information can be computed accurately (e.g., each action value can be computed exactly given a policy), then a series of local decisions can lead to global optimal. However, when the local information is not accurate, it may mislead. In this situation, one should be allowed to explore the non-optimal decision when making the decision so that it is possible to achieve the global, long term optimum.

- Exploitation: Make the best decision given current information
- Exploration: Gather more information

Logistics

- **Prerequisites:** Probability and statistics, numerical optimization
- **Grading policy:** 50% Homework + 10% attendance + 40% Final
- **Homework:**
  - Homework will be assigned via eLearning;
  - Coding language for this course is Python.
- **Course policies:**
  - Final exam is closed book.
  - Cheating in assignments and exams is not tolerated! Any sort of suspected cheating will result in zero grade of the corresponding assignments or exams, followed by penalty subject to university rules.

This course emphasizes basic methods and theory of RL, but hopefully there will be more practical projects.

Questions?