

* Gaussian Pulses :-

input pulse : $A(0, T) = \exp\left(-\frac{T^2}{2T_0^2}\right)$

$$A(0, T) \xrightarrow{\text{FT}} \tilde{A}(0, \omega) \xrightarrow{\text{Propagates in frequency domain}} \tilde{A}(z, \omega) \xrightarrow{\text{IFT}} A(z, T)$$

$$\begin{aligned} \tilde{A}(0, \omega) &= \int_{-\infty}^{\infty} A(0, T) \exp(i\omega T) dT \\ &= \int_{-\infty}^{\infty} \exp\left(-\frac{T^2}{2T_0^2}\right) \exp(i\omega T) dT \\ &= \int_{-\infty}^{\infty} \exp\left(-\frac{T^2}{2T_0^2} - \frac{\omega T}{i}\right) dT \end{aligned}$$

- By using $\int_{-\infty}^{\infty} \exp(-ax^2 - bx) dx = \sqrt{\frac{\pi}{a}} \exp\left(\frac{b^2}{4a}\right)$ --- (1)

- In our case $\rightarrow a = \frac{1}{2T_0^2}$, $b = \frac{\omega}{i}$

So $\tilde{A}(0, \omega) = \sqrt{2\pi T_0^2} \exp\left(-\frac{\omega T_0^2}{2}\right)$ --- (2)

- when the pulse propagates in the frequency domain till z it means : $\tilde{U}(z, \omega) = \tilde{U}(0, \omega) \exp\left(\frac{i}{2} \beta_2 \omega^2 z\right)$

$$\tilde{A}(z, \omega) = \tilde{A}(0, \omega) \exp\left(\frac{i}{2} \beta_2 \omega^2 z\right)$$

- To get the signal in time domain we just take IFT

$$A(z, T) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{A}(z, \omega) \exp(-i\omega T) d\omega$$

$$A(z, T) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{A}(\omega, \omega) \exp\left(\frac{i}{2} \beta_2 \omega^2 z - i\omega T\right) d\omega \quad \text{--- (3)}$$

By substituting (2) into (3)

$$\begin{aligned} A(z, T) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \sqrt{2\pi T_0^2} \exp\left(-\frac{\omega T_0^2}{2} + \frac{i}{2} \beta_2 \omega^2 z - i\omega T\right) d\omega \\ &= \frac{\sqrt{2\pi T_0^2}}{2\pi} \exp\left(-\left[\frac{T_0^2}{2} + \frac{i}{2} \beta_2 z\right] \omega^2 - iT\omega\right) d\omega \end{aligned}$$

- using eq (1) once again $\begin{cases} a = \frac{T_0^2}{2} + \frac{i}{2} \beta_2 z \\ b = iT \end{cases}$

$$A(z, T) = \frac{\sqrt{2\pi T_0^2}}{2\pi} \sqrt{\frac{2\pi}{T_0^2 - i\beta_2 z}} \exp\left(\frac{-T^2}{2(T_0^2 - i\beta_2 z)}\right)$$

$$A(z, T) = \frac{\cancel{\sqrt{2\pi}} T_0}{\cancel{2\pi}} \frac{\cancel{\sqrt{2\pi}}}{\sqrt{T_0^2 - i\beta_2 z}} \exp\left(-\frac{T^2}{2(T_0^2 - i\beta_2 z)}\right)$$

$$A(z, T) = \frac{T_0}{\sqrt{T_0^2 - i\beta_2 z}} \exp\left(-\frac{T^2}{2(T_0^2 - i\beta_2 z)}\right)$$

output pulse

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clear all; close all; clc;

T0 = [12.5, 25]; %ps
for x = 1 : length(T0)
    L = 25; %km
    D = 17;
    lambda = 1550;
    c = 3*10^8;
    B2 = -lambda*lambda*D/(2*pi*c*1e-3);
    T = 512; %FFT window size (ps)
    N = 2048; % number of sampling
    dt = T/N; %time interval
    df = 1/T; %frequency interval
    t = (-N/2:N/2-1)*dt; %time vector
    f = (-N/2:N/2-1)*df; %frequency vector
    w = 2*pi*f;

    % compute dispersion effect
    A0T = exp(-t.*t./(2*T0(x)*T0(x)));
    A0f = fftshift(fft(A0T));

    Disp = exp(1i*B2*w.^2*L/2);

    ALf_1 = A0f .* Disp; %propagation in frequency domain %output spectrum 1
    ALt_1 = fft(fftshift(ALf_1)); %output pulse 1
    ALf_2 = ALf_1 .* Disp; %propagation in frequency %output spectrum 2
    ALt_2 = fft(fftshift(ALf_2)); %output pulse 2

    %Plot in time domain
    figure('Position',[100 100 800 300])
    subplot(121)
    plot(t,abs(A0T).^2)
    grid on;
    hold on;
    plot(t,abs(ALt_1).^2, '--')
    hold on;
    plot(t,abs(ALt_2).^2, '-.')
    ylim([0 1.3])
    xlim([-200 200])
    xticks([-200:50:200])
    legend('0km','25km','50km')
    title("A(z,T) | T0 = "+ T0(x) )
    xlabel('T[Ps]')
    ylabel('P[mW]')

    %Plot PSD
    subplot(122)
    plot(f, 10*log10(abs(N*A0f).^2/N*dt.^2))
    grid on;
    hold on;
    plot(f, 10*log10(abs(N*ALf_1).^2/N*dt.^2), '--')
    hold on;
    plot(f, 10*log10(abs(N*ALf_2).^2/N*dt.^2), '-.')
    ylim([-40 20])
    xlim([-0.05 0.05])
    xticks([-0.05:0.01:0.05])
    legend('0km','25km','50km')
    title("A(z,w) | T0 = "+ T0(x) )
    xlabel('f[THz]')
    ylabel('PSD [dBm/THz]')
end

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