Assignment on the 2nd part of the project

Prof. Akihiro Maruta

281405

Mohammed Alabadsa

Student ID: 28G21130

[1] Calculate the Fourier transform for the following waveforms:

1. $\boldsymbol{f(t) = \mathbf{sech}\,(t)}$

First, let's compute the FT of sech(x) which may be derived using the residue theorem. We simply set up the Fourier integral as usual and convert it into a sum as follows:

$$
\begin{aligned}
F(w) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \operatorname{sech}(t) e^{iwt} dt &= \frac{2}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dt \frac{e^{iwt}}{e^{t}+e^{-t}} \\
&= \frac{2}{\sqrt{2\pi}}\int_{-\infty}^{0} dt \frac{e^{iwt}}{e^{t}+e^{-t}} + \frac{2}{\sqrt{2\pi}}\int_{0}^{\infty} dt \frac{e^{iwt}}{e^{t}+e^{-t}} \\
&= \frac{2}{\sqrt{2\pi}}\sum_{m=0}^{\infty} (-1)^m \left[\int_0^{\infty} dt e^{-[(2m+1)+iw]t} + \int_0^{\infty} dt e^{-[(2m+1)-iw]t}\right] \\
&= \frac{2}{\sqrt{2\pi}}\sum_{m=0}^{\infty} (-1)^m \left[\frac{1}{(2m+1)-iw} + \frac{1}{(2m+1)+iw}\right] \\
&= \frac{4}{\sqrt{2\pi}}\sum_{m=0}^{\infty} \frac{(-1)^m (2m+1)}{(2m+1)^2 + w^2} \\
&= \frac{1}{2\sqrt{2\pi}}\sum_{m=-\infty}^{\infty} \frac{(-1)^m (2m+1)}{\left(m+\frac{1}{2}\right)^2 + \left(\frac{w}{2}\right)^2}
\end{aligned}
$$

By the residue theorem, the sum is equal to the negative sum of the residues at the non-integer poles of

$$\pi \csc(z) \frac{1}{2\sqrt{2\pi}} \frac{2z+1}{\left(z+\frac{1}{2}\right)^2 + \left(\frac{w}{2}\right)^2}$$

which are at $z_{\pm} = -\frac{1}{2} \pm i\frac{w}{2}$. The sum is therefore

$$-\frac{1}{2}\csc(z_+) - \frac{1}{2}\csc(z_-) = -\Re\left[\frac{1}{\sin \pi\left(-\frac{1}{2} + i\frac{w}{2}\right)}\right] = \operatorname{sech}\left(\frac{\pi w}{2}\right)$$

By this reasoning, the FT of sech (t) is $\pi \frac{1}{\sqrt{2\pi}}$ sech $\left(\frac{\pi w}{2}\right)$

$$F[\operatorname{sech}(t)] = \frac{\sqrt{\pi}}{\sqrt{2}} \operatorname{sech}\left(\frac{\pi w}{2}\right)$$

2. $\boldsymbol{f(t) = e^{-t^2}}$

$$F(w) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-t^2}e^{-iwt}dt = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-(t^2+iwt)}dt$$

let b = iw

$$F(w) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-(t^2+bt)}dt$$

Recall that, $\left(t+\frac{b}{2}\right)^2 = t^2+bt+\frac{b^2}{4}$

$$F(w) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\left(t^2+bt+\frac{b^2}{4}-\frac{b^2}{4}\right)}dt$$

$$F(w) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\left(\left(t+\frac{b}{2}\right)^2-\frac{b^2}{4}\right)}dt$$

$$F(w) = \frac{e^{\frac{b^2}{4}}}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\left(t+\frac{b}{2}\right)^2}dt$$

Let $u = t+\frac{b}{2}$, $du = dt$, thus

$$F(w) = \int_{-\infty}^{\infty} e^{-\left(t+\frac{b}{2}\right)^2}dt = \int_{-\infty}^{\infty} e^{-u^2}du$$

$$\int_{-\infty}^{\infty} e^{-u^2}du = \sqrt{\pi}$$

$$F(w) = \sqrt{\pi}\frac{e^{\frac{b^2}{4}}}{\sqrt{2\pi}}$$

$$F[\boldsymbol{e^{-t^2}}] = \frac{e^{\frac{-w^2}{4}}}{\sqrt{2}}$$

3. $\boldsymbol{f(t) = \begin{cases} 1, & |t| \le \frac{1}{2} \\ 0, & otherwise \end{cases}}$

$$F(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-iwt}dt = \frac{1}{\sqrt{2\pi}}\left(-\frac{e^{-iwt}}{iw}\Big|_{-\frac{1}{2}}^{\frac{1}{2}}\right)$$

$$= \frac{2}{\omega\sqrt{2\pi}}\left(\frac{e^{i\omega/2}-e^{-i\omega/2}}{2i}\right)$$

$$= \frac{1}{\sqrt{2\pi}}\frac{\sin\ \omega/2}{\omega/2} = \frac{1}{\sqrt{2\pi}}\mathrm{sinc}\ \frac{\omega}{2}$$

[2] solve the following equations with initial conditions:

1- $i\frac{\partial U}{\partial \xi}+\frac{1}{2}\frac{\partial^2 U}{\partial \tau^2}=0$

2- $i\frac{\partial U}{\partial \xi}-\frac{1}{2}\frac{\partial^2 U}{\partial \tau^2}=0$

3- $i\frac{\partial U}{\partial \xi}+\frac{1}{2}\frac{\partial^2 U}{\partial \tau^2}+|U|^2U=0$

4- $i\frac{\partial U}{\partial \xi}-\frac{1}{2}\frac{\partial^2 U}{\partial \tau^2}+|U|^2U=0$

The solution:

The numerical code of MATLAB will be used to show the nature of propagation for each equation, the initial values such as s, N and m are required, the general form of the normalized NLS equation should be as following

$$\frac{\partial U}{\partial \xi}=-\frac{is}{2}\frac{\partial^2 U}{\partial \tau^2}+iN^2|U|^2U$$

So, we need to change the given equations to make in the general form, in this way we can find the values of s and N for each equation. Where we will identify the fiber length (in units of LD) and m value.

1- $\frac{\partial U}{\partial \xi}=\frac{i}{2}\frac{\partial^2 U}{\partial \tau^2}$ S = -1, N = 0 (anomalous dispersion case)

2- $\frac{\partial U}{\partial \xi}=-\frac{i}{2}\frac{\partial^2 U}{\partial \tau^2}$ S = 1, N = 0 (normal dispersion case)

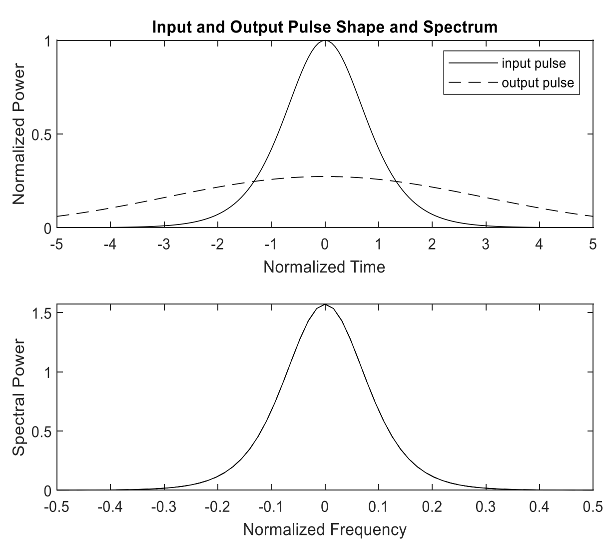
3- $\frac{\partial U}{\partial \xi}=\frac{i}{2}\frac{\partial^2 U}{\partial \tau^2}+i|U|^2U$ S = -1, N = 1 (anomalous dispersion and nonlinearity)

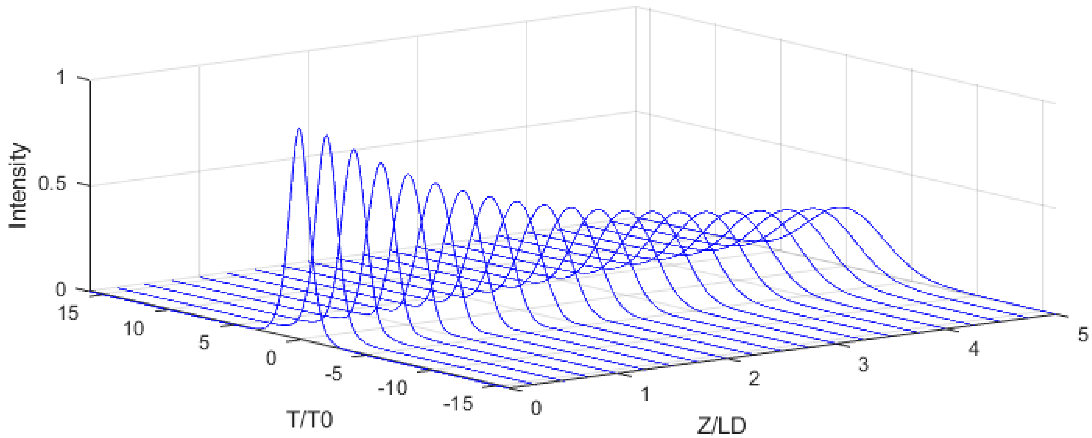
4- $\frac{\partial U}{\partial \xi}=-\frac{i}{2}\frac{\partial^2 U}{\partial \tau^2}+i|U|^2U$ S = 1, N = 1 (normal dispersion and nonlinearity)

We will set the fiber length to 5LD and m will be tested for different values (m=0,1,2).

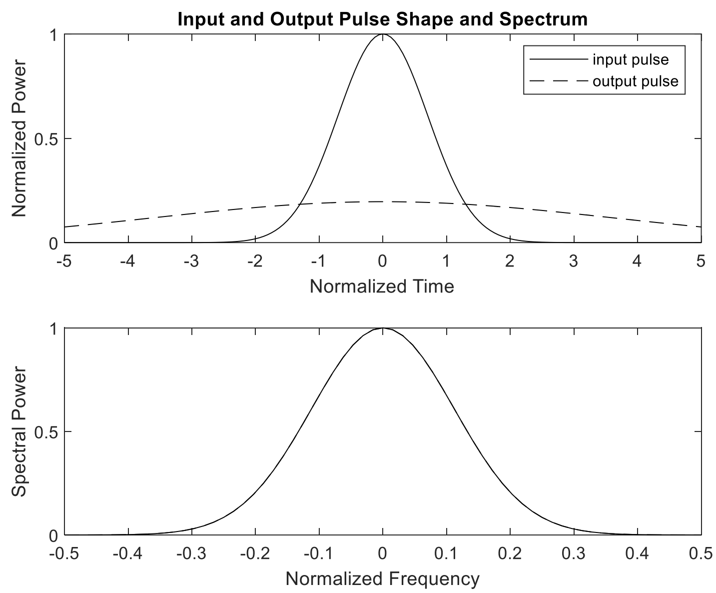
- **anomalous dispersion case (S = -1, N = 0)**

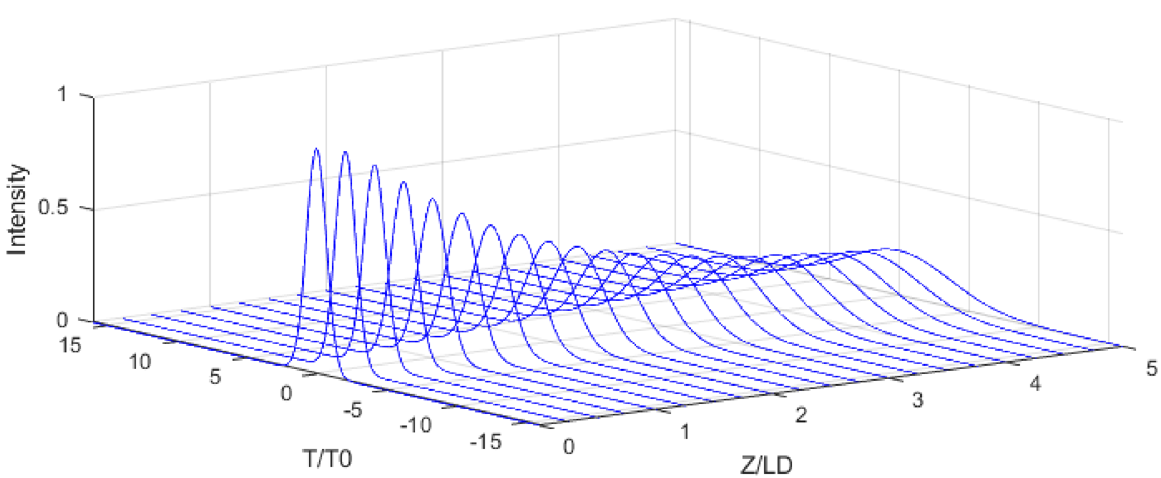
when m = 0 (soliton)



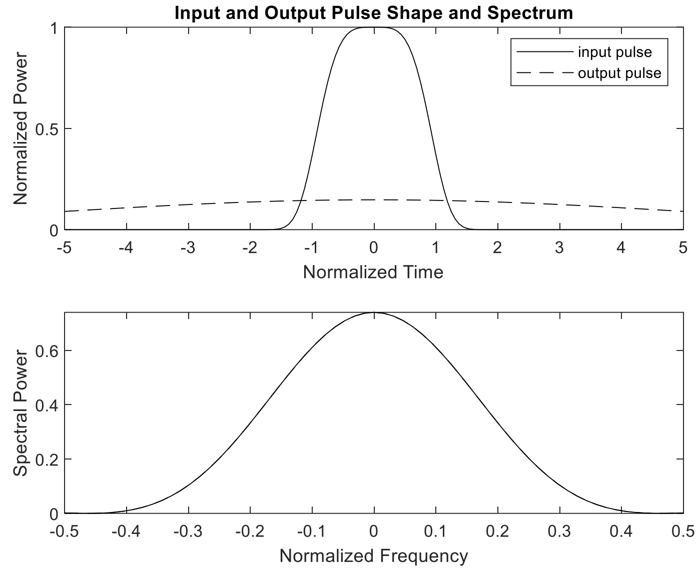

Intensity
T/T0
Z/LD

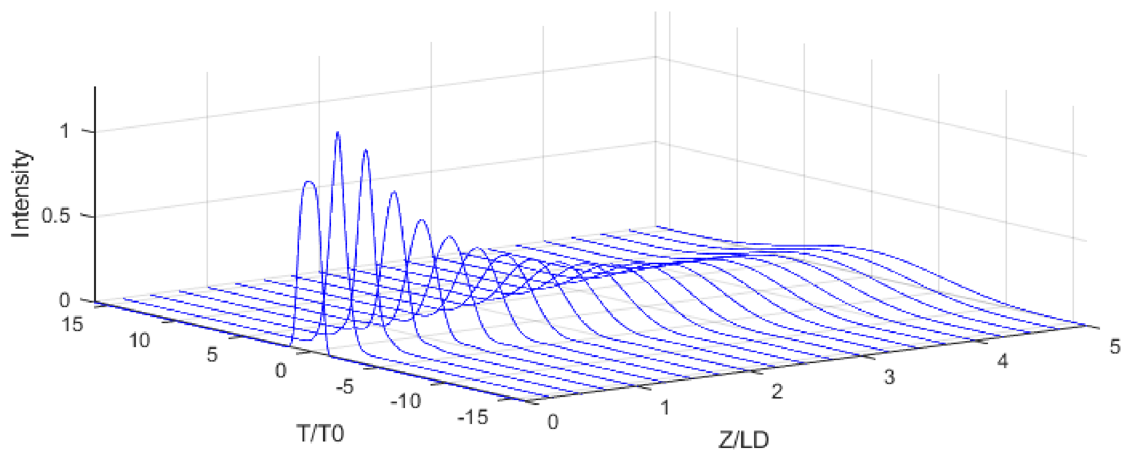
When m = 1


Input and Output Pulse Shape and Spectrum
Normalized Power
input pulse
output pulse
Normalized Time
Spectral Power
Normalized Frequency


Intensity
T/T0
Z/LD

When m = 2

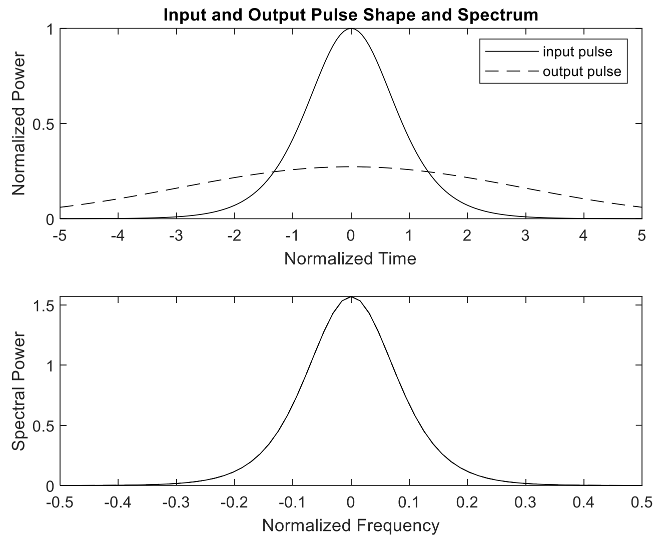


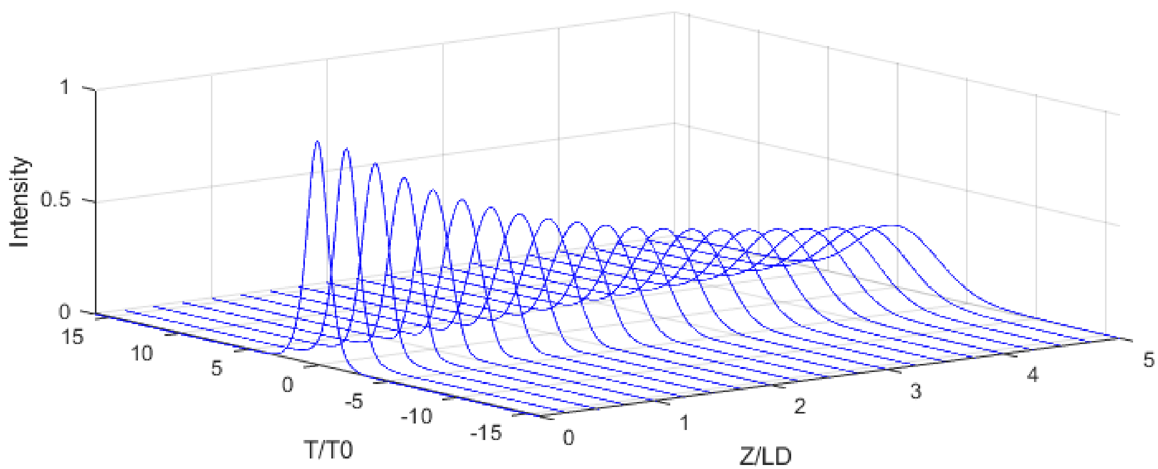


The graphs showed the propagation simulation of different types of input pulses, which were defined with value of m, there were no nonlinearity effect in the simulation, just chromatic dispersion (GVD) was affecting the propagation, this caused a dispersion-induced pulse broadening, it can be see obviously in the graphs above, in was observed that the pulse broadening increases when we change the form of the pulse (when we increase m). it can be seen that there is no spectral broadening due to the absence of nonlinearity.

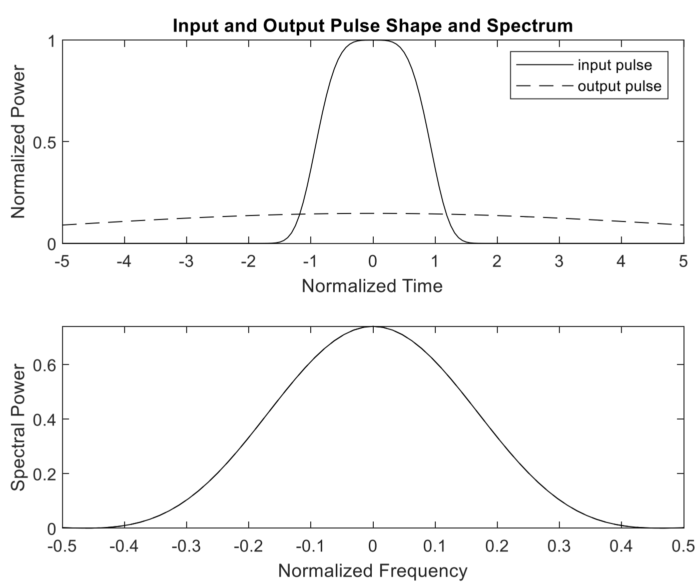
- **normal dispersion case (S = 1, N = 0)**

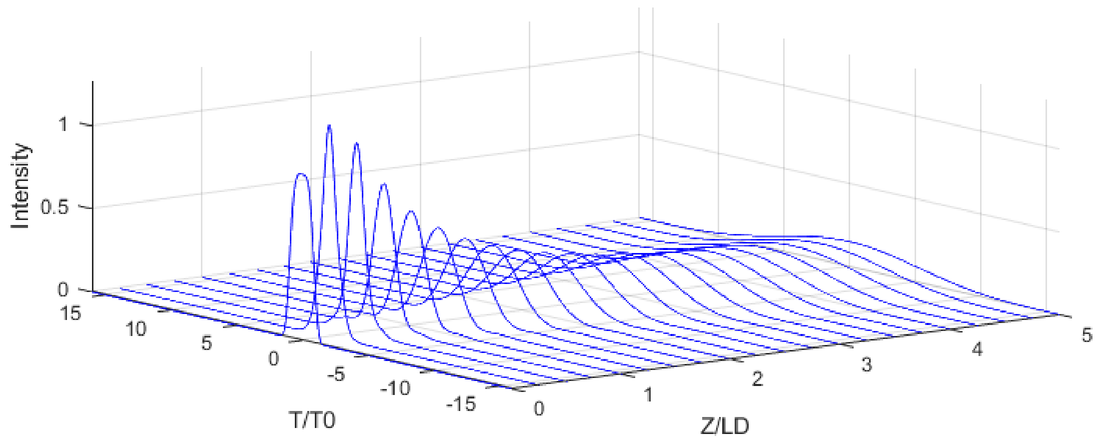
when m = 0





When m = 2

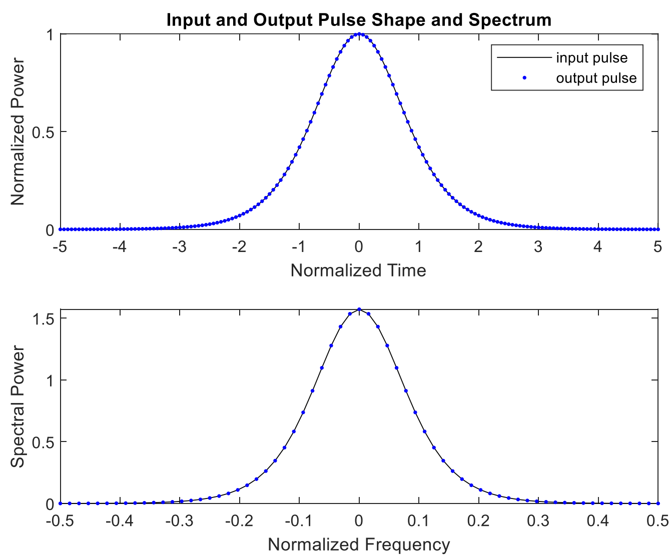


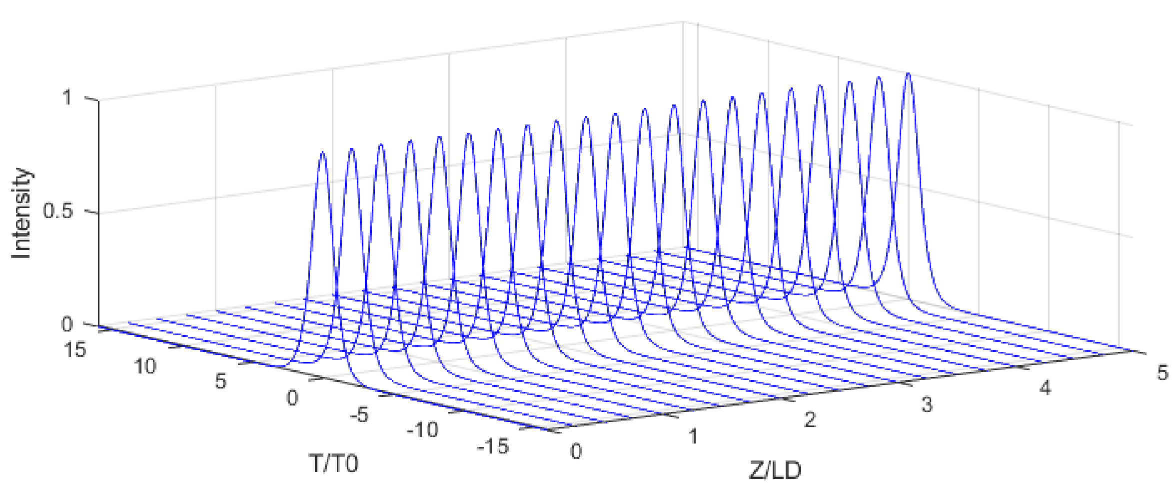


The simulation in this case shows the same nature of propagation of the previous case.

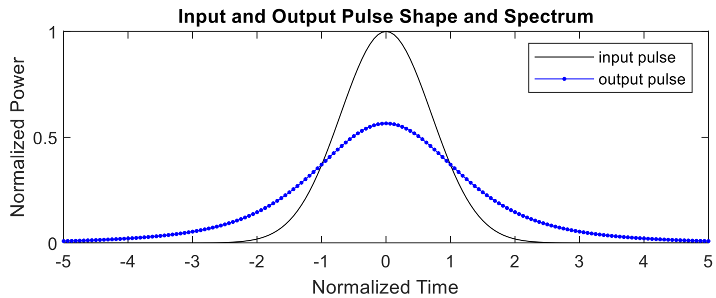
- **anomalous dispersion and nonlinearity (S = -1, N = 1)**

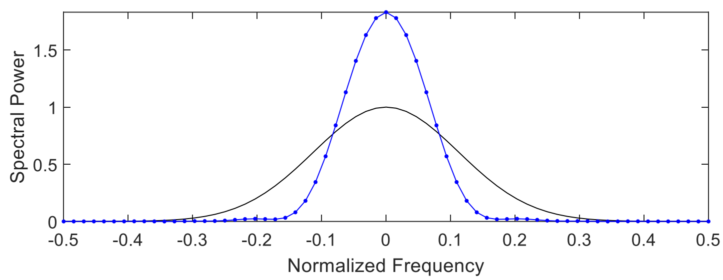
when m = 0 (soliton)

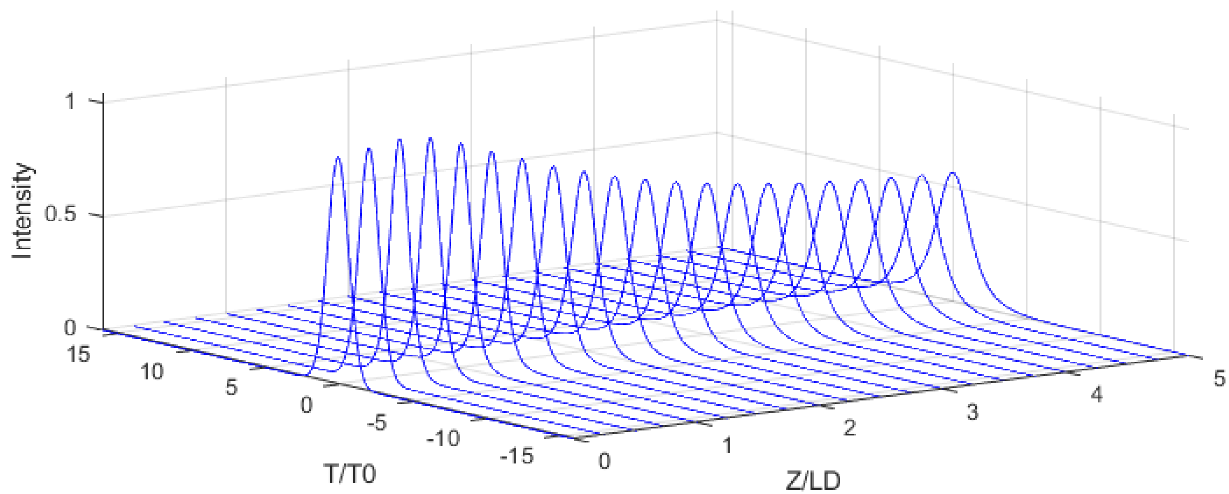




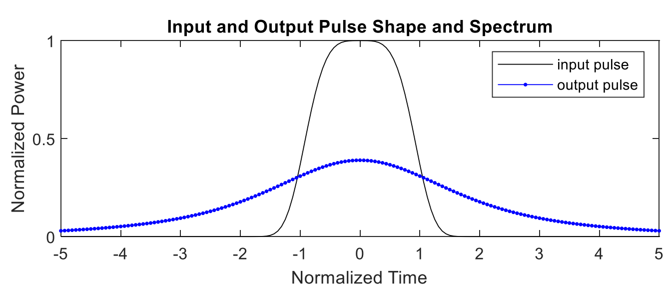
When m = 1,

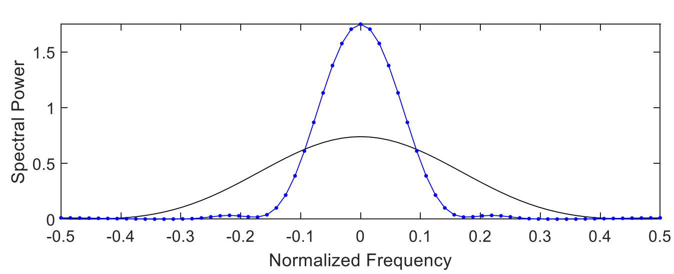


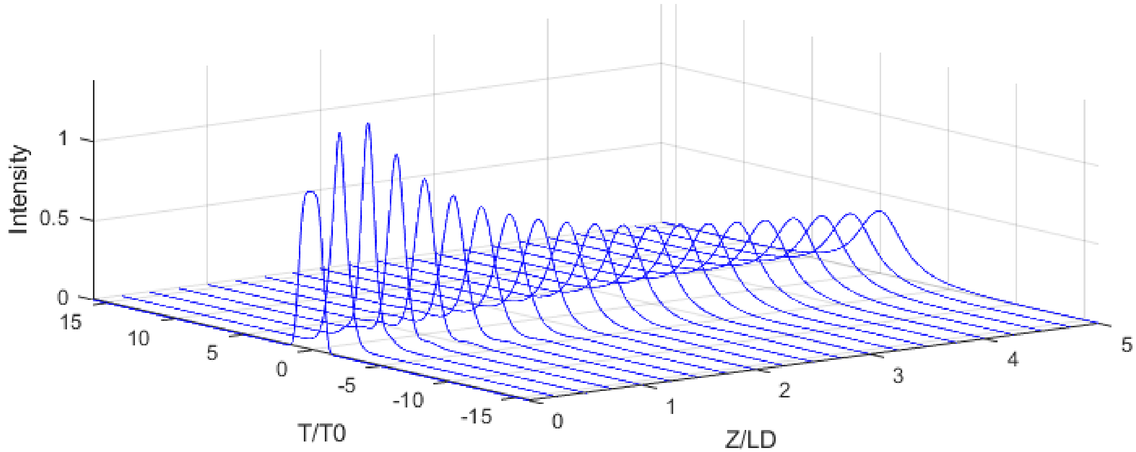




When m = 2



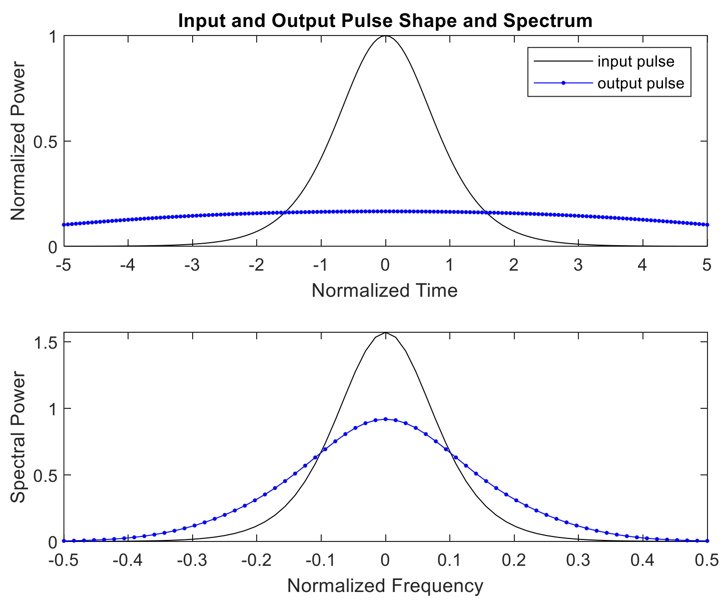


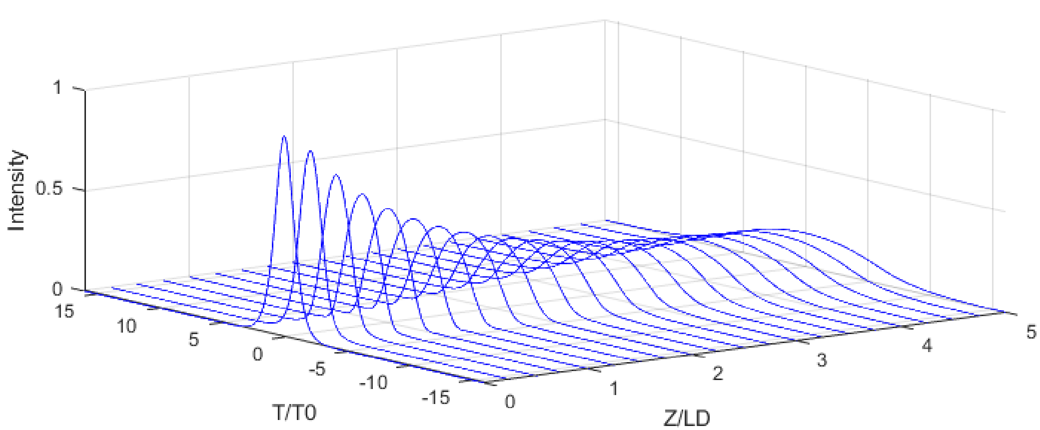


In this case when S=-1 and N=1, when m = 0, the fundamental soliton can be seen, the dispersion effect (pulse broadening) and the nonlinear effect (pulse broadening) are balanced in this case, when the form changes (m increases), we can see more dispersion coming out.

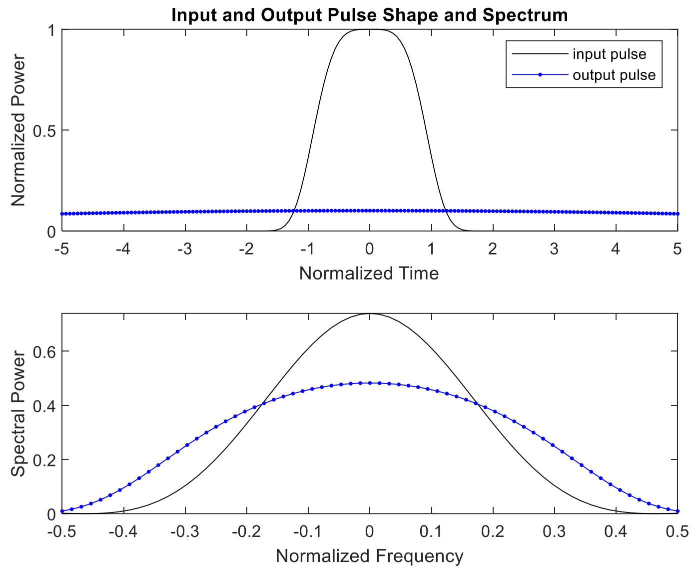
- **normal dispersion and nonlinearity (S = 1, N = 1)**

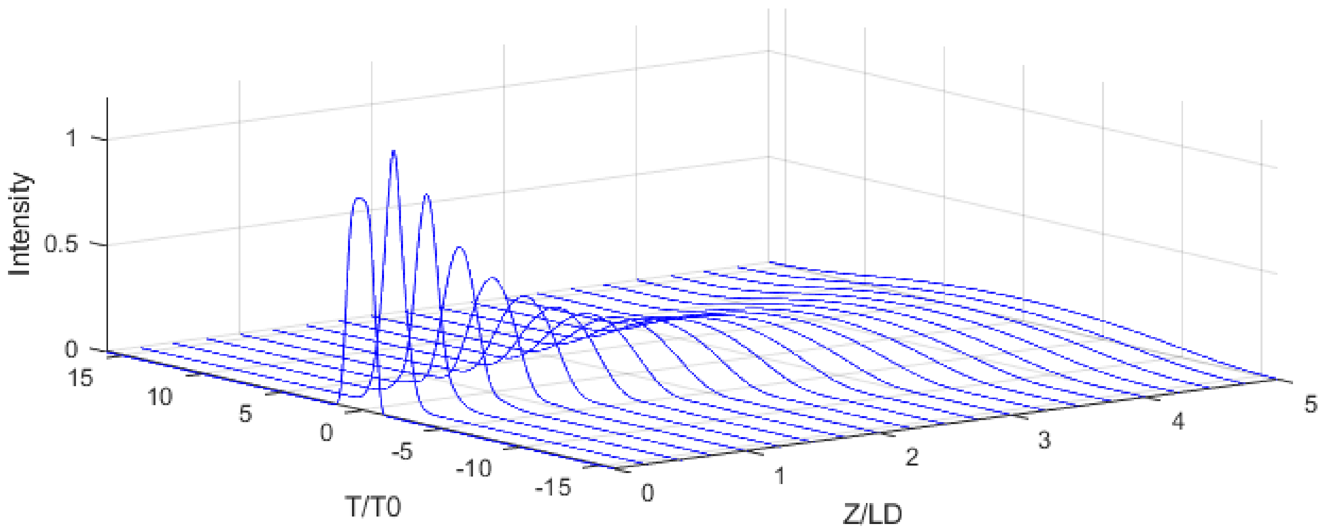
when m = 0 (soliton)





When m = 2,





This case (when S=1 and N=1) is considered the worst case where the pulse broadening is more that any case, it depends of the relation between the dispersion and the nonlinear effect, in this case they do not balance each other they support each other and the distortion will increase.

Appendix:

```
% This code solves the NLS equation with the split-step method
%  idu/dz - sgn(beta2)/2 d^2u/d(tau)^2 + N^2*|u|^2*u = 0
% Written by Govind P. Agrawal in March 2005 for the NLFO book

%---Specify input parameters
clear all; %
close all;
distance = 5 %('Enter fiber length (in units of L_D) = '); %
beta2 = 1 %('dispersion: 1 for normal, -1 for anomalous'); %
N = 1 %('Nonlinear parameter N =  ');  Soliton order% 1-100 1/W km
mshape = 2 %('m = 0 for sech, m > 0 for super-Gaussian = ');%
chirp0 = 0;   % input pulse chirp (default value)%

%---set simulation parameters
nt = 1024; Tmax = 32;   % FFT points and window size
step_num = round(20*distance);   % No. of z steps
deltaz = distance/step_num; % step size in z
dtau = (2*Tmax)/nt;   % step size in tau

%---tau and omega arrays
tau = (-nt/2:nt/2-1)*dtau;   % temporal grid
omega = (pi/Tmax) * [(0:nt/2-1) (-nt/2:-1)]; % frequency grid

%---Input Field profile
if mshape==0   % soliton sech shape
    uu = sech(tau).*exp(-0.5i* chirp0* tau.^2);
else % super-Gaussian
    uu = exp(-0.5*(1+1i*chirp0).*tau.^(2*mshape));
end

%% Plot pulse shape along with z
figure('Position',[100 500 800 300])
z = 0; % Length
plot3(z*ones(1,nt),tau,abs(uu).^2, '-b');
hold on; grid on
axis([0 distance -16 16 0 Inf]);
xlabel('Z/LD')
ylabel('T/T0');
zlabel('Intensity');

%---Plot input pulse shape and spectrum
temp = fftshift(ifft(uu)).*(nt*dtau)/sqrt(2*pi); % spectrum
figure;  subplot(2,1,1);
plot (tau, abs(uu).^2, '-k'); hold on;
axis([-5 5 0 inf]);
xlabel('Normalized Time');
ylabel('Normalized Power');
title('Input and Output Pulse Shape and Spectrum');
subplot(2,1,2);
plot (fftshift(omega)/(2*pi), abs(temp).^2, '-k'); hold on;
axis([-.5 .5 0 inf]);
xlabel('Normalized Frequency');
ylabel('Spectral Power');

%---Store dispersive phase shifts to speedup code
dispersion = exp(0.5i*beta2*omega.^2*deltaz);   % phase factor
hhz = 1i*N^2*deltaz;  % nonlinear phase factor

figure(1)
```

```
% ********* [ Beginning of MAIN Loop] ***********
% scheme: 1/2N -> D -> 1/2N; first half step nonlinear
temp = uu.*exp(abs(uu).^2.*hhz/2);   % note hhz/2
for n=1:step_num

    z=z+deltaz; % Length

    f_temp = ifft(temp).*dispersion;
    uu = fft(f_temp);
    temp = uu.*exp(abs(uu).^2.*hhz);

    if(rem(n,5)==0) % Plot pulse shape along with z
    plot3(z*ones(1,nt),tau,abs(temp).^2, '-b');
    end

end
uu = temp.*exp(-abs(uu).^2.*hhz/2);   % Final field
temp = fftshift(ifft(uu)).* (nt*dtau)/sqrt(2*pi); % Final spectrum
% *************** [ End of MAIN Loop ] **************

figure(2)
%----Plot output pulse shape and spectrum
subplot(2,1,1)
plot (tau, abs(uu).^2, '.-b')
legend('input pulse','output pulse');
subplot(2,1,2)
plot(fftshift(omega)/(2*pi), abs(temp).^2, '.-b')
```