STATISTICS AND PROBABILITY

Implementation in python

Presentation By

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Probability of events

PRESENTED BY:SANIA ARSHAD(098)

1. Introduction to Probability

- **Probability** is a branch of mathematics that deals with the likelihood of different outcomes in uncertain situations. It provides a quantitative framework to evaluate how likely an event is to occur. Understanding probability is essential in various fields, including statistics, finance, science, and everyday decision-making.

Basic Concepts and Types of Probability

- Event: A specific occurrence or outcome.
- Sample Space: Set of all possible outcomes.
- Probability: Number between 0 and 1 representing likelihood.

Types of Probability:

- Theoretical Probability (based on theory).
- Experimental Probability (based on data).
- Conditional Probability (dependent events).

EXAMPLE

- Example 1: A random card is drawn from a deck of 52 cards. What is the probability that it is an ace?
  Solution: E = event of drawing an ace.
  Total number of outcomes = 52
  The favorable number of outcomes = 4 (there are 4 ace cards in a deck of cards. One belonging to each suit).
  P(E) = 4 / 52 = 1 / 13
  Answer: P(E) = 1 / 13
- Example 2: On rolling a fair dice, A is the event of getting a number less than 5, B is the event of getting an odd number and C is the event of getting a multiple of 3. Find the AND event.
  Solution: Sample space of a dice roll = {1, 2, 3, 4, 5, 6}
  A = {1, 2, 3, 4}
  B = {1, 3, 5}
  C = {3, 6}
  $A \cap B \cap C = \{3\}$
  Answer: $A \cap B \cap C = \{3\}$

Probability Rules and Real-World Applications

- Addition Rule: P(A or B) = P(A) + P(B).
- Multiplication Rule: P(A and B) = P(A) × P(B).
- Complement Rule: P(A') = 1 - P(A)
- **Real-World Applications:**
- Insurance.
- Finance.
- Medicine.
- Engineering.

Code Example:

```
import random

def roll_die():
    return random.randint(1, 6)

def simulate_rolls(target, rolls):
    target_count = 0
    roll_counts = [0] * 6

    for _ in range(rolls):
        result = roll_die()
        roll_counts[result - 1] += 1
        if result == target:
            target_count += 1

    probability = target_count / rolls

    print(f"Probability of rolling a {target}: {probability:.2f}")
    print("Roll counts for each number:")
    for i in range(1, 7):
        print(f"  {i}: {roll_counts[i - 1]}")

target_number = int(input("Enter the target number (1 to 6): "))
total_rolls = int(input("Enter the number of rolls: "))

simulate_rolls(target=target_number, rolls=total_rolls)
```

OUTPUT

```
Enter the target number (1 to 6): 4
Enter the number of rolls: 50
Probability of rolling a 4: 0.10
Roll counts for each number:
  1: 8
  2: 11
  3: 6
  4: 5
  5: 10
  6: 10
```

Conditional Probability

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Students will be able to:

find the conditional probability of an event.

Key Vocabulary:

- Independent events
- Dependent events
- Conditional Probability

INDEPENDENT EVENTS

Two events, $\boldsymbol{A}$ and $\boldsymbol{B}$, are **INDEPENDENT EVENTS** if the fact that event $\boldsymbol{A}$ occurs does not affect the probability of $\boldsymbol{B}$ occurring.

- **Rolling a die and getting a 4, and then rolling a second die and getting a 5.**
- **Drawing a card from a deck and getting a jack, replacing it, and then drawing a second card and getting an ace.**

DEPENDENT EVENTS.

- When the outcome or occurrence of the first event ***A*** affects the outcome or occurrence of the second event ***B*** in such a way that the probability is changed, the events are said to be **DEPENDENT EVENTS**.

- **Drawing a card from a deck, not replacing it, and then drawing a second card.**
- **Having a high grades and getting a scholarship.**
- **Selecting a ball from a jar, not replacing it, and then selecting a second ball.**

Conditional Probability

A **CONDITIONAL PROBABILITY** of an event is used when the probability is affected by the knowledge of other circumstances. It can only occur to dependent events.

Conditional Probability

Types of Events

Sample Problem 1

Sample Problem 1: Black and white chips are placed in a box. Jack selects two chips without replacing the first one. If the probability of selecting a black chip and white chip is 30/67, and the probability of selecting a white chip on the first draw is 6/11, find the probability of selecting a black chip on the second draw, given that the first chip is white.

$$P(W) = \frac{6}{11} \qquad P(W\ and\ B) = \frac{30}{67}$$

$$P(B|W) = \frac{P(W\ and\ B)}{P(W)} = \frac{\frac{30}{67}}{\frac{6}{11}} = \frac{30}{67} \cdot \frac{11}{6} = P(B|W) = \frac{55}{67} \cong 82.09\%$$

Sample Problem 2

Sample Problem 2: 9 red balls and 3 green marbles are place in a bag. Find the probability of randomly selecting a red marble on the first draw and a green marble on the second draw.

$$P(R) = \frac{9}{12} = \frac{3}{4} \qquad P(R\ and\ G) = \frac{3}{4} \cdot \frac{3}{11} = \frac{9}{44}$$

$$P(G|R) = \frac{P(R\ and\ G)}{P(R)} = \frac{\frac{9}{44}}{\frac{3}{4}} = \frac{9}{44} \cdot \frac{4}{3}$$

$$P(G|R) = \frac{3}{11} \cong 27.27\%$$

Code Example:

- Let's say we want to calculate the **conditional probability** of getting a sum of 7 given that at least one die shows a 4. This would mean:
- **Event A**: The sum of two dice is 7.
- **Event B**: At least one die shows a 4.
- To calculate $P(A|B)P(A|B)P(A|B)$, we need:
- $P(A\cap B)P($A \cap B)$P(A\cap B)$ – Probability of both events (sum is 7 and one die shows 4).
- $P(B)P(B)P(B)$ – Probability that at least one die shows a 4.

```python
import random

def calculate_conditional_probability(num_rolls):
    """Calculate the probability that the sum is 7 given that one die shows 4."""
    count_A_and_B = 0  # Count of (sum is 7 and one die shows 4)
    count_B = 0        # Count of (one die shows 4)

    for _ in range(num_rolls):
        die1 = random.randint(1, 6)
        die2 = random.randint(1, 6)

        # Check if at least one die is 4
        if die1 == 4 or die2 == 4:
            count_B += 1
            # Check if the sum is 7
            if die1 + die2 == 7:
                count_A_and_B += 1

    # Calculate conditional probability P(A|B)
    return count_A_and_B / count_B if count_B > 0 else 0
```

Code Example:

- **roll_dice()**: Simulates rolling two dice.
- **simulate_rolls()**: Rolls dice multiple times.
- **event_a_and_b()**: Counts occurrences of A∩BA \cap BA∩B.
- **event_b()**: Counts occurrences of event B.
- **calculate_conditional_probability()**:
- Computes conditional probability P(A∣B)P(A|B)P(A∣B).
- **main()**: Runs the program logic.

```python
def main():
    try:
        num_rolls = int(input("Enter the number of rolls to simulate: "))
        if num_rolls <= 0:
            print("Please enter a positive integer for the number of rolls.")
            return
        conditional_prob = calculate_conditional_probability(num_rolls)
        print(f"The conditional probability P(A|B) is: {conditional_prob:.4f}")
    except ValueError:
        print("Invalid input. Please enter an integer.")

if __name__ == "__main__":
    main()
```

OUTPUT:

```
IPython 8.25.0 -- An enhanced Interactive Python.

In [1]: runfile('C:/Users/HP/Desktop/Malaika/Semester 3/Malaika.py', wdir='C:/Users/HP/Desktop/Malaika/
Semester 3')
Enter the number of rolls to simulate: 1
The conditional probability P(A|B) is: 0.0000

In [2]: runfile('C:/Users/HP/Desktop/Malaika/Semester 3/Malaika.py', wdir='C:/Users/HP/Desktop/Malaika/
Semester 3')
Enter the number of rolls to simulate: 8
The conditional probability P(A|B) is: 0.3333
```

INDEPENDENT EVENTS

PRESENTED BY:

EMAN WAHEED(027)

Independence in Statistics and Probability:

- **1. Definition of Independence**
- **Statistical Independence**: Two events A and B are independent if the occurrence of one does not affect the occurrence of the other.

$$P(A \cap B) = P(A)P(B)$$

Example: Consider flipping a fair coin twice. The result of the first flip (heads or tails) does not affect the result of the second flip. In this case, the two events (first flip and second flip) are statistically independent.

SOURCE CODE IN PYTHON:

```
import numpy as np

def check_independence(A, B):
    # Calculate probabilities
    P_A = np.mean(A)  # Probability of A
    P_B = np.mean(B)  # Probability of B
    P_A_and_B = np.mean(A * B)  # Joint probability of A and B

    # Check independence condition
    independence_condition = np.isclose(P_A_and_B, P_A * P_B)

    # Display results
    print(f"P(A): {P_A:.4f}")
    print(f"P(B): {P_B:.4f}")
    print(f"P(A and B): {P_A_and_B:.4f}")
    print(f"P(A) * P(B): {P_A * P_B:.4f}")
    print(f"Are A and B independent? {'Yes' if independence_condition else 'No'}")

def main():
    # In Google Colab, we suggest hardcoding inputs for testing or providing clear instructions for manual entry.
    n_samples = int(input("Enter the number of samples: "))

    # Ensure to guide the user properly
    print("Enter values for event A (space-separated, 0s and 1s):")
    A = np.array(list(map(int, input().split
    ())))
    print("Enter values for event B (space-separated, 0s and 1s):")
    B = np.array(list(map(int, input().split())))

    # Ensure input lengths match
    if len(A) != n_samples or len(B) != n_samples:
```

```python
    # Ensure to guide the user properly
    print("Enter values for event A (space-separated, 0s and 1s):")
    A = np.array(list(map(int, input().split
     ())))
    print("Enter values for event B (space-separated, 0s and 1s):")
    B = np.array(list(map(int, input().split())))

    # Ensure input lengths match
    if len(A) != n_samples or len(B) != n_samples:
        print("Error: The number of samples does not match the input length.")
        return

    check_independence(A, B)

# Corrected main entry point
if __name__ == "__main__":
    main()
```

OUTPUT: WHEN IT IS INDEPENDENT

```
Enter the number of samples: 6
Enter values for event A (space-separated, 0s and 1s):
1 1 0 0 0 0
Enter values for event B (space-separated, 0s and 1s):
0 0 0 0 0 0
P(A): 0.3333
P(B): 0.0000
P(A and B): 0.0000
P(A) * P(B): 0.0000
Are A and B independent? Yes
```

Input Breakdown

1. **Event A:** `[1, 1, 0, 0, 0, 0]`
   - Count of 1s: 2
   - Total samples: 6
   - Probability $P(A) = \frac{2}{6} = 0.3333$
2. **Event B:** `[0, 0, 0, 0, 0, 0]`
   - Count of 1s: 0
   - Probability $P(B) = \frac{0}{6} = 0.0000$

Calculated Probabilities

1. **Joint Probability $P(A \cap B)$:**
   - Here, $P(A \cap B)$ represents the probability that both A and B occur together.
   - In this case, since B has no 1s, there are no instances where both A and B are 1:
     - $P(A \cap B) = 0.$
2. **Product of Individual Probabilities:**
   - $P(A) \cdot P(B) = 0.3333 \cdot 0.0000 = 0.0000.$

Independence Check

The independence condition is defined as:

$$P(A \cap B) = P(A) \cdot P(B)$$

In this case:

- $P(A \cap B) = 0$
- $P(A) \cdot P(B) = 0.0000$

Since both sides equal 0, the independence condition is satisfied, leading to the conclusion that events A and B are independent.

OUTPUT:WHEN IT IS DEPENDENT

```
Enter the number of samples: 6
Enter values for event A (space-separated, 0s and 1s):
1 1 0 0 0 0
Enter values for event B (space-separated, 0s and 1s):
0 0 0 0 1 1
P(A): 0.3333
P(B): 0.3333
P(A and B): 0.0000
P(A) * P(B): 0.1111
Are A and B independent? No
```

Input Breakdown

1. **Event A:** `[1, 1, 0, 0, 0, 0]`
   - Count of 1s: 2
   - Total samples: 6
   - Probability $P(A) = \frac{2}{6} = 0.3333$
2. **Event B:** `[0, 0, 0, 0, 1, 1]`
   - Count of 1s: 2
   - Probability $P(B) = \frac{2}{6} = 0.3333$

Calculated Probabilities

1. **Joint Probability $P(A \cap B)$:**
   - To find $P(A \cap B)$, we look for instances where both A and B are 1 at the same position:
   - Checking pairs:
     - (1, 0), (1, 0), (0, 0), (0, 0), (0, 1), (0, 1)
     - There are **no** instances where both A and B are 1.
   - Therefore, $P(A \cap B) = 0$.
2. **Product of Individual Probabilities:**
   - $P(A) \cdot P(B) = 0.3333 \cdot 0.3333 = 0.1111$.

Independence Check

The independence condition states:

$$P(A \cap B) = P(A) \cdot P(B)$$

In your case:

- $P(A \cap B) = 0$
- $P(A) \cdot P(B) = 0.1111$

Since $0 \neq 0.1111$, the events are **not independent**.

1. Basic Probability Calculation

Question:
Given $P(A) = 0.4$, $P(B) = 0.6$, and $P(A \text{ and } B) = 0.24$, check if events A and B are independent.
Hint: Use the formula:

$$P(A \text{ and } B) = P(A) \times P(B)$$

2. Finding Missing Probability

Question:
If $P(A) = 0.5$, $P(B) = 0.8$, and A and B are independent, find $P(A \text{ and } B)$.

For three events A, B, and C, suppose $P(A) = 0.5$, $P(B) = 0.6$, $P(C) = 0.4$, and $P(A \text{ and } B \text{ and } C) = 0.12$.

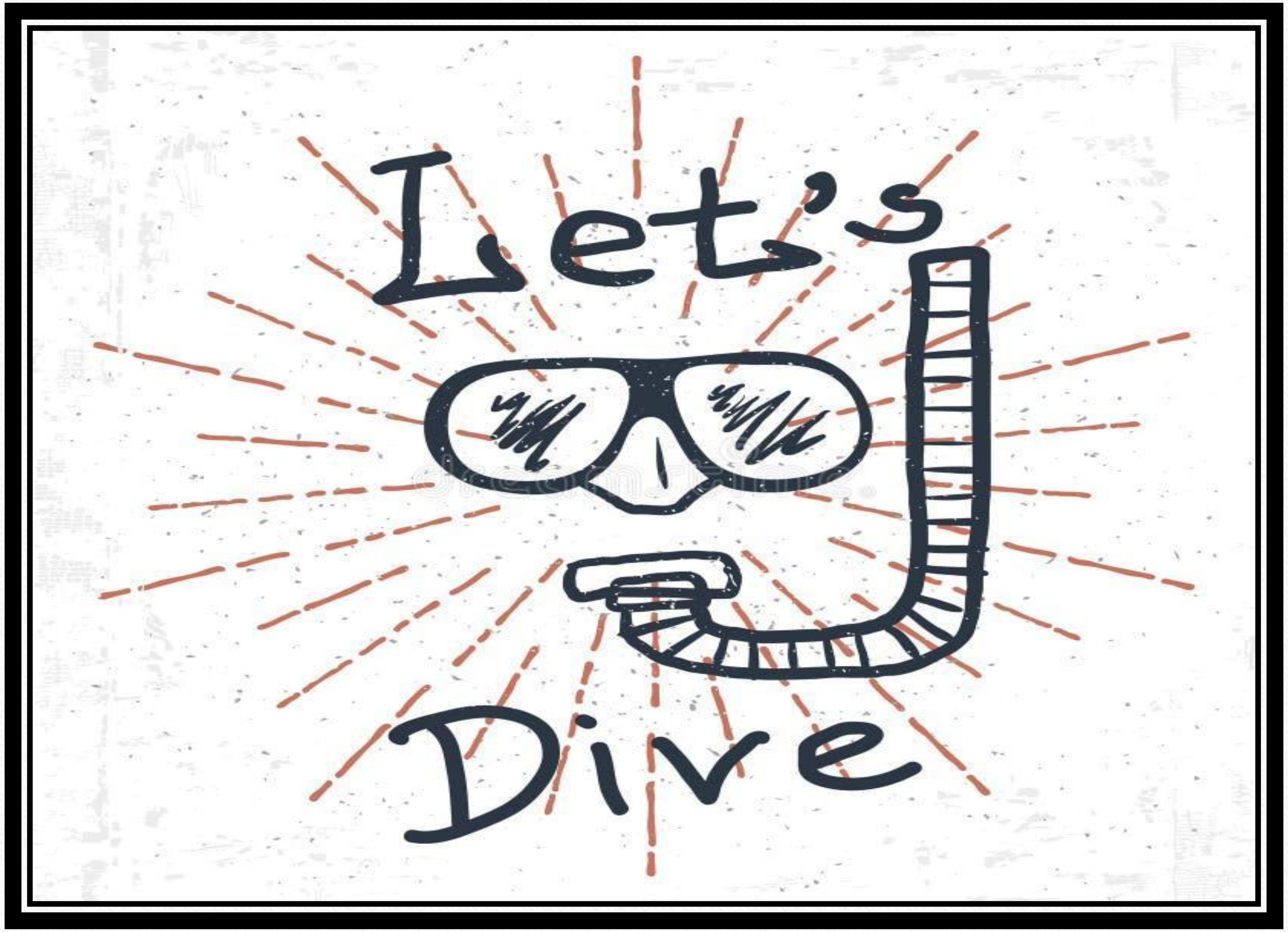
Verify if A, B, and C are jointly independent using:

$$P(A \text{ and } B \text{ and } C) = P(A) \times P(B) \times P(C)$$

Random Variables

PRESENTED BY:MARYAM(061)

TABLE OF CONTENT



Let's
Dive

1. What is a Random Variable?

- "A **random variable** is a numerical outcome of a random process or experiment. It assigns real numbers to every possible outcome of a random phenomenon."

OR

- "A function from a sample space to the real numbers."
- **Essential for** 'understanding probability', 'uncertainty', and 'statistical models'.

TYPES OF RANDOM VARIABLE

- **Discrete Random Variables**
  - Can take only specific, countable values.
  - **Example:** Number of heads in 10 coin tosses.
- **Continuous Random Variables**
  - Can take any value in a range.
  - **Example:** Time taken for a car to complete a race.

LET´S MOVE

Example of Discrete Random Variable

- **Example: Rolling a Fair Die**
- **Random Variable (X):** Outcome of rolling a six-sided die.
- Possible outcomes:
  $X=\{1,2,3,4,5,6\}$
- **Probability Distribution (PMF):**
  All outcomes have an equal probability of 1/6

Example of Continuous Random Variable

- **Example:** **Time to Wait for a Bus**
- **Random Variable (Y):** The time (in minutes) you wait

  for the bus.
- Possible outcomes:
  $Y=[0,\infty)$, Y = [0, infinity)
- **Probability Distribution (PDF):**
  The probabilities are distributed over a continuous interval.

Mathematical Properties of Random Variables

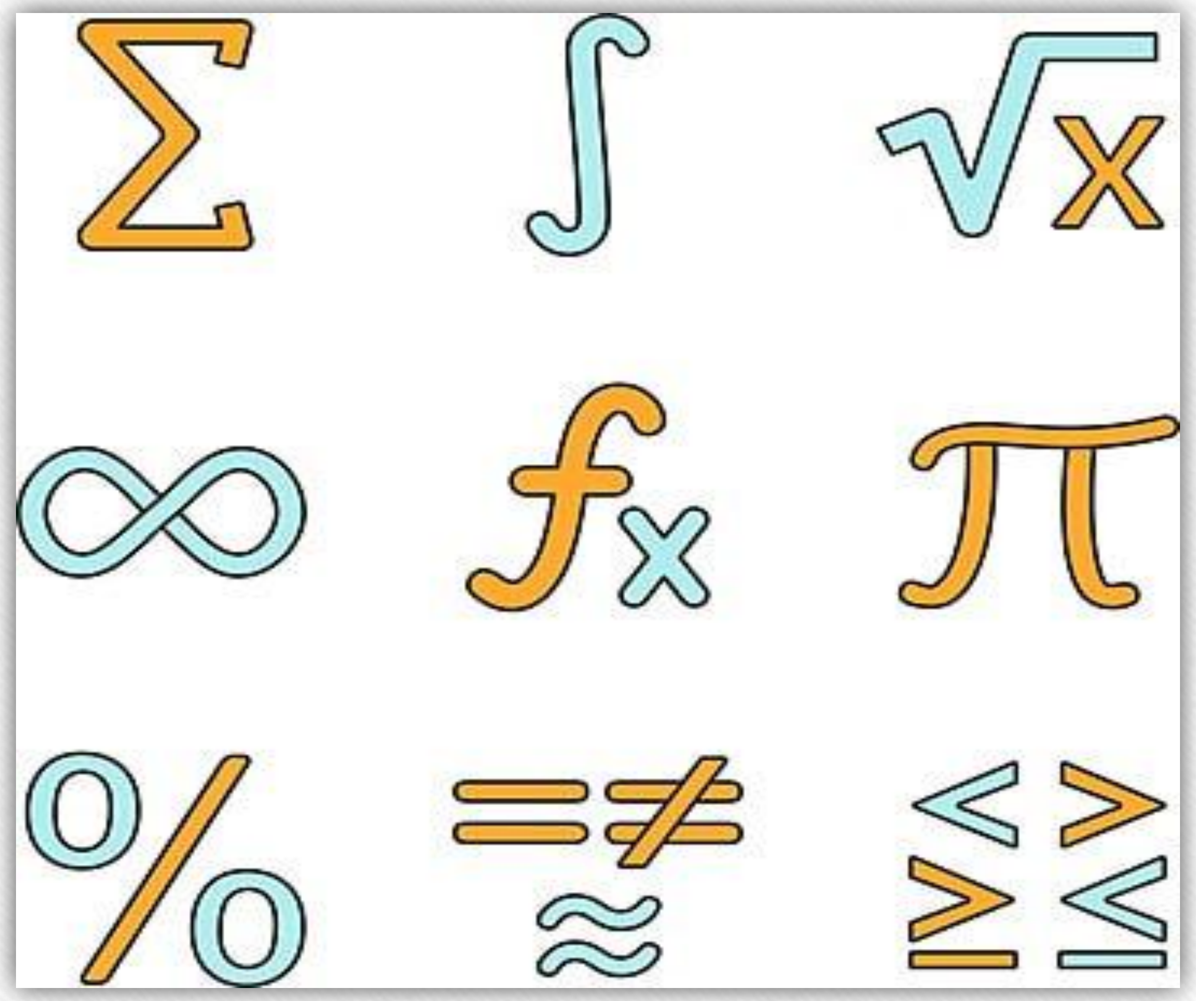
- **Expected Value (E[X]):**
  The weighted average of all possible values:

❑ **FOR DISCRETE:**

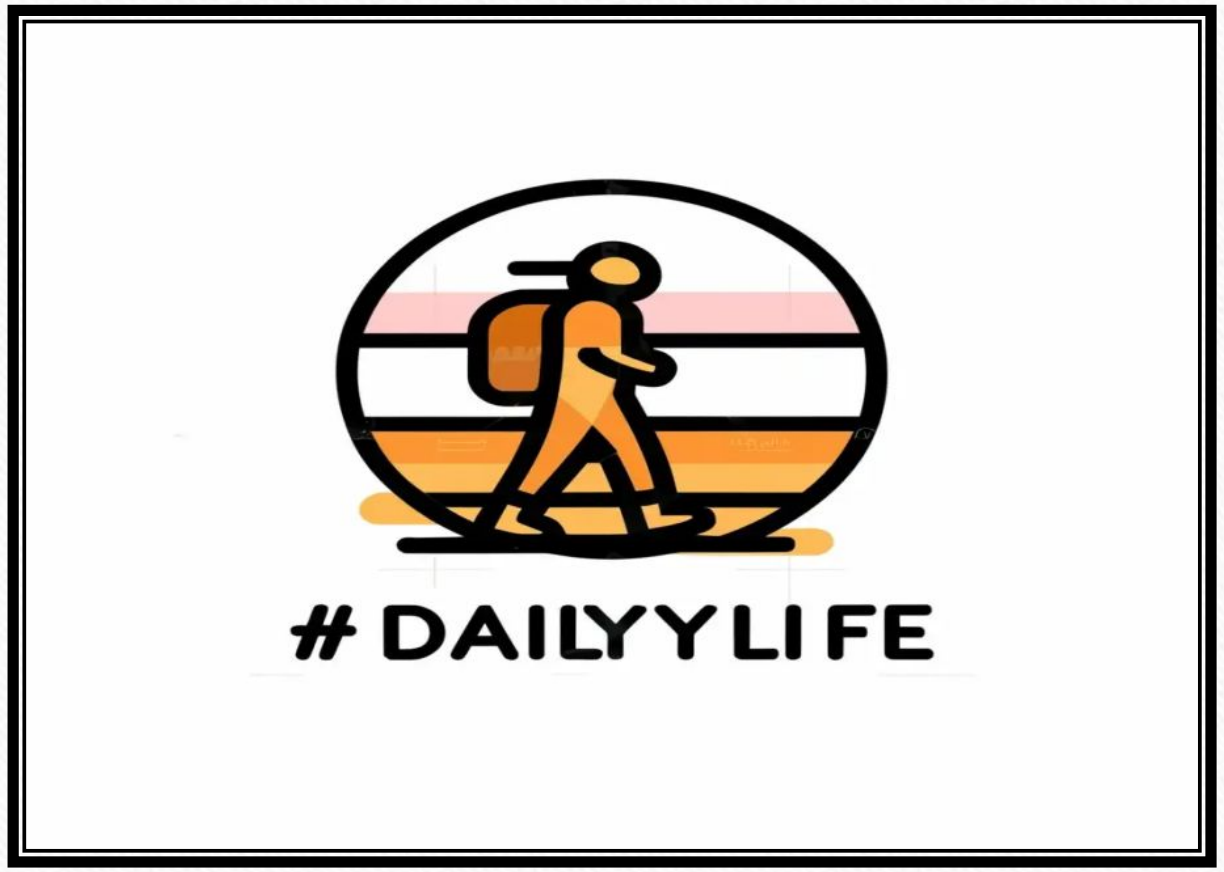
E[X]=∑ xi . P(X=xi) or

❑ **FOR CONTINUOUS:**

E[X]=∫−∞∞ x fX(x)dx (Continuous)



Applications of Random Variables

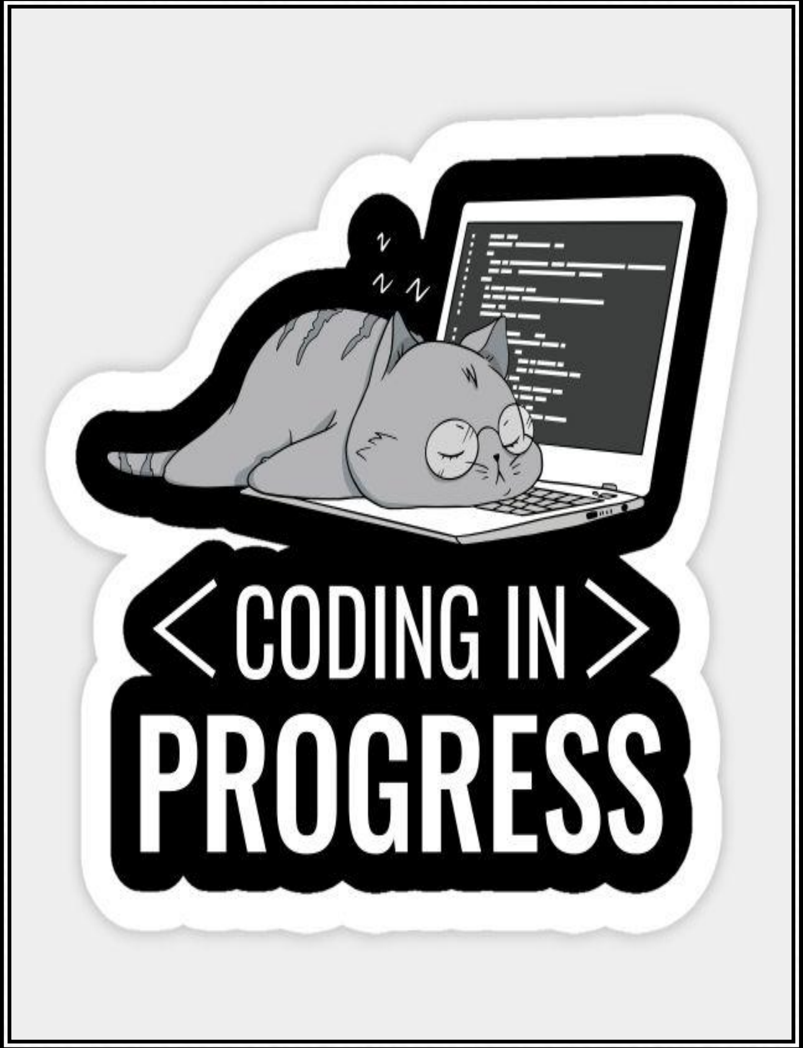


- **Insurance:**
  - Random variables like life expectancy and accident probabilities help calculate insurance premiums.
- **Finance:**
  - Stock price changes, portfolio returns, and risks are modeled using random variables.
- **Weather Forecasting:**
  - Predicting the temperature, precipitation, and storm likelihood uses random variables to model uncertainty.
- **Gaming:**
  - Dice rolls, card shuffling, and random number generation are all modeled using discrete random variables.

AlrightCoders

Python Code Example:
Discrete Random Variable

Lets Consider a Code Example in Python language to calculate mean and variance and then plot the PMF which is shown as graph.




```
import numpy as np
import matplotlib.pyplot as plt

# Define a discrete random variable (Die Roll)
X = np.arange(1, 7)
P_X = np.ones(6) / 6  # Equal probability for each face of a die

# Calculate expected value (mean) and variance
expected_value = np.sum(X * P_X)
variance = np.sum((X - expected_value)**2 * P_X)
# Print results
print("Expected Value:", expected_value)
print("Variance:", variance)

# Plotting the PMF
plt.bar(X, P_X, color='blue')
plt.title('Probability Mass Function (Die Roll)')
plt.xlabel('Outcome')
plt.ylabel('Probability')
plt.show()
```

THE CODE

WHAT WE DID IN OUR CODE?

- **Imported Libraries**:

Imported numpy for numerical operations.

Imported matplotlib.pyplot for plotting.

- **Defined a Discrete Random Variable**:

Represented a die roll with outcomes from 1 to 6 (X).

Assigned equal probabilities (P_X) for each outcome, representing a fair die.

- **Calculated Expected Value and Variance**:

Computed the expected value (mean) of the die roll.

Calculated the variance to measure the spread of outcomes.

- **Printed Results**:

Displayed the expected value and variance in the output.

- **Plotted the Probability Mass Function (PMF)**:

Created a bar chart showing the probabilities of each die roll outcome.

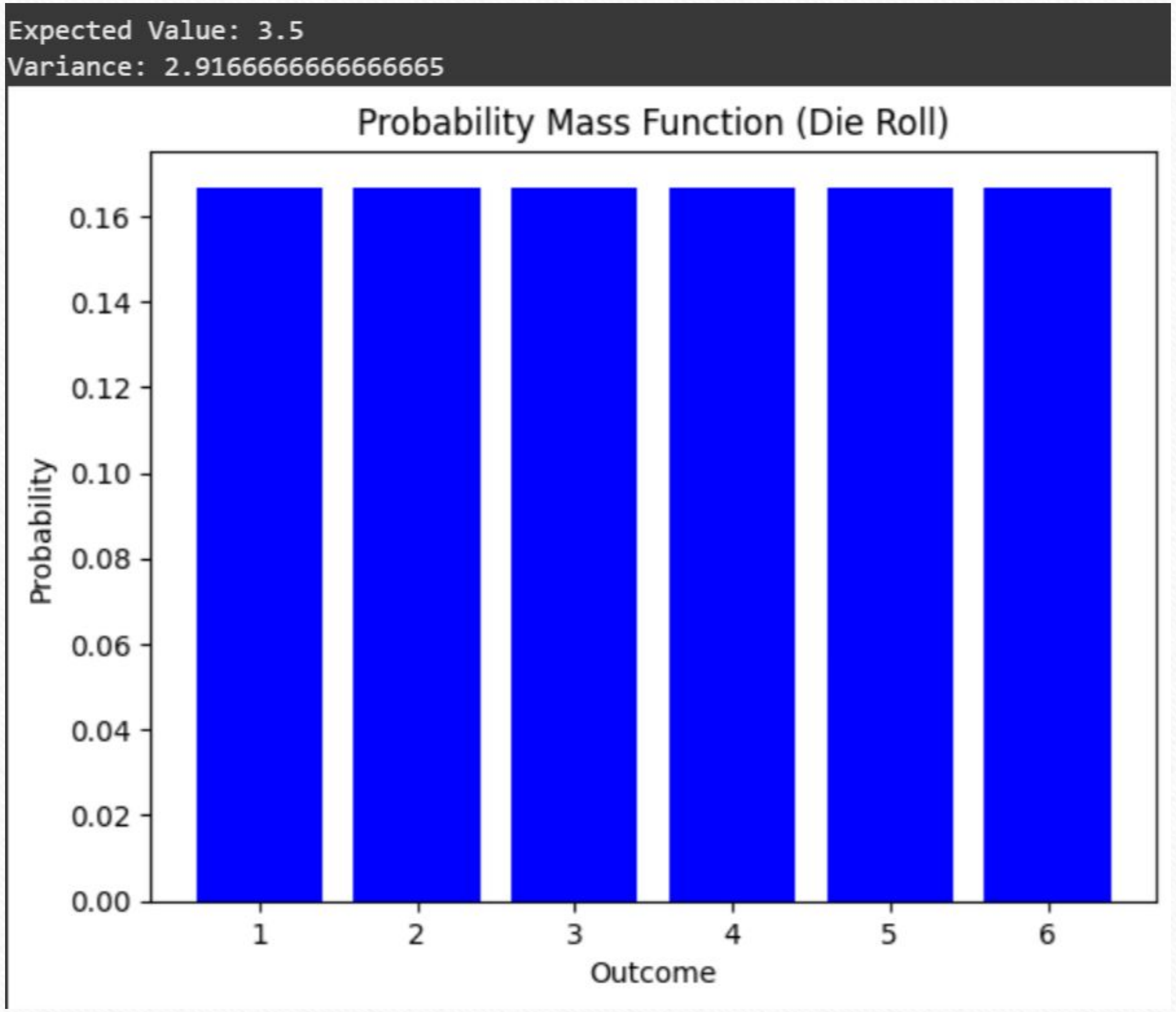
OUTPUT OF OUR CODE

Expected Value:

3.5

Variance:

2.92




THANK YOU FOR YOUR ATTENTION!
ANY QUESTIONS?
DON'T ASK HARD QUESTION
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