Michelson Interferometry

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Abstract

The wavelength of a Helium-Neon Laser was measured by varying the path length of the light beam in a Michelson Interferometer and observing the shift in the interference pattern. Subsequently, the refractive index of a glass slide was measured by introducing it into the experimental setup and varying the angle between the normal to the surface of the glass slide and the incident light beam and again observing how the interference pattern shifted. In both cases, a significant degree of agreement was found between the experimentally measured values and the theoretical predictions.

1 Introduction

The Michelson Interferometer is widely known for its role in the Michelson-Morley experiment which eventually disproved the existence of the ether and paved the way towards a relativistic view of the world. It is an experimental setup which involves a light source (in modern setups mostly a laser), a beam splitter to split the light coming from the source into two different beams and mirrors that reorient the light onto a screen. As a result of the phase difference introduced due to the difference in path lengths of the resultant beams, an interference pattern is observed on the screen.

A number of precise experiments can be conducted using the properties of the interference pattern formed on the screen. For example, very small variations in length can be measured extremely precisely since the interference pattern reacts to the phase difference between the beams incident on the screen, which in turn depends on the number of wavelengths crossed by the light in its path. Due to the wavelengths being of the order of nanometers we can measure changes in the path length upto such orders.

2 Theory

The Michelson interferometer is an amplitude splitting interferometer. This means that the two beams that result from the splitting procedure have half the amplitude of the original beam. This is in contrast to wavefront splitting interferometers which use the wavefronts of the original beam as sources for the wavefronts of the resulting beams.

Before going into the exact details of how an interference pattern arises, we first discuss what requirements need to be satisfied in order for such a pattern to appear in the first place:

2.1 Conditions for Interference [2]

Quasi-monochromaticity

While an interference pattern can be observed with non-monochromatic (say, white) light, the fringes observed under such circumstances are not very sharp. Such a pattern is observed because the individual monochromatic components that the beam is made out of interfere with eachother.

Under ideal circumstances one would use completely monochromatic light of a certain wavelength to observe sharp fringes. This, however, is only a theoretical idealization. What one can actually produce in the lab is light which is only "effectively" monochromatic. Such light is called Quasi-monochromatic light.

Temporal Coherence

An ordinary light source doesn't just produce a single sinusoidal band of light at the same frequency the way you would theoretically expect. Instead "wavetrains" of photons are emitted in spurts, such that at a single point in space, for some amount of time, the wave oscillates sinusoidally when suddenly it encounters another "wave patch" at a different phase.

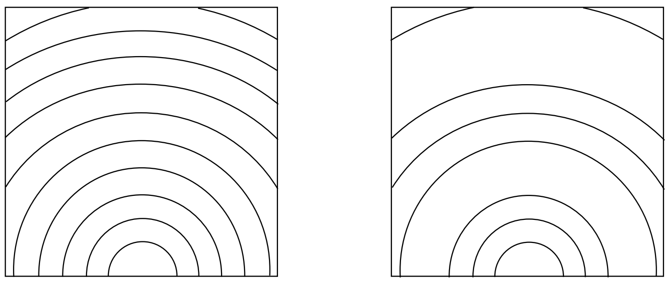

Figure 1: Left: A wave which is infinitely temporally coherent Right: A wave with a more plausible temporal coherence

There is therefore only a certain amount of time for which you can predict the behaviour of the wave at some point in space. This **effective time of predictable sinusoidal behaviour** signifies the "temporal coherence" of the source. Again, under ideal circumstances, one would require the light source to be "infinitely temporally coherent". But this, yet again, is just an idealization. The best one can do is use a light source with a "large enough" temporal coherence.

Spatial Coherence

Quite similar to the idea of Temporal Coherence is the idea of "Spatial" Coherence. One would expect an ideal light source to be spherically symmetric. In actual practice the source has surface features, all of which act as individual sources. The result is a set of overlapping wavefronts being produced from all of these sources. This results in a situation where at some moment in time you cannot predict the sinusoidal behaviour at adjacent points "in space". This predictability (or the lack there) is a measure of the "Spatial Coherence" of the wave.

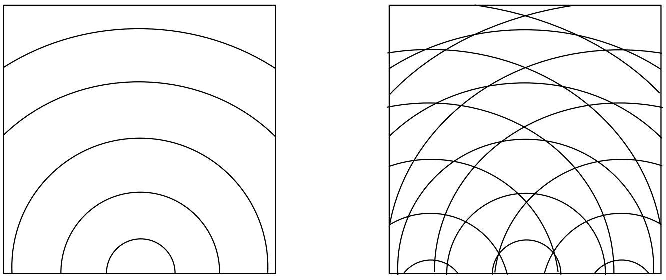

Figure 2: Left: A wave which is infinitely spatially coherent Right: A wave with a more plausible spatial coherence (three sources included in this case).

The more spatially coherent a wave, the sharper and more distinct the interference fringes.

2.2 Interference fringes and the wavelength of light

Interference fringes result from constructive and destructive interference between two light beams. The points at which interference takes place can be varied by changing the path length of one of the beams. This causes the interference pattern to "shift" on the screen. The fringes return exactly to their original positions once the total path length has changed by an amount λ. This is because:

$$sin(\omega t + kx) = sin(\omega t + k(x + \lambda)). \quad (1)$$

Where $k = \frac{2\pi}{\lambda}$. That is, the wave shifted by a length equal to one λ (or any multiple of λ for that matter) is essentially indistinguishable from the original wave with zero wave shift. This phenomenon is used to estimate the wavelength of the light beam.

2.3 Optical Path Length and Refractive Index

An important concept when measuring the refractive index of a medium is that of the optical path length and the optical path difference. In simple words we can define the optical path length as **the distance a light beam must travel through air to create the same phase difference as is created when the same beam travels through a medium of a given thickness and refractive index**. In other words if we place a medium in the way of a light ray, the same situation can be modelled by adding some additional length to the original ray's path. This, precisely, is the optical path length and the difference between the original and new length is the optical path difference.

In the most general setting, the optical path length is the line integral of the refractive index (for a non-homogeneous medium the refractive index is a function of the path parameter) over the path of the beam inside the medium:

$$OPL = \int n \, dt. \quad (2)$$

However for the case of a homogeneous medium (which is what we deal with in the experiment), the integral reduces to a simple product since the refractive index is not a function anymore but constant throughout:

$$OPL = n \times t. \quad (3)$$

Depending on the angle at which a light beam is incident on a piece of material, different optical path differences are introduced. This in turn changes the phase of the wave, thereby shifting the interference fringes on the screen. We can use the formula for the optical path length, Snell's law and some basic trigonometry to derive a relation between the angle of incidence and the refractive index of the material [3]:

$$n_g = \frac{(2t - N\lambda)(1 - cos(\theta))}{2t(1 - cos(\theta)) - N\lambda}. \quad (4)$$

We use this formula to find the refractive index of a glass slide.

3 Experimental Setup

We used a ThorLabs He-Ne Laser as our light source. A beamsplitter was introduced in the path of the light beam. Mirrors were introduced in the path of each of the resulting beams. The mirrors were adjusted until they passed through the beamsplitter again and reconverged on a screen. After some fine adjustment an interference pattern with circular fringes appeared. This was magnified using a plano-convex lens.

Figure 3 shows a top down view of the experimental setup. Figure 4 is a picture of the interference pattern observed.

3.1 Measuring the wavelength of the laser

In order to change the path length of one of the beams precisely by small amounts, we used a ThorLabs Servo Motor connected to a moving stage. On top of this stage we mounted another mirror. We then replaced Mirror 2 (labelled in figure 3) with this setup.

After connecting the servo controller to a computer we used the ThorLabs Kinesis software to move the stage a certain distance and calculated the number of times the fringes shifted and then returned to their original positions.

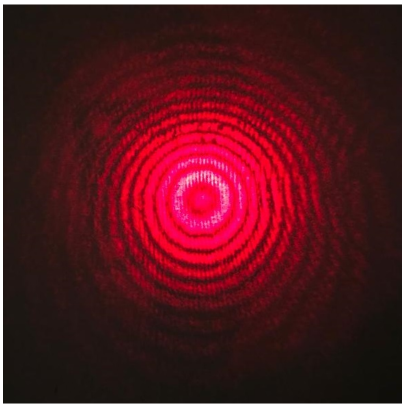

Figure 3: The interference pattern observed

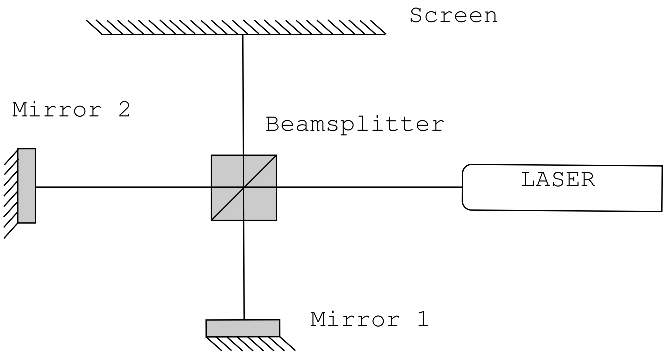

Figure 4: A schematic diagram showing the experimental setup

3.2 Measuring the refractive index of a glass slide

For the measurement of the refractive index, we reverted the experimental setup to its initial configuration (i.e the one shown in the schematic). We measured the thickness of the glass slide whose refractive index was to be measured using a micrometer screw gauge. The glass slide was then placed on rotatable mount with the angles marked on the base. This configuration was placed in the path of the beam right next to Mirror 1 as shown in Figure 5.

We varied the angle between the surface of the slide and the light beam and recorded the number of times the fringes shifted again. The experiment was repeated multiple times and the relation in equation (4) was used to arrive at a value for the refractive index for each trial. The values were then averaged over.

4 Results and Discussion

For the measurement of wavelength we varied the position of the mirror in increments of 0.005 millimeters using the ThorLabs Kinesis software. The following data was obtained:

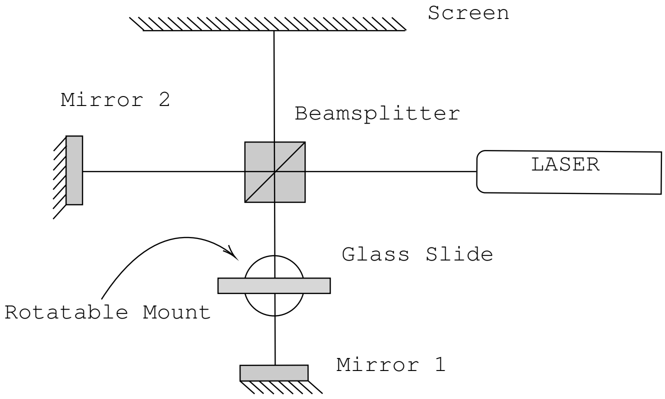

Figure 5: Experimental setup for the measurement of refractive index

Δd (mm)	N
0.005	16
0.01	31
0.015	44
0.02	63
0.025	78

Where Δd was the distance moved by the stage measured in millimeters and N was the number of times the fringes returned to their original locations. We then used the fact that a unit change in the position of the mirror causes double the change in the path length (since both the incoming and outgoing paths are affected) to derive the relation:

$$\frac{\lambda}{2} = \frac{\Delta d}{N},$$

$$\lambda = \frac{2\Delta d}{N},$$

and plotted a line of best fit for the points recorded in the table (Figure 6) and calculated the wavelength from the slope of the line. Using the error in the slope (obtained from python) and then propagating the error to the final value of the wavelength we found that:

$$\lambda = 641 \pm 21\text{nm}.$$

For the measurement of refractive index we varied the angle between the surface of the slide and the light beam in increments of 4 inches and recorded the number of times the fringes shifted again. To compensate for the significant human error involved in counting the number of shifted fringes we repeated the same experiment five times. The following values were recorded:

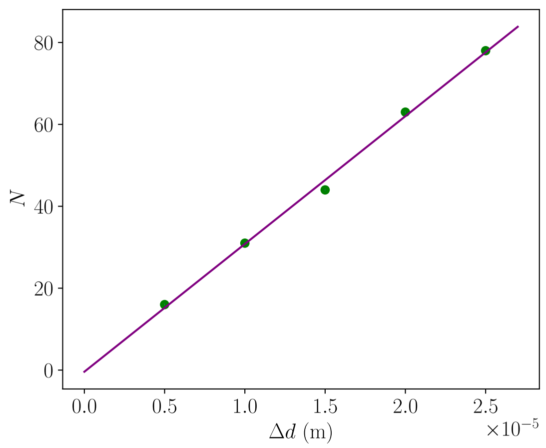

Figure 6: The line of best fit for λ against Δd plotted in Python

	$\theta = 4^\circ$	$\theta = 8^\circ$	$\theta = 12^\circ$
N_1	1	13	22
N_2	2	9	15
N_3	2	9	19
N_4	1	10	21
N_5	1	10	20

Where N_i denotes the number of fringes shifted during the ith trial.The values for larger angles were not measured since the fringes start shifting quite dramatically making any visual analysis with the naked eye impossible. In fact, this phenomenon was evident even at $\theta = 12^\circ$ as can be seen from the wildly varying values recorded for N.

Then by plugging these values into equation (4) and averaging, we obtained the following estimate for the refractive index of the glass slide:

$$n_g = 1.6531 \pm 0.33.$$

5 Conclusion

5.1 The wavelength experiment

The wavelength for the ThorLabs HRR020 Helium Neon Laser, as listed by ThorLabs is 632.8 nanometers [1], which indeed lies within our error bound of 21 nanometers. Infact, the relative error comes out to be about 0.0129, which, though not insignificant, still indicates a great deal of agreement between the theoretical and experimentally measured values.

5.2 The refractive index experiment

The refractive index of ordinary crown glass is usually about 1.52 while that of medium flint glass is around 1.63. Our value for the refractive index was close to the upper end of this range. We note that impurities in the glass may have contributed towards changing the value of the refractive index. Additionally, the high degree of human error involved in counting the fringe shift for larger angles may also have contributed towards a slightly skewed value.

References

[1] Thorlabs HRR020 - HeNe Laser. Accessed: 2023-28-03.

[2] Eugene Hecht. In *Optics: 5th Edition*, 2017.

[3] George S. Monk. In *Light: Principles and Experiments*, 1963.