

Circuits: → synchronous → 1) change of clk 2) memory elements → FF's  
 asynchronous → 1) change in var. 2) unlocked FF's, time delay.

stable: when out. is the same as in., feedback → seq. circuit.



Fundamental mode → only one in. can be changed at a time.

$$Y_1 = xy_1 + x'y_2$$

$$Y_2 = xy_1' + x'y_2'$$

$y_1 y_2 \backslash x$	0	1
00	0	0
01	1	0
11	1	1
10	0	1

$y_1 y_2 \backslash x$	0	1
00	0	1
01	1	1
11	1	0
10	0	0

$y_1 y_2 \backslash x$	0	1
00	00	01
01	11	01
11	11	10
10	00	10

Transition table

• Flow table: letters or symbols

	0	1
a	(a)	b
b	c	(b)
c	(c)	d
d	a	(d)

→ primitive flow table  
 → it has only one steady state table in each row.  
 ↓  
 state.

• race → when we have 2 or more state var.

types: ① noncritical race:

Final state that circuit reaches doesn't depend on the order in which the state var. change.

00 → 11, 00 → 10 → 11, 00 → 01 → 11

② critical race: 00 → 01 → 11, 00 → 11, 00 → 10 → stable state

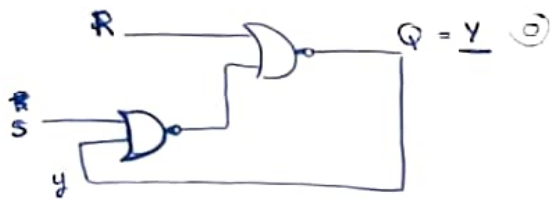
③ cycle: goes through a seq. of unstable states.

Final stable state depends on the order in which state var. change.

• SR Latch:

S	R	Q	Q'
set ← 1	0	1	0
no change ← 0	0	1	0
reset ← 0	1	0	1
no change ← 0	0	0	1
set ← 1	0	1	0
invalid ?? ← 1	1	0	0

$$Y = SR' + R'y$$



SR	00	01	11	10
Y	0	0	0	1
	0	1	0	1

$$Y = YR' + SR'$$

$$Y = YR' + S \text{ when } SR=0$$

Y	Y	S	R
0	0	0	X
0	1	1	0
1	1	X	0
1	0	0	1

excitation table



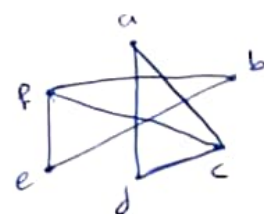
	D	G	Q	
a	0	0	0	after b, e
b	0	1	0	a, X
c	0	0	1	X, f,
d	0	1	1	X
e	1	0	0	
f	1	0	1	
g	1	1	0	X
h	1	1	1	

b	X				
c	✓	X			
d	✓	X	✓		
e	X	✓	X	X	
f	X	✓	✓	X	✓
	a	b	c	d	e

(e, A), (d, a), (f, d), (c, f)  
(e, b), (f, b), (e, f), (c, f)

	D	G	Q	notes
a	0	0	0	e, d
b	0	0	1	e
c	0	1	0	X, b, f
d	1	0	0	X, e
e	1	0	1	b, f
f	1	1	1	c, d, e

	00	01	11	10
a	@, 0	c, -	-, -	d, -
b	@, 1	c, -	-, -	e, -
c	a, -	@, 0	f, -	@, 10
d	a, -	-, -	f, -	@, 10
e	b, -	-, -	f, -	@, 10
f	-, -	c, -	@, 1	f, -



A  
(a, d, c)  
(e, b, e)

## Testing: after fabrication:

### 1) Functional testing:

ex. CPU 1GHz, 32 inputs  $\rightarrow$  # test vectors =  $2^{32} = 4 \times 10^9$  vectors

$$\text{time} = \frac{4 \times 10^9}{1 \times 10^9} = 4 \text{ sec.}$$

64 input  $\rightarrow$  # test vectors =  $2^{64} = 2 \times 10^{19}$  vectors

$$\text{time} = \frac{2 \times 10^{19}}{1 \times 10^9} = 585 \text{ years.}$$

### 2) structural testing:

ex. adder:

A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$\rightarrow$  if we do it by 1  $\rightarrow$  4 vectors

$\rightarrow$  2  $\rightarrow$  2 vectors

①	01	②	00	③	00	④	10
	10		01		11		11

① you can test, S  $\rightarrow$  stuck at 0, C stuck at 1 50%

② S  $\rightarrow$  stuck at 1, C  $\rightarrow$  stuck at 1, S  $\rightarrow$  stuck at 0 75%

③ S  $\rightarrow$  sa1, C  $\rightarrow$  sa1, C  $\rightarrow$  sa0 75%

④ S  $\rightarrow$  sa1, C  $\rightarrow$  sa1, C  $\rightarrow$  sa0, S  $\rightarrow$  sa0 ✓ the best 100%

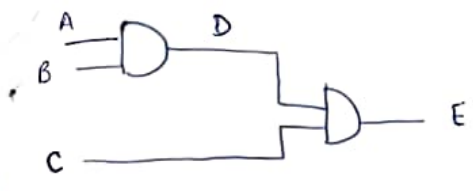
\* Fault modeling: ① node stuck at 0 ② node stuck at 1

\* Path sensitization method (2-value logic): assume each node in the circuit has either (sa1, sa0)

1) Backtrace phase: send the non-fault value under consideration.

2) propagation phase: steer the value of the node under consideration to one of the outputs.

input  $\rightarrow$  control  
output  $\rightarrow$  observe.



test node D:

① assume D sa1

① Backtrace phase: send 0 to D  $\Rightarrow AB = 0 \Rightarrow \overline{AB} = 01$   
 ② propagation phase:  $C = 1 \Rightarrow E = 0$  non-faulty.  
 $E = 1$  sa1 (Fault)

$\overline{ABC}$ 0 0 1	$\overline{ABC}$ 1 0 1	$\overline{ABC}$ 0 1 1
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3-TV

② assume D sa0

① send 1 to D:  $AB = 1 \Rightarrow \overline{AB} = 11$   
 ②  $C = 1 \Rightarrow E = 1$  non-faulty.  
 $E = 0$  (Fault) sa0

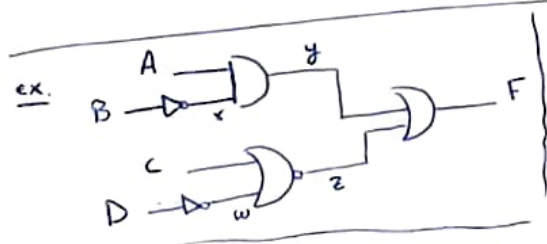
$\overline{ABC}$ 1 1 1
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1-TV

test A sa0

① send 1 to A  
 ②  $B = 1 \Rightarrow D = A = 1$  non-faulty.  
 $D = 0$  Fault (sa0)  
 $\Rightarrow C = 1 \Rightarrow E = 1$  non-faulty.  
 $E = 0$  Fault (sa0)

$\overline{ABC}$ 1 1 1
---------------------------



test vectors

A	B	C	D
1	0	1	0
1	0	1	1
1	0	0	0

• y sa0  
 ① send 1 to y  $\Rightarrow Ax = 1 \Rightarrow \overline{AB} = 10$   
 ② z must be 0  
 $Cw = 1$   
 $F = 1$  - non-faulty  
 $F = 0$  - Fault

C	D
1	0
1	1
0	1

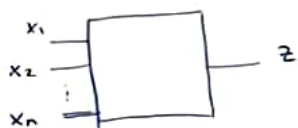
C	D
1	0
1	1
0	0

• D algorithm (5-value logic)  
 1) logic 1  
 2) logic 0  
 3)  $X = \text{unknown}$   
 4)  $D' = (0 = \text{non-faulty})$   
 5)  $D = 1$  (non-faulty), 0 (Faulty)

# Boolean difference:

$$z = f(x_1, x_2, x_3, \dots, x_n)$$

$$\frac{df}{dx_i} = f(x_i=0) \oplus f(x_i=1)$$

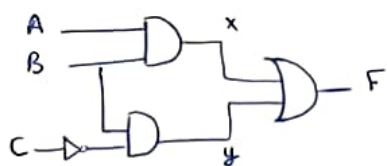


1.  $z$  is sensitive to node  $x_i$  if  $\frac{df}{dx_i} = 1$

2. to test  $x_i$  sa0  $\Rightarrow x_i \frac{dz}{dx_i} \stackrel{??}{=} 1$

3. to test  $x_i$  sa1  $\Rightarrow (x_i)' \frac{dz}{dx_i} \stackrel{??}{=} 1$

Ex:



test node A

$$F(A, B, C) = AB + BC'$$

$$\frac{dF}{dA} = F(A=0) \oplus F(A=1)$$

$$= (BC') \oplus (B + BC')$$

$$= (BC') \oplus B$$

$$= BC'B' + (B' + C)B$$

$$= 0 + CB$$

$$= \boxed{BC}$$

A. F is sensitive to A when  $BC=1 \Rightarrow \overrightarrow{BC} = 11$

B. A sa0 when;  $ABC=1 \Rightarrow \overrightarrow{ABC} = 111$

C. A sa1 when;  $A'BC=1 \Rightarrow \overrightarrow{ABC} = 011$

to test node B:

$$\frac{dF}{dB} = 0 \oplus (A+C') = A+C'$$

F is sensitive to B when

$$A+C' = 1$$

1	0
0	1
1	1

1	1
0	0
1	0

B sa1  $\rightarrow B'(A+C') = 1$

B sa0  $\rightarrow \underline{B(A+C')} = 1$

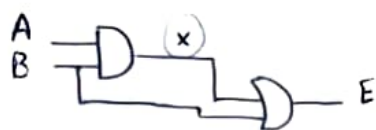
1	1	1
0	1	0
1	0	0

1	0	0
1	0	1
1	0	0



untestable faults: partially untestable / completely untestable.

ex. in boolean.



$$E = x + B$$

$$E = AB + B = (A+1)B = \boxed{B}$$

$$\text{II } \frac{dE}{dx} = E(x=0) \oplus E(x=1)$$

$$= B \oplus 1 = \boxed{B'}$$

• E is sensitive to x when  $B' = 1 \rightarrow \boxed{B=0}$

$$\text{II } x \text{ sa } 0 \rightarrow (x')B' = 1$$

$$(1 \cdot 0) = 1$$

$$\rightarrow B=0$$

$$x = 1$$

must be ~~zero~~ <sup>one</sup> zero

$$x = AB \rightarrow \text{zero}$$

contradiction  $\rightarrow$  x is untestable

$$(A \cdot B)' \cdot B' = ? 1$$

$$A' + B' \cdot B'$$

$$A'B' + B'B'$$

$$(A'+1)B'$$

$$\boxed{B' = 1}$$

$\rightarrow \boxed{B=0}$ , A is don't care  $\rightarrow$  either 1 or zero

$$\text{III } x \text{ sa } 1 \rightarrow x' \cdot B' = 1$$

$$\boxed{B=0}$$

$$\boxed{x=0}$$

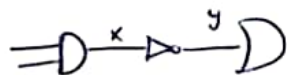
$$x = AB$$

don't care  $\leftarrow \boxed{A=0, B=0} \checkmark \rightarrow$  partially testable.

• Fault collapsing: 1) fault equivalence: 2 or more faults said to be eq. if they have the same test vectors.

$$\begin{matrix} a \\ b \end{matrix} \Rightarrow D \rightarrow c$$

$$a \text{ sa } 0: \begin{bmatrix} a & b \\ 1 & 1 \end{bmatrix} \equiv \begin{bmatrix} b \text{ sa } 0 \\ a & b \\ 1 & 1 \end{bmatrix} \equiv \begin{bmatrix} c \text{ sa } 0 \\ a & b \\ 1 & 1 \end{bmatrix}$$



$$x \text{ sa } 0 \equiv y \text{ sa } 1$$

$$y \text{ sa } 0 \equiv x \text{ sa } 1$$

$$\begin{matrix} a \\ b \end{matrix} \Rightarrow D \rightarrow c$$

$$\underline{a \text{ sa } 0 \equiv b \text{ sa } 0 \equiv c \text{ sa } 1}$$

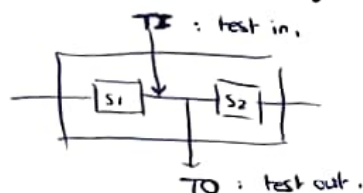
Fault dominance: Fault  $F_1$  is said to dominate Fault  $F_2$  if the test vectors of  $F_2$  are subset of test vectors of  $F_1$ .

$F_1$				$F_2$		
a	b	c		a	b	c
0		0	}	0	0	0
1		1		1	1	1
1	0	1				

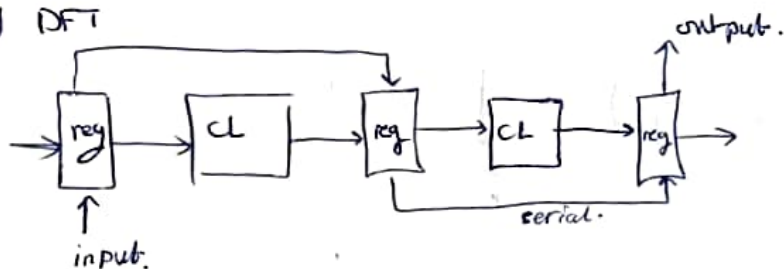
$F_1$  dominates  $F_2 \rightarrow$  for the minimum test vectors  $\rightarrow$  we remove  $F_2$

• Design for testability (DFT):

1) ADHOC DFT: minor changes on the circuit, for controllability & observability



2) structured DFT



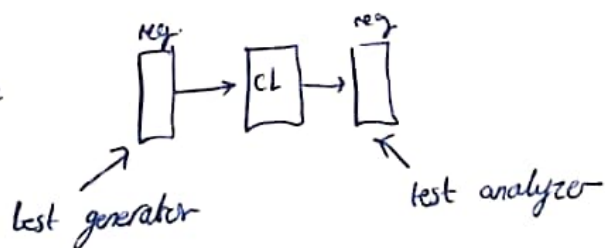
reg  $\rightarrow$  for scan design (shifting.  $\rightarrow$  reg. either scan reg. or shift reg. to insert data) in scan mode. (serially)

3) Built-in self test (BIST): auto. do the test



in normal mode  $\rightarrow$  work in parallel. (registers.)

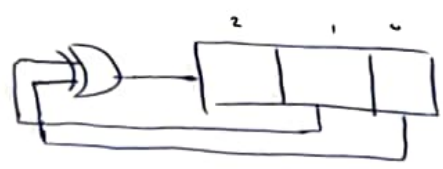
in test mode



$\hookrightarrow$  we don't use as a counter, we use linear feedback shift reg. (LFSR)

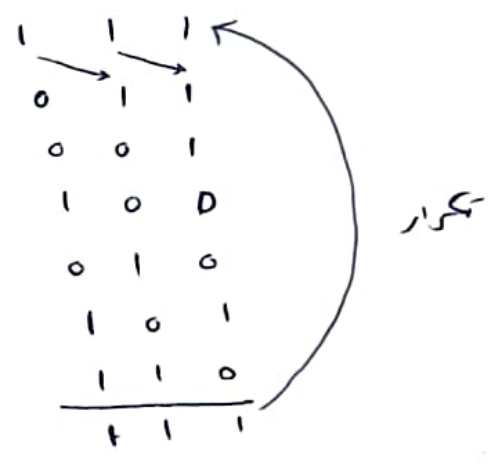
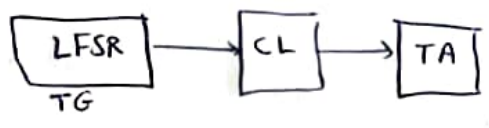
• LFSR :

• 3-bit LFSR :



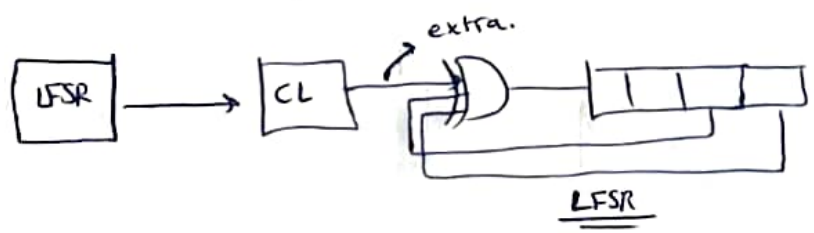
$x^3 + x^2 + 1 \rightarrow$  prime polynomial.

Comp  $\left( x^3 + x + 1 \right)$



- TA : 1) single signature input reg. (SISR)
- 2) multiple input signature reg. (MISR)

• SISR :



LFSR with extra input to the LFSR coming from the circuit under Test (CUT)