DISCRETE MATHEMATICS

Subject Description: This subject deals with discrete structures like set theory, mathematical logic, relations, languages, graphs and trees.

Goal: To learn about the discrete structures for computer based applications.

Objective: On successful completion of this subject the students should have: - Understanding the concepts of discrete mathematics - Learning applications of discrete structures in Computer Science.

UNIT I: Set theory-Introduction-Set & its Elements-Set Description-Types of sets-Venn-Euler Diagrams- Set operations & Laws of set theory-Fundamental products-partitions of sets-minsets- Algebra of sets and Duality-Inclusion and Exclusion principle

UNIT II: Mathematical logic – Introduction- prepositional calculus –Basic logical operations- Tautologies-Contradiction-Argument-Method o f proof- Predicate calculus.

UNIT III: Relations – Binary Relations – Set operation on relations-Types of Relations – Partial order relation – Equivalence relation – Composition of relations – Functions – Types of functions – Invertible functions – Composition of functions.

UNIT IV: Languages – Operations on languages – Regular Expressions and regular languages – Grammar – Types of grammars – Finite state machine – Finite – State automata

UNIT V: Graph Theory – Basic terminology – paths, cycle & Connectivity – Sub graphs – Types of graphs – Representation of graphs in computer memory - Trees – Properties of trees – Binary trees – traversing Binary trees – Computer Representation of general trees.

TEXT BOOKS:

1. Discrete Mathematics, J.K. Sharma, 2nd edition, 2005, Macmillan India Ltd. (UNIT I TO V)

REFERENCE BOOKS:

1. Discrete Mathematics Structures with Applications to Computer Science, J. P. Tremblay, R Manohar, McGraw Hill International Edition
2. Discrete Mathematics, M. K. Venkataraman, N.Sridharan, N.Chandarasekaran, National Publishing Company, Chennai

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SET THEORY

1. Definition

A set is a collection of well defined objects. The objects can be anything - people, alphabets, numerals, etc. The objects are known as elements or members of the set.

2. Notations

Sets are denoted by capital letters, A, B, C .. . The elements are denoted by small letters a, b, c . . . .

If an object x is a member of a set A, it is written as $x \in A$. $x \in A$ is read as 'x belongs to A' or 'x is in A' or 'x is a member of A'. If there is another object y which is not a member of A, it is written as $y \notin A$. $y \notin A$ is read as 'y does not belong to A' or 'y is not in A' or 'y is not a member of A'. The symbol $\in$ means 'belongs to' or 'a member of' and the symbol $\notin$ means 'does not belong to' or 'not a member of'.

3. Methods of Description of Sets

There are two methods by which a set is described. They are :

1. Tabulation Method (Roster Method) and
2. Set Builder Method (The Rule Method)

1. Tabulation Method : This is also called Roster Method. Under this method, the elements are listed, separated by commas and enclosed by braces { }.

Examples : 1. The set of vowels in English Alphabet is written as

A = {a, e, i, o, u}

2. The set of days is written as

A = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

3. The set of all natural numbers is written as N = {1, 2, 3, ... }

Note : (i) When a set has large number of elements or infinite number of elements (as the set N), we can not write all the elements.

(ii) Although any alphabet can be used to represent a set in general, certain sets are represented by specific alphabets.

For example, the set of all natural numbers is denoted by N,
the set of all rational numbers is denoted by Q,
the universal set is denoted by $\cup$,
the null set is denoted by ϕ or { }, ...

2. Set Builder Method : This is also called The Rule Method. Under this method, the letter x is used to represent a typical element. The properties of the elements are spelt out. All the objects which possess the properties are the elements of the set. x and the properties are written within braces as seen below.

Examples : 1. The set of vowels in English Alphabet is written as

A = {x: x is a vowel in English Aphabet} or
= {x | x is a vowel in English Alphabet

Each of these two is read in the same manner as 'A is the set of all x such that x is a vowel in English Alphabet'. This means 'A is the set of all the vowels in English'. Vertical line and the symbol : are interchangeable and they are read as 'such that'.

2. The set of all days in a week is written as
$A = \{x : x \text{ is a day in a week}\}$

3. A set $S = \{1, 2, 3, 4, 5\}$ may be written as
$S = \{x : x \in N,\ x < 6\}$.

4. Kinds or Types of Sets

1. Finite Set : If there are a specific or a finite number of elements in a set, the set is a finite set.

Examples : 1. The set of all books in the library of a College.

2. The set of all share holders of State Bank of India.

3. The set of all Chartered Accountants of India.

2. Infinite Set : If there are countless or infinite number of elements in a set, the set is an infinite set.

Examples : 1. The set of all natural numbers.

2. The set of all points in a line.

3. Singleton Set : If there is only one element in a set, the set is a singleton set.

Examples : 1. The set of the present Principal of a College.

2. A = {0}.

3. B = {1}.

4. The Empty or Null or Void Set : A set which has no element is called the Empty or the Null or the Void set. It is denoted by the Greek letter ϕ (read, Phi) or braces without any elements, { }.

Examples : 1. ϕ = {x : x is an odd number, x is an even number}.

2. ϕ = {x : x is a student who studies for more than 24 hours on a holiday}.

5. Subset and Superset : If every element of a set A is also an element of a set B, A is said to be a subset of B. It is denoted by $A \subseteq B$. $A \subseteq B$ is read as 'A is a subset of B' or 'A is contained in B'. The symbol $\subseteq$ looks like $\leq$ and gives a similar meaning.

If $x \in B \ \forall \ x \in A$, $A \subseteq B$. In this context, B is called a superset of A. In symbols, it is $B \supseteq A$. $B \supseteq A$ is read as 'B is a superset of A' or 'B contains A'. The symbol $\supseteq$ looks like $\geq$ and gives a similar meaning.

Examples for $A \subseteq B$ and $B \supseteq A$: 1. A = {1, 2, 3, 4} and B = {1, 2, 3, 4, 5}

2. A = {a, b, c} and B = {x, y, z, a, b, c}

3. A = {5, 6} and B = {6, 5}

6. Proper and Improper Subsets : There are two kinds of subsets, viz.,

1. Proper subsets and 2. Improper Subsets.

Proper Subsets : Set A is a proper subset of set B if every element of A is also an element of B and B has at least one element which is not an element of A. This is denoted by $A \subset B$. $A \subset B$ is read as 'A is a proper subset of B'.

Examples : 1. If $A = \{a, e, i, o, u\}$ and
$B = \{a, b, c, d, e, i, o, u\}$, $A \subset B$.

2. If $A = \{2, 4, 6, 8, \ldots\}$ and $N = \{1, 2, 3, 4, \ldots\}$, $A \subset N$.

Equal Sets or Improper Subsets : If A is a subset of B and B is a subset of A, A is an improper subset of B and B is an improper suhset of A. That is, A and B are equal. In symbols, if $A \subseteq B$ and $B \subseteq A$, $A = B$.

Examples : 1. If $A = \{1, 3, 5, 7\}$ and $B = \{3, 7, 1, 5\}$ then $A \subseteq B$, $B \subseteq A$ and $A = B$.

2. If $A = \{a, e, i, o, u\}$ and $B = \{a, a, i, i, i, e, u, o\}$, $A \subseteq B$, $B \subseteq A$ and $A = B$.

3. If $A = \{1, 2, 3, \ldots, 100000\}$ and $B = \{100000, 99999, 99998, \ldots 1\}$, $A \subseteq B$, $B \subseteq A$ and $A = B$

Note : (i) Order of the elements in a set is immaterial (as seen in examples 1 and 3 above).

(ii) Repetition of any member does not alter a set (as seen in example 2 above)

(iii) Any set is an improper subset of itself.

(iv) It is to be remembered that **the null set is a subset of every set.**

(v) If A is a subset of B and B is a subset of C, A is a subset of C.

7. Universal Set : The set which is a superset of all the sets under consideration is called the universal set. It is usually denoted by U.

Examples : 1. Let A be the set of vowels and B be the set of consonants. Then U = {a, b, c, d, . . . x, y, z}. That is, the set of all alphabets is the universal set.

2. If A = {1, 3, 5, ...}, B = {2, 4, 6, . . . },
C = {1, 2, 3, 4, 5} and D = {1, 2, 3, 4, 5, 6, 7, 8, 9}, the universal set, U = N = {1, 2, 3, . . . }.

8. Disjoint Sets : Two sets A and B are said to be disjoint if they have no element in common.

Examples : 1. A = {5, 10, 15, 20, 25, 30} and
B = {0, 2, 4, 6, 8} are disjoint sets.

2. If A = {x, y, z} and B = {l, m, n, o, p}, A and B are disjoint sets.

9. Power Set : The set of all the subsets of a sct is the power set of the set.

The power set of a set A is denoted by 2^A.

$\therefore 2^A = \{x \mid x \subseteq A\}$.

Examples: 1. If A = {a}, the power set of A, $2^A = \{\phi, \{a\}\}$. It is to be remembered that the null set ϕ is a subset of every set.

2. If A = {a, b}, $2^A = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

3. If A = {a, b, c}, $2^A = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \{a, b, c\}\}$.

Note : 1. A set whose elements are sets is called a family of sets or a class of sets. Any power set is an exampie for a family of sets.

2. If there are n elements in A, there are 2^n elements in 2^A.

5. Venn Diagram

Diagrams which represent the relations between sets are called Venn diagrams. The English mathematician John Venn (1834 - 1923) first introduced these diagrams. A rectangle represents the universal set U and the circles inside the rectangle represent the different sets under consideration.

6. Set Operations

1. Intersection of Sets : The intersection of two sets A and B is the set of all the elements which are elements of both A and B. It is denoted by $A \cap B$ and it is read as 'A intersection B' or 'A cap B'.

$\therefore A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

$\therefore A \cap B = \phi$ if A and B are disjoint sets.

Note : (i) $A \cap B$ is the same as $B \cap A$. i.e., $A \cap B = B \cap A$.

(ii) $A \cap B$ is a subset of A as well as B.

(iii) If $A \subseteq B$, $A \cap B = A$.

(iv) If $B \subseteq A$, $A \cap B = B$.

(v) $A \cap U = A$; $A \cap \phi = \phi$; $A \cap A = A$.

Examples : 1. If $A = \{1, 2, 4, 6, 8\}$, $B = \{2, 3, 4, 5, 6\}$ and $C = \{3, 6, 9, 12, 15\}$, find (a) $A \cap B$ (b) $B \cap C$ and (c) $C \cap A$.

(B.Com. Madras, M 91)

Answer : (a) $A \cap B = \{2, 4, 6\}$

(b) $B \cap C = \{3, 6\}$

(c) $C \cap A = \{6\}$

2. If $A = \{1, 2, 4, 6, 8\}$ and $B = \{3, 5, 7\}$, $A \cap B = \phi$.

3. Given that $A = \{0, 1, 3, 5\}$, $B = \{1, 2, 4, 7\}$ and $C = \{1, 2, 3, 5, 8\}$, prove that $(A \cap B) \cap C = A \cap (B \cap C)$.

(B.Com. / B.C.S. Bharathiar, A 2K)

Solution : $(A \cap B) = \{1\}$ and $(A \cap B) \cap C = \{1\}$ .... (1)

$(B \cap C) = \{1, 2\}$ and $A \cap (B \cap C) = \{1\}$ ..... (2)

From (1) and (2), $(A \cap B) \cap C = A \cap (B \cap C)$

Note : $A \cap B \cap C = (A \cap B) \cap C = A \cap (B \cap C)$

Prove $(A \cap B) \cap C = A \cap (B \cap C)$

$x \in (A \cap B) \cap C$ $\Rightarrow [x \in (A \cap B)]$ and $[x \in C]$

$\Rightarrow [x \in A$ and $x \in B]$ and $[x \in C]$

$\Rightarrow [x \in A]$ and $[x \in B$ and $x \in C]$

$\Rightarrow [x \in A]$ and $[x \in (B \cap C)]$

$\Rightarrow x \in A \cap (B \cap C)$

$\therefore (A \cap B) \cap C \subseteq A \cap (B \cap C)$ ..... (1)

Consider y, any element of the set $A \cap (B \cap C)$

$y \in A \cap (B \cap C)$ $\Rightarrow [y \in A]$ and $[y \in (B \cap C)]$

$\Rightarrow [y \in A]$ and $[y \in B$ and $y \in C]$

$\Rightarrow [y \in A$ and $y \in B]$ and $[y \in C]$

$\Rightarrow [y \in (A \cap B)]$ and $[y \in C]$

$\Rightarrow [y \in (A \cap B) \cap C]$

$\therefore A \cap (B \cap C) \subseteq (A \cap B) \cap C$ ..... (2)

From (1) and (2), $(A \cap B) \cap C = A \cap (B \cap C)$

2. Union of Sets : The union of two sets A and B is the set of all the elements which are elements of either A or B (or both). In symbols, it is denoted by $A \cup B$ and it is read as 'A union B' or 'A cup B'.

$\therefore (A \cup B) = \{x \mid x \in A \text{ or } x \in B\}$ (or $x \in A \cap B$ is implied)

Note : (i) $A \cup B$ is the same as $B \cup A$. i.e., $A \cup B = B \cup A$.

(ii) A and B are subsets of $A \cup B$.

(iii) If $A \subseteq B$, $A \cup B = B$.

(iv) If $B \subseteq A$, $A \cup B = A$.

(v) $A \cup U = U$; $A \cup \phi = A$; $A \cup A = A$.

Examples : 1. If $A = \{1, 3, 4, 5\}$ and $B = \{1, 7, 8, 10\}$, find $A \cup B$ and $A \cap B$. (B.Com./B.C.S. Bharathiar, N 2K)

Answer : $A \cup B = \{1, 3, 4, 5, 7, 8, 10\}$;

$A \cap B = \{1\}$.

2. If $A = \{0, 1, 5, 7, 9\}$ and $B = \{2, 3, 6, 10\}$,

$A \cup B = \{0, 1, 2, 3, 5, 6, 7, 9, 10\}$.

3. If $A = \{a, b, c, d, e, f, g, h, i\}$ and $B = \{a, c, e, g\}$,

$A \cup B = \{a, b, c, d, e, f, g, h, i\}$.

Prove

~~**Note**~~ **:** $(A \cup B) \cup C = A \cup (B \cup C)$ can be proved as follows also :

Let x be an element of the set $(A \cup B) \cup C$

Then, $x \in (A \cup B) \cup C \Rightarrow [x \in (A \cup B)]$ or $[x \in C]$

$\Rightarrow [x \in A$ or $x \in B]$ or $[x \in C]$

$\Rightarrow [x \in A]$ or $[x \in B$ or $x \in C]$

$\Rightarrow [x \in A]$ or $[x \in (B \cup C)]$

$\Rightarrow x \in A \cup (B \cup C)$

$\therefore (A \cup B) \cup C \subseteq A \cup (B \cup C)$ ....(1)

Consider y, any element of the set $A \cup (B \cup C)$

Then, $y \in A \cup (B \cup C) \Rightarrow [y \in A]$ or $[y \in (B \cup C)]$

$\Rightarrow [y \in A]$ or $[y \in B$ or $y \in C]$

$\Rightarrow [y \in A$ or $y \in B]$ or $[y \in C]$

$\Rightarrow [y \in (A \cup B)]$or $[y \in C]$

$\Rightarrow y \in (A \cup B) \cup C$

$\therefore A \cup (B \cup C) \subseteq (A \cup B) \cup C)$ ....(2)

From (1) and (2), $(A \cup B) \cup C = A \cup (B \cup C)$

Examples : 1. If $A = \{0, 1, 2\}$, $B = \{1, 3, 4\}$, $C = \{7, 8\}$ are sets, find $A \cup (B \cup C)$ and $(A \cup B) \cap C$.

(B.Com. / B.C.S. Bharathiar, N 2K)

Solution : $B \cup C = \{1, 3, 4, 7, 8\}$ and

$A \cup (B \cup C) = \{0, 1, 2, 3, 4, 7, 8\}$.

$A \cup B = \{0, 1, 2, 3, 4\}$ and

$(A \cup B) \cap C = \{\ \ \}$, the null set.

Note : $(A \cup B) = \{0, 1, 2, 3, 4\}$ and

$(A \cup B) \cup C = \{0, 1, 2, 3, 4, 7, 8\} = A \cup (B \cup C)$

$\therefore\ A \cup B \cup C = (A \cup B) \cup C = A \cup (B \cup C)$

2. If the sets A, B and C are defined as follows:
$A = \{5, 6, 7, 8, 9\}$, $B = \{8, 9, 10\}$ and $C = \{5, 7, 11\}$, write down the following sets:-

(i) $A \cup (B \cap C)$ (ii) $(A \cup B) \cap (B \cup C)$ (iii) $(A \cap B) \cup C$

(I.C.W.A. Foundation, J 94)

Solution : (i) $B \cap C = \phi$ and $A \cup (B \cap C) = \{5, 6, 7, 8, 9\}$.

(ii) $A \cup B = \{5, 6, 7, 8, 9, 10\}$, $B \cup C = \{5, 7, 8, 9, 10, 11\}$ and $(A \cup B) \cap (B \cup C) = \{5, 7, 8, 9, 10\}$.

(iii) $A \cap B = \{8, 9\}$ and $(A \cap B) \cup C = \{5, 7, 8, 9, 11\}$.

3. Difference of Sets : The difference of the sets A and B is the set of those elements of A which are not the elements of B. In symbols, it is denoted by A–B and it is read as 'A difference B' or 'A minus B'.

That is, $A-B = \{x \mid x \in A, x \notin B\}$

Similarly, $B-A = \{x \mid x \in B, x \notin A\}$

Examples : 1. If $A = \{1, 2, 4, 6, 8\}$, $B = \{2, 3, 4, 5, 6\}$, $C = \{3, 6, 9, 12, 15\}$, find $A - B$, $B-C$ and $C-A$.

(B.Com. Madras, M 91)

Answer : $A-B=\{1, 8\}$, $B-C=\{2, 4, 5\}$ and $C-A=\{3, 9, 12, 15\}$.

2. If $A=\{10, 20, 30, 40, 50\}$ and $B=\{10, 20, 30\}$, $A-B=\{40, 50\}$.

3. If $A = \{a, b, c, d\}$ and $B = \{x, y, z\}$, $A-B = \{a, b, c, d\}$.

Note : 1. Generally, $A-B \neq B-A$.

In example 1, $B-A = \{3, 5\} \neq A-B$,

$C-B = \{9, 12, 15\} \neq B-C$ and

$A-C = \{1, 2, 4, 8\} \neq C-A$.

In example 2, $B-A = \phi$ and in example 3, $B-A = \{x, y, z\} = B$.

2. $A-B$ is a subset of A and $B-A$ is a subset of B.

3. $A-B$, $A \cap B$ and $B-A$ are mutually disjoint.

4. $A-B = A-(A \cap B)$ and $B-A = B-(A \cap B)$.

4. Complement of a Set : The set of all those elements of the universal set, U, which are not the elements of a set A is the complement of the set A and is denoted by A^c or $\overline{A}$ or A'.

i.e., $A^c = \overline{A} = A' = \{x \mid x \in U, x \notin A\}$.

That is, $A' = U - A$.

Examples : 1. If U = {1, 2, 3, . . .} and A = {1, 3, 5, . . .}, A^c or $\overline{A}$ or A' = {2, 4, 6, ...}.

2. If U is the set of all alphabets and A is the set of all consonants, A^c is the set of all vowels.

3. If U = {0, 2, 4, 6, 8, 10} and A = {0, 2, 4, 6, 8, 10}, $A^c = \phi$.

4. If U = {0, 5, 10, 15} and A = ϕ, A^c = U.

5. A universal set is U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Sets A, B and C are as given below:

A = {1, 2, 4, 6}, B = {3, 4, 7, 8} and C = {2, 3, 9, 10}

Find (i) $(A \cup B \cup C)'$ (ii) $A' - (B \cap C)$ (iii) $A - (B \cup C)'$ where A' denotes the complementary set of A and so on.

(I.C.W.A. Foundation, J 95)

Solution : (i) $B \cup C = \{2, 3, 4, 7, 8, 9, 10\}$

$A \cup B \cup C = A \cup (B \cup C) = \{1,2,3,4,6,7,8,9,10\}$

$\therefore \quad (A \cup B \cup C)' = \{5\}$

(ii) $A' = \{3, 5, 7, 8, 9, 10\}$

$B \cap C = \{3\}$

$\therefore \quad A' - (B \cap C) = \{5, 7, 8, 9, 10\}$

(iii) $(B \cup C)' = \{1, 5, 6\} \quad \because B \cup C = \{2,3,4,7,8,9,10\}$

$A - (B \cup C)' = \{2, 4\}$

7. Laws and Properties of Sets

Sets have the following properties.

1. Commutative Laws :

For any two sets A and B, $A \cup B = B \cup A$ and

$A \cap B = B \cap A$.

Note :

But $A - B \neq B - A$.

2. Associative Laws :

For any three sets A, B and C,

$A \cup (B \cup C) = (A \cup B) \cup C$ and

$A \cap (B \cap C) = (A \cap B) \cap C$.

3. Distributive Laws :

For any three sets A, B and C,

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Prove

Note : $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$ can be proved as follows also :

Let x be any element of the set $A\cup(B\cap C)$

Then, $x \in A\cup(B\cap C) \Rightarrow [x \in A]$ or $[x \in (B\cap C]$

$\Rightarrow [x \in A]$ or $[x \in B$ and $x \in C]$

$\Rightarrow [x\in A$ or $x\in B]$ and $[x\in A$ or $x\in C]$

$\Rightarrow [x \in (A\cup B)]$ and $[x \in (A\cup C)]$

$\Rightarrow x \in (A\cup B)\cap(A\cup C)$

$\therefore A\cup(B\cap C) \subseteq (A\cup B)\cap(A\cup C)$ .... (1)

Consider y, any element of the set $(A\cup B)\cap(A\cup C)$

Then, $y\in(A\cup B)\cap(A\cup C) \Rightarrow [y\in(A\cup B)]$ and $[y\in(A\cup C)]$

$\Rightarrow [y\in A$ or $y\in B)]$ and $[y\in A$ or $y\in C]$

$\Rightarrow [y\in A]$ or $[y\in B$ and $y\in C]$

$\Rightarrow [y\in A]$ or $[y\in(B\cap C]$

$\Rightarrow y\in A\cup(B\cap C)$

$\therefore (A\cup B)\cap(A\cup C)\subseteq A\cup(B\cap C)$ .... (2)

From (1) and (2), $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$

i.e., union is found to be distributive over intersection of sets.

2. Verify $A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$ by means of a Venn diagram.

(B.Com./B.C.S. Bharathiar N 99; B.B.M. Bharathiar, A 2K)

Prove: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Then. $x \in A \cap (B \cup C) \Rightarrow [x \in A]$ and $[x \in (B \cup C)]$

$\Rightarrow [x \in A]$ and $[x \in B$ or $x \in C]$

$\Rightarrow [x \in A$ and $x \in B]$ or $[x \in A$ and $x \in C]$

$\Rightarrow [x \in (A \cap B)]$ or $[x \in (A \cap C)]$

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$

$\therefore A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ . . . . (1)

Consider y, any element of the set $(A \cap B) \cup (A \cap C)$

Then, $y \in (A \cap B) \cup (A \cap C) \Rightarrow [y \in (A \cap B)]$ or $[y \in (A \cap C)]$

$\Rightarrow [y \in A$ and $y \in B]$ or $[y \in A$ and $y \in C]$

$\Rightarrow [y \in A]$ and $[y \in B$ or $y \in C]$

$\Rightarrow [y \in A]$ and $[y \in (B \cup C)]$

$\Rightarrow y \in A \cap (B \cup C)$

$\therefore (A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$ . . . . (2)

From (1) and (2), $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

i.e., intersection is found to be distributive over union of sets.

Example 1 : If $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 5, 6\}$ and $C = \{1, 3, 5\}$, verify that

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (B.Com. M.K., N 96)

Solution : We find $B \cup C = \{1, 2, 3, 4, 5, 6\}$

and $A \cap (B \cup C) = \{1, 2, 3, 4\}$ . . . . (1)

$A \cap B = \{2, 4\}$

$A \cap C = \{1, 3\}$

and $(A \cap B) \cup (A \cap C) = \{1, 2, 3, 4\}$ . . . . (2)

From (1) and (2), it is found that

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Example 2 : If $A = \{1, 2, 3, 4\}$, $B = \{3. 4, 5, 6\}$ and $C = \{1, 5, 6, 7, 8\}$, verify that

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(C.A. Entrance, N 76; B.B.M. Bharathiar, A 01)

Solution : We find $(B \cap C) = \{5, 6\}$

and $A \cup (B \cap C) = \{1, 2, 3, 4, 5, 6\} \quad \ldots\ldots (1)$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

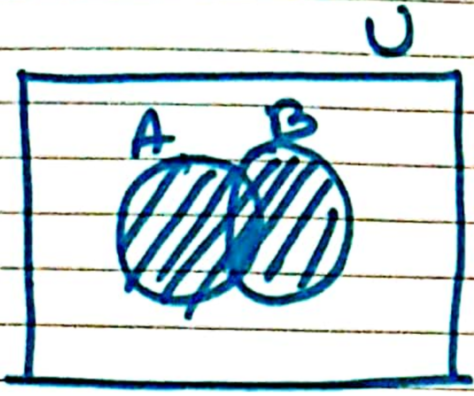
$A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$

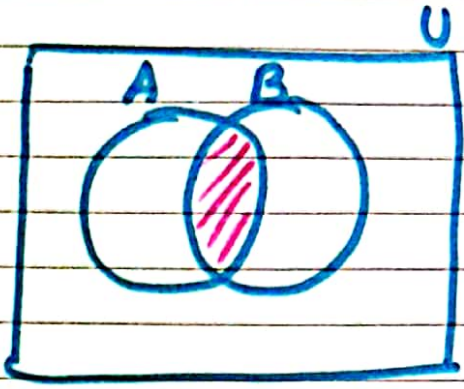
and $(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6\} \quad \ldots\ldots (2)$

From (1) and (2), it is found that

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

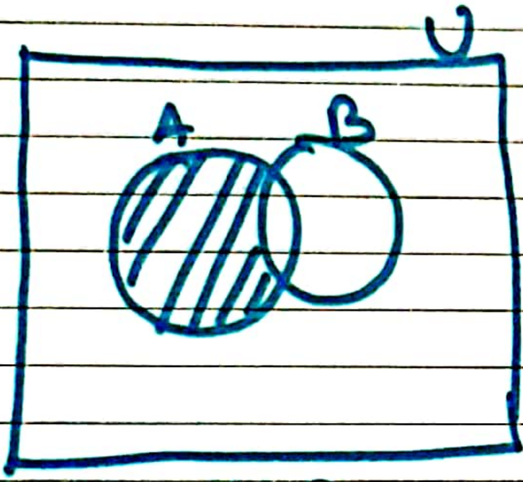
Venn Diagram

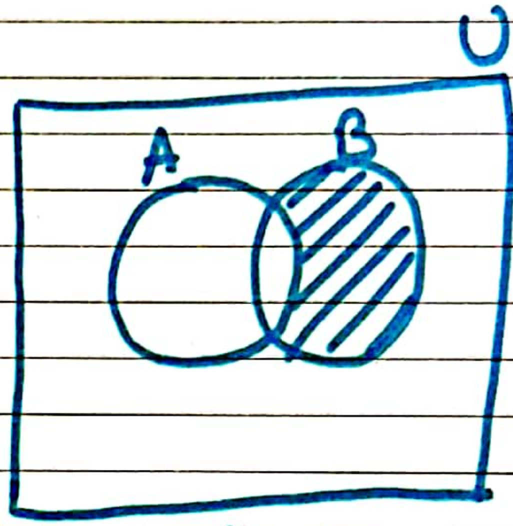




~~Intersection~~ $A \cup B$ ↑ Union of sets

$A \cap B$ → Intersection

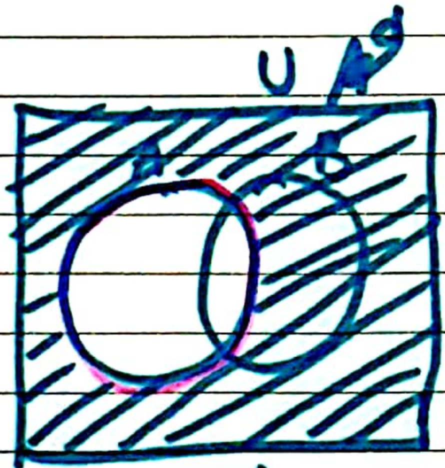




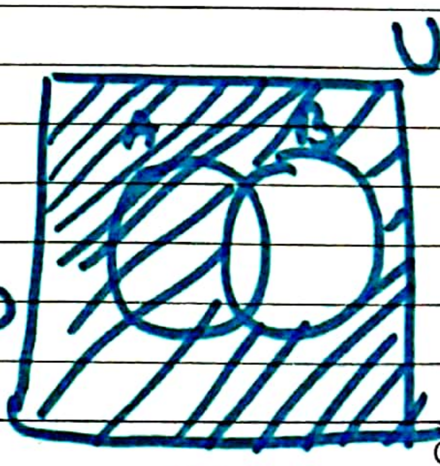
$A - B$

$B - A$

Difference of sets



Complement of sets



A'

B'

Date

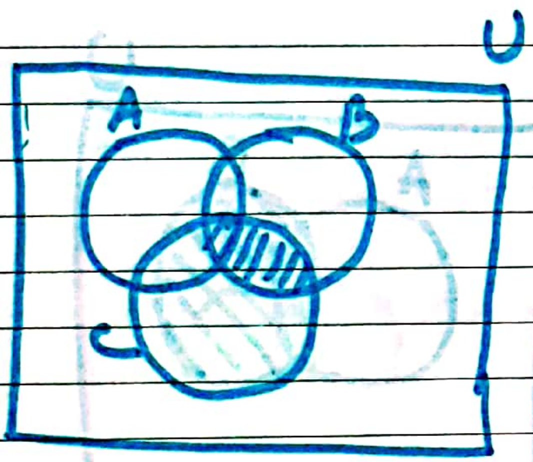
Prove Distributive law using Venn Diagram for any 3 sets

1) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ &

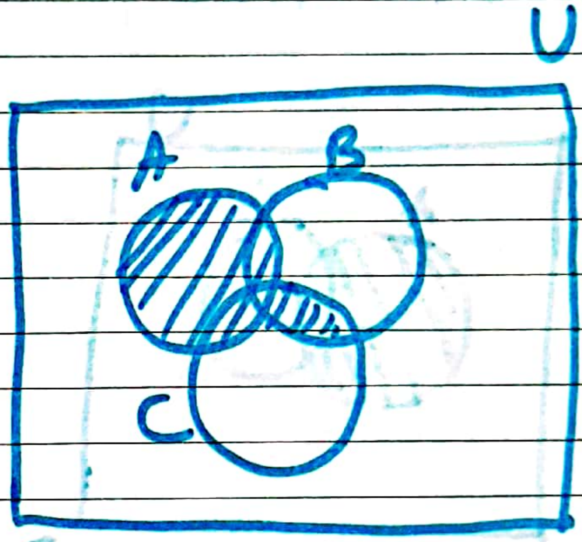
2) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

I^{st} law $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

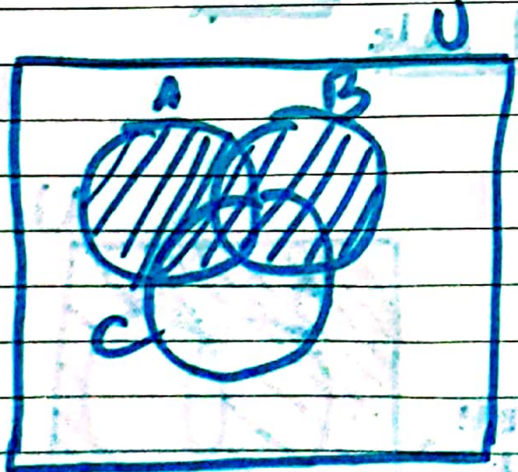
L.H.S $A \cup (B \cap C)$



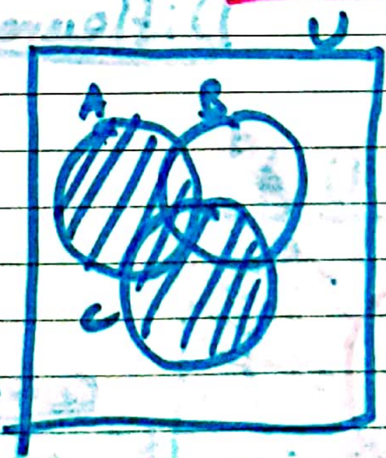
$B \cap C$



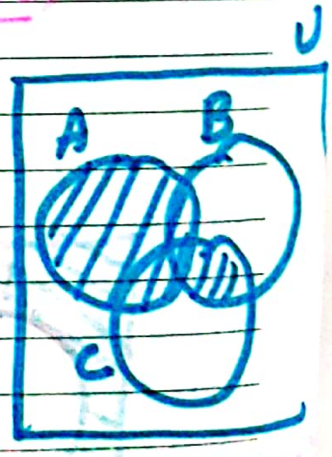
$A \cup (B \cap C)$ → ①



$A \cup B$



$A \cup C$

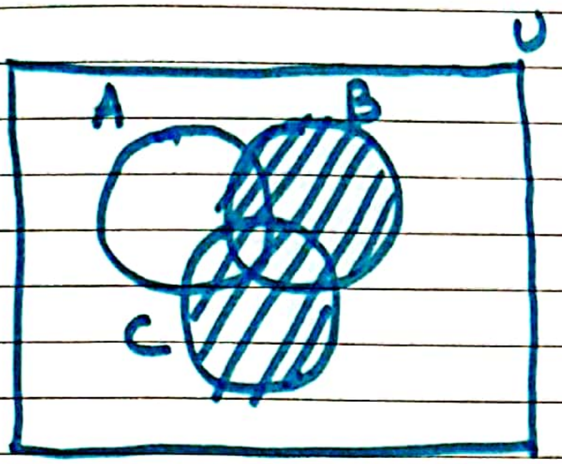


$(A \cup B) \cap (A \cup C)$ → ②

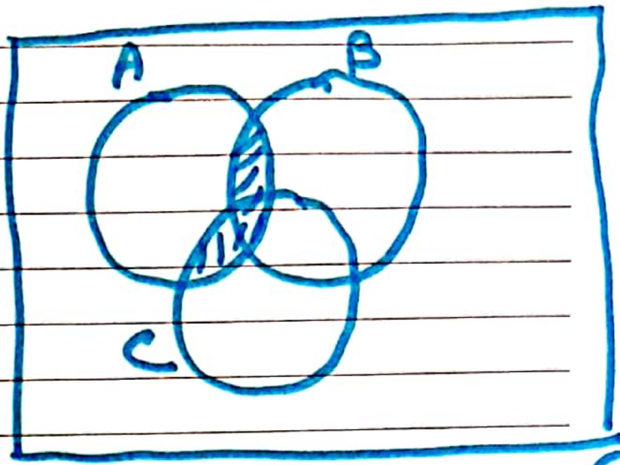
$\therefore A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

2nd law: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S $A \cap (B \cup C)$

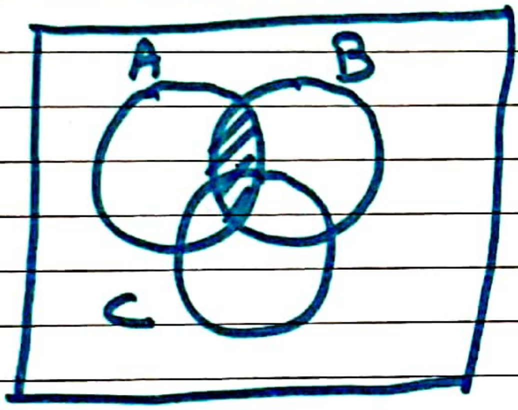


$B \cup C$

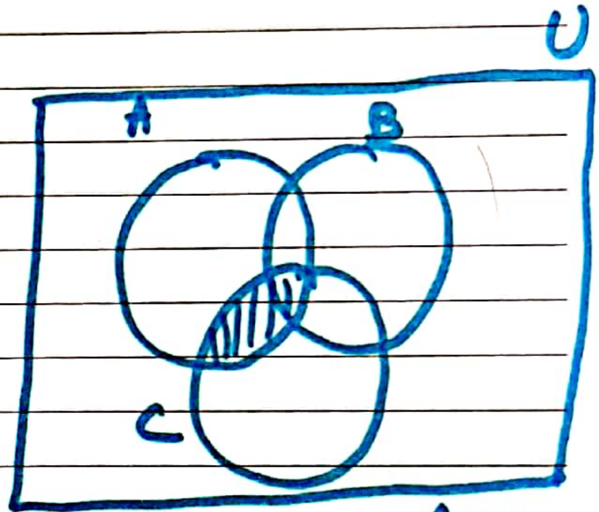


$A \cap B \cup C \rightarrow (1)$

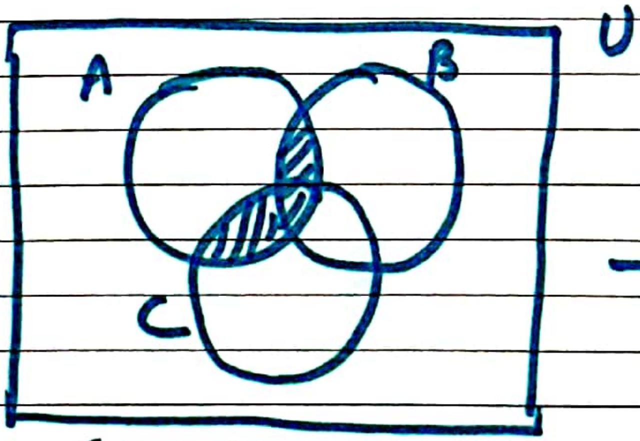
R.H.S $(A \cap B) \cup (A \cap C)$



$A \cap B$



$A \cap C$



$\rightarrow (2)$

$(A \cap B) \cup (A \cap C)$

$\therefore$ From (1) & (2)

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

4. De Morgan's Laws :

For any two sets A and B,

$$(A \cup B)' = A' \cap B' \text{ and}$$

$$(A \cap B)' = A' \cup B'.$$

For any three sets A, B and C,

$$A - (B \cup C) = (A-B) \cap (A-C) \text{ and}$$

$$A - (B \cap C) = (A-B) \cup (A-C).$$

Prove **Note :** $(A \cup B)' = A' \cap B'$ can be proved as follows also :

Let x be any element of the set $(A \cup B)'$

Then, $x \in (A \cup B)' \Rightarrow [x \in U]$ and $[x \notin (A \cup B)]$

$\Rightarrow [x \in U]$ and $[x \notin A$ **and** $x \notin B]$

$\Rightarrow [x \in U$ and $x \notin A]$ and $[x \in U$ and $x \notin B]$

$\Rightarrow [x \in A']$ and $[x \in B']$

$\Rightarrow x \in [A' \cap B']$

$\therefore (A \cup B)' \subseteq A' \cap B'$ .... (1)

Consider y, any element of the set $A' \cap B'$.

Then, $y \in [A' \cap B'] \Rightarrow [y \in A']$ and $[y \in B']$

$\Rightarrow [y \in U$ and $y \notin A]$ and $[y \in U$ and $y \notin B]$

$\Rightarrow [y \in U]$ and $[y \notin A$ and $y \notin B]$

$\Rightarrow [y \in U]$ and $[y \notin (A \cup B)]$

$\Rightarrow y \in (A \cup B)'$

$\therefore A' \cap B' \subseteq (A \cup B)'$ .... (2)

From (1) and (2), the conclusion is $(A \cup B)' = A' \cap B'$.

Prove ~~**Note**~~ **:** $(A \cap B)' = A' \cup B'$ can be proved as follows also :

Let x be any element of the set $(A \cap B)'$

Then, $x \in (A \cap B)' \Rightarrow [x \in U]$ and $[x \notin (A \cap B)]$

$\Rightarrow [x \in U]$ and $[x \notin A$ **or** $x \notin B]$

$\Rightarrow [x \in U$ and $x \notin A]$ or $[x \in U$ and $x \notin B]$

$\Rightarrow [x \in A']$ or $[x \in B']$

$\Rightarrow x \in [A' \cup B']$

$\therefore (A \cap B)' \subseteq A' \cup B'$ . . . . (1)

Consider y, any element of the set $A' \cup B'$

Then, $y \in A' \cup B' \Rightarrow [y \in A']$ or $[y \in B']$

$\Rightarrow [y \in U$ and $y \notin A]$ or $[y \in U$ and $y \notin B]$

$\Rightarrow [y \in U]$ and $[y \notin A$ or $y \notin B]$

$\Rightarrow [y \in U]$ and $[y \notin (\mathbf{A} \cap \mathbf{B})]$

$\Rightarrow y \in (A \cap B)'$

$\therefore A' \cup B' \subseteq (A \cap B)'$ . . . . (2)

From (1) and (2), the conclusion is $(A \cap B)' = A' \cup B'$.

Example 3 : If U = {0, 1,2; 3, 4, 5}, A = {0, 1, 2} and B = {2, 4}, prove that

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

(B.Com. Bharathiar, A 95)

Solution : (i) We find $A \cup B = \{0, 1, 2, 4\}$ and

$$(A \cup B)' = \{3, 5\} \quad \ldots.(1)$$

$$A' = \{3, 4, 5\}$$

$$B' = \{0, 1, 3, 5\} \text{ and}$$

$$A' \cap B' = \{3, 5\} \quad \ldots.(2)$$

From (1) and (2), it is found that $(A \cup B)' = A' \cap B'$.

(ii) We find $A \cap B = \{2\}$ and

$$(A \cap B)' = \{0, 1, 3, 4, 5\} \quad \ldots.(3)$$

$$A' = \{3, 4, 5\}$$

$$B' = \{0, 1, 3, 5\} \text{ and}$$

$$A' \cup B' = \{0, 1, 3, 4, 5\} \quad \ldots.(4)$$

From (3) and (4), it is found th

Example 4 : If $A = \{0, 1, 3, 4, 6, 7, 9, 10\}$, $B = \{2, 3, 4, 5, 6\}$ and $C = \{4, 5, 6, 7, 8, 9\}$, prove that

(i) $A - (B \cup C) = (A-B) \cap (A-C)$ and

(ii) $A - (B \cap C) = (A-B) \cup (A-C)$.

Solution : (i) $B \cup C = \{2, 3, 4, 5, 6, 7, 8, 9\}$

$A - (B \cup C) = \{0, 1, 10\}$ .... (1)

$A - B = \{0, 1, 7, 9, 10\}$,

$A - C = \{0, 1, 3, 10\}$ and

$(A-B) \cap (A-C) = \{0, 1, 10\}$ .... (2)

From (1) and (2), $A - (B \cup C) = (A-B) \cap (A-C)$

(ii) $B \cap C = \{4, 5, 6\}$

$A - (B \cap C) = \{0, 1, 3, 7, 9, 10\}$ .... (3)

$A - B = \{0, 1, 7, 9, 10\}$

$A - C = \{0, 1, 3, 10\}$ and

$(A-B) \cup (A-C) = \{0, 1, 3, 7, 9, 10\}$ .... (4)

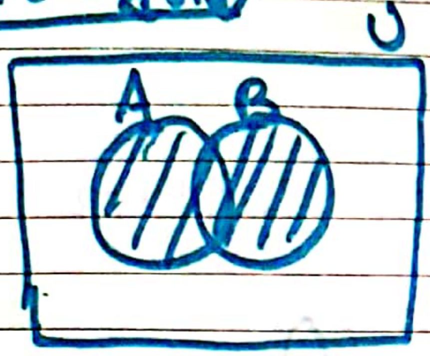
From (3) and (4), $A-(B \cap C) = (A-B) \cup (A-C)$

8. Number of Elements

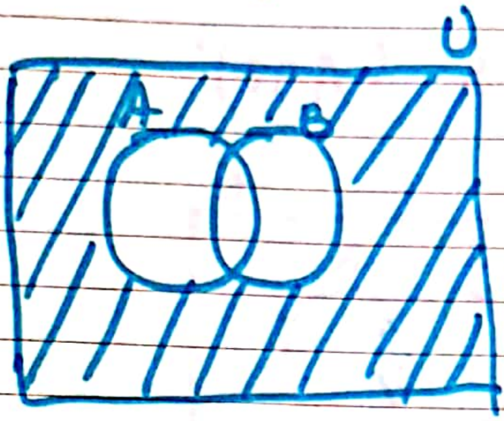
Demorgans law - venn diagram

1) $(A \cup B)' = A' \cap B'$ (2 sets)

L.H.S $(A \cup B)'$



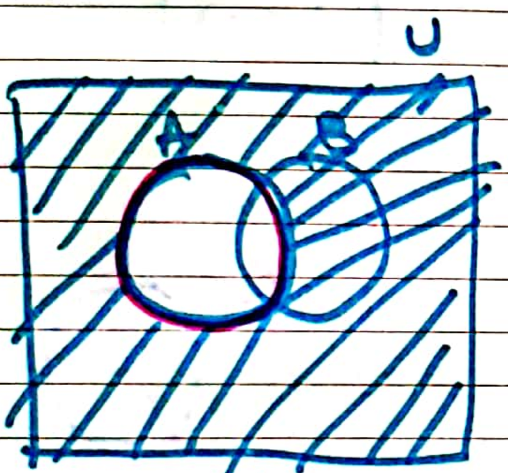
$(A \cup B)$



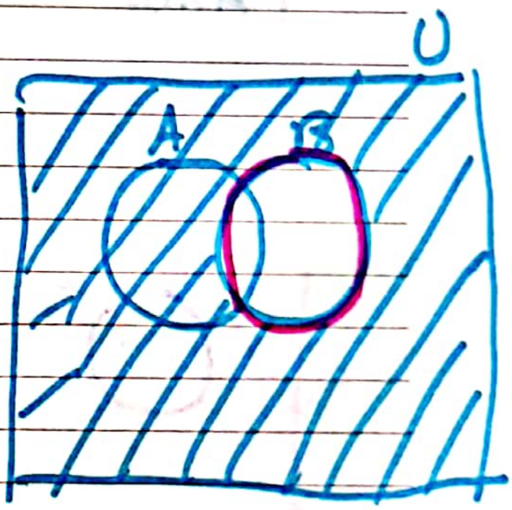
→ (1)

$(A \cup B)'$

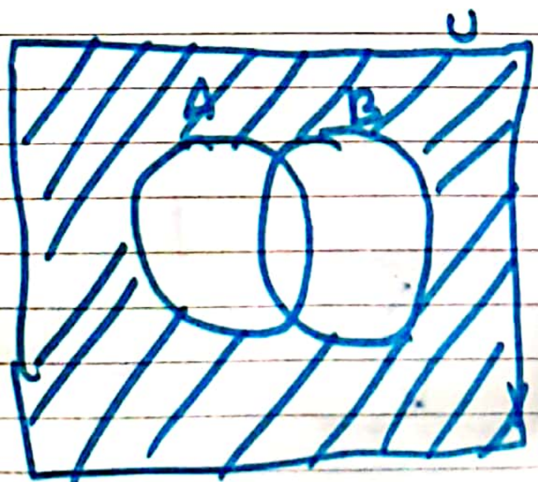
R.H.S $A' \cap B'$



A'



B'



→ (2)

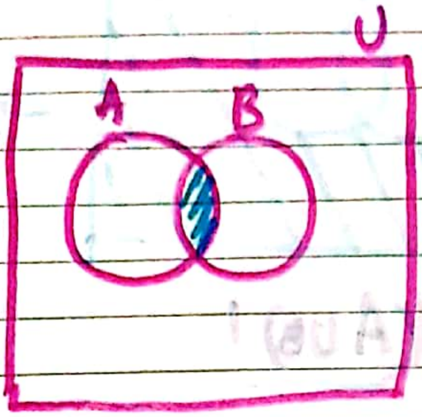
$A' \cap B'$ (From (1) & (2))

L.H.S = R.H.S

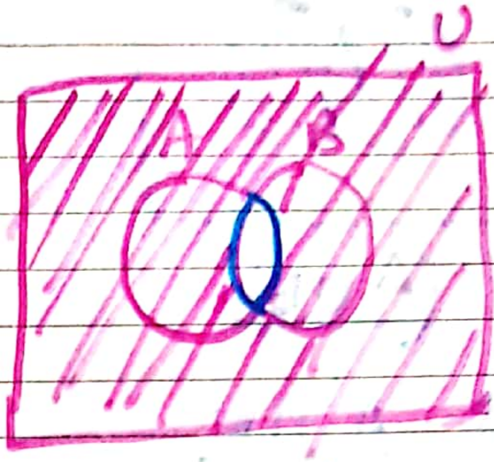
$(A \cup B)' = A' \cap B'$ → Hence proved

2) $(A \cap B)' = A' \cup B'$

L.H.S $(A \cap B)'$

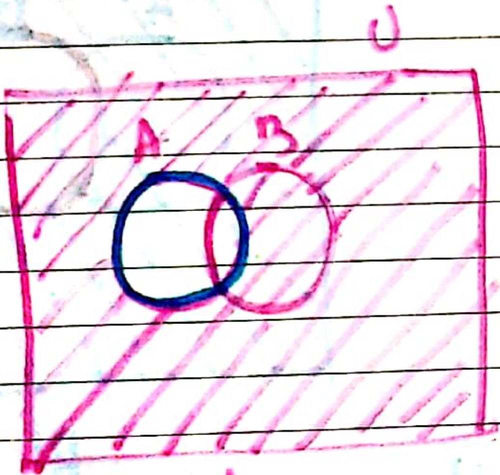


$(A \cap B)$

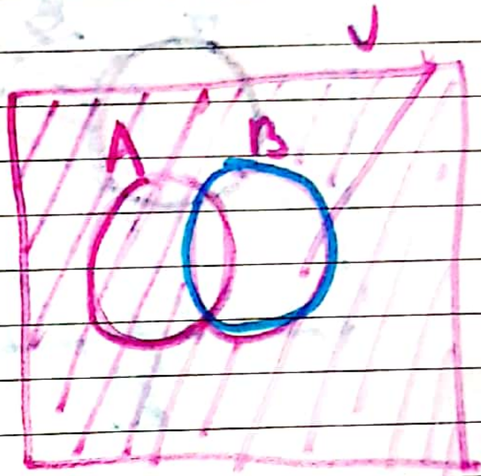


→ (1)

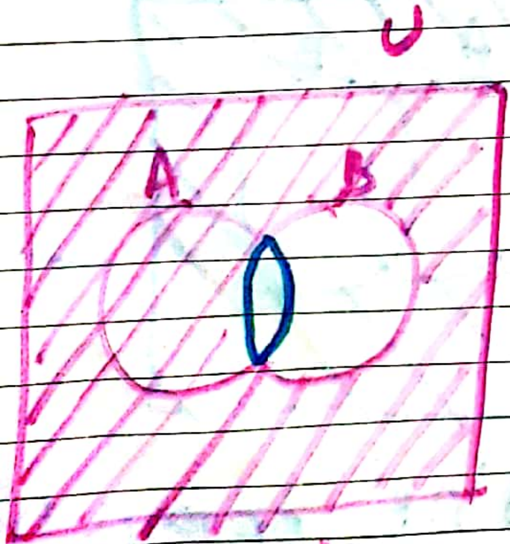
$(A \cap B)'$



A'



B'



→ (2)

$A' \cup B'$

L.H.S = R.H.S

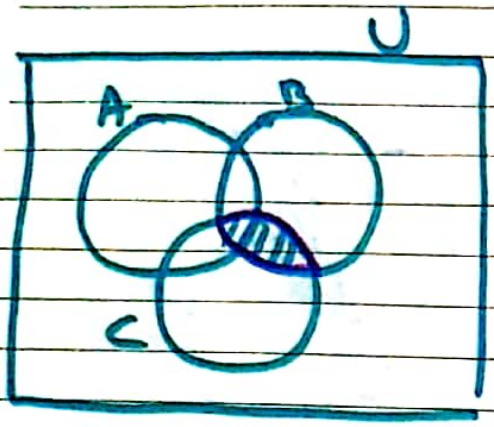
$(A \cap B)' = A' \cup B'$

Hence proved

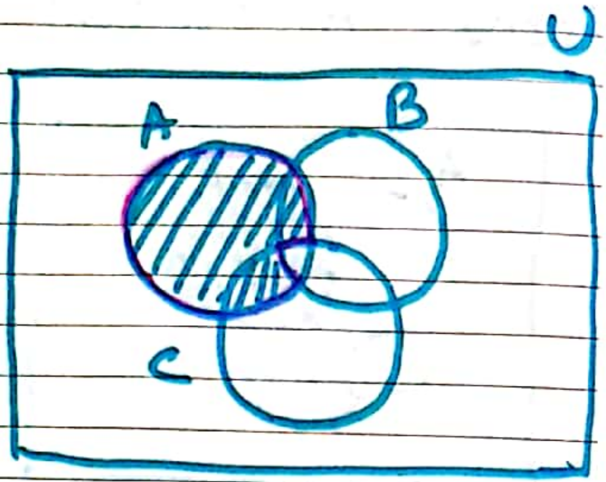
Date

3) $A-(B\cap C) = (A-B)\cup(A-C)$

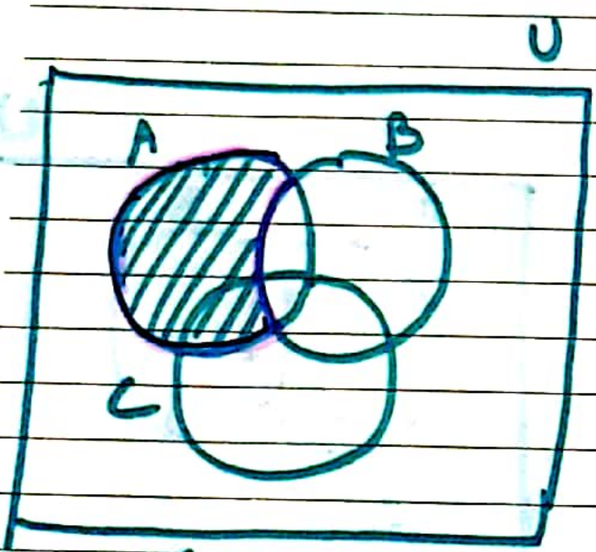
L.H.S $= A-(B\cap C)$



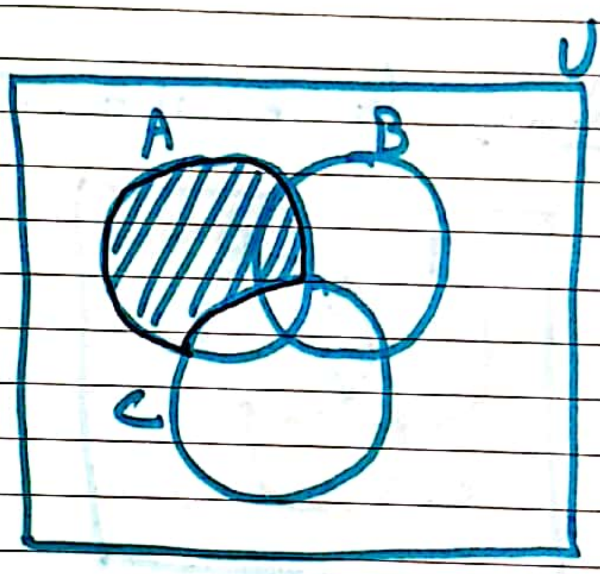
$B\cap C$



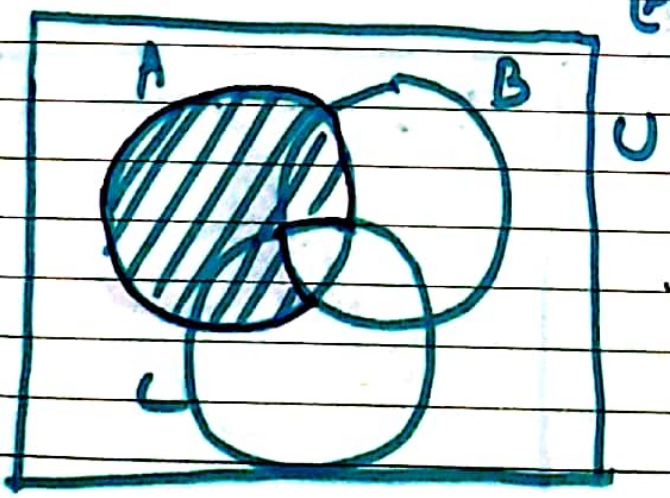
$(A-B\cap C) \longrightarrow ①$



$(A-B)$



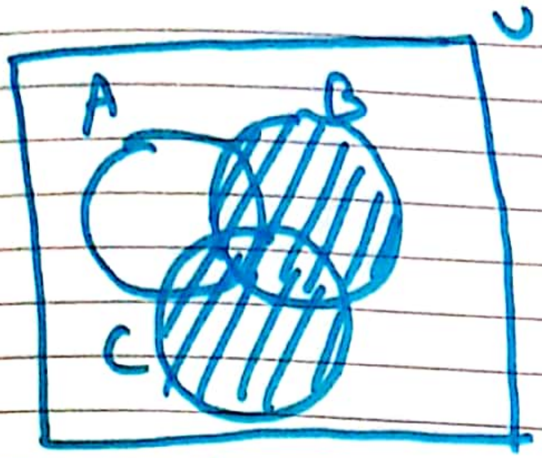
$(A-C)$



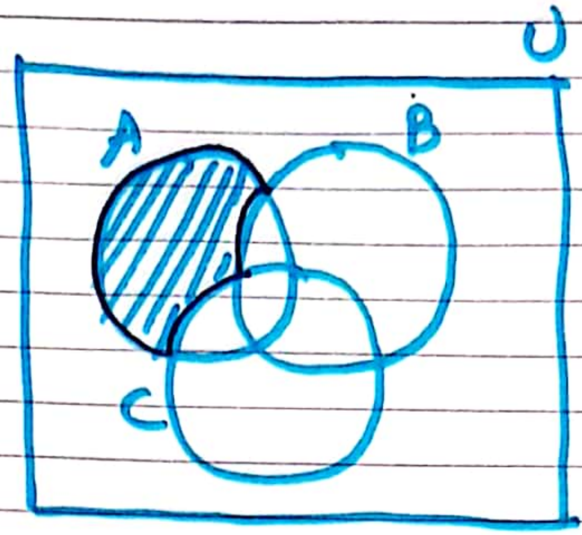
$\longrightarrow ②$

$\therefore$ ① & ② $A-(B\cap C) = (A-B)\cup(A-C)$

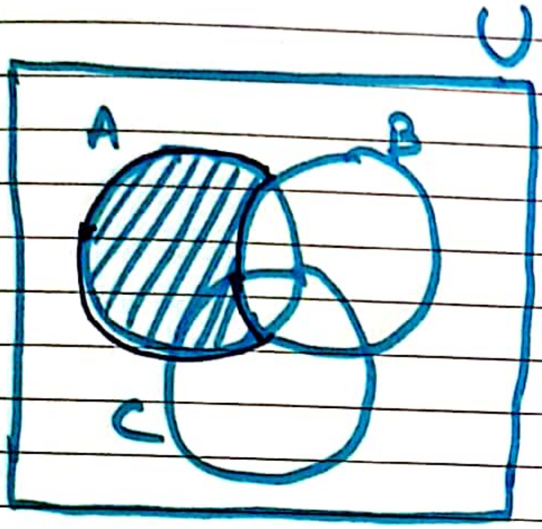
4) $A - (B \cup C) = (A - B) \cap (A - C)$



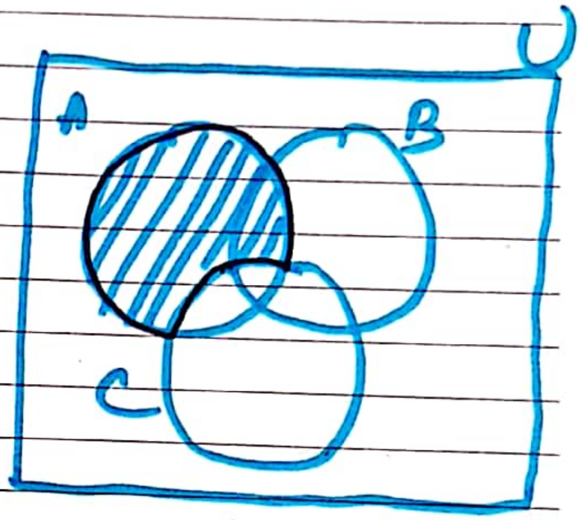
$\underline{B \cup C}$



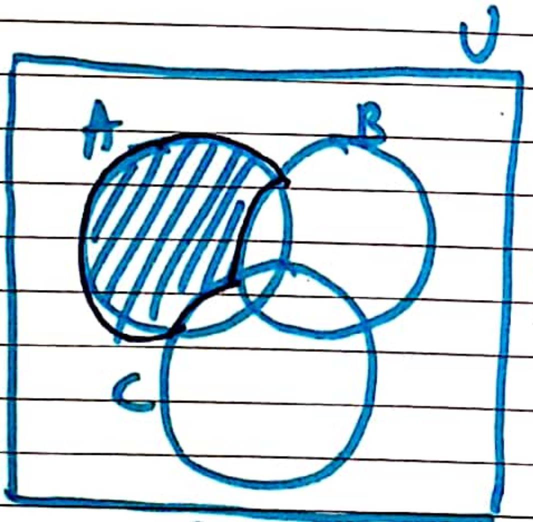
$A - (B \cup C) \longrightarrow (1)$



$(A - B)$



$(A - C)$



$\longrightarrow (2)$

$(A - B) \cap (A - C)$

$\therefore (1) = (2)$

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